CBSE Test Paper 04

CH-2 Polynomials

---

1. If $x+y+z=9$ and $xy+yz+zx=23$, then the value of $x^3+y^3+z^3-3xyz$ is
   a. 108
   b. 209
   c. 144
   d. 180
2. The coefficient of x^3 in $2x+x^2-5x^3+x^4$ is
   a. -1
   b. 2
   c. 1
   d. -5
3. If $f(x)=x^2-5x+1$, then the value of f(2) + f(-1) is
   a. 2
   b. 1
   c. -2
   d. -1
4. The value of $(102)^3$ is
   a. 1820058
   b. 1001208
   c. 1061280

d. 1061208

5. The possible expressions for the length, breadth and height of the cuboid whose volume is given by $3x^3 - 12x$ is

   a. 3x, (x + 2) and (x – 2)

   b. x, (3x + 2) and (x – 2)

   c. x, (x + 2) and (3x – 2)

   d. none of these

6. Fill in the blanks:
   If the number of terms of polynomial are 2 and 3, then the corresponding polynomials are called ________ and ________.
7. Fill in the blanks:
   The degree of the zero polynomial is ________.

8. Whether the following are zero of the polynomial, indicated against them.p(x) = 2x + 1, x = $\frac{1}{2}$.

9. Classify as linear, quadratic and cubic polynomials:
   3t

10. If x - $\frac{1}{x}$ = -1, find the value of x^2 + $\frac{1}{x^2}$

11. Write the expanded form of : $\left(x - \frac{2}{3}y\right)^3$

12. Find the value of $x^2 + \frac{1}{x^2}$, if $x - \frac{1}{x} = \sqrt{3}$.

13. Find the remainder when f(x) = $x^3 - 6x^2 + 2x - 4$ is divided by g(x) = 3x - 1.

14. Without actual division, prove that $2x^4 + x^3 - 14x^2 - 19x - 6$ is exactly divisible by $x^2 + 3x + 2$.

15. Find m and n, if (x + 2) and (x + 1) are the factors of $x^3 + 3x^2 - 2mx + n$.

CBSE Test Paper 04

CH-2 Polynomials

Solution

1. (a) 108

   Explanation:

   Given: $x+y+z=9$ and $xy+yz+zx=23$

   $x^3+y^3+z^3-3xyz$ = $(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$

   = $(x+y+z)\left[(x+y+z)^2-2xy-2yz-2zx-xy-yz-zx\right]$

   = $(x+y+z)\left[(x+y+z)^2-3xy-3yz-3zx\right]$

   = $(x+y+z)\left[(x+y+z)^2-3(xy+yz+zx)\right]$

   = $(9)\left[(9)^2-3(23)\right]$

   = $9\times[81-69]$

   = 9 x 12

   = 108

2. (d) -5

   Explanation:

   The coefficient of x^3 in $2x+x^2-5x^3+x^4$ is -5.

3. (a) 2

   Explanation:

   $f(x)=x^2-5x+1$

   $f(2)+f(-1)$

   = $(2)^2-5\times2+1+(-1)^2-5\times(-1)+1$

= $4 - 10 + 1 + 1 + 5 + 1$

= $12 - 10$

= 2

4. (d) 1061208

   Explanation:

   $(102)^3 = (100 + 2)^3$

   = $(100)^3 + (2)^3 + 3 \times 100 \times 2\,(100 + 2)$

   = 1000000+8+60000+1200

   = 1061208

5. (a) 3x, (x + 2) and (x – 2)

   Explanation:

   To find the length, breadth and height, we will factorize the given polynomial.

   $3x^3 - 12x$

   = $3x\left[x^2 - 4\right]$

   = $3x\left[x^2 - (2)^2\right]$

   = $3x\,(x + 2)\,(x - 2)$

   Therefore, the possible expressions for the length, breadth and height of the cuboid whose volume is given by $3x^3 - 12x$ are $3x, (x + 2)$ and $(x - 2)$.

6. binomial, trinomial

7. not defined

8. p($\frac{1}{2}$) = 2($\frac{1}{2}$) + 1 = 1 + 1 = 2 $\neq$ 0

   $\therefore$ $\frac{1}{2}$ is not a zero of p(x).

9. 3t

We can observe that the degree of the polynomial $(3t)$is 1. Therefore, we can conclude that the polynomial 3t is a linear polynomial.

10. We have,

$(x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x}$

$\Rightarrow (x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$

$\Rightarrow (-1)^2 = x^2 + \frac{1}{x^2} - 2$ [$\because x - \frac{1}{x} = -1$]

$\Rightarrow 1 = x^2 + \frac{1}{x^2} - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 1 + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 3$

11. $\left(x-\frac{2}{3}y\right)^3 = x^3-\left(\frac{2}{3}y\right)^3-3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$= x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right) = x^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$

$= x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3$

12. According to the question,

$x-\frac{1}{x}=\sqrt{3}$

Squaring both the sides,

$\Rightarrow \left(x-\frac{1}{x}\right)^2=(\sqrt{3})^2$

$\Rightarrow \quad x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}=3$

$\Rightarrow \quad x^2+\frac{1}{x^2}=3+2$

$\Rightarrow x^2+\frac{1}{x^2}=5$

13. We have, g(x) = 3x - 1 = $3\left(x-\frac{1}{3}\right)$

Therefore, by remainder theorem when f(x) is divided by $g(x)=3\left(x-\frac{1}{3}\right)$, the remainder is equal to $f\left(\frac{1}{3}\right)$.

Now, f(x) = $x^3 - 6x^2 + 2x - 4$

$\Rightarrow \quad f\left(\frac{1}{3}\right)=\left(\frac{1}{3}\right)^3-6\left(\frac{1}{3}\right)^2+2\left(\frac{1}{3}\right)-4$

$$\Rightarrow \quad f\left(\frac{1}{3}\right) = \frac{1}{27} - \frac{6}{9} + \frac{2}{3} - 4 = \frac{1-18+18-108}{27} = -\frac{107}{27}$$

Hence, required remainder $= -\frac{107}{27}$

14. Let $p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ and $q(x) = x^2 + 3x + 2$

Then, $q(x) = x^2 + 3x + 2 = x^2 + 2x + x + 2$

$= x(x+2) + 1(x+2) = (x+2)(x+1)$

Now, p(-1) = $2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6$

= 2 - 1 - 14 + 19 - 6 = 21 -21

p(-1) = 0

and, p(-2) = $2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6$

= 32 - 8 - 56 + 38 - 6 = 70 - 70

p(-2) = 0

$\Rightarrow$ (x + 1) and (x + 2) are the factors of p(x), so p(x) is divisible by (x + 1) and (x + 2).

Hence, p(x) is divisible by (x + 1)(x + 2) = $x^2 + 3x + 2$.

15. Let f(x) = $x^3 + 3x^2 - 2mx + n$

Since, (x + 2) and (x + 1) are the factors of f(x).

$\therefore$ f(-2) = 0 and f(-1) = 0

$\Rightarrow$ f(-2) = $(-2)^3 + 3(-2)^2 - 2m(-2) + n = 0$ and f(-1) = $(-1)^3 + 3(-1)^2 - 2m(-1) + n = 0$

$\Rightarrow$ -8 + 12 + 4m + n = 0 and -1 + 3 + 2m + n = 0

$\Rightarrow$ 4m + n = -4 ...(i) and 2m + n = -2 ...(ii)

On multiplying Eq. (ii) by 2 and then subtracting Eq. (i) from Eq. (ii), we get,

4m + 2n - (4m + n) = -4 - (-4) $\Rightarrow$ n = 0

On putting n = 0 in Eq. (i), we get 4m + 0= -4 $\Rightarrow$ m = -1

Hence, m = -1 and n = 0.