19.SOLUTION OF TRIANGLE

1. Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

2. Cosine Formula: (i) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (ii) $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$ (iii) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

3. Projection Formula: (i) $a = b \cos C + c \cos B$ (ii) $b = c \cos A + a \cos C$ (iii) $c = a \cos B + b \cos A$

4. Napier's Analogy - tangent rule:

(i) $\tan \frac{B - C}{2} = \frac{b - c}{b + c} \cot \frac{A}{2}$ (ii) $\tan \frac{C - A}{2} = \frac{c - a}{c + a} \cot \frac{B}{2}$ (iii) $\tan \frac{A - B}{2} = \frac{a - b}{a + b} \cot \frac{C}{2}$

5. Trigonometric Functions of Half Angles:

(i) $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$; $\sin \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$; $\sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(ii) $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$; $\cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$; $\cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(iii) $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$ where $s = \frac{a + b + c}{2}$ is semi perimetre of triangle.

(iv) $\sin A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)} = \frac{2\Delta}{bc}$

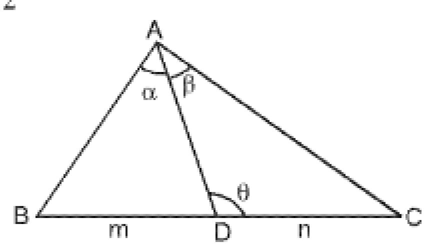
6. Area of Triangle (Δ) : $\Delta = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \sqrt{s(s-a)(s-b)(s-c)}$

7. m - n Rule:

If BD : DC = m : n, then

$(m + n) \cot \theta = m \cot \alpha - n \cot \beta$

$= n \cot B - m \cot C$



8. Radius of Circumcirlce :

$R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C} = \frac{abc}{4\Delta}$

9. Radius of The Incircle :

(i) $r = \frac{\Delta}{s}$

(ii) $r = (s - a) \tan \frac{A}{2} = (s - b) \tan \frac{B}{2} = (s - c) \tan \frac{C}{2}$

(iii) $r = \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\cos \frac{A}{2}}$ & so on

(iv) $r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

10. Radius of The Ex-Circles :

(i) $r_1 = \frac{\Delta}{s - a}$; $r_2 = \frac{\Delta}{s - b}$; $r_3 = \frac{\Delta}{s - c}$

(ii) $r_1 = s \tan \frac{A}{2}$; $r_2 = s \tan \frac{B}{2}$; $r_3 = s \tan \frac{C}{2}$

(iii) $r_1 = \frac{a \cos \frac{B}{2} \cos \frac{C}{2}}{\cos \frac{A}{2}}$ & so on

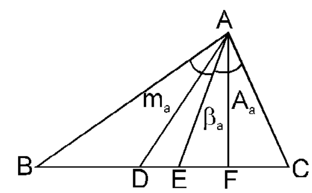
(iv) $r_1 = 4 R \sin \frac{A}{2} . \cos \frac{B}{2} . \cos \frac{C}{2}$

11. Length of Angle Bisectors, Medians & Altitudes :

(i) Length of an angle bisector from the angle A = $\beta_a = \dfrac{2bc\cos\frac{A}{2}}{b+c}$;

(ii) Length of median from the angle A = $m_a = \dfrac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

& (iii) Length of altitude from the angle A = $A_a = \dfrac{2\Delta}{a}$



12. The Distances of The Special Points from Vertices and Sides of Triangle:

(i) Circumcentre (O) : OA = R & O_a = R cos A **(ii)** Incentre (I) : IA = $r \operatorname{cosec}\dfrac{A}{2}$ & I_a = r

(iii) Excentre (I_1) : $I_1 A = r_1 \operatorname{cosec}\dfrac{A}{2}$ **(iv)** Orthocentre : HA = 2R cos A & H_a = 2R cos B cos C

(v) Centroid (G) : $GA = \dfrac{1}{3}\sqrt{2b^2+2c^2-a^2}$ & $G_a = \dfrac{2\Delta}{3a}$

13. Orthocentre and Pedal Triangle:

The triangle KLM which is formed by joining the feet of the altitudes is called the Pedal Triangle.

(i) Its angles are $\pi - 2A$, $\pi - 2B$ and $\pi - 2C$.

(ii) Its sides are a cosA = R sin 2A,
b cosB = R sin 2B and
c cosC = R sin 2C

(iii) Circumradii of the triangles PBC, PCA, PAB and ABC are equal.

14. Excentral Triangle:

The triangle formed by joining the three excentres I_1, I_2 and I_3 of Δ ABC is called the excentral or excentric triangle.

(i) Δ ABC is the pedal triangle of the $\Delta\, I_1 I_2 I_3$.

(ii) Its angles are $\dfrac{\pi}{2} - \dfrac{A}{2}$, $\dfrac{\pi}{2} - \dfrac{B}{2}$ & $\dfrac{\pi}{2} - \dfrac{C}{2}$.

(iii) Its sides are $4R\cos\dfrac{A}{2}$, $4R\cos\dfrac{B}{2}$ & $4R\cos\dfrac{C}{2}$.

(iv) $I\,I_1 = 4R\sin\dfrac{A}{2}$; $I\,I_2 = 4R\sin\dfrac{B}{2}$; $I\,I_3 = 4R\sin\dfrac{C}{2}$.

(v) Incentre I of Δ ABC is the orthocentre of the excentral $\Delta\, I_1 I_2 I_3$.

15. Distance Between Special Points :

(i) Distance between circumcentre and orthocentre $OH^2 = R^2(1 - 8\cos A\cos B\cos C)$

(ii) Distance between circumcentre and incentre $OI^2 = R^2\left(1 - 8\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}\right) = R^2 - 2Rr$

(iii) Distance between circumcentre and centroid $OG^2 = R^2 - \dfrac{1}{9}(a^2 + b^2 + c^2)$