Chapter 4. Graphing Relations and Functions

Ex. 4.8

Answer 1CU.

By looking the pattern first find a general rule and then write an algebraic expression of the pattern.

Answer 2CU.

Consider a sequence with first term as 4 and second terms as 8.

It can be observed that the second term is double of the first term.

So, the third term will be double of the second term, that is third term will be 16.

Hence, the required sequence will be as below

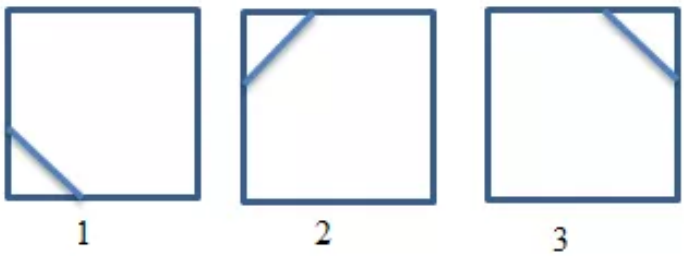
$4,8,16,32,\ldots$

Answer 3CU.

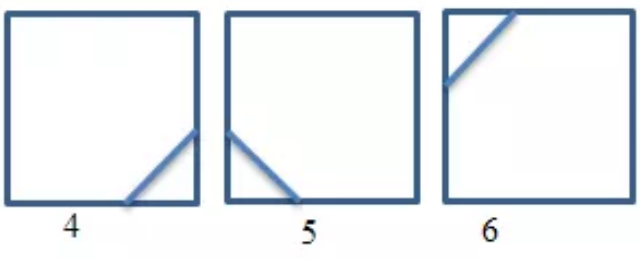
To determine whether an equation correctly represents a relation given in a table, first substitute the values of domain in the equation. If these values coincide with the range, then the equation correctly represents a relation given in a table.

Answer 4CU.

Consider the following pattern



The above pattern consists of squares with a triangle in corner. The triangle in the square is rotated in a clockwise direction. The next three figures are as shown below

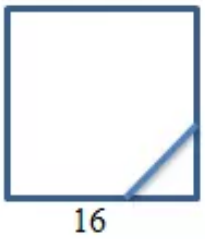


It can be observed that the figure 1 and figure 2 are similar to figure 5 and figure 6 respectively.

This means that the pattern repeats after every 4th figure.

Therefore designs 4, 8, 12, 16, and so on, will be the same.

Hence, the figure 16 in the pattern is



Answer 5CU.

Consider the sequence

$1,2,4,7,11,\ldots$

The object is to find the next three terms of the sequence.

The differences between two consecutive terms of the sequence are

$$2-1=1$$
$$4-2=2$$
$$7-4=3$$
$$11-7=4$$

It can be observed that the difference between each term is increased by 1 in each successive term.

Thus to find the next three terms in the sequence $1,2,4,7,11,\ldots$, continue adding 1 to each successive difference. That is adding 5, 6, and 7.

Hence, the next three terms of the sequence are $\boxed{16,22,\text{and } 29}$.

Answer 6CU.

Consider the sequence

$5,9,6,10,7,11,\ldots$

The object is to find the next three terms of the sequence.

The differences between two consecutive terms of the sequence are

$$9-5=4$$
$$6-9=-3$$
$$10-6=4$$
$$7-10=-3$$

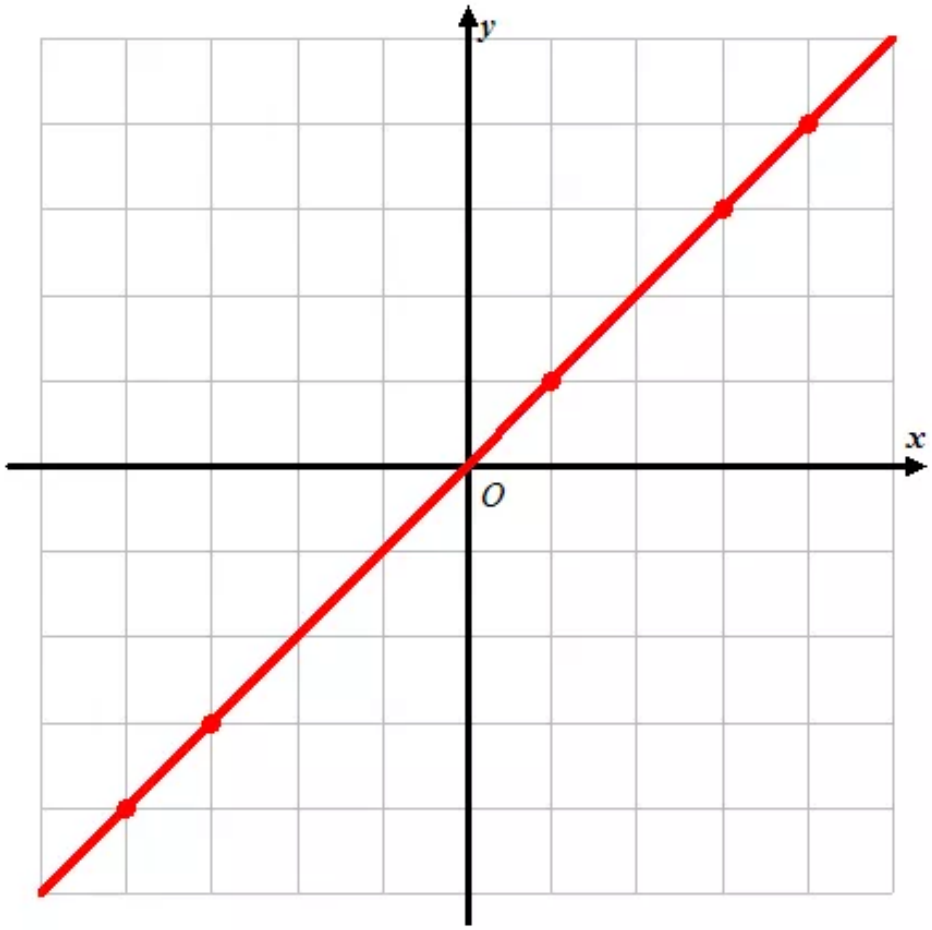
It can be observed that the second, fourth, sixth, so on terms are increased by 4 and third, fifth, so on terms are decreased by 3.

Thus to find the next three terms in the sequence $1,2,4,7,11,\ldots$, continue subtracting 3 from 11 and adding 4 to the resulting term and repeat the process. That is adding $-3,4,\text{ and}-3$.

Hence, the next three terms of the sequence are $\boxed{8,12,\text{and } 9}$.

Answer 7CU.

Consider the graph of the function



Construct a table of ordered pairs which lies on the above graph.

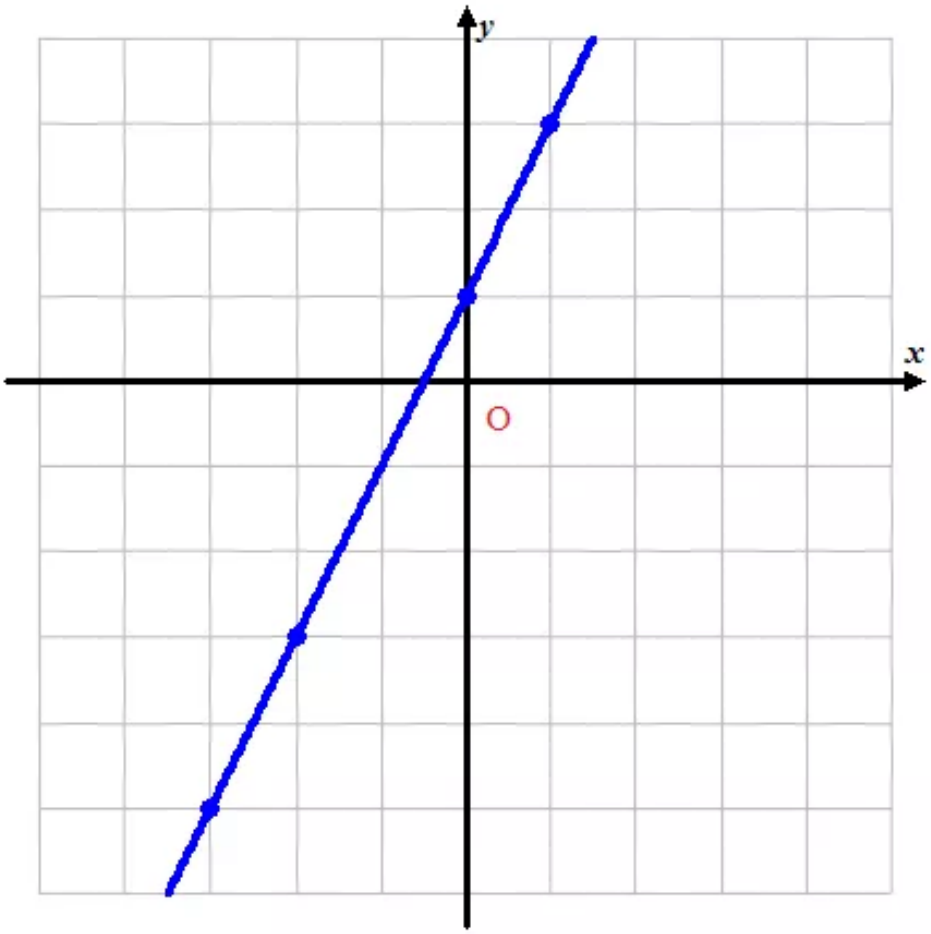
x	y
1	1
3	3
4	4
−3	−3
−4	−4

From the above table it can be observed that the values of x and y are identical. So, the required equation in function notation is

$\boxed{y = x}$ or $\boxed{f(x) = x}$

Answer 8CU.

Consider the graph of the function



Construct a table of ordered pairs which lies on the above graph.

x	y
0	1
1	3
-2	-3
-3	-5

The differences between two consecutive values of x are $1, -3, -1$.

The differences between two consecutive values of y are $2, -6, -2$.

Thus, the difference in y values is twice the difference of x values.

Therefore, the suitable equation might be $y = 2x$.

Before concluding the result first check this equation.

Check: If $x=0$, then $y=2(0)$ or 0. But the *y* value for $x=0$ is 1.

This is a difference of 1. Try some other values in the domain to see if the same difference occurs.

x	$2x$	y
0	0	1
1	2	3
−2	−4	−3
−3	−6	−5

From above table it can be observed that *y* is always 1 more than $2x$.

This pattern suggests that 1 should be added to one side of the equation in order to correctly describe the relation.

Check $y=2x+1$.

If $x=-2$, then $y=2(-2)+1$ or -3 ✓

If $x=-3$, then $y=2(-3)+1$ or -5 ✓

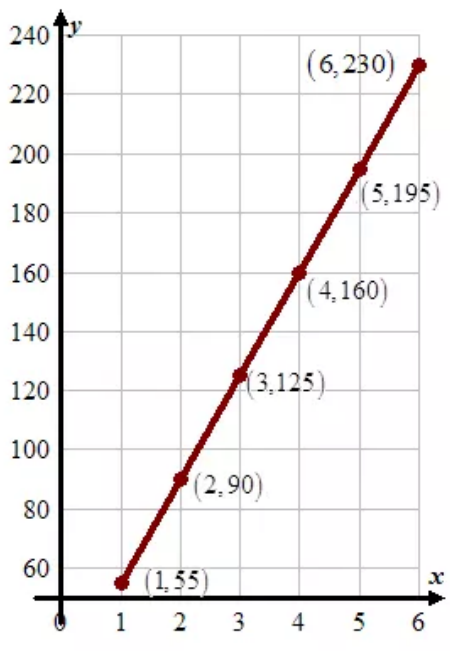
Answer 9CU..

Consider the following table that shows the underground temperature of rocks at various depths below Earth's surface.

Depth(km)	1	2	3	4	5	6
Temperature $(^\circ C)$	55	90	125	160	195	230

The object is to graph the above data.

For this plot the points $(1,55),(2,90),(3,125),(4,160),(5,195),\text{and}(6,230)$ on the graph and connect them by a smooth curve.



Answer 10CU.

Consider the following table that shows the underground temperature of rocks at various depths below Earth's surface.

Depth(km)	1	2	3	4	5	6
Temperature $(^\circ C)$	55	90	125	160	195	230

The object is to find an equation in function notation for the relation given in above table.

The difference between any two consecutive values of *d* is 1.

The difference between any two consecutive values of *t* is 35.

Thus, the difference in *d* values is equal to 35 times of difference of *t* values.

Therefore, the suitable equation might be $t = 35d.$

Before concluding the results first check this equation.

Check: If $d = 1,$ then $t = 35$. But the *t* value for $d = 1,$ is 55.

This is an increment of 20. Try some other values in the domain to see if the same increment occurs.

d	1	2	3	4	5	6
35*d*	35	70	105	140	175	210
t	55	90	125	160	195	230

From above table it can be observed that *t* is always 20 more than $35d$

This pattern suggests that 20 should be added to one side of the equation in order to correctly describe the relation.

Check $t = 35d + 20.$

If $d = 1$, then $t = 35(1) + 20$ or 55 ✓

If $t = 5$, then $t = 35(5) + 20$ or 195 ✓

Hence, an equation in function notation for the given relation is $\boxed{t(d) = 35d + 20.}$

Answer 11CU.

Consider the following table that shows the underground temperature of rocks at various depths below Earth's surface.

Depth(km)	1	2	3	4	5	6
Temperature $(^\circ C)$	55	90	125	160	195	230

The object is to find the temperature of a rock that is 10 kilometers below the surface.

For that first find an equation in function notation for the relation given in above table.

The difference between any two consecutive values of *d* is 1.

The difference between any two consecutive values of *t* is 35.

Thus, the difference in *d* values is equal to 35 times of difference of *t* values.

Therefore, the suitable equation might be $t = 35d.$

Before concluding the results first check this equation.

Check: If $d = 1,$ then $t = 35$. But the *t* value for $d = 1,$ is 55.

This is an increment of 20. Try some other values in the domain to see if the same increment occurs.

d	1	2	3	4	5	6
35*d*	35	70	105	140	175	210
t	55	90	125	160	195	230

From above table it can be observed that *t* is always 20 more than $35d$

This pattern suggests that 20 should be added to one side of the equation in order to correctly describe the relation.

Check $t = 35d + 20.$

If $d = 1$, then $t = 35(1) + 20$ or 55 ✓

If $t = 5$, then $t = 35(5) + 20$ or 195 ✓

Hence, an equation in function notation for the given relation is $t(d) = 35d + 20.$

Replacing *d* by 10 in the function $t(d)=35d+20.$

$t(d)=35d+20$ Original function

$t(10)=35(10)+20$ Replace *d* by 10

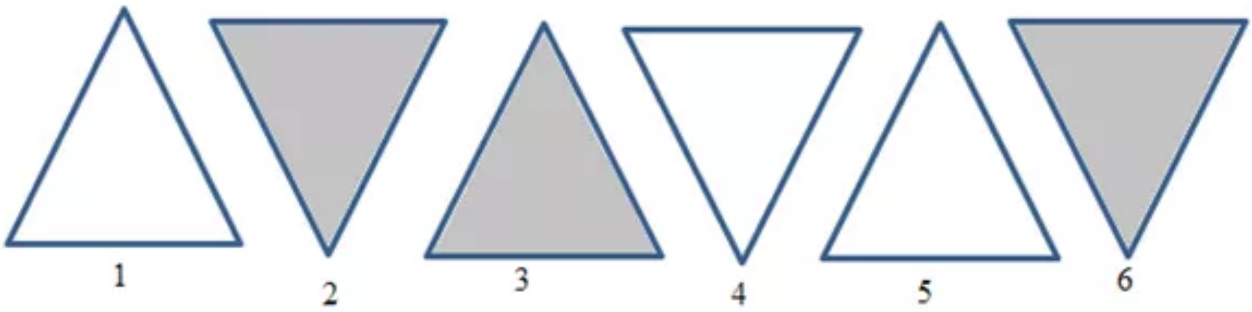
$t(10)=350+20$ Simplify

$t(10)=370$ Add

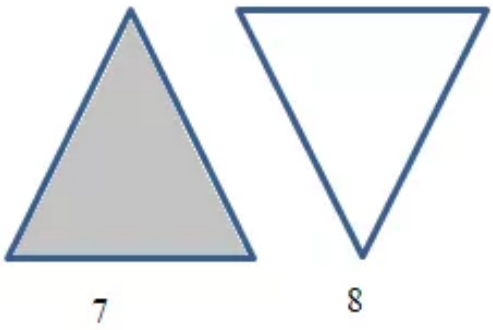
Thus, temperature of a rock below 10 kilometers of Earth's surface is $\boxed{370^\circ C}$.

Answer 12PA.

Consider the following pattern



The above pattern consists of triangles. From second triangle every triangle is generated by rotating the previous triangle over 180°. The second and third triangles are shaded whereas the next two are un-shaded. So, the 6th and 7th triangle should be shaded triangles and 8th triangle is un- shaded. The next two figures are as shown below.



It can be observed that the figure 1 and figure 2 are similar to figure 5 and figure 6 respectively.

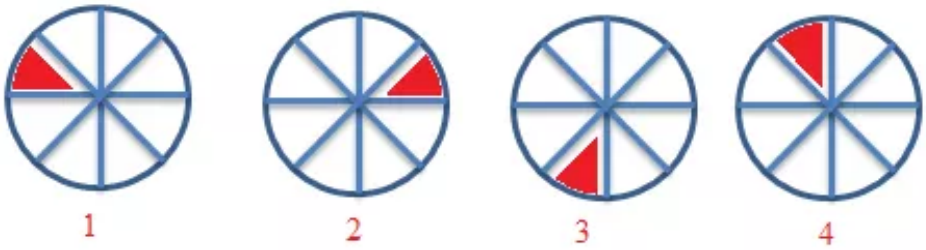
This means that the pattern repeats after every 4th figure.

Therefore designs 4, 8, 12, 16, 20, and so on, will be the same.

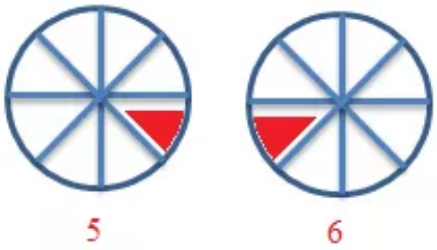
Hence, the figure 21 in the pattern is

Answer 13PA.

Consider the following pattern



The above pattern consists of circles which are divided into 8 sectors. In the first figure one of the sectors is shaded. The same shaded sector when shown in the second figure is placed on the same diameter but opposite to sector in figure one. The same process repeats to form the next circles. The next two figures are as shown below.



It can be observed from the pattern that the given pattern repeats after every 8th figure.

Therefore designs , 5, 13, 21, and so on, will be the same.

Hence, the figure 21 in the pattern is

Answer 14PA.

Consider the sequence

$0,2,6,12,20,\ldots$

The object is to find the next three terms of the sequence.

The differences between two consecutive terms of the sequence are

$$2-0=2$$
$$6-2=4$$
$$12-6=6$$
$$20-12=8$$

It can be observed that the difference between each term is increased by 2 in each successive term.

Thus to find the next three terms in the sequence $0,2,6,12,20,\ldots$, continue adding 2 to each successive difference. That is adding 10, 12, and 14.

Hence, the next three terms of the sequence are $\boxed{30,42,\text{and } 56}$.

Answer 15PA.

Consider the sequence

$9,7,10,8,11,9,12,\ldots$

The object is to find the next three terms of the sequence.

The differences between two consecutive terms of the sequence are

$7-9=-2$

$10-7=3$

$8-10=-2$

$11-8=3$

$9-11=-2$

$12-9=3$

It can be observed that the second, fourth, sixth, so on terms are decreased by 2 with the previous number and third, fifth, so on terms are increased by 3.

Thus to find the next three terms in the sequence $9,7,10,8,11,9,12,\ldots$, continue subtracting 2 from 12 and adding 3 to the resulting term and repeat the process. That is adding $-2,3,\text{ and}-2$.

Hence, the next three terms of the sequence are $\boxed{10,13,\text{and } 11}$.

Answer 16PA.

Consider the sequence

$1,4,9,16,\ldots$

The object is to find the next three terms of the sequence.

The numbers in the given can be written as

$1=1^2$

$4=2^2$

$9=3^2$

$16=4^2$

It can be observed that the 1st term of the sequence is 1^2, second term of the sequence is 2^2, the 3rd term of the sequence is 3^2, and so on.

Thus the 5th, 6th, and 7th terms of the sequence are $5^2,6^2,\text{and } 7^2$ respectively.

Hence, the next three terms of the sequence are $\boxed{25,36,\text{and } 49}$.

Answer 17PA.

Consider the sequence

$0,2,5,9,14,20,\ldots$

The object is to find the next three terms of the sequence.

The differences between two consecutive terms of the sequence are

$2-0=2$

$5-2=3$

$9-5=4$

$14-9=5$

$20-14=6$

It can be observed that the difference between each term is increased by 1 in each successive term.

Thus to find the next three terms in the sequence $0,2,5,9,14,20,\ldots$, continue adding 1 to each successive difference. That is adding 7, 8, and 9.

Hence, the next three terms of the sequence are $\boxed{27,35,\text{and } 44}$.

Answer 18PA.

Consider the sequence

$a+1,a+2,a+3,\ldots$

The object is to find the next three terms of the sequence.

The differences between two consecutive terms of the sequence are

$a+2-(a+1)=a+2-a-1$ or 1

$a+3-(a+2)=a+3-a-2$ or 1

It can be observed that the difference between each term is 1.

Thus to find the next three terms in the sequence $a+1,a+2,a+3,\ldots$ continue adding 1 to each term.

Hence, the next three terms of the sequence are $\boxed{a+4,a+5,\text{and } a+6}$.

Answer 19PA.

Consider the sequence

$x+1, 2x+1, 3x+1, \ldots$

The object is to find the next three terms of the sequence.

The differences between two consecutive terms of the sequence are

$2x+1-(x+1)=2x+1-x-1$ or x

$3x+1-(2x+1)=3x+1-2x-1$ or x

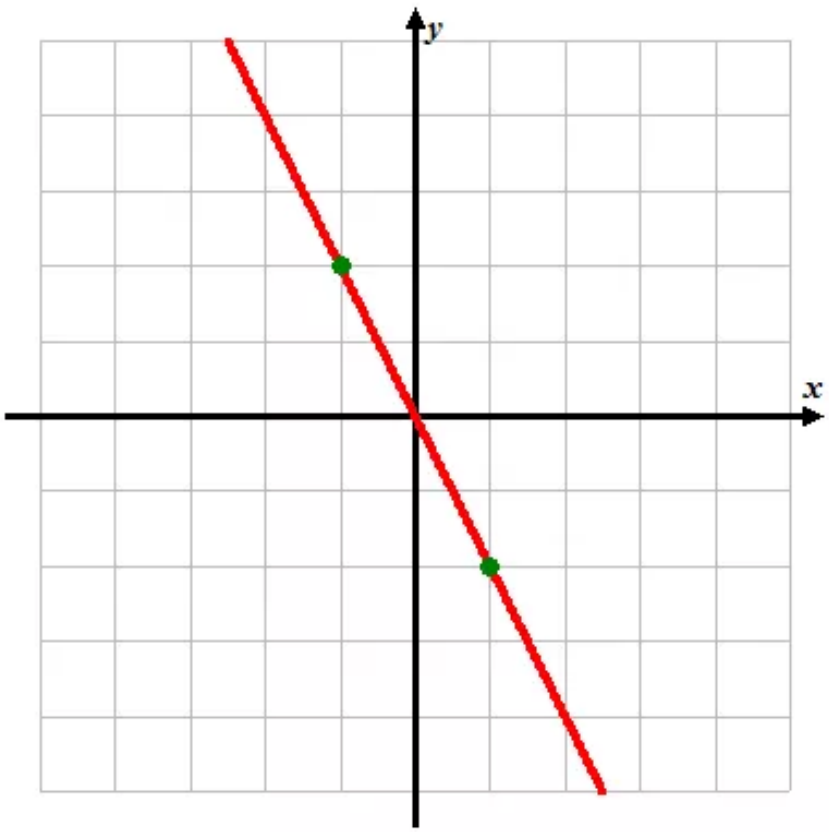
It can be observed that the difference between each term is x.

Thus to find the next three terms in the sequence $x+1, 2x+1, 3x+1, \ldots$ continue adding x to each term.

Hence, the next three terms of the sequence are $\boxed{4x+1, 5x+1, 6x+1}$.

Answer 20PA.

Consider the graph of the function



Construct a table of ordered pairs which lies on the above graph.

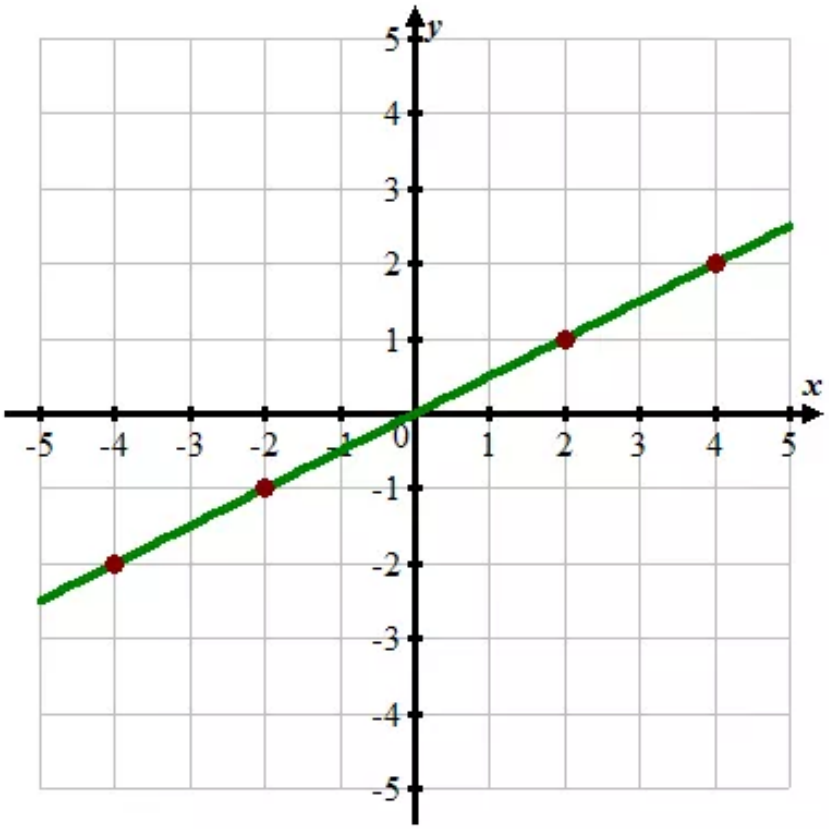
x	y
-1	2
1	-2

From the above table it can be observed that the values of y are -2 times the values of x. So, the required equation in function notation is

$\boxed{y=-2x}$ or $\boxed{f(x)=-2x}$

Answer 21PA.

Consider the graph of the function



Construct a table of ordered pairs which lies on the above graph.

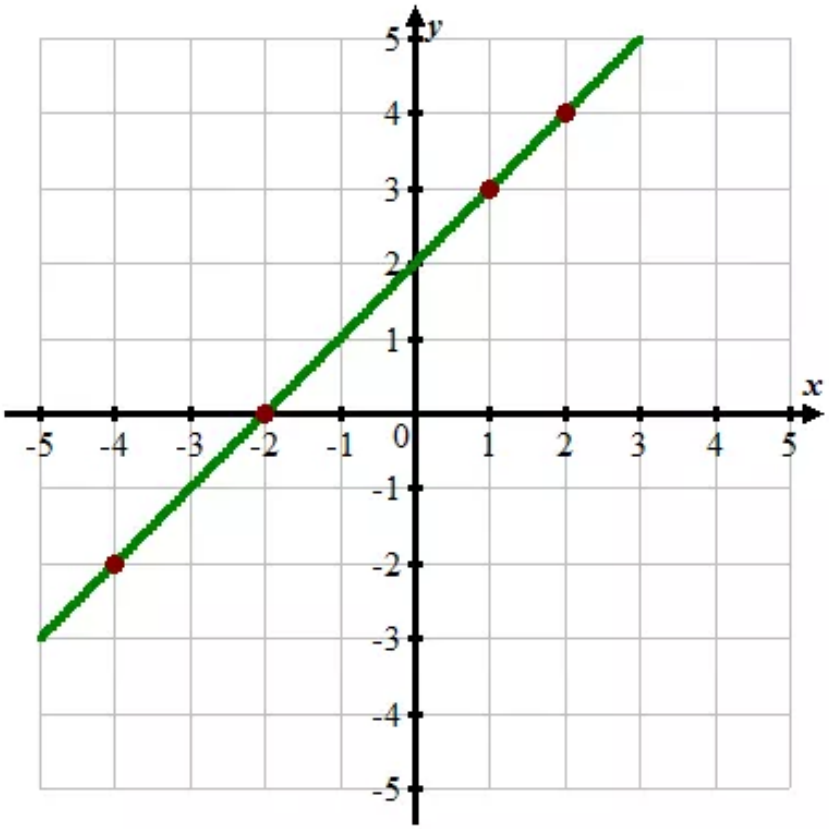
x	y
2	1
4	2
-2	-1
-4	-2

From the above table it can be observed that the values of y are $\frac{1}{2}$ times the values of x. So, the required equation in function notation is

$\boxed{y=\frac{1}{2}x}$ or $\boxed{f(x)=\frac{1}{2}x}$

Answer 22PA.

Consider the graph of the function



Construct a table of ordered pairs which lies on the above graph.

x	y
-2	0
-4	-2
1	3
2	4

The differences between two consecutive values of x are $-2, 5, 1.$

The differences between two consecutive values of y are $-2, 5, 1.$

Thus, the difference in y values is equal difference of x values.

Therefore, the suitable equation might be $y = x.$

Before concluding the results first check this equation.

Check: If $x = 1,$ then $y = 1.$ But the y value for $x = 1,$ is 3.

This is a difference of 2. Try some other values in the domain to see if the same difference occurs.

x	x	y
-2	-2	0
-4	-4	-2
1	1	3
2	2	4

From above table it can be observed that y is always 2 more than $x.$

This pattern suggests that 2 should be added to one side of the equation in order to correctly describe the relation.

Check $y = x + 2.$

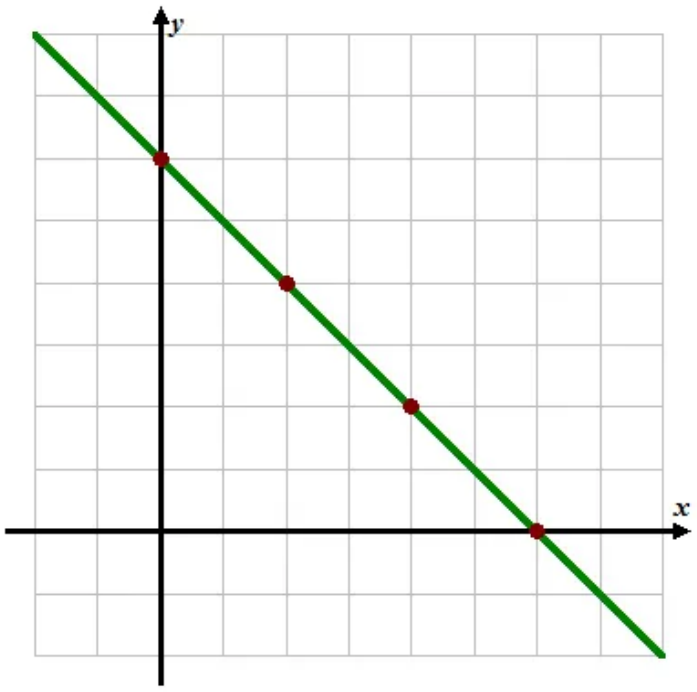
If $x = -2,$ then $y = -2 + 2$ or 0 ✓

If $x = 2,$ then $y = 2 + 2$ or 4 ✓

Hence, an equation in function notation for the given relation is $\boxed{y = x + 2}$

Answer 23PA.

Consider the graph of the function



Construct a table of ordered pairs which lies on the above graph.

x	y
0	6
2	4
4	2
6	0

The difference between any two consecutive values of x is 2.

The difference between any two consecutive values of y is $-2.$

Thus, the difference in y values is equal to negative difference of x values.

Therefore, the suitable equation might be $y = -x.$

Before concluding the results first check this equation.

Check: If $x = 2,$ then $y = -2$. But the y value for $x = 2,$ is 4.

This is an increment of 6. Try some other values in the domain to see if the same increment occurs.

x	x	y
0	0	6
2	−2	4
4	−4	2
6	−6	0

From above table it can be observed that y is always 6 more than $-x.$

This pattern suggests that 6 should be added to one side of the equation in order to correctly describe the relation.

Check $y = -x + 6.$

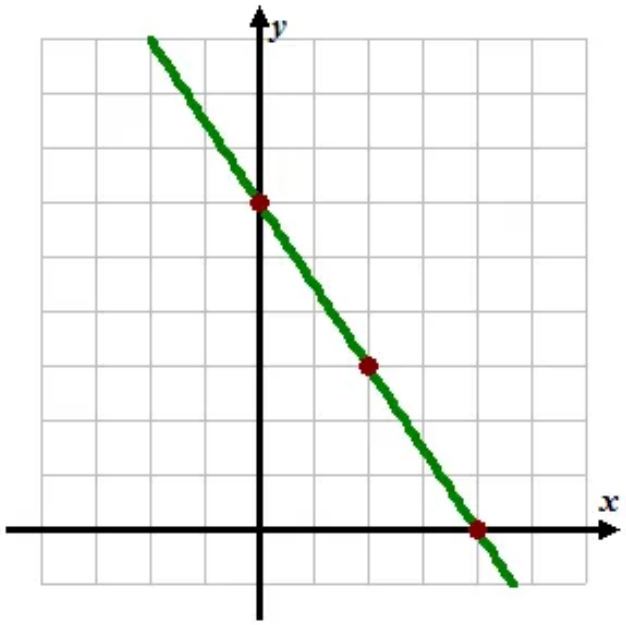
If $x = 0$, then $y = 0 + 6$ or 6 ✓

If $x = 2$, then $y = -2 + 6$ or 4 ✓

Hence, an equation in function notation for the given relation is $\boxed{y = -x + 6}$

Answer 24PA.

Consider the graph of the function



Construct a table of ordered pairs which lies on the above graph.

x	y
0	6
2	3
4	0

The difference between any two consecutive values of x is 2.

The difference between any two consecutive values of y is -3.

Thus, the difference in y values is equal to $-\frac{3}{2}$ times of difference of x values.

Therefore, the suitable equation might be $y = -\frac{3}{2}x$.

Before concluding the results first check this equation.

Check: If $x = 0,$ then $y = 0$. But the *y* value for $x = 0,$ is 6.

This is an increment of 6. Try some other values in the domain to see if the same increment occurs.

x	$-\frac{3}{2}x$	y
0	0	6
2	−3	3
4	−6	0

From above table it can be observed that *y* is always 6 more than $-\frac{3}{2}x.$

This pattern suggests that 6 should be added to one side of the equation in order to correctly describe the relation.

Check $y = -\frac{3}{2}x + 6.$

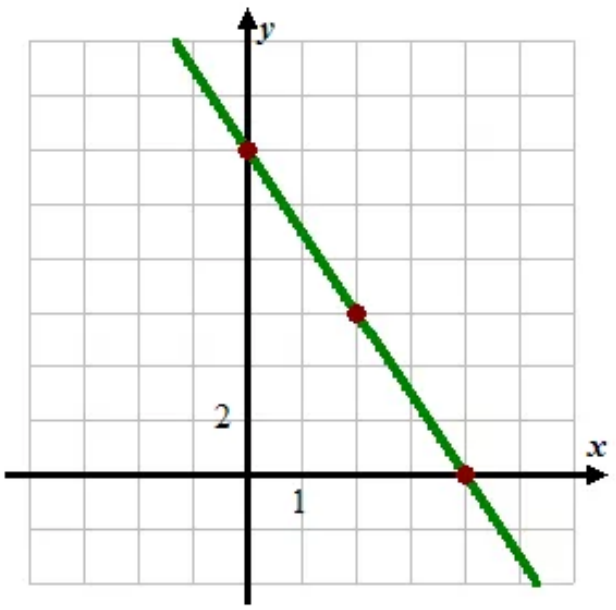
If $x = 0$, then $y = 0 + 6$ or 6 ✓

If $x = 2$, then $y = -3 + 6.$ or 3 ✓

Hence, an equation in function notation for the given relation is $\boxed{y = -\frac{3}{2}x + 6}$

Answer 25PA.

Consider the graph of the function



Construct a table of ordered pairs which lies on the above graph.

x	y
0	12
2	6
4	0

The difference between any two consecutive values of x is 2.

The difference between any two consecutive values of y is $-6.$

Thus, the difference in y values is equal to -3 times of difference of x values.

Therefore, the suitable equation might be $y=-3x.$

Before concluding the results first check this equation.

Check: If $x=0,$ then $y=0$. But the y value for $x=0,$ is 12.

This is an increment of 12. Try some other values in the domain to see if the same increment occurs.

x	$-3x$	y
0	0	12
2	−6	6
4	−12	0

From above table it can be observed that y is always 12 more than $-3x.$

This pattern suggests that 12 should be added to one side of the equation in order to correctly describe the relation.

Check $y=-3x+12.$

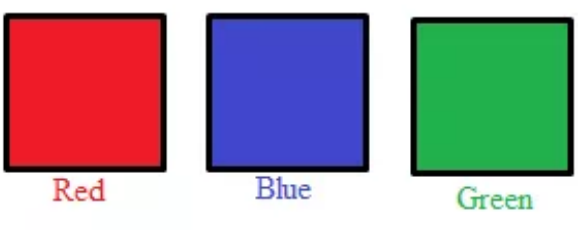
If $x=0$, then $y=0+12$ or 12 ✓

If $x=2$, then $y=-6+12.$ or 6 ✓

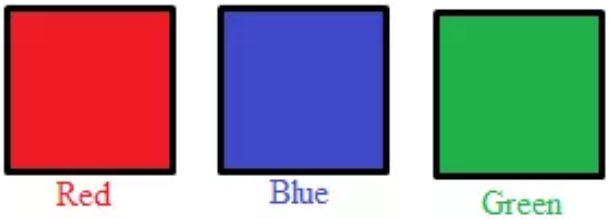
Hence, an equation in function notation for the given relation is $\boxed{y=-3x+12}$

Answer 26PA.

Consider the following pattern



The above pattern c is chain of three colors in that order. The next three figures are as shown below

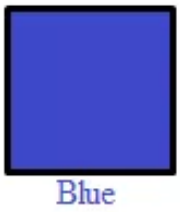


It can be observed that the color of figure 1 and figure 4 are similar.

This means that the pattern repeats after every 3rd figure.

Therefore designs 3, 6, 9,12,15,18, 21..., 51 and so on, will be the same color green.

Hence, the figure 50 in the pattern is



Hence, the 50th person receives $\boxed{\text{Blue}}$ color.

Answer 27PA.

Consider the Fibonacci sequence

1

$1+1=2$

$2+1=3$

$3+2=5$

The first term of the Fibonacci sequence is 1. From second term onwards each number is obtained by adding the two numbers that precede it.

Thus, the first 12 terms of the sequence are

$a_1=1$

$a_2=1$

$a_3=1+1 \text{ or } 2$

$a_4=2+1 \text{ or } 3$

$a_5=3+2 \text{ or } 5$

$a_6=5+3 \text{ or } 8$

$a_7=8+5 \text{ or } 13$

$a_8=13+8 \text{ or } 21$

$a_9=21+13 \text{ or } 34$

$a_{10}=34+21 \text{ or } 55$

$a_{11}=55+34 \text{ or } 89$

$a_{12}=89+55 \text{ or } 144$

Hence, $\boxed{1,1,2,3,5,8,13,21,34,55,89,\text{and } 144}$ are the first 12 terms of the Fibonacci sequence.

Answer 28PA.

Consider the Fibonacci sequence

1

$1+1=2$

$2+1=3$

$3+2=5$

$\vdots$

The first term of the Fibonacci sequence is 1. From second term onwards each number is obtained by adding the two numbers that precede it.

The following are some terms in the Fibonacci sequence

$a_1=1$

$a_2=1$

$a_3=1+1 \text{ or } 2$

$a_4=2+1 \text{ or } 3$

$a_5=3+2 \text{ or } 5$

$a_6=5+3 \text{ or } 8$

$a_7=8+5 \text{ or } 13$

$a_8=13+8 \text{ or } 21$

$a_9=21+13 \text{ or } 34$

$a_{10}=34+21 \text{ or } 55$

$a_{11}=55+34 \text{ or } 89$

$a_{12}=89+55 \text{ or } 144$

The 3rd number, 6th number, 9th number, and so on are $2,8,34,144,\ldots$

It can be observed that each term in $2,8,34,144,\ldots$ are multiple of 2.

Therefore, these terms are divisible by 2.

The 4th number, 8th number, 12th number, and so on are $3,21,144,\ldots$

It can be observed that each term in $3,21,144,\ldots$ are multiple of 3.

Therefore, these terms are divisible by 3.

The 5th number, 10th number, 15th number, and so on are $5,55,610,\ldots$

It can be observed that each term in $5,55,610,\ldots$ are multiple of 5.

Therefore, these terms are divisible by 5.

Hence, every fourth term of the Fibonacci sequence is $\boxed{\text{divisible by 3}}$ and every fifth term is $\boxed{\text{divisible by 5}}$.

Answer 29PA.

Consider the following table that shows the maximum heart rate to maintain, for different ages during aerobic activities.

Age(yr)	20	30	40	50	60	70
Pluse rate (beats/min)	175	166	157	148	139	130

The difference between any two consecutive values of A is 10.

The difference between any two consecutive values of P is $-9.$

Thus, the difference in P values is equal to $-\frac{9}{10}$ times of difference of A values.

Therefore, the suitable equation might be $P=-\frac{9}{10}A.$

Before concluding the results first check this equation.

Check: If $A=20,$ then $P=-\frac{9}{10}(20)$ or -18. But the P value for $A=20,$ is 175.

This is an increment of 193. Try some other values in the domain to see if the same increment occurs.

A	20	30	40	50	60	70
$-\frac{9}{10}A$	-18	-27	-36	-45	-54	-63
P	175	166	157	148	139	130

From above table it can be observed that P is always 193 more than $-\frac{9}{10}A.$

This pattern suggests that 193 should be added to one side of the equation in order to correctly describe the relation.

Check $P=-\frac{9}{10}A+193.$

If $A=20,$, then $P=-\frac{9}{10}(20)+193$ or 175 ✓

If $A=50,$, then $P=-\frac{9}{10}(50)+193$ or 148 ✓

Hence, an equation in function notation for the given relation is $\boxed{P(A)=-\frac{9}{10}A+193}$

Answer 30PA.

Consider the following table that shows the maximum heart rate to maintain, for different ages during aerobic activities.

Age(yr)	20	30	40	50	60	70
Pulse rate (beats/min)	175	166	157	148	139	130

The object is to find the maximum heart rate to maintain of a 10-year old and 80-year old person in aerobic training.

For this first find the function notation for the relation given in the above table.

The difference between any two consecutive values of *A* is 10.

The difference between any two consecutive values of *P* is $-9.$

Thus, the difference in *P* values is equal to $-\frac{9}{10}$ times of difference of *A* values.

Therefore, the suitable equation might be $P=-\frac{9}{10}A.$

Before concluding the results first check this equation.

Check: If $A=20,$ then $P=-\frac{9}{10}(20)$ or -18. But the *P* value for $A=20,$ is 175.

This is an increment of 193. Try some other values in the domain to see if the same increment occurs.

A	20	30	40	50	60	70
$-\frac{9}{10}A$	-18	-27	-36	-45	-54	-63
P	175	166	157	148	139	130

From above table it can be observed that *P* is always 193 more than $-\frac{9}{10}A.$

This pattern suggests that 193 should be added to one side of the equation in order to correctly describe the relation.

Check $P=-\frac{9}{10}A+193.$

If $A=20,$, then $P=-\frac{9}{10}(20)+193$ or 175 ✓

If $A=50,$, then $P=-\frac{9}{10}(50)+193$ or 148 ✓

Hence, an equation in function notation for the given relation is $P(A)=-\frac{9}{10}A+193$

Replacing *A* by 10 in the function $P(A)=-\frac{9}{10}A+193$

$P(A)=-\frac{9}{10}A+193$ Original function

$P(10)=-\frac{9}{10}(10)+193$ Replace *A* by 10

$P(10)=-9+193$ Simplify

$P(10)=184$ Add

Thus, a 10-year old person should maintain $\boxed{184 \text{ beats/min}}$ maximum heart rate in aerobic training.

Replacing *A* by 80 in the function $P(A)=-\frac{9}{10}A+193$

$P(A)=-\frac{9}{10}A+193$ Original function

$P(10)=-\frac{9}{10}(80)+193$ Replace *A* by 80

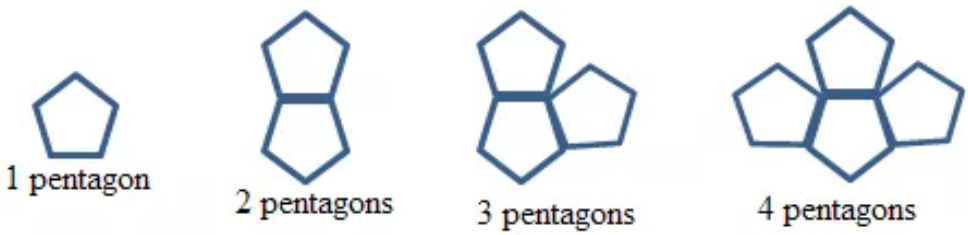
$P(10)=-72+193$ Simplify

$P(10)=121$ Add

Thus, an 80-year old person should maintain $\boxed{121 \text{ beats/min}}$ maximum heart rate in aerobic

Answer 31PA.

Consider the following arrangement of regular pentagons where each side of each pentagon is 1 centimeter.



The object is to find the perimeter of each arrangement of pentagons.

Since each side of pentagon is 1 centimeter, so the perimeter of pentagon in first arrangement is $1+1+1+1+1=5$ centimeters.

In the 2nd diagram of the arrangement of pentagons, there are 2 pentagons so that only one side touches to each other. So the perimeter of the 2 pentagons in the arrangement is

$1+1+1+1+1+1+1+1=8$ centimeters.

In the 3rd diagram of the arrangement of pentagons, there are 3 pentagons so that only one side touches to each other. So the perimeter of the 3 pentagons in the arrangement is

$1+1+1+1+1+1+1+1+1+1+1=11$ centimeters.

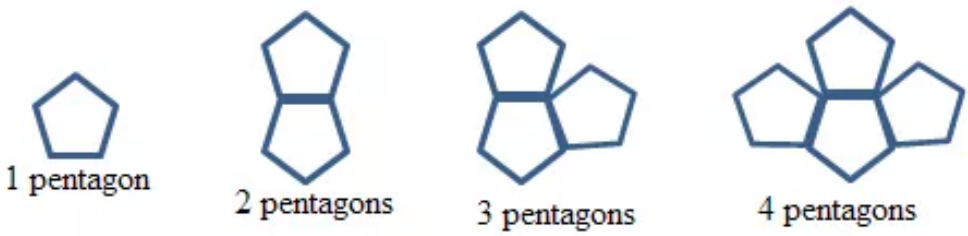
In the 4th diagram of the arrangement of pentagons, there are 4 pentagons so that only one side touches to each other. So the perimeter of the 4 pentagons in the arrangement is

$1+1+1+1+1+1+1+1+1+1+1+1+1+1=14$ centimeters.

Hence, the perimeters for each arrangement of pentagons are $\boxed{5\text{cms},8\text{cms},11\text{cms},\text{and } 14\text{cms}}$ respectively.

Answer 32PA.

Consider the following arrangement of regular pentagons where each side of each pentagon is 1 centimeter.



The object is to find an equation in function form to represent the perimeter $f(n)$ on n pentagons.

First find the perimeter of each arrangement of pentagons shown above.

Since each side of pentagon is 1 centimeter, so the perimeter of pentagon in first arrangement is $1+1+1+1+1=5$ centimeters.

In the 2nd diagram of the arrangement of pentagons, there are 2 pentagons so that only one side touches to each other. So the perimeter of the 2 pentagons in the arrangement is

$1+1+1+1+1+1+1+1=8$ centimeters.

In the 3rd diagram of the arrangement of pentagons, there are 3 pentagons so that only one side touches to each other. So the perimeter of the 3 pentagons in the arrangement is

$1+1+1+1+1+1+1+1+1+1+1=11$ centimeters.

In the 4th diagram of the arrangement of pentagons, there are 4 pentagons so that only one side touches to each other. So the perimeter of the 4 pentagons in the arrangement is

$1+1+1+1+1+1+1+1+1+1+1+1+1+1=14$ centimeters.

Hence, the perimeters for each arrangement of pentagons are 5cms, 8cms, 11cms, and 14cms respectively.

The differences between two consecutive terms of the sequence are

$$8-5=3$$
$$11-8=3$$
$$14-11=3$$

It can be observed that the difference between the terms is constant and equal to 3.

Therefore, the common difference is 3 and the first term is $a_1=5$

Using the formula for the *n*th term of an arithmetic sequence $5,8,11,14,\ldots$

$a_n=a_1+(n-1)d$ Formula for nth term

$a_n=5+(n-1)3$ $a_1=5, d=3$

$a_n=5+3n-3$ Distributive Property

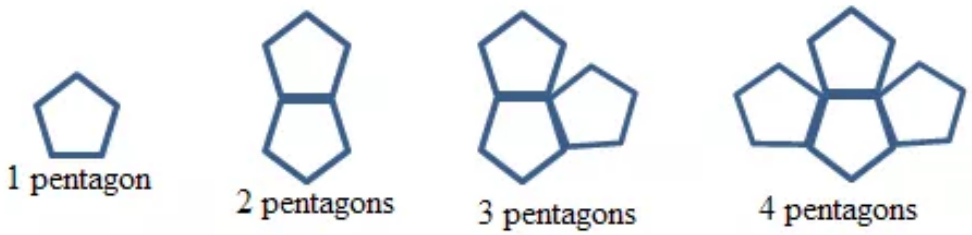
$a_n=3n+2$ Simplify

Hence equations in function form to represent the perimeter $f(n)$ on *n* pentagons.

is $\boxed{f(n)=3n+2}$

Answer 33PA.

Consider the following arrangement of regular pentagons where each side of each pentagon is 1 centimeter.



The object is to find the perimeter if 24 pentagons are used.

First find an equation in function form to represent the perimeter $f(n)$ on *n* pentagons.

Before finding the equation in function, find the perimeter of each arrangement of pentagons shown above.

Since each side of pentagon is 1 centimeter, so the perimeter of pentagon in first arrangement is $1+1+1+1+1=5$ centimeters.

In the 2nd diagram of the arrangement of pentagons, there are 2 pentagons so that only one side touches to each other. So the perimeter of the 2 pentagons in the arrangement is

$1+1+1+1+1+1+1+1=8$ centimeters.

In the 3rd diagram of the arrangement of pentagons, there are 3 pentagons so that only one side touches to each other. So the perimeter of the 3 pentagons in the arrangement is

$1+1+1+1+1+1+1+1+1+1+1=11$ centimeters.

In the 4th diagram of the arrangement of pentagons, there are 4 pentagons so that only one side touches to each other. So the perimeter of the 4 pentagons in the arrangement is

$1+1+1+1+1+1+1+1+1+1+1+1+1+1=14$ centimeters.

Hence, the perimeters for each arrangement of pentagons are 5cms, 8cms, 11cms, and 14cms respectively.

The differences between two consecutive terms of the sequence are

$$8-5=3$$
$$11-8=3$$
$$14-11=3$$

It can be observed that the difference between the terms is constant and equal to 3.

Therefore, the common difference is 3 and the first term is $a_1=5$

Using the formula for the *n*th term of an arithmetic sequence $5,8,11,14,\ldots$

$a_n=a_1+(n-1)d$	Formula for nth term
$a_n=5+(n-1)3$	$a_1=5, d=3$
$a_n=5+3n-3$	Distributive Property
$a_n=3n+2$	Simplify

Hence equations in function form to represent the perimeter $f(n)$ on *n* pentagons.

is $f(n)=3n+2$

Replacing *n* by 24 in the function $f(n)=3n+2$

$f(n)=3n+2$ Original function

$f(24)=3(24)+2$ Replace *n* by 24

$f(24)=72+2$ Simplify

$f(24)=74$ Add

Thus the perimeter if 24 pentagons are used is $\boxed{74 \text{ cms}}$.

Answer 34PA.

Writing the equations from patterns is important in science because in scientific experiments it is necessary to find a relation or develop a formula from observing the results of an experiment.

Consider the table which shows the different volumes of water and the corresponding volumes of ice.

Volume of Water $\left(\text{ft}^3\right)$	11	22	33	44	55
Volume of Ice $\left(\text{ft}^3\right)$	12	24	36	48	60

The relationship between the volume of water and the volume of ice is for every 11 cubic feet the volume of water increase, the volume of ice increases 12 cubic feet. So, the next five values of the above table can be obtained from the following table.

Volume of Water $\left(\text{ft}^3\right)$	66	77	88	99	110
Volume of Ice $\left(\text{ft}^3\right)$	72	84	96	108	120

Hence, the container should have volume of at least $\boxed{\text{108 cubic feet}}$

Answer 35PA.

Consider the sequence

$3,4,6,9,\ldots$

The object is to find the next two terms of the sequence.

The differences between two consecutive terms of the sequence are

$4-3=1$

$6-4=2$

$9-6=3$

It can be observed that the difference between each term is increased by 1 in each successive term.

Thus to find the next two terms in the sequence $3,4,6,9,\ldots$, continue adding 1 to each successive difference. That is adding 4 and 5.

Hence, the next two terms of the sequence are 13, 18.

Therefore, the correct option is $\boxed{B}$.

Answer 36PA.

Suppose *x* number of candies are distributed to each of 5 children from *p* pieces of candy.

If 4 pieces remain after this distribution, then *p* can be expressed as

$p = x + 4$, ----- (1)

where *x* is a multiple of 5.

The object is to find the number of pieces of candy remain after $p + 4$ pieces of candy were distributed equally among 5 children.

Add 4 each side of equation (1)

$p = x + 4$ Original equation

$p + 4 = (x + 4) + 4$ Add 4 each side

$p + 4 = x + 8$ Add

$p + 4 = (x + 5) + 3$ Write $x + 8$ as $(x + 5) + 3$

$p + 4 = y + 3 \quad y = x + 5$

In the last equation, *y* is multiple of 5 because *x* is multiple of 5 implies that $x + 5$ is also a multiple of 5.

Thus, after $p + 4$ pieces of candy were distributed equally among 5 children there always remain 3 candies.

Therefore, the correct option is $\boxed{D}$.

Answer 37MYS.

Consider an arithmetic sequence

$1, 4, 7, 10, \ldots$

The object is to find the next three terms of the sequence.

First find the common difference of the sequence.

The differences between two consecutive numbers in the sequence

$4 - 1 = 3$

$7 - 4 = 3$

$10 - 7 = 3$

It can be observed that the difference between the terms is constant and equal to 3.

Therefore, the common difference is 3.

The next three term of the sequence $1,4,7,10,\ldots$ can be obtained by adding the common difference 3 to the last term 10 and continue adding 3 until the next terms are found.

$$10+3=13$$
$$13+3=16$$
$$16+3=19$$

Hence, the next three terms of the given arithmetic sequence are $\boxed{13,16,19}$.

Answer 38MYS.

Consider an arithmetic sequence

$$9,5,1,-3,\ldots$$

The object is to find the next three terms of the sequence.

First find the common difference of the sequence.

The differences between two consecutive numbers in the sequence

$$5-9=-4$$
$$1-5=-4$$
$$-3-1=-4$$

It can be observed that the difference between the terms is constant and equal to -4.

Therefore, the common difference is -4.

The next three term of the sequence $9,5,1,-3,\ldots$ can be obtained by adding the common difference -4 to the last term -3 and continue adding -4 until the next terms are found.

$$-3+(-4)=-7$$
$$-7+(-4)=-11$$
$$-11+(-4)=-15$$

Hence, the next three terms of the given arithmetic sequence are $\boxed{-7,-11,-15}$.

Answer 39MYS.

Consider an arithmetic sequence

$$-25,-19,-13,-7,\ldots$$

The object is to find the next three terms of the sequence.

First find the common difference of the sequence.

The differences between two consecutive numbers in the sequence

$$-19-(-25)=-19+25 \text{ or } 6$$
$$-13-(-19)=-13+19 \text{ or } 6$$
$$-7-(-13)=-7+13 \text{ or } 6$$

It can be observed that the difference between the terms is constant and equal to 6.

Therefore, the common difference is 6.

The next three term of the sequence $-25,-19,-13,-7,\ldots$ can be obtained by adding the common difference 4 to the last term -7 and continue adding 6 until the next terms are found.

$-7+6=-1$

$-1+6=5$

$5+6=11$

Hence, the next three terms of the given arithmetic sequence are $\boxed{-1,5,11}$.

Answer 40MYS.

Consider an arithmetic sequence

$22,34,46,58,\ldots$

The object is to find the next three terms of the sequence.

First find the common difference of the sequence.

The differences between two consecutive numbers in the sequence

$34-22=12$

$46-34=12$

$58-46=12$

It can be observed that the difference between the terms is constant and equal to 12.

Therefore, the common difference is 12.

The next three term of the sequence $22,34,46,58,\ldots$ can be obtained by adding the common difference 12 to the last term 58 and continue adding 12 until the next terms are found.

$58+12=70$

$70+12=82$

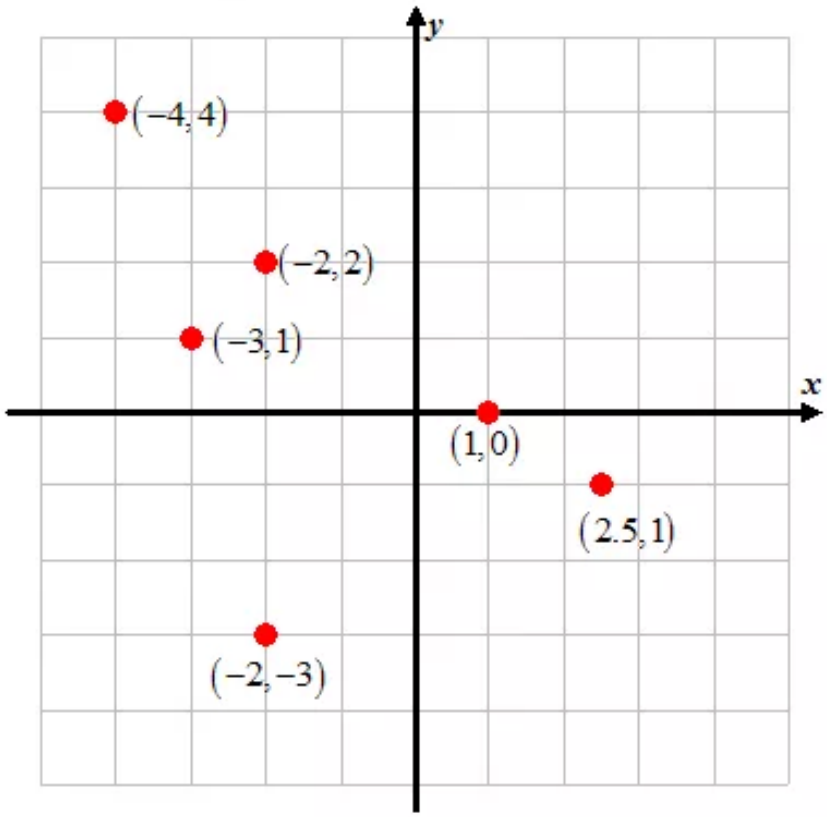
$82+12=94$

Hence, the next three terms of the given arithmetic sequence are $\boxed{70,82,94}$.

Answer 41MYS.

A function is a relation in which each first component (element of domain) in the ordered pairs corresponds to exactly one second component (element of range).

Consider the graph:



The set of ordered pairs of above graph is

$$\{(1,0),(2.5,1),(-2,2),(-2,-3),(-3,1),(-4,4)\}$$

The ordered pairs $(-2,2)$ and $(-2,-3)$ have the same *x*-value. Thus, the *x*-value -2 is assigned to two *y*-values, 2 and -3. So, the relation

$\{(1,0),(2.5,1),(-2,2),(-2,-3),(-3,1),(-4,4)\}$ is not a function.

Hence the relation represented by given graph is $\boxed{\text{not a function}}$.

Answer 42MYS.

The height of world's tallest waterfall A is 3212 feet and it is 102 feet higher than the waterfall T.

Thus, the height of waterfall T is 102 feet less than 3212 feet.

That is, the height of waterfall T is

$$3212-102=3110$$

Therefore, the height of waterfall T is $\boxed{3110 \text{ feet}}$