

# BITSAT : SOLVED PAPER 2016

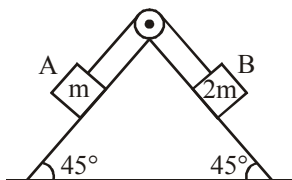
## (memory based)

### INSTRUCTIONS

- This question paper contains total 150 questions divided into four parts:
  - Part I : Physics Q. No. 1 to 40
  - Part II : Chemistry Q. No. 41 to 80
  - Part III : (A) English Proficiency Q. No. 81 to 95  
(B) Logical Reasoning Q. No. 96 to 105
  - Part IV : Mathematics Q. No. 106 to 150
- All questions are multiple choice questions with four options, only one of them is correct.
- Each correct answer awarded 3 marks and  $-1$  for each incorrect answer.
- Duration of paper 3 Hours

### PART - I : PHYSICS

- What should be the velocity of rotation of earth due to rotation about its own axis so that the weight of a person becomes  $\frac{3}{5}$  of the present weight at the equator. Equatorial radius of the earth is 6400 km.
  - (a)  $8.7 \times 10^{-7}$  rad/s
  - (b)  $7.8 \times 10^{-4}$  rad/s
  - (c)  $6.7 \times 10^{-4}$  rad/s
  - (d)  $7.4 \times 10^{-3}$  rad/s
- Block A of mass  $m$  and block B of mass  $2m$  are placed on a fixed triangular wedge by means of a massless, inextensible string and a frictionless pulley as shown in figure.



The wedge is inclined at  $45^\circ$  to the horizontal on both the sides. If the coefficient of friction between the block A and the wedge is  $\frac{2}{3}$  and that between the block B and the wedge is  $\frac{1}{3}$  and both the blocks A and B are released from rest, the acceleration of A will be

- (a)  $-1 \text{ ms}^{-2}$
  - (b)  $1.2 \text{ ms}^{-2}$
  - (c)  $0.2 \text{ ms}^{-2}$
  - (d) zero
- The surface charge density of a thin charged disc of radius  $R$  is  $\sigma$ . The value of the electric field at the centre of the disc is  $\frac{\sigma}{2\epsilon_0}$ . With respect to

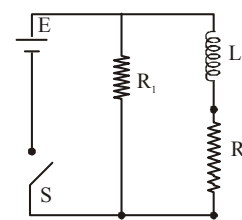
the field at the centre, the electric field along the axis at a distance  $R$  from the centre of the disc

- (a) reduces by 70.7%
  - (b) reduces by 29.3%
  - (c) reduces by 9.7%
  - (d) reduces by 14.6%
- The molecules of a given mass of a gas have r.m.s. velocity of  $200 \text{ ms}^{-1}$  at  $27^\circ\text{C}$  and  $1.0 \times 10^5 \text{ Nm}^{-2}$  pressure. When the temperature and pressure of the gas are respectively,  $127^\circ\text{C}$  and  $0.05 \times 10^5 \text{ Nm}^{-2}$ , the r.m.s. velocity of its molecules in  $\text{ms}^{-1}$  is :

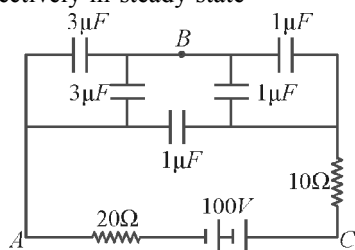
- (a)  $100\sqrt{2}$
- (b)  $\frac{400}{\sqrt{3}}$
- (c)  $\frac{100\sqrt{2}}{3}$
- (d)  $\frac{100}{3}$

- An inductor of inductance  $L = 400 \text{ mH}$  and resistors of resistance  $R_1 = 2\Omega$  and  $R_2 = 2\Omega$  are connected to a battery of emf  $12 \text{ V}$  as shown in the figure. The internal resistance of the battery is negligible. The switch  $S$  is closed at  $t = 0$ . The potential drop across  $L$  as a function of time is

- (a)  $\frac{12}{t} e^{-3t} \text{ V}$
- (b)  $6(1 - e^{-t/0.2}) \text{ V}$
- (c)  $12e^{-5t} \text{ V}$
- (d)  $6e^{-5t} \text{ V}$



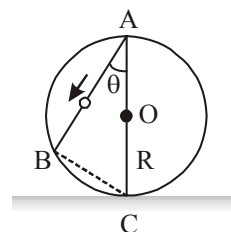
6. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area  $A$  and wire 2 has cross-sectional area  $3A$ . If the length of wire 1 increases by  $\Delta x$  on applying force  $F$ , how much force is needed to stretch wire 2 by the same amount?  
 (a)  $4F$  (b)  $6F$  (c)  $9F$  (d)  $F$
7. Two spheres of different materials one with double the radius and one-fourth wall thickness of the other are filled with ice. If the time taken for complete melting of ice in the larger sphere is 25 minute and for smaller one is 16 minute, the ratio of thermal conductivities of the materials of larger spheres to that of smaller sphere is  
 (a) 4 : 5 (b) 5 : 4 (c) 25 : 8 (d) 8 : 25
8. A biconvex lens has a radius of curvature of magnitude 20 cm. Which one of the following options best describe the image formed of an object of height 2 cm placed 30 cm from the lens?  
 (a) Virtual, upright, height = 1 cm  
 (b) Virtual, upright, height = 0.5 cm  
 (c) Real, inverted, height = 4 cm  
 (d) Real, inverted, height = 1 cm
9. In the figure below, what is the potential difference between the point  $A$  and  $B$  and between  $B$  and  $C$  respectively in steady state



- (a)  $V_{AB} = V_{BC} = 100V$   
 (b)  $V_{AB} = 75V, V_{BC} = 25V$   
 (c)  $V_{AB} = 25V, V_{BC} = 75V$   
 (d)  $V_{AB} = V_{BC} = 50V$
10. A radioactive element  $X$  converts into another stable element  $Y$ . Half life of  $X$  is 2 hrs. Initially only  $X$  is present. After time  $t$ , the ratio of atoms of  $X$  and  $Y$  is found to be 1 : 4, then  $t$  in hours is  
 (a) 2 (b) 4  
 (c) between 4 and 6 (d) 6
11. The approximate depth of an ocean is 2700 m. The compressibility of water is  $45.4 \times 10^{-11} \text{ Pa}^{-1}$  and density of water is  $10^3 \text{ kg/m}^3$ . What fractional compression of water will be obtained at the bottom of the ocean ?

- (a)  $1.0 \times 10^{-2}$  (b)  $1.2 \times 10^{-2}$   
 (c)  $1.4 \times 10^{-2}$  (d)  $0.8 \times 10^{-2}$
12. A frictionless wire  $AB$  is fixed on a sphere of radius  $R$ . A very small spherical ball slips on this wire. The time taken by this ball to slip from  $A$  to  $B$  is

- (a)  $\frac{\sqrt{2gR}}{g \cos \theta}$   
 (b)  $2\sqrt{gR} \cdot \frac{\cos \theta}{g}$   
 (c)  $2\sqrt{\frac{R}{g}}$   
 (d)  $\frac{gR}{\sqrt{g \cos \theta}}$



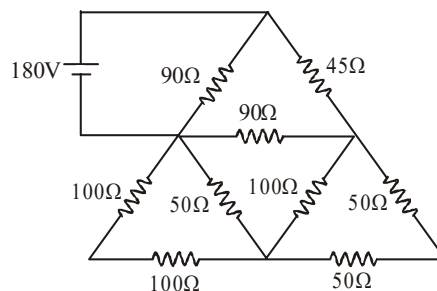
13. A string of length  $\ell$  is fixed at both ends. It is vibrating in its 3<sup>rd</sup> overtone with maximum amplitude 'a'. The amplitude at a distance  $\ell/3$  from one end is

- (a)  $a$  (b) 0 (c)  $\frac{\sqrt{3}a}{2}$  (d)  $\frac{a}{2}$

14. A deuteron of kinetic energy 50 keV is describing a circular orbit of radius 0.5 metre in a plane perpendicular to the magnetic field  $B$ . The kinetic energy of the proton that describes a circular orbit of radius 0.5 metre in the same plane with the same  $B$  is

- (a) 25 keV (b) 50 keV  
 (c) 200 keV (d) 100 keV

15. In the circuit shown in the figure, find the current in  $45 \Omega$ .



- (a) 4 A (b) 2.5 A  
 (c) 2 A (d) None of these
16. Kepler's third law states that square of period of revolution ( $T$ ) of a planet around the sun, is proportional to third power of average distance  $r$

between sun and planet i.e.  $T^2 = Kr^3$  here  $K$  is constant. If the masses of sun and planet are  $M$  and  $m$  respectively then as per Newton's law of gravitation force of attraction between them is

$$F = \frac{GMm}{r^2}, \text{ here } G \text{ is gravitational constant. The}$$

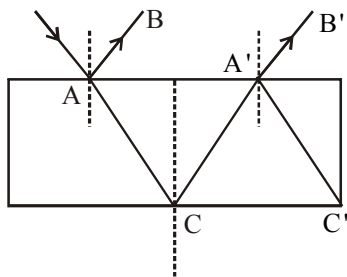
relation between  $G$  and  $K$  is described as

- (a)  $GMK = 4\pi^2$  (b)  $K = G$   
(c)  $K = \frac{1}{G}$  (d)  $GK = 4\pi^2$

17. Find the number of photon emitted per second by a 25 watt source of monochromatic light of wavelength  $6600 \text{ \AA}$ . What is the photoelectric current assuming 3% efficiency for photoelectric effect ?

- (a)  $\frac{25}{3} \times 10^{19} \text{ J}, 0.4 \text{ amp}$   
(b)  $\frac{25}{4} \times 10^{19} \text{ J}, 6.2 \text{ amp}$   
(c)  $\frac{25}{2} \times 10^{19} \text{ J}, 0.8 \text{ amp}$   
(d) None of these

18. A ray of light of intensity  $I$  is incident on a parallel glass slab at point A as shown in diagram. It undergoes partial reflection and refraction. At each reflection, 25% of incident energy is reflected. The rays AB and A'B' undergo interference. The ratio of  $I_{\max}$  and  $I_{\min}$  is :



- (a) 49:1 (b) 7:1 (c) 4:1 (d) 8:1

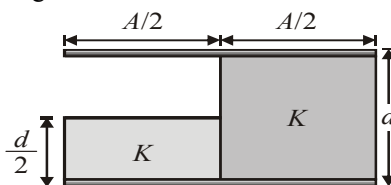
19. A capillary tube of radius  $r$  is immersed vertically in a liquid such that liquid rises in it to height  $h$  (less than the length of the tube). Mass of liquid in the capillary tube is  $m$ . If radius of the capillary tube is increased by 50%, then mass of liquid that will rise in the tube, is

- (a)  $\frac{2}{3}m$  (b)  $m$  (c)  $\frac{3}{2}m$  (d)  $\frac{9}{4}m$

20. The drift velocity of electrons in silver wire with cross-sectional area  $3.14 \times 10^{-6} \text{ m}^2$  carrying a current of 20 A is. Given atomic weight of Ag = 108, density of silver =  $10.5 \times 10^3 \text{ kg/m}^3$ .

- (a)  $2.798 \times 10^{-4} \text{ m/sec.}$   
(b)  $67.98 \times 10^{-4} \text{ m/sec.}$   
(c)  $0.67 \times 10^{-4} \text{ m/sec.}$   
(d)  $6.798 \times 10^{-4} \text{ m/sec.}$

21. A parallel plate capacitor of area ' $A$ ' plate separation ' $d$ ' is filled with two dielectrics as shown. What is the capacitance of the arrangement ?



- (a)  $\frac{3K\epsilon_0 A}{4d}$  (b)  $\frac{4K\epsilon_0 A}{3d}$   
(c)  $\frac{(K+1)\epsilon_0 A}{2d}$  (d)  $\frac{K(K+3)\epsilon_0 A}{2(K+1)d}$

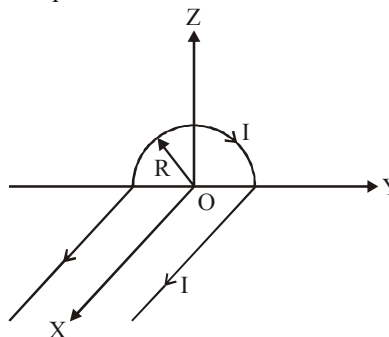
22. In the Young's double-slit experiment, the intensity of light at a point on the screen where the path difference is  $\lambda$  is  $K$ , ( $\lambda$  being the wavelength of light used). The intensity at a point where the path difference is  $\lambda/4$ , will be :

- (a)  $K$  (b)  $K/4$  (c)  $K/2$  (d) Zero

23. The mass of  ${}^7\text{N}^{15}$  is 15.00011 amu, mass of  ${}^8\text{O}^{16}$  is 15.99492 amu and  $m_p = 1.00783$  amu. Determine binding energy of last proton of  ${}^8\text{O}^{16}$ .

- (a) 2.13 MeV (b) 0.13 MeV  
(c) 10 MeV (d) 12.13 MeV

24. A wire carrying current  $I$  has the shape as shown in adjoining figure. Linear parts of the wire are very long and parallel to X-axis while semicircular portion of radius  $R$  is lying in Y-Z plane. Magnetic field at point O is :



$$(a) \quad \vec{B} = -\frac{\mu_0}{4\pi} \frac{I}{R} (\hat{i} \times 2\hat{k})$$

$$(b) \quad \vec{B} = -\frac{\mu_0}{4\pi} \frac{I}{R} (\hat{i} + 2\hat{k})$$

$$(c) \quad \vec{B} = \frac{\mu_0}{4\pi} \frac{I}{R} (\hat{i} - 2\hat{k})$$

$$(d) \quad \vec{B} = \frac{\mu_0}{4\pi} \frac{I}{R} (\hat{i} + 2\hat{k})$$

25. A stone projected with a velocity  $u$  at an angle  $\theta$  with the horizontal reaches maximum height  $H_1$ . When it is projected with velocity  $u$  at an angle

$\left(\frac{\pi}{2} - \theta\right)$  with the horizontal, it reaches maximum height  $H_2$ . The relation between the horizontal range  $R$  of the projectile, heights  $H_1$  and  $H_2$  is

$$(a) \quad R = 4\sqrt{H_1 H_2} \quad (b) \quad R = 4(H_1 - H_2)$$

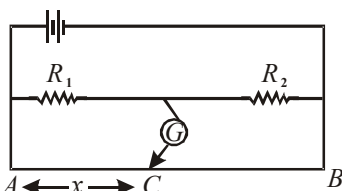
$$(c) \quad R = 4(H_1 + H_2) \quad (d) \quad R = \frac{H_1^2}{H_2^2}$$

26. If the series limit wavelength of Lyman series for the hydrogen atom is  $912 \text{ \AA}$ , then the series limit wavelength for Balmer series of hydrogen atoms is

$$(a) \quad 912 \text{ \AA} \quad (b) \quad 912 \times 2 \text{ \AA}$$

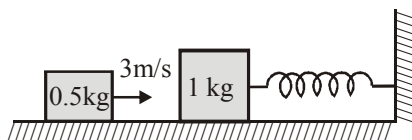
$$(c) \quad 912 \times 4 \text{ \AA} \quad (d) \quad \frac{912}{2} \text{ \AA}$$

27. In the shown arrangement of the experiment of the meter bridge if  $AC$  corresponding to null deflection of galvanometer is  $x$ , what would be its value if the radius of the wire  $AB$  is doubled?



$$(a) \quad x \quad (b) \quad x/4 \quad (c) \quad 4x \quad (d) \quad 2x$$

28. A  $1 \text{ kg}$  mass is attached to a spring of force constant  $600 \text{ N/m}$  and rests on a smooth horizontal surface with other end of the spring tied to wall as shown in figure. A second mass of  $0.5 \text{ kg}$  slides along the surface towards the first at  $3 \text{ m/s}$ . If the masses make a perfectly inelastic collision, then find amplitude and time period of oscillation of combined mass.



$$(a) \quad 5 \text{ cm}, \frac{\pi}{10} \text{ s}$$

$$(b) \quad 5 \text{ cm}, \frac{\pi}{5} \text{ s}$$

$$(c) \quad 4 \text{ cm}, \frac{2\pi}{5} \text{ s}$$

$$(d) \quad 4 \text{ cm}, \frac{\pi}{3} \text{ s}$$

29. The frequency of vibration of string is given by

$$v = \frac{p}{2l} \left[ \frac{F}{m} \right]^{1/2}$$

Here  $p$  is number of segments in the string and  $l$  is the length. The dimensional formula for  $m$  will be

$$(a) \quad [M^0 L T^{-1}] \quad (b) \quad [M L^0 T^{-1}]$$

$$(c) \quad [M L^{-1} T^0] \quad (d) \quad [M^0 L^0 T^0]$$

30. For the angle of minimum deviation of a prism to be equal to its refracting angle, the prism must be made of a material whose refractive index :

$$(a) \quad \text{lies between } \sqrt{2} \text{ and } 1$$

$$(b) \quad \text{lies between } 2 \text{ and } \sqrt{2}$$

$$(c) \quad \text{is less than } 1$$

$$(d) \quad \text{is greater than } 2$$

31. Consider elastic collision of a particle of mass  $m$  moving with a velocity  $u$  with another particle of the same mass at rest. After the collision the projectile and the struck particle move in directions making angles  $\theta_1$  and  $\theta_2$  respectively with the initial direction of motion. The sum of the angles  $\theta_1 + \theta_2$ , is :

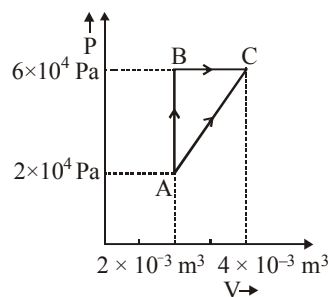
$$(a) \quad 45^\circ \quad (b) \quad 90^\circ \quad (c) \quad 135^\circ \quad (d) \quad 180^\circ$$

32. A conducting circular loop is placed in a uniform magnetic field of  $0.04 \text{ T}$  with its plane perpendicular to the magnetic field. The radius of the loop starts shrinking at  $2 \text{ mm/s}$ . The induced emf in the loop when the radius is  $2 \text{ cm}$  is

$$(a) \quad 4.8 \mu\text{V} \quad (b) \quad 0.8 \mu\text{V}$$

$$(c) \quad 1.6 \mu\text{V} \quad (d) \quad 3.2 \mu\text{V}$$

33. Figure below shows two paths that may be taken by a gas to go from a state A to a state (c)



In process AB, 400 J of heat is added to the system and in process BC, 100 J of heat is added to the system. The heat absorbed by the system in the process AC will be

- (a) 500 J (b) 460 J (c) 300 J (d) 380 J

34. Two resistances at  $0^\circ \text{C}$  with temperature coefficient of resistance  $\alpha_1$  and  $\alpha_2$  joined in series act as a single resistance in a circuit. The temperature coefficient of their single resistance will be

- (a)  $\alpha_1 + \alpha_2$  (b)  $\frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$   
(c)  $\frac{\alpha_1 - \alpha_2}{2}$  (d)  $\frac{\alpha_1 + \alpha_2}{2}$

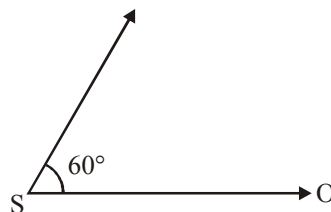
35. Two identical charged spheres suspended from a common point by two massless strings of lengths  $l$ , are initially at a distance  $d$  ( $d \ll l$ ) apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity  $v$ . Then  $v$  varies as a function of the distance  $x$  between the spheres, as :

- (a)  $v \propto x^{\frac{1}{2}}$  (b)  $v \propto x$   
(c)  $v \propto x^{-\frac{1}{2}}$  (d)  $v \propto x^{-1}$

36. A point particle of mass 0.1 kg is executing S.H.M. of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is  $8 \times 10^{-3}$  Joule. Obtain the equation of motion of this particle if this initial phase of oscillation is  $45^\circ$ .

- (a)  $y = 0.1 \sin\left(\pm 4t + \frac{\pi}{4}\right)$   
(b)  $y = 0.2 \sin\left(\pm 4t + \frac{\pi}{4}\right)$   
(c)  $y = 0.1 \sin\left(\pm 2t + \frac{\pi}{4}\right)$   
(d)  $y = 0.2 \sin\left(\pm 2t + \frac{\pi}{4}\right)$

37. A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other. The source is moving with a speed of  $19.4 \text{ ms}^{-1}$  at an angle of  $60^\circ$  with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer is (velocity of sound in air  $330 \text{ ms}^{-1}$ )



- (a) 103 Hz (b) 106 Hz  
(c) 97 Hz (d) 100 Hz

38. A resistor of resistance  $R$ , capacitor of capacitance  $C$  and inductor of inductance  $L$  are connected in parallel to AC power source of voltage  $\varepsilon_0 \sin \omega t$ . The maximum current through the resistance is half of the maximum current through the power source. Then value of  $R$  is

- (a)  $\frac{\sqrt{3}}{\left|\omega C - \frac{1}{\omega L}\right|}$  (b)  $\sqrt{3} \left|\frac{1}{\omega C} - \omega L\right|$   
(c)  $\sqrt{5} \left|\frac{1}{\omega C} - \omega L\right|$  (d) None of these

39. A lens having focal length  $f$  and aperture of diameter  $d$  forms an image of intensity  $I$ . Aperture of diameter  $\frac{d}{2}$  in central region of lens is covered by a black paper. Focal length of lens and intensity of image now will be respectively:

- (a)  $f$  and  $\frac{I}{4}$  (b)  $\frac{3f}{4}$  and  $\frac{I}{2}$   
(c)  $f$  and  $\frac{3I}{4}$  (d)  $\frac{f}{2}$  and  $\frac{I}{2}$

40. A circular disc of radius  $R$  and thickness  $\frac{R}{6}$  has moment inertia  $I$  about an axis passing through its centre perpendicular to its plane. It is melted and recasted into a solid sphere. The moment of inertia of the sphere about its diameter is

- (a)  $I$  (b)  $\frac{2I}{8}$  (c)  $\frac{I}{5}$  (d)  $\frac{I}{10}$

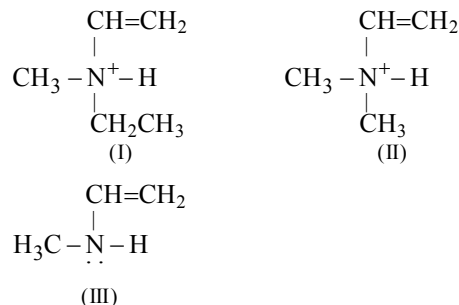
## PART - II: CHEMISTRY

41. In  $\text{PO}_4^{3-}$ , the formal charge on each oxygen atom and the P - O bond order respectively are
- (a)  $-0.75, 0.6$  (b)  $-0.75, 1.0$   
(c)  $-0.75, 1.25$  (d)  $-3, 1.25$

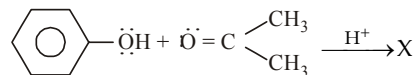
42. The decreasing order of the ionization potential of the following elements is  
 (a)  $\text{Ne} > \text{Cl} > \text{P} > \text{S} > \text{Al} > \text{Mg}$   
 (b)  $\text{Ne} > \text{Cl} > \text{P} > \text{S} > \text{Mg} > \text{Al}$   
 (c)  $\text{Ne} > \text{Cl} > \text{S} > \text{P} > \text{Mg} > \text{Al}$   
 (d)  $\text{Ne} > \text{Cl} > \text{S} > \text{P} > \text{Al} > \text{Mg}$
43. Knowing that the chemistry of lanthanoids (Ln) is dominated by its +3 oxidation state, which of the following statements is incorrect?  
 (a) The ionic size of Ln (III) decrease in general with increasing atomic number  
 (b) Ln (III) compounds are generally colourless.  
 (c) Ln (III) hydroxide are mainly basic in character.  
 (d) Because of the large size of the Ln (III) ions the bonding in its compounds is predominantly ionic in character.
44. Which of the following arrangements does not represent the correct order of the property stated against it?  
 (a)  $\text{V}^{2+} < \text{Cr}^{2+} < \text{Mn}^{2+} < \text{Fe}^{2+}$  : paramagnetic behaviour  
 (b)  $\text{Ni}^{2+} < \text{Co}^{2+} < \text{Fe}^{2+} < \text{Mn}^{2+}$  : ionic size  
 (c)  $\text{Co}^{3+} < \text{Fe}^{3+} < \text{Cr}^{3+} < \text{Sc}^{3+}$  : stability in aqueous solution  
 (d)  $\text{Sc} < \text{Ti} < \text{Cr} < \text{Mn}$  : number of oxidation states
45. Which of the following is paramagnetic?  
 (a)  $[\text{Fe}(\text{CN})_6]^{4-}$  (b)  $[\text{Ni}(\text{CO})_4]$   
 (c)  $[\text{Ni}(\text{CN})_4]^{2-}$  (d)  $[\text{CoF}_6]^{3-}$
46. The hypothetical complex chloro-diaquatriamminecobalt (III) chloride can be represented as  
 (a)  $[\text{CoCl}(\text{NH}_3)_3(\text{H}_2\text{O})_2]\text{Cl}_2$   
 (b)  $[\text{Co}(\text{NH}_3)_3(\text{H}_2\text{O})\text{Cl}_3]$   
 (c)  $[\text{Co}(\text{NH}_3)_3(\text{H}_2\text{O})_2\text{Cl}]$   
 (d)  $[\text{Co}(\text{NH}_3)_3(\text{H}_2\text{O})_3]\text{Cl}_3$
47. The normality of 26% (wt/vol) solution of ammonia (density = 0.855) is approximately:  
 (a) 1.5 (b) 0.4 (c) 15.3 (d) 4
48. 1.25 g of a sample of  $\text{Na}_2\text{CO}_3$  and  $\text{Na}_2\text{SO}_4$  is dissolved in 250 ml solution. 25 ml of this solution neutralises 20 ml of 0.1N  $\text{H}_2\text{SO}_4$ . The % of  $\text{Na}_2\text{CO}_3$  in this sample is  
 (a) 84.8% (b) 8.48% (c) 15.2% (d) 42.4%
49. Which of the following compound has all the four types ( $1^\circ$ ,  $2^\circ$ ,  $3^\circ$  and  $4^\circ$ ) of carbon atoms?  
 (a) 2, 3, 4-Trimethylpentane

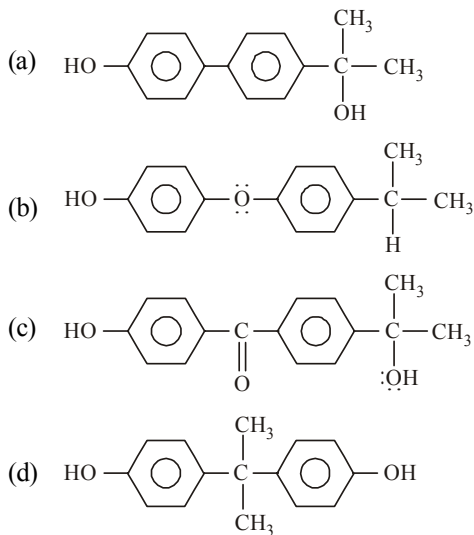
- (b) *neo*-Pentane  
 (c) 2, 2, 4-Trimethylpentane  
 (d) None of the three

50. Which of the following has two stereoisomers?

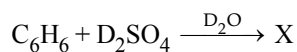


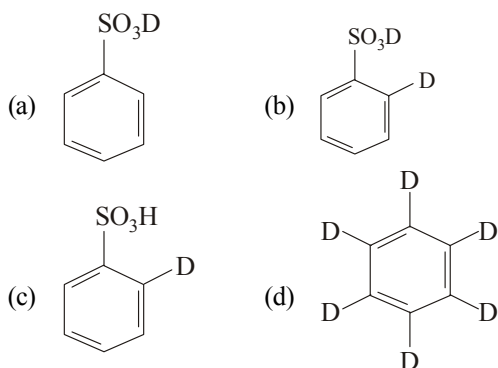
- (a) None of these (b) Only I  
 (c) Only III (d) I and III

51.   $\xrightarrow{\text{H}^+} \text{X}$   
 [X] is



52.  $\text{CH}_3\text{C}\equiv\text{CCH}_3 \xrightarrow{\text{H}_2/\text{Pt}} \text{A} \xrightarrow{\text{D}_2/\text{Pt}} \text{B}$   
 The compounds A and B, respectively are  
 (a) *cis*-butene-2 and *rac*-2, 3-dideuterobutane  
 (b) *trans*-butene-2 and *rac*-2, 3-dideuterobutane  
 (c) *cis*-butene-2 and *meso*-2, 3-dideuterobutane  
 (d) *trans*-butene-2 and *meso*-2, 3-dideuterobutane
53. Give the possible structure of X in the following reaction:

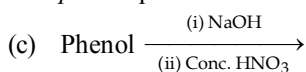
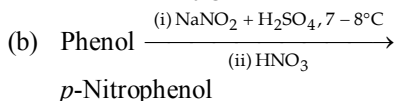
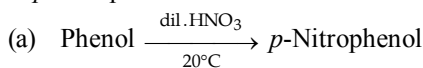




54. An aromatic compound has molecular formula  $C_7H_7Br$ . Give the possible isomers and the appropriate method to distinguish them.

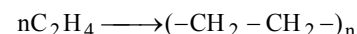
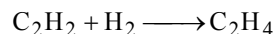
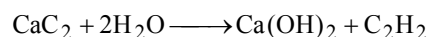
- (a) 3 isomers; by heating with  $AgNO_3$  solution  
 (b) 4 isomers; by treating with  $AgNO_3$  solution  
 (c) 4 isomers; by oxidation  
 (d) 5 isomers; by oxidation

55. Which of the following method gives better yield of *p*-nitrophenol?



(d) None of the three.

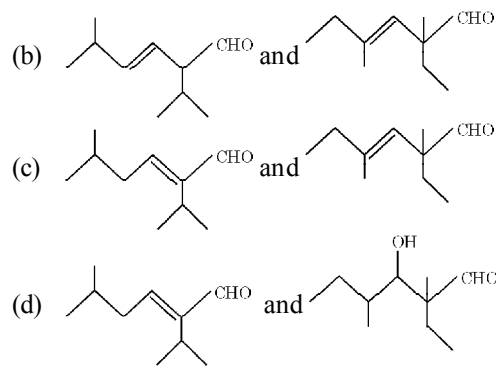
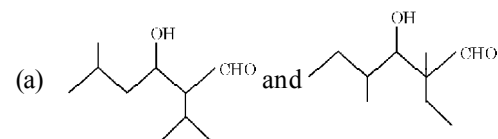
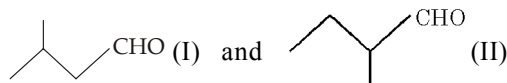
56. Formation of polyethylene from calcium carbide takes place as follows



The amount of polyethylene obtained from 64.1 kg of  $CaC_2$  is

- (a) 7 kg (b) 14 kg (c) 21 kg (d) 28 kg

57. The most likely acid-catalysed aldol condensation products of each of the two aldehydes I and II will respectively be



58. Sometimes, the colour observed in Lassaigne's test for nitrogen is green. It is because

- (a) of green colour of ferrous sulphate  
 (b) ferric ferrocyanide is also green  
 (c) of green colour of copper sulphate  
 (d) of excess of  $Fe^{3+}$  ions whose yellow colour makes the blue colour of ferric ferrocyanide to appear green.

59. Fructose on reduction gives a mixture of two alcohols which are related as

- (a) diastereomers (b) epimers  
 (c) both (a) and (b) (d) anomers.

60. What will happen when D-(+)-glucose is treated with methanolic  $HCl$  followed by Tollens' reagent?

- (a) A black ppt. will be formed  
 (b) A red ppt. will be formed  
 (c) A green colour will appear  
 (d) No characteristic colour or ppt. will be formed.

61. Which of the followings forms the base of talcum powder?

- (a) Zinc stearate  
 (b) Sodium aluminium silicate  
 (c) Magnesium hydrosilicate  
 (d) Chalk

62. The important antioxidant used in food is

- (a) BHT (b) BHC  
 (c) BTX (d) All the three

63. The first emission line in the atomic spectrum of hydrogen in the Balmer series appears at

- (a)  $\frac{9R}{400} cm^{-1}$  (b)  $\frac{7R}{144} cm^{-1}$   
 (c)  $\frac{3R}{4} cm^{-1}$  (d)  $\frac{5R}{36} cm^{-1}$



64. An  $e^-$  has magnetic quantum number as  $-3$ , what is its principal quantum number?

- (a) 1 (b) 2 (c) 3 (d) 4

65. At what temperature, the rate of effusion of  $N_2$  would be 1.625 times than that of  $SO_2$  at  $50^\circ C$ ?

- (a) 110 K (b) 173 K (c) 373 K (d) 273 K

66. The average kinetic energy of an ideal gas per molecule in SI unit at  $25^\circ C$  will be

- (a)  $6.17 \times 10^{-21} \text{ kJ}$  (b)  $6.17 \times 10^{-21} \text{ J}$   
(c)  $6.17 \times 10^{-20} \text{ J}$  (d)  $7.16 \times 10^{-20} \text{ J}$

67. The degree of dissociation of  $PCl_5$  ( $\alpha$ ) obeying the equilibrium  $PCl_5 \rightleftharpoons PCl_3 + Cl_2$  is related to the equilibrium pressure by

- (a)  $\alpha \propto \frac{1}{P^4}$  (b)  $\alpha \propto \frac{1}{\sqrt{P}}$   
(c)  $\alpha \propto \frac{1}{P^2}$  (d)  $\alpha \propto P$

68. In a closed system,  $A(s) \rightleftharpoons 2B(g) + 3C(g)$ , if partial pressure of C is doubled, then partial pressure of B will be

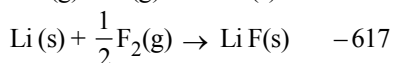
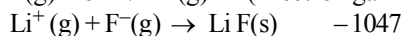
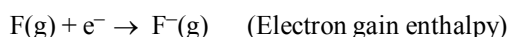
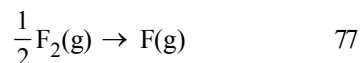
- (a)  $2\sqrt{2}$  times the original value  
(b)  $\frac{1}{2}$  times the original value  
(c) 2 times the original value  
(d)  $\frac{1}{2\sqrt{2}}$  times the original value

69. For a particular reversible reaction at temperature  $T$ ,  $\Delta H$  and  $\Delta S$  were found to be both +ve. If  $T_e$  is the temperature at equilibrium, the reaction would be spontaneous when

- (a)  $T_e > T$  (b)  $T > T_e$   
(c)  $T_e$  is 5 times  $T$  (d)  $T = T_e$

70. Given

**Reaction** **Energy Change (in kJ)**



Based on data provided, the value of electron gain enthalpy of fluorine would be :

- (a)  $-300 \text{ kJ mol}^{-1}$  (b)  $-350 \text{ kJ mol}^{-1}$   
(c)  $-328 \text{ kJ mol}^{-1}$  (d)  $-228 \text{ kJ mol}^{-1}$

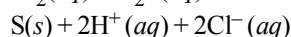
71. The percentage hydrolysis of 0.15 M solution of ammonium acetate,  $K_a$  for  $CH_3COOH$  is  $1.8 \times 10^{-5}$  and  $K_b$  for  $NH_3$  is  $1.8 \times 10^{-5}$

- (a) 0.556 (b) 4.72 (c) 9.38 (d) 5.56

72. For a sparingly soluble salt  $A_p B_q$ , the relationship of its solubility product  $L_s \rightarrow K_{sp}$  with its solubility (S) is

- (a)  $L_s \rightarrow K_{sp} = S^{pq} (pq)^{p+q}$   
(b)  $L_s = S^{p+q} \cdot p^p q^q$   
(c)  $L_s \rightarrow K_{sp} = S^{p+q} \cdot p^q q^p$   
(d)  $L_s \rightarrow K_{sp} = S^{pq} p^p q^q$

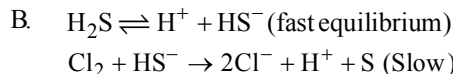
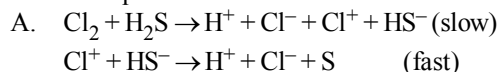
73. Consider the reaction :



The rate equation for this reaction is

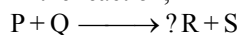
$\text{rate} = k[Cl_2][H_2S]$

Which of these mechanisms is/are consistent with this rate equation?

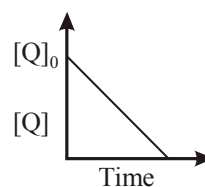


- (a) B only (b) Both A and B  
(c) Neither A nor B (d) A only

74. In the reaction,



The time taken for 75% reaction of P is twice the time taken for 50% reaction of P. The concentration of Q varies with reaction time as shown in the figure. The overall order of the reaction is



- (a) 2 (b) 3 (c) 0 (d) 1

75. The EMF of the cell  $Tl/Tl^+ (0.001M) \parallel Cu^{2+} (0.01M)/Cu$  is 0.83. The cell EMF can be increased by

- (a) Increasing the concentration of  $Tl^+$  ions.  
(b) Increasing the concentration of  $Cu^{2+}$  ions.  
(c) Increasing the concentration of  $Tl^+$  and  $Cu^{2+}$  ions.  
(d) None of these



76. Electrolysis is carried out in three cells  
(A) 1.0 M  $\text{CuSO}_4$  Pt electrode  
(B) 1.0 M  $\text{CuSO}_4$  copper electrodes  
(C) 1.0 M  $\text{KCl}$  Pt electrodes  
If volume of electrolytic solution is maintained constant in each of the cell, which is correct set of pH changes in (A), (B) and (C) cell respectively?  
(a) decrease in all the three  
(b) increase in all the three  
(c) decrease, constant, increase  
(d) increase, constant, increase
77. The equilibrium constant for the disproportionation reaction  
 $2\text{Cu}^+(\text{aq}) \longrightarrow \text{Cu}(\text{s}) + \text{Cu}^{2+}(\text{aq})$  at  $25^\circ\text{C}$   
( $E^\circ \text{Cu}^+/\text{Cu} = 0.52\text{V}$ ,  $E^\circ \text{Cu}^{2+}/\text{Cu} = 0.16\text{V}$ ) is  
(a)  $6 \times 10^4$  (d)  $6 \times 10^6$   
(c)  $1.2 \times 10^6$  (d)  $1.2 \times 10^{-6}$
78. The non stoichiometric compound  $\text{Fe}_{0.94}\text{O}$  is formed when x % of  $\text{Fe}^{2+}$  ions are replaced by as many  $\frac{2}{3}\text{Fe}^{3+}$  ions, x is  
(a) 18 (b) 12 (c) 15 (d) 6
79. Al (at. wt 27) crystallizes in the cubic system with a cell edge of  $4.05 \text{ \AA}$ . Its density is  $2.7 \text{ g per cm}^3$ . Determine the unit cell type calculate the radius of the Al atom  
(a) fcc,  $2.432 \text{ \AA}$  (b) bcc,  $2.432 \text{ \AA}$   
(c) bcc,  $1.432 \text{ \AA}$  (d) fcc,  $1.432 \text{ \AA}$
80. A compound of Xe and F is found to have 53.5% of Xe. What is oxidation number of Xe in this compound?  
(a) -4 (b) 0 (c) +4 (d) +6

### PART - III(A): ENGLISH PROFICIENCY

**DIRECTIONS (Qs. 81 & 82) :** Choose the word which best expresses the meaning of the given word.

81. CORPULENT  
(a) Lean (b) Gaunt  
(c) Emaciated (d) Obese
82. EMBEZZLE  
(a) Misappropriate (b) Balance  
(c) Remunerate (d) Clear

**DIRECTIONS (Qs. 83 & 84) :** Choose the word which is the exact OPPOSITE of the given words.

83. ARROGANT  
(a) Humble (b) Cowardly  
(c) Egotistic (d) Gentlemanly

84. EXODUS  
(a) Influx (b) Home-coming  
(c) Return (d) Restoration

**DIRECTIONS (Qs. 85-88) :** Read the following passage and answer the questions that follows.

At this stage of civilisation, when many nations are brought in to close and vital contact for good and evil, it is essential, as never before, that their gross ignorance of one another should be diminished, that they should begin to understand a little of one another's historical experience and resulting mentality. It is the fault of the English to expect the people of other countries to react as they do, to political and international situations. Our genuine goodwill and good intentions are often brought to nothing, because we expect other people to be like us. This would be corrected if we knew the history, not necessarily in detail but in broad outlines, of the social and political conditions which have given to each nation its present character.

85. According to the author of 'Mentality' of a nation is mainly product of its  
(a) History  
(b) international position  
(c) Politics  
(d) present character
86. The need for a greater understanding between nations  
(a) was always there  
(b) is no longer there  
(c) is more today than ever before  
(d) will always be there
87. The character of a nation is the result of its  
(a) Mentality  
(b) cultural heritage  
(c) gross ignorance  
(d) socio-political conditions
88. According to the author his countrymen should  
(a) read the story of other nations  
(b) have a better understanding of other nations  
(c) not react to other actions  
(d) have vital contacts with other nations

**DIRECTIONS (Qs. 89-90) :** In questions below, each passage consist of six sentences. The first and sixth sentence are given in the begining. The middle four sentences in each have been removed and jumbled up. These are labelled as P, Q, R and S. Find out the proper order for the four sentences.

89. S1 : A force of exists between everybody in the universe.  
P : Normally it is very small but when the one of the bodies is a planet, like earth, the force is considerable.

Q: It has been investigated by many scientists including Galileo and Newton.

R: Everything on or near the surface of the earth is attracted by the mass of earth.

S: This gravitational force depends on the mass of the bodies involved.

S6: The greater the mass, the greater is the earth's force of attraction on it. We can call this force of attraction gravity.

The Proper sequence should be:

- (a) PRQS (b) PRSQ  
(c) QSRP (d) QSPR

90. S1: Calcutta unlike other cities kepts its trams.

P: As a result there horrendous congestion.

Q: It was going to be the first in South Asia.

R: They run down the centre of the road

S: To ease in the city decided to build an underground railway line.

S6: The foundation stone was laid in 1972.

The Proper sequence should be:

- (a) PRSQ (b) PSQR  
(c) SQRP (d) RPSQ

**DIRECTIONS (Qs. 91-92) :** Pick out the most effective word from the given words to fill in the blank to make the sentence meaningfully complete.

91. The miser gazed ..... at the pile of gold coins in front of him.

- (a) Avidly (b) Admiringly  
(c) Thoughtfully (d) Earnestly

92. I saw a ..... of cows in the field.

- (a) Group (b) Herd  
(c) Swarm (d) Flock

**DIRECTIONS (93-95) :** Read the each sentence to find out whether there is any grammatical error in it. The error, if any will be in one part of the sentence. The letter of that part is the answer. If there is no error, the answer is 'd'. (Ignore the errors of punctuation, if any).

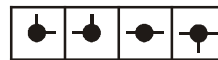
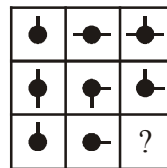
93. (a) We discussed about the problem so thoroughly  
(b) on the eve of the examination  
(c) that I found it very easy to work it out.  
(d) No error.

94. (a) An Indian ship  
(b) laden with merchandise  
(c) got drowned in the Pacific Ocean.  
(d) No error.

95. (a) I could not put up in a hotel  
(b) because the boarding and lodging charges  
(c) were exorbitant.  
(d) No error.

## PART - III (B): LOGICAL REASONING

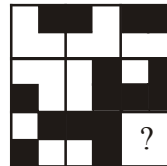
96. Select a suitable figure from the four alternatives that would complete the figure matrix.



(1) (2) (3) (4)

- (a) 1 (b) 2 (c) 3 (d) 4

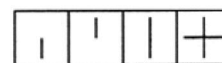
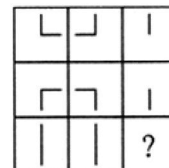
97. Select a suitable figure from the four alternatives that would complete the figure matrix.



(1) (2) (3) (4)

- (a) 1 (b) 2 (c) 3 (d) 4

98. Select a suitable figure from the four alternatives that would complete the figure matrix.



(1) (2) (3) (4)

- (a) 1 (b) 2 (c) 3 (d) 4

**DIRECTION (Q. 99) :** Choose the correct alternative that will continue the same pattern and replace the question mark in the given series.

99. 3, 4, 7, 7, 13, 13, 21, 22, 31, 34, ?

- (a) 42 (b) 43 (c) 51 (d) 52

100. Introducing a boy, a girl said, "He is the son of the daughter of the father of my uncle." How is the boy related to the girl?

- (a) Brother (b) Nephew  
(c) Uncle (d) Son-in-law

**DIRECTIONS (Qs. 101-102) :** In these series, you will be looking at both the letter pattern and the number pattern. Fill the blank in the middle of the series or end of the series.

101. QAR, RAS, SAT, TAU, \_\_\_\_\_  
 (a) UAV (b) UAT (c) TAS (d) TAT
102. DEF, DEF<sub>2</sub>, DE2F<sub>2</sub>, \_\_\_\_\_, D<sub>2</sub>E<sub>2</sub>F<sub>3</sub>  
 (a) DEF<sub>3</sub> (b) D<sub>3</sub>EF<sub>3</sub>  
 (c) D<sub>2</sub>E<sub>3</sub>F (d) D<sub>2</sub>E<sub>2</sub>F<sub>2</sub>

**DIRECTIONS (Qs. 103-104) :** In each question below are given two statements followed by two conclusions numbered I and II. You have to take the given two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusion and then decide which of the given conclusions logically follows from the two given statements, disregarding commonly known facts.

**Give answer:**

- (a) If only conclusion I follows  
 (b) If only conclusion II follows  
 (c) If neither I nor II follows and  
 (d) If both I and II follow.

103. **Statements :** Raman is always successful. No fool is always successful.

**Conclusions :**

- I. Raman is a fool.  
 II. Raman is not a fool.





104. **Statements :** Some desks are caps. No cap is red.

**Conclusions :**

- I. Some caps are desks.  
 II. No desk is red.

105. Choose the set of figures which follows the given rule.

**Rule :** Closed figures losing their sides and open figures gaining their sides.

- (1) 
- (2) 
- (3) 
- (4) 
- (a) 1 (b) 2 (c) 3 (d) 4

## PART - IV : MATHEMATICS

106. Let  $f(x) = \frac{ax+b}{cx+d}$ , then  $f(f(x)) = x$ , provided that :

- (a)  $d = -a$  (b)  $d = a$   
 (c)  $a = b = 1$  (d)  $a = b = c = d = 1$

107. Two finite sets have  $m$  and  $n$  elements. The number of subsets of the first set is 112 more than that of the second set. The values of  $m$  and  $n$  respectively are,

- (a) 4, 7 (b) 7, 4 (c) 4, 4 (d) 7, 7

108. If  $A$  and  $B$  are positive acute angles satisfying

$$3 \cos^2 A + 2 \cos^2 B = 4 \text{ and } \frac{3 \sin A}{\sin B} = \frac{2 \cos B}{\cos A},$$

Then the value of  $A + 2B$  is equal to :

- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{3}$  (d)  $\frac{\pi}{4}$

109. If  $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$ , then  $\cos \theta_1 + \cos \theta_2 + \cos \theta_3 =$

- (a) 0 (b) 1 (c) 2 (d) 3

110. If  $\tan(\cot x) = \cot(\tan x)$ , then  $\sin 2x$  is equal to :

- (a)  $\frac{2}{(2n+1)\pi}$  (b)  $\frac{4}{(2n+1)\pi}$   
 (c)  $\frac{2}{n(n+1)\pi}$  (d)  $\frac{4}{n(n+1)\pi}$

111. The general solution of the equation

$$\sin 2x + 2 \sin x + 2 \cos x + 1 = 0 \text{ is}$$

- (a)  $3n\pi - \frac{\pi}{4}$   
 (b)  $2n\pi + \frac{\pi}{4}$   
 (c)  $2n\pi + (-1)^n \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$   
 (d)  $n\pi - \frac{\pi}{4}$

112. In a  $\triangle ABC$ , if  $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ , and the side  $a = 2$ , then area of the triangle is

- (a) 1 (b) 2 (c)  $\frac{\sqrt{3}}{2}$  (d)  $\sqrt{3}$

113. If  $\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ , then what is the value of  $x$ ?

- (a)  $a/b$  (b)  $ab$   
 (c)  $b/a$  (d)  $\frac{a-b}{1+ab}$

114. The arithmetic mean of numbers  $a, b, c, d, e$  is  $M$ . What is the value of  $(a - M) + (b - M) + (c - M) + (d - M) + (e - M)$ ?

- (a)  $M$  (b)  $a + b + c + d + e$   
(c)  $0$  (d)  $5M$

115. The fourth term of an A.P. is three times of the first term and the seventh term exceeds the twice of the third term by one, then the common difference of the progression is

- (a)  $2$  (b)  $3$  (c)  $\frac{3}{2}$  (d)  $-1$

116. The sum to  $n$  terms of the series

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$$

- (a)  $n - 1 - 2^{-n}$  (b)  $1$   
(c)  $n - 1 + 2^{-n}$  (d)  $1 + 2^{-n}$

117. If  $\log a, \log b$ , and  $\log c$  are in A.P. and also  $\log a - \log 2b, \log 2b - \log 3c, \log 3c - \log a$  are in A.P., then

- (a)  $a, b, c$ , are in H.P.  
(b)  $a, 2b, 3c$  are in A.P.  
(c)  $a, b, c$  are the sides of a triangle  
(d) none of the above

118.  $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 +$

$$\left(x^3 + \frac{1}{x^3}\right)^2 \dots \text{upto } n \text{ terms is}$$

- (a)  $\frac{x^{2n} - 1}{x^2 - 1} \times \frac{x^{2n+2} + 1}{x^{2n}} + 2n$   
(b)  $\frac{x^{2n} + 1}{x^2 + 1} \times \frac{x^{2n+2} - 1}{x^{2n}} - 2n$   
(c)  $\frac{x^{2n} - 1}{x^2 - 1} \times \frac{x^{2n} - 1}{x^{2n}} - 2n$   
(d) None of these

119. If  $z_1 = \sqrt{3} + i\sqrt{3}$  and  $z_2 = \sqrt{3} + i$ , then the

complex number  $\left(\frac{z_1}{z_2}\right)^{50}$  lies in the :

- (a) first quadrant (b) second quadrant  
(c) third quadrant (d) fourth quadrant

120. If the matrix  $\begin{bmatrix} 1 & 3 & \lambda + 2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$  is singular, then  $\lambda =$

- (a)  $-2$  (b)  $4$  (c)  $2$  (d)  $-4$

121. Let  $\alpha_1, \alpha_2$  and  $\beta_1, \beta_2$  be the roots of  $ax^2 + bx + c = 0$  and  $px^2 + qx + r = 0$  respectively. If the system of equations  $\alpha_1 y + \alpha_2 z = 0$  and  $\beta_1 y + \beta_2 z = 0$  has a non-trivial solution, then

- (a)  $\frac{b^2}{q^2} = \frac{ac}{pr}$  (b)  $\frac{c^2}{r^2} = \frac{ab}{pq}$   
(c)  $\frac{a^2}{p^2} = \frac{bc}{qr}$  (d) None of these

122. If  $[ ]$  denotes the greatest integer less than or equal to the real number under consideration and  $-1 \leq x < 0; 0 \leq y < 1; 1 \leq z < 2$ , then the value of the determinant

$$\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$$

- (a)  $[z]$  (b)  $[y]$   
(c)  $[x]$  (d) None of these

123. If  $\alpha, \beta$  are the roots of the equations  $x^2 - 2x - 1 = 0$ , then what is the value of  $\alpha^2 \beta^{-2} + \alpha^{-2} \beta^2$

- (a)  $-2$  (b)  $0$  (c)  $30$  (d)  $34$

124. If  $a, b$  and  $c$  are real numbers then the roots of the equation  $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$  are always

- (a) real (b) imaginary  
(c) positive (d) negative

125.  $\lim_{n \rightarrow \infty} \frac{a^n + b^n}{a^n - b^n}$ , where  $a > b > 1$ , is equal to

- (a)  $-1$  (b)  $1$  (c)  $0$  (d) None

126. The number of points at which the function

$$f(x) = \frac{1}{\log |x|}$$

- (a)  $1$  (b)  $2$  (c)  $3$  (d)  $4$

127. If  $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  then  $f(x)$  is

- (a) continuous as well as differentiable at  $x = 0$   
(b) continuous but not differentiable at  $x = 0$   
(c) differentiable but not continuous at  $x = 0$   
(d) neither continuous nor differentiable at  $x = 0$

128. For any differentiable function  $y$  of  $x$ ,

$$\frac{d^2x}{dy^2} \left( \frac{dy}{dx} \right)^3 + \frac{d^2y}{dx^2} =$$

- (a) 0 (b)  $y$  (c)  $-y$  (d)  $x$

129. The set of all values of  $a$  for which the function  $f(x) = (a^2 - 3a + 2)(\cos^2 x/4 - \sin^2 x/4) + (a-1)x + \sin 1$  does not possess critical points is

- (a)  $[1, \infty)$  (b)  $(0, 1) \cup (1, 4)$   
(c)  $(-2, 4)$  (d)  $(1, 3) \cup (3, 5)$

130. Match List I with List II and select the correct answer using the code given below the lists:

**List I**

(A)  $f(x) = \cos x$

(B)  $f(x) = \ln x$

(C)  $f(x) = x^2 - 5x + 4$

(D)  $f(x) = e^x$

**List II**

1. The graph cuts y-axis in infinite number of points

2. The graph cuts x-axis in two points

3. The graph cuts y-axis in only one point

4. The graph cuts x-axis in only one point

5. The graph cuts x-axis in infinite number of points

**Codes:**

(A)	(B)	(C)	(D)
(a) 1	4	5	3
(b) 1	3	5	4
(c) 5	4	2	3
(d) 5	3	2	4

131. What is the x-coordinate of the point on the curve  $f(x) = \sqrt{x}(7x - 6)$ , where the tangent is parallel to x-axis?

- (a)  $-\frac{1}{3}$  (b)  $\frac{2}{7}$  (c)  $\frac{6}{7}$  (d)  $\frac{1}{2}$

132. A wire 34 cm long is to be bent in the form of a quadrilateral of which each angle is  $90^\circ$ . What is the maximum area which can be enclosed inside the quadrilateral?

- (a)  $68 \text{ cm}^2$  (b)  $70 \text{ cm}^2$   
(c)  $71.25 \text{ cm}^2$  (d)  $72.25 \text{ cm}^2$

133. Consider the following statements in respect of the function  $f(x) = x^3 - 1$ ,  $x \in [-1, 1]$

I.  $f(x)$  is increasing in  $[-1, 1]$

II.  $f(x)$  has no root in  $(-1, 1)$ .

Which of the statements given above is/are correct?

- (a) Only I (b) Only II  
(c) Both I and II (d) Neither I nor II

134. At an extreme point of a function  $f(x)$ , the tangent to the curve is

- (a) parallel to the x-axis  
(b) perpendicular to the x-axis  
(c) inclined at an angle  $45^\circ$  to the x-axis  
(d) inclined at an angle  $60^\circ$  to the x-axis

135. The curve  $y = xe^x$  has minimum value equal to

- (a)  $-\frac{1}{e}$  (b)  $\frac{1}{e}$  (c)  $-e$  (d)  $e$

136. A ray of light coming from the point  $(1, 2)$  is reflected at a point A on the x-axis and then passes through the point  $(5, 3)$ . The co-ordinates of the point A is

- (a)  $\left(\frac{13}{5}, 0\right)$  (b)  $\left(\frac{5}{13}, 0\right)$   
(c)  $(-7, 0)$  (d) None of these

137. The equation

$$x^2 - 2\sqrt{3}xy + 3y^2 - 3x + 3\sqrt{3}y - 4 = 0$$
 represents

- (a) a pair of intersecting lines  
(b) a pair of parallel lines with distance between them  $\frac{5}{2}$   
(c) a pair of parallel lines with distance between them  $5\sqrt{2}$   
(d) a conic section, which is not a pair of straight lines

138. The line joining  $(5, 0)$  to  $((10 \cos \theta, 10 \sin \theta))$  is divided internally in the ratio  $2 : 3$  at P. If  $\theta$  varies, then the locus of P is

- (a) a pair of straight lines  
(b) a circle  
(c) a straight line  
(d) None of these

139. The number of integral values of  $\lambda$  for which  $x^2 + y^2 + \lambda x + (1 - \lambda)y + 5 = 0$  is the equation of a circle whose radius cannot exceed 5, is

- (a) 14 (b) 18 (c) 16 (d) None

140. The lengths of the tangent drawn from any point

on the circle  $15x^2 + 15y^2 - 48x + 64y = 0$  to the two circles

$$5x^2 + 5y^2 - 24x + 32y + 75 = 0 \text{ and } 5x^2 + 5y^2 - 48x + 64y + 300 = 0$$
 are in the ratio of

- (a)  $1 : 2$  (b)  $2 : 3$  (c)  $3 : 4$  (d) None

141. The length of the chord  $x + y = 3$  intercepted by the circle  $x^2 + y^2 - 2x - 2y - 2 = 0$  is  
 (a)  $\frac{7}{2}$  (b)  $\frac{3\sqrt{3}}{2}$  (c)  $\sqrt{14}$  (d)  $\frac{\sqrt{7}}{2}$
142. The locus of the point of intersection of two tangents to the parabola  $y^2 = 4ax$ , which are at right angle to one another is  
 (a)  $x^2 + y^2 = a^2$  (b)  $ay^2 = x$   
 (c)  $x + a = 0$  (d)  $x + y \pm a = 0$
143. The parabola having its focus at  $(3, 2)$  and directrix along the y-axis has its vertex at  
 (a)  $(2, 2)$  (b)  $\left(\frac{3}{2}, 2\right)$   
 (c)  $\left(\frac{1}{2}, 2\right)$  (d)  $\left(\frac{2}{3}, 2\right)$
144. The number of values of  $r$  satisfying the equation  ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$  is  
 (a) 1 (b) 2 (c) 3 (d) 4
145. If  $\sum_{r=0}^n \frac{r+2}{r+1} {}^nC_r = \frac{2^8 - 1}{6}$ , then  $n =$   
 (a) 8 (b) 4 (c) 6 (d) 5
146. All the words that can be formed using alphabets A, H, L, U and R are written as in a dictionary (no alphabet is repeated). Rank of the word RAHUL is  
 (a) 71 (b) 72 (c) 73 (d) 74
147. If the sum of odd numbered terms and the sum of even numbered terms in the expansion of  $(x + a)^n$  are A and B respectively, then the value of  $(x^2 - a^2)^n$  is  
 (a)  $A^2 - B^2$  (b)  $A^2 + B^2$   
 (c)  $4AB$  (d) None
148. If the third term in the expansion of  $[x + x^{\log_{10} x}]^5$  is  $10^6$ , then  $x$  may be  
 (a) 1 (b)  $\sqrt{10}$  (c) 10 (d)  $10^{-2/5}$
149. If three vertices of a regular hexagon are chosen at random, then the chance that they form an equilateral triangle is :  
 (a)  $\frac{1}{3}$  (b)  $\frac{1}{5}$  (c)  $\frac{1}{10}$  (d)  $\frac{1}{2}$
150. A man takes a step forward with probability 0.4 and backward with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is  
 (a)  $\frac{2^5 \cdot 3^5}{5^{10}}$  (b)  $462 \times \left(\frac{6}{25}\right)^5$   
 (c)  $231 \times \frac{3^5}{5^{10}}$  (d) none of these

# SOLUTIONS

## PART - I : PHYSICS

1. (b) True weight at equator,  $W = mg$   
Observed weight at equator,

$$W' = mg' = \frac{3}{5}mg$$

At equator, latitude  $\lambda = 0$

Using the formula,

$$mg' = mg - mR\omega^2 \cos^2 \lambda$$

$$\frac{3}{5}mg = mg - mR\omega^2 \cos^2 0 = mg - mR\omega^2$$

$$\Rightarrow mR\omega^2 = mg - \frac{3}{5}mg = \frac{2}{5}mg$$

$$\therefore \omega = \left( \frac{2}{5} \frac{g}{R} \right)^{1/2}$$

$$= \left( \frac{2 \times 9.8}{5 \times 6.4 \times 10^6} \right)^{1/2} = 7.8 \times 10^{-4} \text{ rad/s.}$$

2. (d) 3. (a)

4. (b) Here  $v_1 = 200 \text{ m/s}$ ;  
temperature  $T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$   
temperature  $T_2 = 127^\circ\text{C} = 127 + 273 = 400 \text{ K}$ ,  
 $V = ?$

R.M.S. Velocity,  $V \propto \sqrt{T}$

$$\Rightarrow \frac{v}{200} = \sqrt{\frac{400}{300}}$$

$$\Rightarrow v = \frac{200 \times 2}{\sqrt{3}} \text{ m/s}$$

$$\Rightarrow v = \frac{400}{\sqrt{3}} \text{ m/s}$$

5. (c) Growth in current in  $LR_2$  branch when switch is closed is given by

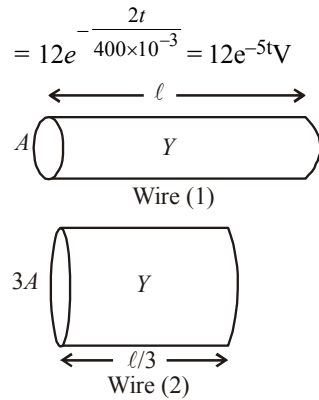
$$i = \frac{E}{R_2} [1 - e^{-R_2 t / L}]$$

$$\Rightarrow \frac{di}{dt} = \frac{E}{R_2} \cdot \frac{R_2}{L} \cdot e^{-R_2 t / L} = \frac{E}{L} e^{-R_2 t / L}$$

Hence, potential drop across

$$L = \left( \frac{E}{L} e^{-R_2 t / L} \right) L = E e^{-R_2 t / L}$$

6. (c)



As shown in the figure, the wires will have the same Young's modulus (same material) and the length of the wire of area of cross-section  $3A$  will be  $\ell/3$  (same volume as wire 1).

$$\text{For wire 1, } Y = \frac{F/A}{\Delta x / \ell} \quad \dots(i)$$

$$\text{For wire 2, } Y = \frac{F'/3A}{\Delta x / (\ell/3)} \quad \dots(ii)$$

$$\text{From (i) and (ii), } \frac{F}{A} \times \frac{\ell}{\Delta x} = \frac{F'}{3A} \times \frac{\ell}{3\Delta x}$$

$$\Rightarrow F' = 9F$$

7. (d)

Radius of small sphere =  $r$

Thickness of small sphere =  $t$

Radius of bigger sphere =  $2r$

Thickness of bigger sphere =  $t/4$

Mass of ice melted = (volume of sphere)  $\times$  (density of ice)

Let  $K_1$  and  $K_2$  be the thermal conductivities of larger and smaller sphere.

For bigger sphere,

$$\frac{K_1 4\pi (2r)^2 \times 100}{t/4} = \frac{\frac{4}{3}\pi (2r)^3 \rho L}{25 \times 60}$$

For smaller sphere,

$$\frac{K_2 \times 4\pi r^2 \times 100}{t} = \frac{\frac{4}{3}\pi r^3 \rho L}{16 \times 60}$$

$$\therefore \frac{K_1}{K_2} = \frac{8}{25}$$

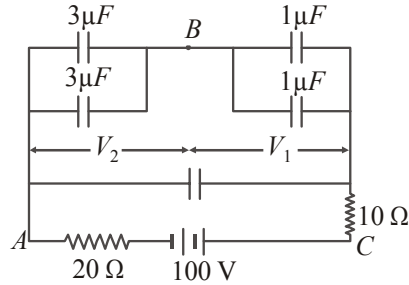


8. (c)

9. (c) The equivalent circuit is shown in figure.

$$V_1 + V_2 = 100$$

$$\text{and } 2V_1 = 6V_2$$



On solving above equations, we get

$$V_1 = 75V, V_2 = 25V$$

10. (c) Let  $N_0$  be the number of atoms of X at time  $t=0$ .

Then at  $t = 4$  hrs (two half lives)

$$N_x = \frac{N_0}{4} \text{ and } N_y = \frac{3N_0}{4}$$

$$\therefore N_x/N_y = 1/3$$

and at  $t = 6$  hrs (three half lives)

$$N_x = \frac{N_0}{8} \text{ and } N_y = \frac{7N_0}{8}$$

$$\text{or } \frac{N_x}{N_y} = \frac{1}{7}$$

The given ratio  $\frac{1}{4}$  lies between  $\frac{1}{3}$  and  $\frac{1}{7}$ .

Therefore,  $t$  lies between 4 hrs and 6 hrs.

11. (b) Compressibility of water,

$$K = 45.4 \times 10^{-11} \text{ Pa}^{-1}$$

$$\text{density of water } P = 10^3 \text{ kg/m}^3$$

$$\text{depth of ocean, } h = 2700 \text{ m}$$

$$\text{We have to find } \frac{\Delta V}{V} = ?$$

As we know, compressibility,

$$K = \frac{1}{B} = \frac{(\Delta V/V)}{P} \quad (P = Pgh)$$

$$\text{So, } (\Delta V/V) = KPgh$$

$$= 45.4 \times 10^{-11} \times 10^3 \times 10 \times 2700$$

$$= 1.2258 \times 10^{-2}$$

12. (c) Acceleration of body along AB is  $g \cos \theta$

Distance travelled in time  $t$  sec =

$$AB = \frac{1}{2}(g \cos \theta)t^2$$

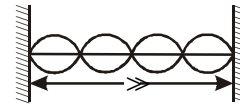
From  $\triangle ABC$ ,  $AB = 2R \cos \theta$

$$\text{Thus, } 2R \cos \theta = \frac{1}{2}g \cos \theta t^2$$

$$\Rightarrow t^2 = \frac{4R}{g} \Rightarrow t = 2\sqrt{\frac{R}{g}}$$

13. (c) For a string vibrating in its  $n^{\text{th}}$  overtone  $(n+1)^{\text{th}}$  harmonic)

$$y = 2A \sin \left( \frac{(n+1)\pi x}{L} \right) \cos \omega t$$



$$\text{For } x = \frac{\ell}{3} \quad 2A = a \text{ \& } n = 3;$$

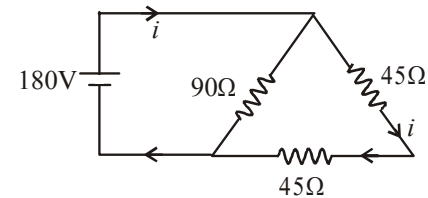
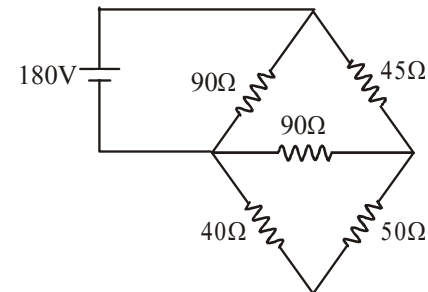
$$y = \left[ a \sin \left( \frac{4\pi}{\ell} \cdot \frac{\ell}{3} \right) \right] \cos \omega t$$

$$= a \sin \frac{4\pi}{3} \cos \omega t = -a \left( \frac{\sqrt{3}}{2} \right) \cos \omega t$$

$$\text{i.e. at } x = \frac{\ell}{3}, \text{ the amplitude is } \frac{\sqrt{3}a}{2}$$

14. (d)

15. (c)



$$i = \frac{180}{90} = 2A$$

16. (a) As we know, orbital speed,  $V_{\text{orb}} = \sqrt{\frac{GM}{r}}$

$$\text{Time period } T = \frac{2\pi r}{v_{\text{orb}}} = \frac{2\pi r}{\sqrt{GM}} \sqrt{r}$$

Squarring both sides,

$$T^2 = \left( \frac{2\pi r \sqrt{r}}{\sqrt{GM}} \right)^2 = \frac{4\pi^2}{GM} \cdot r^3$$

$$\Rightarrow \frac{T^2}{r^3} = \frac{4\pi^2}{GM} = K \Rightarrow GMK = 4\pi^2$$

17. (a)  $P_{in} = 25W, \lambda = 6600 \text{ \AA} = 6600 \times 10^{-10} \text{ m}$   
 $nh\nu = P$

$\Rightarrow$  Number of photons emitted/sec,

$$n = \frac{P}{hc} = \frac{P\lambda}{hc} = \frac{25 \times 6600 \times 10^{-10}}{6.64 \times 10^{-34} \times 3 \times 10^8}$$

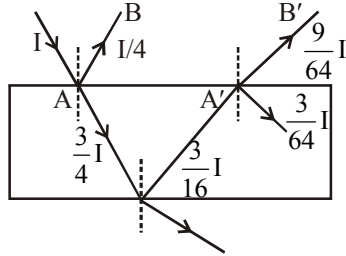
$$= 8.28 \times 10^{19} = \frac{25}{3} \times 10^{19}$$

3% of emitted photons are producing current

$$\therefore I = \frac{3}{100} \times ne$$

$$= \frac{3}{100} \times \frac{25}{3} \times 10^{19} \times 1.6 \times 10^{-19} = 0.4A$$

18. (a)



From figure  $I_1 = \frac{I}{4}$  and  $I_2 = \frac{9I}{64}$

$$\Rightarrow \frac{I_2}{I_1} = \frac{9}{16}$$

By using  $\frac{I_{\max}}{I_{\min}} = \left( \frac{\sqrt{\frac{I_2}{I_1}} + 1}{\sqrt{\frac{I_2}{I_1}} - 1} \right)^2$

$$= \left( \frac{\sqrt{\frac{9}{16}} + 1}{\sqrt{\frac{9}{16}} - 1} \right)^2 = \frac{49}{1}$$

19. (c)  $h = \frac{2T \cos \theta}{r \rho g} \Rightarrow h \propto \frac{1}{r} \Rightarrow \frac{h_2}{h_1} = \frac{r_1}{r_2} = \frac{2}{3}$

$$\left( \because r_1 = r, r_2 = r + 50\% \text{ of } r = \frac{3}{2}r \right)$$

New mass  $m_2 = \pi r_2^2 h_2 \rho = \pi \left( \frac{3}{2} r_1 \right)^2 \left( \frac{2}{3} h_1 \right) \rho$

$$= \frac{3}{2} \left( \pi r_1^2 h_1 \right) \rho = \frac{3}{2} m$$

20. (d) Number of electrons per kg of silver

$$= \frac{6.023 \times 10^{26}}{108}$$

Number of electrons per unit volume of silver

$$n = \frac{6.023 \times 10^{26}}{108} \times 10.5 \times 10^3$$

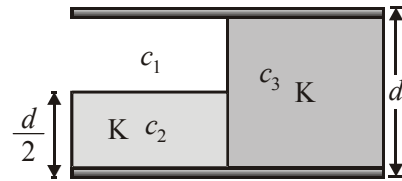
$$v_d = \frac{I}{neA}$$

$$= \frac{20}{6.023 \times 10^{26} \times 10.5 \times 10^3 \times 1.6 \times 10^{-19} \times 3.14 \times 10^{-6}} \times 108$$

$$= 6.798 \times 10^{-4} \text{ m/sec.}$$

21. (d)

$$c_1 = \frac{(A/2)\epsilon_0}{d/2} = \frac{A\epsilon_0}{d}, c_2 = K \frac{A\epsilon_0}{d}, c_3 = K \frac{A\epsilon_0}{2d}$$



$$\therefore c_{eq.} = \frac{c_1 \times c_2}{c_1 + c_2} + c_3 = \frac{(3+K)KA\epsilon_0}{2d(K+1)}$$

( $\because C_1$  and  $C_2$  are in series and resultant of these two in parallel with  $C_3$ )

22. (c) For path difference  $\lambda$ , phase difference  $= 2\pi \text{ rad.}$

For path difference  $\frac{\lambda}{4}$ , phase difference

$$= \frac{\pi}{2} \text{ rad.}$$

As  $K = 4I_0$  so intensity at given point where

path difference is  $\frac{\lambda}{4}$

$$K' = 4I_0 \cos^2\left(\frac{\pi}{4}\right) \left(\cos \frac{\pi}{4} = \cos 45^\circ\right)$$

$$= 2I_0 = \frac{K}{2}$$

23. (d)  $M({}_8\text{O}^{16}) = M({}_7\text{N}^{15}) + 1m_p$   
 binding energy of last proton  
 $= M({}_7\text{N}^{15}) + m_p - M({}_8\text{O}^{16})$   
 $= 15.00011 + 1.00783 - 15.99492$   
 $= 0.01302 \text{ amu} = 12.13 \text{ MeV}$

24. (b)

25. (a)  $H_1 = \frac{u^2 \sin^2 \theta}{2g}$

and  $H_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$

$$H_1 H_2 = \frac{u^2 \sin^2 \theta}{2g} \times \frac{u^2 \cos^2 \theta}{2g} = \frac{(u^2 \sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$$

$$\therefore R = 4\sqrt{H_1 H_2}$$

26. (c)  $\frac{1}{\lambda} = R \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

For limiting wavelength of Lyman series

$$n_1 = 1, n_2 = \infty \quad \frac{1}{\lambda_L} = R$$

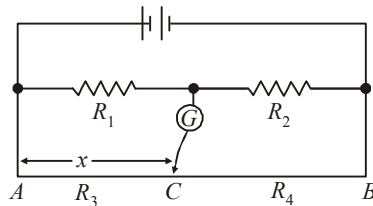
For limiting wavelength of Balmer series

$$n_1 = 2, n_2 = \infty$$

$$\frac{1}{\lambda_B} = R \left( \frac{1}{4} \right) \Rightarrow \lambda_B = \frac{4}{R}$$

$$\therefore \lambda_B = 4\lambda_L = 4 \times 912 \text{ \AA}$$

27. (a)



At null point  $\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{x}{100-x}$

If radius of the wire is doubled, then the resistance of AC will change and also the resistance of CB will change. But since

$$\frac{R_1}{R_2} \text{ does not change so, } \frac{R_3}{R_4} \text{ should also}$$

not change at null point. Therefore the point C does not change.

28. (a)

29. (c)  $v = \frac{p}{2\ell} \left[ \frac{F}{m} \right]^{1/2}$

$$v^2 = \frac{p}{4\ell^2} \frac{F}{m} \Rightarrow m = \frac{p^2 F}{4\ell^2 v^2}$$

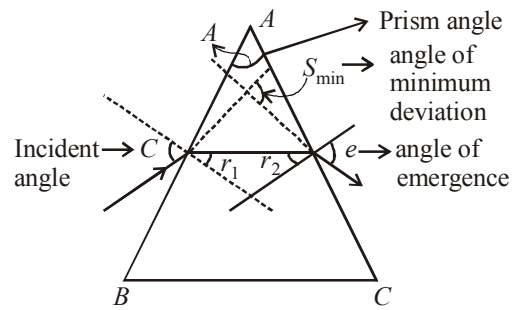
Now, dimensional formula of R.H.S.

$$= \frac{MLT^{-2}}{L^2 \left( \frac{1}{T} \right)^2}$$

[p will have no dimension as it is an integer]  
 $= ML^{-1}T^0$ .

So, dimensions of m will be  $ML^{-1}T^0$ .

30. (b)



The angle of minimum deviation is given as

$$\delta_{\min} = i + e - A$$

for minimum deviation

$$\delta_{\min} = A \text{ then}$$

$$2A = i + e$$

in case of  $\delta_{\min}$   $i = e$

$$2A = 2i \quad r_1 = r_2 = \frac{A}{2}$$

$$i = A = 90^\circ$$

from Snell's law

$$1 \sin i = n \sin r_1$$

$$\sin A = n \sin \frac{A}{2}$$

$$2 \sin \frac{A}{2} \cos \frac{A}{2} = n \sin \frac{A}{2}$$

$$2 \cos \frac{A}{2} = n$$

when  $A = 90^\circ = i_{\min}$

then  $n_{\min} = \sqrt{2}$

$i = A = 0$   $n_{\max} = 2$

31. (b)

32. (d) Induced emf in the loop is given by

$$e = -B \cdot \frac{dA}{dt} \text{ where } A \text{ is the area of the loop.}$$

$$e = -B \cdot \frac{d}{dt} (\pi r^2) = -B \pi 2r \frac{dr}{dt}$$

$$r = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$dr = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$dt = 1 \text{ s}$$

$$\begin{aligned} e &= -0.04 \times 3.14 \times 2 \times 2 \times 10^{-2} \times \frac{2 \times 10^{-3}}{1} \text{ V} \\ &= 0.32 \pi \times 10^{-5} \text{ V} \\ &= 3.2 \pi \times 10^{-6} \text{ V} \\ &= 3.2 \pi \mu \text{ V} \end{aligned}$$

33. (b) In cyclic process ABCA

$$Q_{\text{cycle}} = W_{\text{cycle}}$$

$$Q_{AB} + Q_{BC} + Q_{CA} = \text{ar. of } \Delta ABC$$

$$+400 + 100 + Q_{C \rightarrow A} = \frac{1}{2} (2 \times 10^{-3}) (4 \times 10^4)$$

$$\Rightarrow Q_{C \rightarrow A} = -460 \text{ J}$$

$$\Rightarrow Q_{A \rightarrow C} = +460 \text{ J}$$

34. (d)  $R_1 = R_0(1 + \alpha_1 t) + R_0(1 + \alpha_2 t)$

$$= 2R_0 \left( 1 + \frac{\alpha_1 + \alpha_2}{2} t \right)$$

$$\text{Comparing with } R = R_0(1 + \alpha t)$$

$$\alpha = \frac{\alpha_1 + \alpha_2}{2}$$

35. (c) From figure  $\tan \theta = \frac{F_e}{mg} \approx \theta$

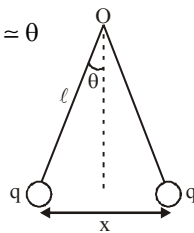
$$\frac{kq^2}{x^2 mg} = \frac{x}{2\ell}$$

$$\text{or } x^3 \propto q^2 \dots (1)$$

$$\text{or } x^{3/2} \propto q \dots (2)$$

Differentiating eq. (1) w.r.t. time

$$3x^2 \frac{dx}{dt} \propto 2q \frac{dq}{dt} \text{ but } \frac{dq}{dt} \text{ is constant}$$



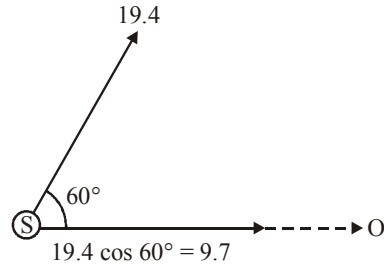
so  $x^2(v) \propto q$  Replace q from eq. (2)

$$x^2(v) \propto x^{3/2} \text{ or } v \propto x^{-1/2}$$

36. (a)

37. (a) Here, original frequency of sound,  $f_0 = 100 \text{ Hz}$

$$\text{Speed of source } V_s = 19.4 \cos 60^\circ = 9.7$$



From Doppler's formula

$$f^l = f_0 \left( \frac{V - V_0}{V - V_s} \right)$$

$$f^l = 100 \left( \frac{V - 0}{V - (+9.7)} \right)$$

$$f^l = 100 \frac{V}{V \left( 1 - \frac{9.7}{V} \right)}$$

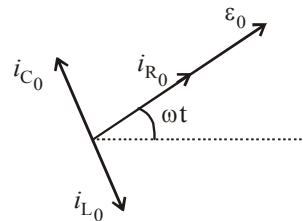
$$f^l = 100 \left( 1 + \frac{9.7}{330} \right) = 103 \text{ Hz}$$

Apparent frequency  $f^l = 103 \text{ Hz}$

$$38. (a) \frac{i_{R_0}}{\sqrt{(i_{R_0})^2 + (i_{C_0} - i_{L_0})^2}} = \frac{1}{2}$$

$$\Rightarrow \frac{\epsilon_0 / R}{\sqrt{(\epsilon_0 / R)^2 + \left( \epsilon_0 \omega C - \frac{\epsilon_0}{\omega L} \right)^2}} = \frac{1}{2}$$

$$\Rightarrow R = \frac{\sqrt{3}}{\left( \omega C - \frac{1}{\omega L} \right)}$$



39. (c) By covering aperture, focal length does not change. But intensity is reduced by  $\frac{1}{4}$  times, as aperture diameter  $\frac{d}{2}$  is covered.

$$\therefore I' = I - \frac{I}{4} = \frac{3I}{4}$$

$$\therefore \text{New focal length} = f \text{ and intensity} = \frac{3I}{4}$$

40. (c) According to problem disc is melted and recasted into a solid sphere so their volume will be same.

$$V_{\text{Disc}} = V_{\text{Sphere}} \Rightarrow \pi R_{\text{Disc}}^2 t = \frac{4}{3} \pi R_{\text{Sphere}}^3$$

$$\Rightarrow \pi R_{\text{Disc}}^2 \left( \frac{R_{\text{Disc}}}{6} \right) = \frac{4}{3} \pi R_{\text{Sphere}}^3 \left[ t = \frac{R_{\text{Disc}}}{6}, \text{ given} \right]$$

$$\Rightarrow R_{\text{Disc}}^3 = 8 R_{\text{Sphere}}^3 \Rightarrow R_{\text{Sphere}} = \frac{R_{\text{Disc}}}{2}$$

Moment of inertia of disc

$$I_{\text{Disc}} = \frac{1}{2} M R_{\text{Disc}}^2 = I \text{ (given)}$$

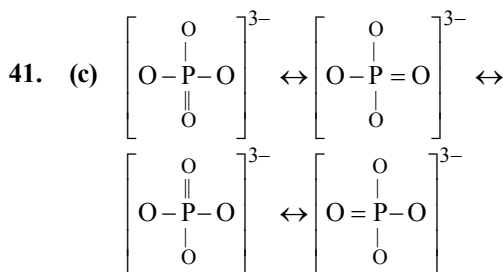
$$\therefore M (R_{\text{Disc}})^2 = 2I$$

Moment of inertia of sphere

$$I_{\text{Sphere}} = \frac{1}{2} M R_{\text{Sphere}}^2$$

$$= \frac{2}{5} M \left( \frac{R_{\text{Disc}}}{2} \right)^2 = \frac{M}{10} (R_{\text{Disc}})^2 = \frac{2I}{10} = \frac{I}{5}$$

## PART - II: CHEMISTRY



Bond order

$$= \frac{\text{Number of bonds}}{\text{Number of Resonating structures}}$$

$$= \frac{5}{4} = 1.25$$

Three unit negative charge is being shared by four O atoms. Formal charge =  $-3/4$

42. (b) Closed shell (Ne), half filled (P) and completely filled configuration (Mg) are the cause of higher value of I.E.

43. (b) Most of the  $\text{Ln}^{3+}$  compounds except  $\text{La}^{3+}$  and  $\text{Lu}^{3+}$  are coloured due to the presence of  $f$ -electrons.

44. (a)

45. (d)  $\text{Fe}^{2+}$  electronic configuration is  $[\text{Ar}] 3d^6$ . Since CN is strong field ligand  $d$  electrons are paired. In  $\text{Ni}(\text{CO})_4$  O. S. of Ni is zero electronic configuration is  $[\text{Ar}] 3d^8 4s^2$ . In presence of CO it is  $[\text{Ar}] 3d^{10} 4s^0$ , electrons are paired. Electronic configuration of  $\text{Ni}^{2+}$   $[\text{Ar}] 3d^8 4s^0$ , due to  $\text{CN}^-$  ligand all electrons are paired.  $\text{Co}^{3+}$  is  $[\text{Ar}] 3d^6$  since F is weak ligand hence paramagnetic.

46. (a) The complex chlorodiaquatriammine cobalt (III) chloride can have the structure  $[\text{CoCl}(\text{NH}_3)_3(\text{H}_2\text{O})_2]\text{Cl}_2$

47. (c) Wt. of  $\text{NH}_3 = 26 \text{ g} = \frac{26}{17} \text{ g eq} = 1.53 \text{ g eq}$

$$\text{Vol. of soln.} = 100 \text{ ml} = 0.1 \text{ L}$$

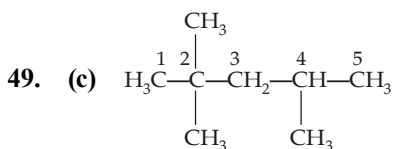
$$\therefore \text{Normality} = \frac{1.53}{0.1} = 15.3 \text{ N}$$

48. (a) Let the amount of  $\text{Na}_2\text{CO}_3$  present in the mixture be  $x \text{ g}$ .  $\text{Na}_2\text{SO}_4$  will not react with  $\text{H}_2\text{SO}_4$ . Then

$$\frac{x}{53} = \frac{20 \times 0.1 \times 10}{1000} \therefore x = 1.06 \text{ g}$$

$$\therefore \text{Percentage of } \text{Na}_2\text{CO}_3 =$$

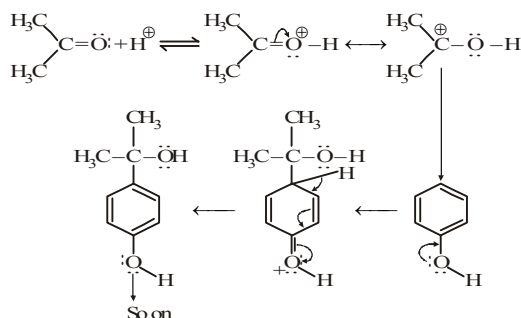
$$\frac{1.06 \times 100}{1.25} = 84.8\%$$



$\text{C}_1$  and  $\text{C}_5$  are  $1^\circ$ ,  $\text{C}_3$  is  $2^\circ$ ,  $\text{C}_4$  is  $3^\circ$ , and  $\text{C}_2$  is  $4^\circ$ .

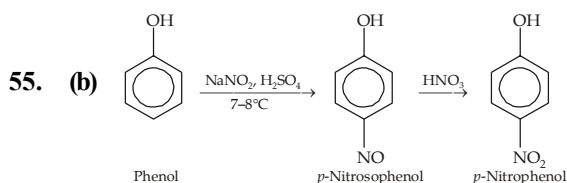
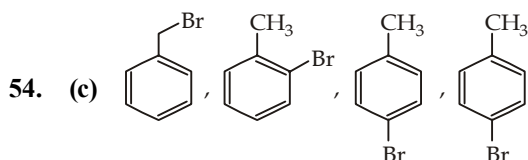
50. (b) A rapid umbrella type inversion rapidly converts the structure III to its enantiomer; hence the two enantiomers are not separable.

51. (d)



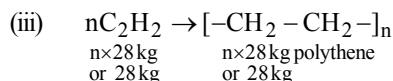
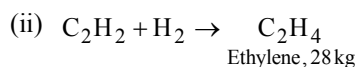
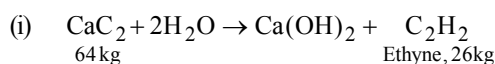
52. (c) Catalytic hydrogenation of alkynes gives *cis-alkene* which in turn adds deuterium atoms in presence of  $H_2$  again in *cis*-manner forming *meso*-2, 3-dideuterobutane.

53. (d)  $D_2SO_4$  transfers  $D^+$ , an electrophile, to form hexadeuterobenzene.



Thus here, oxidation of phenol is minimised by forming *p*-nitrosophenol.

56. (d) The concerned chemical reactions are



Thus 64 kg of  $CaC_2$  gives 26 kg of acetylene which in turn gives 28 kg of ethylene whose 28 kg gives 28 kg of the polymer, polythene.

57. (d)

58. (d) Although blue coloured ferric ferrocyanide is formed but due to the presence of yellow coloured  $Fe^{3+}$  salts, the blue colour gives the shade of green.

59. (c) Ketoses on reduction produce a new chiral carbon leading to the formation of two isomeric alcohols which are diastereomeric as well as C-2 epimers.

60. (d) Reaction of D-(+)-glucose with methanolic  $HCl$  leads to formation of methyl glucoside ( $C_1-OH$  group is methylated) which, being acetal, is not hydrolysable by base, so it will not respond Tollens' reagent.

61. (c) Magnesium hydrosilicate forms base of Talcum powder.

62. (a) BHT is the important antioxidant used in food.

63. (d) For Balmer  $n_1 = 2$  and  $n_2 = 3$ ;

$$\nu = R \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{5R}{36} \text{ cm}^{-1}$$

64. (d) When  $m = -3$ ,  $l = 3$ ,  $\therefore n = 4$ .

65. (c)  $r \propto U$  and  $U = \sqrt{\frac{3RT}{M}}$

$$\therefore \frac{r_1}{r_2} = \sqrt{\frac{T_1 M_2}{T_2 M_1}} \text{ or } \frac{r_{N_2}}{r_{SO_2}} = \sqrt{\frac{T_1 \times 64}{323 \times 28}} = 1.625$$

or  $T_2 = 373 \text{ K}$

66. (b)

$$\overline{KE} = \frac{3}{2} kT = \frac{3}{2} \times \frac{8.313}{6.023 \times 10^{23}} \times 298 = 6.17 \times 10^{-21} \text{ J}$$

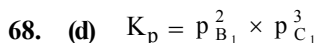
$$(\text{Average Kinetic energy } \overline{KE} = \frac{3}{2} kT = \frac{3}{2} \frac{R}{N} T)$$



$$\therefore K_p = \frac{\frac{\alpha}{1+\alpha} P \times \frac{\alpha}{1+\alpha} P}{\frac{1-\alpha}{1+\alpha} P} = \frac{\alpha^2 P}{1-\alpha^2}$$

$$\text{or, } K_p = \alpha^2 P$$

$$\therefore \alpha = \sqrt{\frac{K_p}{P}} \text{ when } 1 - \alpha^2 = 1$$



$$\text{Again, } K_p = p_{B_2}^2 \times (2p_{C_1})^3$$

$$\therefore p_{B_1}^2 \times p_{C_1}^3 = p_{B_2}^2 \times 8 p_{C_1}^3$$

$$\therefore \frac{p_{B_1}^2}{8} = p_{B_2}^2 \quad \text{or,} \quad \frac{p_{B_1}}{2\sqrt{2}} = p_{B_2}$$

69. (b) At equilibrium  $\Delta G = 0$

$$\text{Hence, } \Delta G = \Delta H - T_e \Delta S = 0$$

$$\therefore \Delta H = T_e \Delta S \quad \text{or} \quad T_e = \frac{\Delta H}{\Delta S}$$

For a spontaneous reaction

$\Delta G$  must be negative which is possible only if  $\Delta H - T\Delta S < 0$

$$\therefore \Delta H < T\Delta S$$

$$\text{or } T > \frac{\Delta H}{\Delta S}; \quad T_e < T$$

70. (c) Applying Hess's Law

$$\Delta_f H^\circ = \Delta_{\text{sub}} H + \frac{1}{2} \Delta_{\text{diss}} H + \text{I.E.} + \text{E.A.} + \Delta_{\text{lattice}} H$$

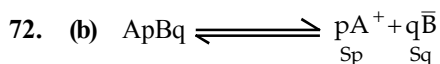
$$-617 = 161 + 520 + 77 + \text{E.A.} + (-1047)$$

$$\text{E.A.} = -617 + 289 = -328 \text{ kJ mol}^{-1}$$

$$\therefore \text{electron affinity of fluorine} = -328 \text{ kJ mol}^{-1}$$

71. (a)

$$\alpha = \sqrt{\frac{K_w}{K_a \times K_b}} = \sqrt{\frac{1 \times 10^{-14}}{1.8 \times 10^{-5} \times 1.8 \times 10^{-5}}} = 0.55$$



Let the solubility be  $S$  mol/liter

Thus,

$$K_{\text{sp}} = [\text{A}^+]^p [\text{B}^-]^q$$

$$= [\text{Sp}]^p [\text{Sq}]^q = p^p q^q (S)^{p+q}$$

73. (d)

74. (d) For P, if  $t_{50\%} = x$

$$\text{then } t_{75\%} = 2x$$

This is true only for first order reaction.

So, order with respect to P is 1.

Further the graph shows that concentration of Q decreases with time. So rate, with respect to Q, remains constant. Hence, it is zero order wrt Q.

$$\text{So, overall order is } 1 + 0 = 1$$

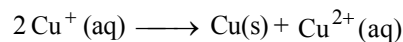
75. (b) The oxidation potential

$$\propto \frac{1}{\text{Concentration of ions}} \quad \text{and reduction}$$

potential  $\propto$  concentration of ions. The cell voltage can be increased by decreasing the concentration of ions around anode or by increasing the concentration of ions around cathode

76. (c)

77. (c) The reaction



$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.0592}{1} \log \frac{[\text{Cu}^{2+}]}{[\text{Cu}^+]^2}$$

$$\text{At equilibrium } E_{\text{cell}} = 0$$

$$\therefore E^\circ_{\text{cell}} = 0.0592 \log K_c$$

$$\text{or, } \log K_c = \frac{0.52 - 0.16}{0.0592}$$

$$\therefore K_c = 1.2 \times 10^6$$

78. (a) The number of  $\text{Fe}^{3+}$  ions replacing  $x \text{Fe}^{2+}$

$$\text{ions} = \frac{2x}{3} \text{ vacancies of cations}$$

$$= x - \frac{2x}{3} = x/3$$

$$\text{But } x/3 = 1 - 0.94 = 0.06, \quad x = 0.06 \times 3 = 0.18 = 18\%$$

79. (d)  $\rho = \frac{Z \times M}{N_o \times a^3},$

$$2.7 = \frac{Z \times 27}{6.02 \times 10^{23} \times (4.05)^3 \times 10^{-24}}$$

$$\therefore Z = 4$$

Hence it is face centred cubic unit lattice.

$$\text{Again } 4r = a\sqrt{2} = 5.727 \text{ \AA}$$

$$\therefore r = 1.432 \text{ \AA}$$

80. (d) Xe = 53.5%  $\therefore$  F = 46.5%

Relative number of atoms Xe

$$= \frac{53.5}{131.2} = 0.4 \quad \text{and} \quad F = \frac{46.5}{19} = 2.4$$

Simple ratio Xe = 1 and F = 6 ; Molecular formula is  $\text{XeF}_6$

O.N. of Xe is +6

### PART - III (A): ENGLISH PROFICIENCY

- |         |         |         |         |
|---------|---------|---------|---------|
| 81. (d) | 82. (a) | 83. (a) | 84. (a) |
| 85. (a) | 86. (c) | 87. (d) | 88. (b) |
| 89. (d) | 90. (d) | 91. (a) | 92. (b) |
| 93. (a) | 94. (c) | 95. (a) |         |



### PART - III (B): LOGICAL REASONING

96. (a) In each row, the third figure comprises of a black circle and only those line segments which are not common to the first and the second figures.
97. (d) In each row (as well as each column), the third figure is a combination of all the elements of the first and the second figures.
98. (c) In each row, the third figure is a collection of the common elements (line segments) of the first and the second figures.
99. (b) The given sequence is a combination of two series :  
I. 3, 7, 13, 21, 31, ? and II. 4, 7, 13, 22, 34  
The pattern in I is + 4, + 6, + 8, + 10, ....  
The pattern in II is + 3, + 6, + 9, + 12, ....  
So, missing term = 31 + 12 = 43.
100. (a) The father of the boy's uncle is the grandfather of the boy and daughter of the grandfather is sister of father.
101. (a) In this series, the third letter is repeated as the first letter of the next segment. The middle letter, A, remains static. The third letters are in alphabetical order, beginning with R.
102. (d) In this series, the letters remain the same: DEF.  
The subscript numbers follow this series:  
111, 112, 122, 222, 223, 233, 333, ...
103. (b) Since both the premises are universal and one premise is negative, the conclusion must be universal negative and should not contain the middle term. So, only II follows.
104. (a) Since one premise is particular and the other premise is negative, the conclusion must be particular negative and should not contain the middle term. So, it follows that 'Some desks are not red'. However, I is the converse of the first premise and thus it holds.
105. (c)

### PART - IV : MATHEMATICS

106. (a)  $f(x) = \frac{ax+b}{cx+d}$   
$$\text{fof}(x) = \frac{a\left\{\frac{ax+b}{cx+d}\right\}+b}{c\left\{\frac{ax+b}{cx+d}\right\}+d} \Rightarrow \frac{a^2x+ab+bcx+bd}{acx+bc+cdx+d^2} = x$$
$$\Rightarrow (ac+dc)x^2 + (bc+d^2-bc-a^2)x - ab-bd = 0, \forall x \in \mathbb{R}$$
$$\Rightarrow (a+d)c = 0, d^2 - a^2 = 0 \text{ and } (a+d)b = 0$$
$$\Rightarrow a+d = 0$$
107. (b)  $2^m - 2^n = 112 \Rightarrow 2^n(2^{m-n} - 1) = 16 \cdot 7$   
 $\therefore 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$   
Comparing we get  $n = 4$  and  $m - n = 3$   
 $\Rightarrow n = 4$  and  $m = 7$
108. (b) Given,  $3 \cos^2 A + 2 \cos^2 B = 4$   
 $\Rightarrow 2 \cos^2 B - 1 = 4 - 3 \cos^2 A - 1$   
 $\Rightarrow \cos 2B = 3(1 - \cos^2 A) = 3 \sin^2 A \dots (1)$   
and  $2 \cos B \sin B = 3 \sin A \cos A$   
 $\sin 2B = 3 \sin A \cos A \dots (2)$   
Now,  $\cos(A + 2B) = \cos A \cos 2B - \sin A \sin 2B$   
 $= \cos A (3 \sin^2 A) - \sin A (3 \sin A \cos A) = 0$   
[using eqs. (1) and (2)]  
 $\Rightarrow A + 2B = \frac{\pi}{2}$
109. (a) Since,  $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$   
 $\therefore \sin \theta_1 = \sin \theta_2 = \sin \theta_3 = 1 \Rightarrow \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2}$   
 $\therefore \cos \theta_1 + \cos \theta_2 + \cos \theta_3 = 0$
110. (b) Given,  
 $\tan(\cot x) = \cot(\tan x) = \tan\left(\frac{\pi}{2} - \tan x\right)$   
 $\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x$   
 $\Rightarrow \cot x + \tan x = n\pi + \frac{\pi}{2}$

$$\Rightarrow \frac{1}{\sin x \cos x} = n\pi + \frac{\pi}{2} \Rightarrow \frac{1}{\sin 2x} = \frac{n\pi}{2} + \frac{\pi}{4}$$

$$\Rightarrow \sin 2x = \frac{1}{\frac{n\pi}{2} + \frac{\pi}{4}} = \frac{4}{(2n+1)\pi}$$

**111. (d)** Given,  $\sin 2x + 2 \sin x + 2 \cos x + 1 = 0$

$$\Rightarrow 1 + \sin 2x + 2(\sin x + \cos x) = 0$$

$$\Rightarrow (\sin x + \cos x)^2 + 2(\sin x + \cos x) = 0$$

$$\Rightarrow (\sin x + \cos x)(\sin x + \cos x + 2) = 0$$

$$\therefore \sin x + \cos x = 0 \quad \text{or} \quad \sin x + \cos x = -2$$

But,  $\sin x + \cos x = -2$  is inadmissible.

$$\text{Since, } |\sin x| \leq 1, |\cos x| \leq 1$$

$$\therefore \sin x + \cos x = 0 \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = 0$$

$$\Rightarrow x + \frac{\pi}{4} = n\pi \Rightarrow x = n\pi - \frac{\pi}{4}$$

**112. (d)**  $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$$\Rightarrow \frac{\cos A}{2R \sin A} = \frac{\cos B}{2R \sin B} = \frac{\cos C}{2R \sin C}$$

$$\Rightarrow \cot A = \cot B = \cot C$$

$$\Rightarrow A = B = C = 60^\circ \Rightarrow \Delta ABC \text{ is equilateral}$$

$$\text{Hence, area of } \Delta = \frac{\sqrt{3}}{4} a^2 = \sqrt{3}.$$

**113. (d)** Given,

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\therefore 2\tan^{-1} a - 2\tan^{-1} b = 2 \tan^{-1} x$$

$$\Rightarrow \tan^{-1} a - \tan^{-1} b = \tan^{-1} x$$

$$\Rightarrow \tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1} x$$

$$\Rightarrow x = \frac{a-b}{1+ab}$$

**114. (c)** Given  $M = \frac{a+b+c+d+e}{5}$

$$\Rightarrow a+b+c+d+e = 5M$$

$$\Rightarrow a+b+c+d+e-5M = 0$$

$$\Rightarrow (a-M) + (b-M) + (c-M) + (d-M) + (e-M) = 0$$

Hence, required value = 0

**115. (a)** Let the progression be  $a, a+d, a+2d,$

$$\text{Then } x_4 = 3x_1 \Rightarrow a+3d = 3a \Rightarrow 3d = 2a \dots(i)$$

$$\text{Again } x_7 = 2x_3 + 1$$

$$\Rightarrow a+6d = 2(a+2d)+1 \Rightarrow 2d = a+1 \dots(ii)$$

Solving (i) and (ii), we get

$$a = 3, d = 2$$

**116. (c)**  $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$

$$= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \left(1 - \frac{1}{16}\right) + \dots$$

$$= n - \frac{\frac{1}{2} \left\{1 - \frac{1}{2^n}\right\}}{1 - \frac{1}{2}} = n - 1 + 2^{-n}$$

**117. (c)**

**118. (a)** The series is

$$(x^2 + x^4 + x^6 + \dots) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \dots\right) + (2 + 2 + \dots)$$

$$= \frac{x^2(x^{2n}-1)}{x^2-1} + \frac{\frac{1}{x^2}\left(1 - \frac{1}{x^{2n}}\right)}{1 - \frac{1}{x^2}} + 2n$$

$$= \frac{x^2(x^{2n}-1)}{x^2-1} + \frac{x^{2n}-1}{(x^2-1)x^{2n}} + 2n$$

$$= \frac{x^{2n}-1}{x^2-1} \times \frac{x^{2n+2}+1}{x^{2n}} + 2n$$

**119. (a)**  $\left(\frac{z_1}{z_2}\right)^{50} = \left(\frac{\sqrt{3}+i\sqrt{3}}{\sqrt{3}+i}\right)^{50}$

$$= \left[\left(\frac{\sqrt{3}(1+i)}{\sqrt{3}+i}\right)^2\right]^{25} = \left[\frac{3(2i)}{3-1+2\sqrt{3}i}\right]^{25}$$

$$= \left(\frac{3i}{1+\sqrt{3}i}\right)^{25} = \frac{3^{25}i^{25}}{(-2\omega^2)^{25}}$$

$$= -i \cdot \omega \left( \frac{3}{2} \right)^{25} = -i \left( \frac{-1 + \sqrt{3}i}{2} \right) \left( \frac{3}{2} \right)^{25}$$

$$= \left( \frac{3}{2} \right)^{25} \left( \frac{\sqrt{3}}{2} + \frac{1}{2}i \right)$$

Hence,  $\left( \frac{z_1}{z_2} \right)^{50}$  lies in the first quadrant as both real and imaginary parts of this number are positive.

**120. (b)**  $|A| = 0$  as the matrix A is singular

$$\therefore |A| = \begin{vmatrix} 1 & 3 & \lambda + 2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$$

Apply  $R_2 \rightarrow R_2 - 2R_1$  and  $R_3 \rightarrow R_3 - 3R_1$  and expand.

$$-2(4 - 3\lambda) + 4(4 - 2\lambda) = 0$$

$$\Rightarrow 8 - 2\lambda = 0 \Rightarrow \lambda = 4$$

For  $\lambda = 4$ , the second and the third column are proportional.

**121. (a)** Since  $\alpha_1, \alpha_2$  and  $\beta_1, \beta_2$  are the roots of  $ax^2 + bx + c = 0$  and  $px^2 + qx + r = 0$  respectively, therefore

$$\alpha_1 + \alpha_2 = \frac{-b}{a}, \alpha_1 \alpha_2 = \frac{c}{a} \quad \dots(1)$$

$$\text{and } \beta_1 + \beta_2 = \frac{-q}{p}, \beta_1 \beta_2 = \frac{r}{p} \quad \dots(2)$$

Since the given system of equation has a non-trivial solution

$$\therefore \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0 \text{ i.e. } \alpha_1 \beta_2 - \alpha_2 \beta_1 = 0$$

$$\text{or } \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \frac{\sqrt{\alpha_1 \alpha_2}}{\sqrt{\beta_1 \beta_2}}$$

$$\Rightarrow \frac{pb}{qa} = \sqrt{\frac{pc}{ra}} \Rightarrow \frac{b^2}{q^2} = \frac{ac}{pr}$$

**122. (a)** Since,  $-1 \leq x < 0$

$$\therefore [x] = -1$$

$$0 \leq y < 1 \quad \therefore [y] = 0$$

$$1 \leq z < 2 \quad \therefore [z] = 1$$

$$\therefore \text{ Given determinant} = \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 1 = [z]$$

**123. (d)**

**124. (a)** Given equation is  $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$$\Rightarrow 3x^2 - 2(b + a + c)x + ab + bc + ca = 0$$

Now, here  $A = 3, B = -2(a + b + c)$

$$C = ab + bc + ca$$

$$\therefore D = \sqrt{B^2 - 4AC}$$

$$= \sqrt{(-2(a + b + c))^2 - 4(3)(ab + bc + ca)}$$

$$= \sqrt{4(a + b + c)^2 - 12(ab + bc + ca)}$$

$$= 2\sqrt{a^2 + b^2 + c^2 - ab - bc - ca}$$

$$= 2\sqrt{\frac{1}{2}\{(a - b)^2 - (b - c)^2 + (c - a)^2\}} \geq 0$$

$$\text{125. (b) } \text{limit} = \lim_{n \rightarrow \infty} \frac{1 + \left(\frac{b}{a}\right)^n}{1 - \left(\frac{b}{a}\right)^n} = 1,$$

because  $0 < \frac{b}{a} < 1$  implies

$$\left(\frac{b}{a}\right)^n \rightarrow 0 \text{ as } n \rightarrow \infty$$

**126. (c)** The function  $\log|x|$  is not defined at  $x = 0$ , so,  $x = 0$  is a point of discontinuity. Also for  $f(x)$  to be defined,  $\log|x| \neq 0 \Rightarrow x \neq \pm 1$ . Hence, 0, 1, -1 are three points of discontinuity.

**127. (a)** We have,

$$Lf'(0) = \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{-h} = \lim_{h \rightarrow 0} \frac{-h \log \cosh}{-h \log(1 + h^2)}$$

$$= \lim_{h \rightarrow 0} \frac{\log \cosh}{\log(1 + h^2)} \quad \left( \frac{0}{0} \text{ form} \right)$$

$$= \lim_{h \rightarrow 0} \frac{-\tan h}{2h/(1 + h^2)} = -1/2$$

$$Rf'(0) = \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h \log \cosh}{h \log(1 + h^2)}$$

$$= \lim_{h \rightarrow 0} \frac{\log \cosh}{\log(1 + h^2)} \quad \left( \frac{0}{0} \text{ form} \right)$$

$$= \lim_{h \rightarrow 0} \frac{-\tan h}{2h/(1 + h^2)} = \frac{-1}{2}$$

Since  $Lf'(0) = Rf'(0)$ , therefore  $f(x)$  is differentiable at  $x = 0$

Since differentiability  $\Rightarrow$  continuity, therefore  $f(x)$  is continuous at  $x = 0$ .

$$\begin{aligned}
 128. (a) \quad \frac{dy}{dx} &= \left( \frac{dx}{dy} \right)^{-1} \\
 &\Rightarrow \frac{d^2y}{dx^2} = -1 \left( \frac{dx}{dy} \right)^{-2} \left\{ \frac{d}{dx} \left( \frac{dx}{dy} \right) \right\} \\
 &\Rightarrow \frac{d^2y}{dx^2} = (-1) \left( \frac{dx}{dy} \right)^{-2} \left\{ \frac{d}{dy} \left( \frac{dx}{dy} \right) \frac{dy}{dx} \right\} \\
 &= (-1) \left( \frac{dy}{dx} \right)^2 \left\{ \frac{d^2x}{dy^2} \cdot \frac{dy}{dx} \right\} = - \left( \frac{dy}{dx} \right)^3 \left\{ \frac{d^2x}{dy^2} \right\} \\
 &\Rightarrow \frac{d^2x}{dy^2} \left( \frac{dy}{dx} \right)^3 + \frac{d^2y}{dx^2} = 0
 \end{aligned}$$

129. (b)

130. (c) (A) Graph of  $f(x) = \cos x$  cuts  $x$ -axis at infinite number of points. (5 of list II)  
 (B) Graph of  $f(x) = \ln x$  cuts  $x$ -axis in only one point. (4 of list II)  
 (C) Graph of  $f(x) = x^2 - 5x + 4$  cuts  $x$  axis in two points (2 of list II)  
 (D) Graph of  $f(x) = e^x$  cuts  $y$ -axis in only one point. (3 of list II)

131. (b)  $f(x) = \sqrt{x} (7x - 6) = 7x^{3/2} - 6x^{1/2}$

$$f'(x) = 7 \times \frac{3}{2} x^{1/2} - 6 \times \frac{1}{2} x^{-1/2}$$

When tangent is parallel to  $x$  axis  $f'(x) = 0$

$$\text{or, } \frac{21}{2} x^{1/2} - 3x^{-1/2} = 0$$

$$\frac{21}{2} \sqrt{x} = \frac{3}{\sqrt{x}}$$

$$\text{or, } 7x = 2 \Rightarrow x = \frac{2}{7}$$

132. (d) Let one side of quadrilateral be  $x$  and another side be  $y$

$$\text{so, } 2(x + y) = 34$$

$$\text{or, } (x + y) = 17 \quad \dots(i)$$

We know from the basic principle that for a given perimeter square has the maximum area, so,  $x = y$  and putting this value in equation (i)

$$x = y = \frac{17}{2}$$

$$\text{Area} = x \cdot y = \frac{17}{2} \times \frac{17}{2} = \frac{289}{4} = 72.25$$

133. (a) Since  $f(x)$  is an increasing function in  $[-1, 1]$  and it has a root in  $(-1, 1)$ .

$\therefore$  Only statement I is correct.

134. (a) At an extreme point of a function  $f(x)$ , slope is always zero.

Thus, At an extreme point of a function  $f(x)$ , the tangent to the curve is parallel to the  $x$ -axis.

135. (a) Let  $y = xe^x$ .

Differentiate both side w.r.t. ' $x$ '.

$$\Rightarrow \frac{dy}{dx} = e^x + xe^x = e^x (1 + x)$$

$$\text{Put } \frac{dy}{dx} = 0$$

$$\Rightarrow e^x (1 + x) = 0$$

$$\Rightarrow x = -1$$

$$\text{Now, } \frac{d^2y}{dx^2} = e^x + e^x (1 + x) = e^x (x + 2)$$

$$\left( \frac{d^2y}{dx^2} \right)_{(x=-1)} = \frac{1}{e} + 0 > 0$$

Hence,  $y = xe^x$  is minimum function and

$$y_{\min} = -\frac{1}{e}.$$

136. (a)

137. (b) We have  $a = 1$ ,  $h = -\sqrt{3}$ ,  $b = 3$ ,  $g = -\frac{3}{2}$ ,

$$f = \frac{3\sqrt{3}}{2}, c = -4.$$

$$\text{Thus } abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

Hence the equation represents a pair of straight lines.

$$\text{Again } \frac{a}{h} = \frac{h}{b} = \frac{g}{f} = -\frac{1}{\sqrt{3}}$$

$\therefore$  the lines are parallel. The distance between them

$$= 2 \sqrt{\frac{g^2 - ac}{a(a+b)}} = 2 \sqrt{\frac{\frac{9}{4} + 4}{1(1+3)}} = \frac{5}{2}.$$

138. (b) Let  $P(x, y)$  be the point dividing the join of  $A$  and  $B$  in the ratio  $2 : 3$  internally, then

$$x = \frac{20 \cos \theta + 15}{5} = 4 \cos \theta + 3 \Rightarrow \cos \theta = \frac{x-3}{4} \dots(i)$$

$$y = \frac{20 \sin \theta + 0}{5} = 4 \sin \theta \Rightarrow \sin \theta = \frac{y}{4} \dots(ii)$$

Squaring and adding (i) and (ii), we get the required locus  $(x-3)^2 + y^2 = 16$ , which is a circle.

139. (c) Radius  $\leq 5$

$$\sqrt{\frac{\lambda^2}{4} + \frac{(1-\lambda)^2}{4}} - 5 \leq 5 \Rightarrow \lambda^2 + (1-\lambda)^2 - 20 \leq 100$$

$$\Rightarrow 2\lambda^2 - 2\lambda - 119 \leq 0$$

$$\therefore \frac{1-\sqrt{239}}{2} \leq \lambda \leq \frac{1+\sqrt{239}}{2} \Rightarrow -7.2 \leq \lambda \leq 8.2$$

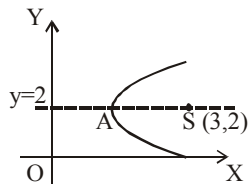
(approx.)

$\therefore \lambda = -7, -6, -5, \dots, 7, 8$ , in all 16 values

140. (a) 141. (c) 142. (c)

143. (b) Vertex of the parabola is a point which lies on the axis of the parabola, which is a line  $\perp$  to the directrix through the focus, i.e.,  $y=2$  and equidistant from the focus and directrix

$x=0$ , so that the vertex is  $\left(\frac{3}{2}, 2\right)$ .



$$144. (b) {}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$$

$$\Rightarrow {}^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2-1} + {}^{39}C_{r^2}$$

$$\Rightarrow {}^{40}C_{3r} = {}^{40}C_{r^2}$$

$$\Rightarrow r^2 = 3r \text{ or } r^2 = 40 - 3r$$

$$\Rightarrow r = 0, 3 \text{ or } -8, 5$$

3 and 5 are the values as the given equation is not defined by  $r=0$  and  $r=-8$ . Hence, the number of values of  $r$  is 2.

$$145. (d) \sum_{r=0}^n \frac{r+2}{r+1} {}^nC_r = \frac{2^8-1}{6}$$

$$\Rightarrow \sum_{r=0}^n \left[1 + \frac{1}{r+1}\right] {}^nC_r = \frac{2^8-1}{6}$$

$$\Rightarrow 2^n + \sum_{r=0}^n \frac{1}{n+1} \cdot {}^{n+1}C_{r+1} = \frac{2^8-1}{6}$$

$$\Rightarrow 2^n + \frac{2^{n+1}-1}{n+1} = \frac{2^8-1}{6} \Rightarrow \frac{2^n(n+3)-1}{n+1} = \frac{2^8-1}{6}$$

$$\Rightarrow \frac{2^n(n+1+2)-1}{n+1} = \frac{2^5(6+2)-1}{6}$$

Comparing we get  $n+1=6 \Rightarrow n=5$

146. (d) No. of words starting with A are  $4! = 24$

No. of words starting with H are  $4! = 24$

No. of words starting with L are  $4! = 24$

These account for 72 words

Next word is RAHLU and the 74<sup>th</sup> word RAHUL.

147. (a)

$$(x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + {}^nC_3 x^{n-3} a^3 + {}^nC_4 x^{n-4} a^4 + \dots$$

$$= ({}^nC_0 x^n + {}^nC_2 x^{n-2} a^2 + {}^nC_4 x^{n-4} a^4 + \dots) + ({}^nC_1 x^{n-1} a + {}^nC_3 x^{n-3} a^3 + {}^nC_5 x^{n-5} a^5 + \dots)$$

$$= A + B \dots(1)$$

$$\text{Similarly, } (x-a)^n = A - B \dots(2)$$

Multiplying eqns. (1) and (2), we get

$$(x^2 - a^2)^n = A^2 - B^2$$

148. (c) Put  $\log_{10} x = y$ , the given expression becomes  $(x + x^y)^5$ .

$$T_3 = {}^5C_2 \cdot x^3 (x^y)^2 = 10x^{3+2y} = 10^6 \text{ (given)}$$

$$\Rightarrow (3+2y) \log_{10} x = 5 \log_{10} 10 = 5$$

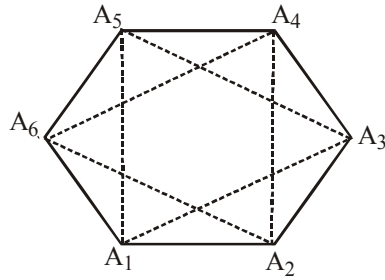
$$\Rightarrow (3+2y)y = 5$$

$$\Rightarrow y = 1, -\frac{5}{2}$$

$$\Rightarrow \log_{10} x = 1 \text{ or } \log_{10} x = -\frac{5}{2}$$

$$\therefore x = 10 \text{ or } x = (10)^{-5/2}$$

149. (c) Three vertices can be selected in  ${}^6C_3$  ways.



The only equilateral triangles possible are  $A_1A_3A_5$  and  $A_2A_4A_6$

$$p = \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$$

150. (b) As  $0.4 + 0.6 = 1$ , the man either takes a step forward or a step backward. Let a step forward be a success and a step backward be a failure.

Then, the probability of success in one step

$$= p = 0.4 = \frac{2}{5}$$

The probability of failure in one step

$$= q = 0.6 = \frac{3}{5}$$

In 11 steps he will be one step away from the starting point if the numbers of successes and failures differ by 1.

So, the number of successes = 6

The number of failures = 5

or the number of successes = 5,

The number of failures = 6

$\therefore$  the required probability

$$\begin{aligned} &= {}^{11}C_6 p^6 q^5 + {}^{11}C_5 p^5 q^6 \\ &= {}^{11}C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^5 + {}^{11}C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^6 \\ &= \frac{11!}{6!5!} \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^5 \left\{ \frac{2}{5} + \frac{3}{5} \right\} \\ &= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{120} \cdot \frac{2^5 \cdot 3^5}{5^{10}} = 462 \times \left(\frac{6}{25}\right)^5 \end{aligned}$$