Sample Question Paper - 7
Mathematics (041)
Class- XII, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Evaluate $\int_{-1}^{1} 5x^4\sqrt{x^5+1}dx$ **[2]**

OR

Find: $\int \frac{x^2+x}{x^3-x^2+x-1}dx$

2. Solve the differential equation: $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$ **[2]**

3. If $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$, show that $\vec{a} = \vec{0}$ or $\vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b}-\vec{c})$. **[2]**

4. If a line makes angles α , β and γ with the coordinate axes, prove that (cos 2α + cos 2β + cos 2 γ) = -1. **[2]**

5. One card is drawn at random from a well shuffled deck of 52 cards. **[2]**
In which of the following cases are the events E and F independent?
E: the card drawn is black
F: the card drawn is a king

6. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected. **[2]**

Section B

7. Evaluate the definite integral: $\int_1^2 e^{2x}\left(\frac{1}{x} - \frac{1}{2x^2}\right)dx$ **[3]**

8. Find the particular solution of the differential equation (1 - y^2)(1 + log|x|)dx + 2xy dy = 0 given that y = 0, when x = 1. **[3]**

OR

Find the general solution of the differential equation: $(x+y+1)\frac{dy}{dx} = 1$

9. If with reference to the right handed system of mutually $\perp$ unit vectors $\hat{i}, \hat{j}, \hat{k}$ and $\vec{\alpha} = 3\hat{i} - \hat{j},\ vec\beta = 2\hat{i} + \hat{j} - 3\hat{k}$then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is || to $\vec{\alpha}$ and $\vec{\beta}_2$ is $\perp$ to $\vec{\alpha}$ **[3]**

10. Find the shortest distance between the pairs of lines whose vector equations are: **[3]**

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} - 5\hat{j} + 2\hat{k})$ and $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$

OR

Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$. Also, find the image of P in this line.

Section C

11. Evaluate: $\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$. **[4]**

12. Find the area of the region enclosed by the parabola $y^2 = x$ and the line x + y = 2. **[4]**

OR

Using integration, find the area of the triangular region whose sides have the equations y = 2x + 1, y = 3x + 1 and x = 4.

13. Find the equation of the plane through the line of intersection of $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - \hat{j}) + 4 = 0$ and perpendicular to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$. Hence, find whether the plane thus obtained contains the line x - 1 = 2y - 4 = 3z - 12. **[4]**

CASE-BASED/DATA-BASED

14. In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class. **[4]**

Based on the above information, answer the following questions.

i. The probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics?

ii. The probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics?

Solution

MATHEMATICS 041

Class 12 - Mathematics

Section A

1. Put t = x^5 + 1, then dt = $5x^4$ dx.
   Therefore, $\int 5x^4\sqrt{x^5+1}dx$ = $\int \sqrt{t}dt = \frac{2}{3}t^{\frac{3}{2}} = \frac{2}{3}\left(x^5+1\right)^{\frac{3}{2}}$
   Hence, $\int_{-1}^{1} 5x^4\sqrt{x^5+1}dx$ = $\frac{2}{3}\left[\left(x^5+1\right)^{\frac{3}{2}}\right]_{-1}^{1}$
   = $\frac{2}{3}\left[\left(1^5+1\right)^{\frac{3}{2}} - \left((-1)^5+1\right)^{\frac{3}{2}}\right]$
   = $\frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}(2\sqrt{2}) = \frac{4\sqrt{2}}{3}$

   OR

   Let I = $\int \frac{x^2+x}{x^3-x^2+x-1}dx$
   Now let $\frac{x^2+x}{(x-1)(x^2+1)}$=$\frac{A}{x-1} + \frac{Bx+C}{x^2+1}$
   Getting A = 1, B = 0, C = 1
   Therefore, I = $\int \frac{1}{x-1}dx$+$\int \frac{1}{x^2+1}dx$
   = log$|x-1|$+$\tan^{-1}$ x + C
2. The given differential equation is $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$
   $\Rightarrow$ y - ay^2 = $\frac{dy}{dx}$ (a + x)
   $\Rightarrow$ (y - ay^2) dx = (a + x) dy
   $\Rightarrow$ $\frac{dx}{a+x} = \frac{dy}{y-ay^2}$ [separating the variables]
   $\Rightarrow$ $\int \frac{1}{a+x}dx = \int \frac{1}{y-ay^2}dy$ [Integrating both sides]
   $\Rightarrow$ $\int \frac{1}{a+x}dx = \int \left(\frac{1}{y} + \frac{a}{1-ay}\right)dy$ [By using partial fractions of RHS]
   $\Rightarrow$ log |x + a| = log |y| - log |1 - ay| + log C
   $\Rightarrow$ log$\left|\frac{(x+a)(1-ay)}{y}\right|$ = log C
   $\Rightarrow$ $\frac{(x+a)(1-ay)}{y}$ = C
   $\Rightarrow$ (x + a) (1 - ay) = Cy,) = Cy, which is the general solution of the given differential equation.
3. $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} \Rightarrow \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c}$ = 0 $\Rightarrow \vec{a}\cdot(\vec{b}-\vec{c})$ = 0
   $\Rightarrow \vec{a} = \vec{0}$ or $(\vec{b}-\vec{c}) = \vec{0}$ or $\vec{a} \perp (\vec{b}-\vec{c})$
   $\Rightarrow \vec{a} = \vec{0}$ or $\vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b}-\vec{c})$
   Th, $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ $\Rightarrow \vec{a} = \vec{0}$ or $\vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b}-\vec{c})$
4. Let the direction cosines of the given line be l,m,n. Then,we have
   l = cos α , m = cos β and n = cos γ
   $\therefore$ (I^2 + m^2 + n^2) = 1 $\Rightarrow$ $\cos^2\alpha$ + $\cos^2\beta$ + $\cos^2\gamma$ = 1
   $\Rightarrow$ $2\cos^2\alpha$ + $2\cos^2\beta$ + $2\cos^2\gamma$ = 2
   $\Rightarrow$ (1 + cos2α) + (1 + cos2β) + (1 + cos2γ) = 2
   $\Rightarrow$ (cos2α + cos2β + cos2γ) = -1
   Therefore, (cos 2α + cos 2β + cos 2γ) = -1
5. Given: A deck of 52 cards.
   In a deck of 52 cards, 26 cards are black and 4 cards are king and only 2 cards are black and King both.
   Hence, P(E) = The card drawn is black = $\frac{26}{52} = \frac{1}{2}$
   P(F) = The card drawn is a king = $\frac{4}{52} = \frac{1}{13}$

P(E ∩ F) = The card drawn is a black and king both = $\frac{2}{52} = \frac{1}{26}$ ......(i)

And P(E).P(F) = $\frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$ ......(ii)

From (i) and (ii)

P (E ∩ F) = P(E).P(F)

Hence, E and F are independent events.

6. Required probability is given by,

$\frac{{}^3C_2 \times {}^5C_2}{{}^8C_4} = \frac{3 \times 10}{70}$

$= \frac{3}{7}$

Section B

7. We have,

I = $\int_1^2 e^{2x}\left(\frac{1}{x} - \frac{1}{2x^2}\right) dx$

I = $\int_1^2 \frac{1}{x} \cdot e^{2x}$ - $\int_1^2 \frac{1}{2x^2} \cdot e^{2x} dx$

$\Rightarrow$ I = I_1 - I_2

Now, $I_1 = \int_1^2 \frac{1}{x} e^{2x}$ (By parts we have)

$\Rightarrow I_1 = [\frac{1}{x}]_1^2 \cdot \int_1^2 e^{2x} dx$ - $\int_1^2 -\frac{1}{x^2} \frac{e^{2x}}{2} dx$

$\Rightarrow I_1 = \left[\frac{1}{x} \cdot \frac{e^{2x}}{2}\right]_1^2 + \int_1^2 \frac{1}{2x^2} e^{2x} dx$

$\Rightarrow I_1 = \left[\frac{1}{2x} e^{2x}\right]_1^2 + I_2$

As, I = I_1 - I_2

$\Rightarrow I = \left[\frac{1}{2x} e^{2x}\right]_1^2 - I_2 + I_2$

$\Rightarrow I = \left[\frac{1}{2x} e^{2x}\right]_1^2 = \frac{1}{2}\left[\frac{1}{2} e^4 - e^2\right]$

$\Rightarrow I = \frac{1}{4} e^2 (e^2 - 1)$

8. We have,

$(1 - y^2)(1 + \log|x|)\, dx + 2xy\, dy = 0.$

On separating the variables, we get

$\frac{(1+\log|x|)}{x} dx + \frac{2y}{1-y^2} dy = 0$ [dividing both sides by $x(1 - y^2)$]

On integrating, we get

$\int\left(\frac{1}{x} + \frac{\log|x|}{x}\right) dx + \int \frac{2y}{1-y^2} dy = 0$

$\Rightarrow \log|x| + \frac{(\log|x|)^2}{2} - \log|1 - y^2| = \log C$ ...(i)

Also, given y = 0 and x = 1

$\therefore \log 1 + \frac{(\log 1)^2}{2} - \log|1 - 0| = \log C$

$\Rightarrow 0 + 0 - 0 = \log C \Rightarrow \log C = 0$

On putting log C = 0 in Eq. (i), we get

$\log|x| + \frac{(\log|x|)^2}{2} - \log|1 - y^2| = 0$

Which is the required solution of given differential equation.

OR

The given equation may be written as $\frac{dx}{dy} = \frac{x+y+1}{1} \Rightarrow \frac{dx}{dy} - x = (1 + y)$

This is of the form $\frac{dx}{dy} + Px = Q$, where P = -1, Q = 1 + y

$\text{IF} = e^{-\int dy} = e^{-y}$

Therefore, the required solution is given by, $x \times 1.F = \int Q \times (1.F) dy \Rightarrow xe^{-y} = \int e^{-y}(y + 1) dy$

$\Rightarrow xe^{-y} = \int e^{-y} dy + \int ye^{-y} dt \Rightarrow xe^{-y} = -e^{-y} + y\int e^{-y} dy - \int\left[\frac{dy}{dx}\int e^{-y} dy\right] dy$

$\Rightarrow xe^{-y} = -e^{-y} - ye^{-y} + \int 1.e^{-y} dy$

$\Rightarrow xe^{-y} = -e^{-y} - ye^{-y} - e^{-y} + c$

$\Rightarrow xe^{-y} = -2e^{-y} - ye^{-y} + c$

$\therefore x = ce^{y} - y - 2$

9. Let $\vec{\beta}_1 = \lambda\vec{\alpha}\ \left[\because \vec{\beta}_1 || to\ \vec{\alpha}\right]$

$\vec{\beta}_1 = \lambda\left(3\hat{i} - \hat{j}\right)$

$= 3\lambda\hat{i} - \lambda\hat{j}$

$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$= \left(2\hat{i} + \hat{j} - 3\hat{k}\right) - \left(3\lambda\hat{i} - \lambda\hat{j}\right)$

$= (2 - 3\lambda)\,\hat{i} + (1 + \lambda)\,\hat{j} - 3\hat{k}$

$\vec{\alpha}.\vec{\beta}_2 = 0\ \left[\because \vec{\beta}_2 \perp \vec{\alpha}\right]$

$3\,(2 - 3\lambda) - (1 + \lambda) = 0$

$\lambda = \frac{1}{2}$

$\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$

$\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

10. We know that $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} - 5\hat{j} + 2\hat{k})$ and $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$

Comparing the given equations with the equations

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$

We get,

$\overrightarrow{a_1} = 2\hat{i} - \hat{j} - \hat{k}$

$\overrightarrow{a_2} = \hat{i} + 2\hat{j} + \hat{k}$

$\overrightarrow{b_1} = 2\hat{i} - 5\hat{j} + 2\hat{k}$

$\overrightarrow{b_2} = \hat{i} - \hat{j} + \hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = -\hat{i} + 3\hat{j} + 2\hat{k}$

and $|\overrightarrow{b_1} \times \overrightarrow{b_2}|$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 2 \\ 1 & -1 & 1 \end{vmatrix}$

= $-3\hat{i} + 3\hat{k}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}|$ = $\sqrt{(-3)^2 + 3^2}$

= $\sqrt{9 + 9}$

= $3\sqrt{2}$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})$ = $(-\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (-3\hat{i} + 3\hat{k})$

= 3 + 6 = 9

Hence the shortest distance between the lines

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by

d = $\left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

= $\left|\frac{9}{3\sqrt{2}}\right|$

= $\frac{3}{\sqrt{2}}$

OR

According to question , the vector equation of the given line is

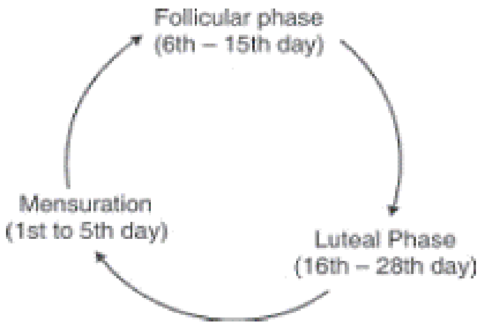
$\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$

Clearly, it passes through the point (-1, 3, 1) and it has direction ratios 2, 3, -1.

So, its Cartesian equations are

$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = r$ (say)

The general point on this line is (2r -1, 3r + 3, - r +1)
Suppose N be the foot of the perpendicular drawn from the point P(5, 4, 2) on the given line.
Then, this point is N(2r -1, 3r + 3, - r +1) for some fixed value of r. D.r.'s of PN are(2r - 6, 3r -1, - r -1).
Direction ratios of the given line are 2, 3, -1.
Since PN is perpendicular to the given line (i), we have
2(2r - 6) + 3(3r -1) -1 • (-r -1) = 0 $\Rightarrow$ 14r = 14 $\Rightarrow$ r = 1
So, the point N is given by N(l, 6, 0).
Hence, the foot of the perpendicular from the given point P(5,4,2) on the given line is N(1, 6,0).
Suppose Q (α, β, γ) be the image of P(5,4, 2) in the given line.
Then, N(1, 6, 0) is the midpoint of PQ
$\therefore \frac{5+\alpha}{2} = 1, \frac{4+\beta}{2} = 6$ and $\frac{2+y}{2} = 0 \Rightarrow \alpha = -3, \beta = 8$ and $\gamma = -2$



Section C

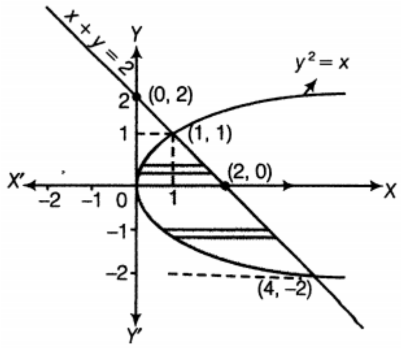
11. Let, $I = \int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$
Put, t = sin x
dt = cos x dx
$I = \int \frac{dt}{(1+t)(2+t)}$
Using partial fractions,
$\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$ .... (1)
A(2 + t) + B(1 + t) = 1
Now put t + 1 = 0
Therefore, t = -1
A(2 - 1) + B(0) = 1
A = 1
Now put t + 2 = 0
Therefore, t = -2
A(0) + B(-2 + 1) = 1
B = -1
Now From equation (1) we get
$\frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$
$\int \frac{1}{(1+t)(2+t)} dt = \int \frac{1}{1+t} dt - \int \frac{1}{2+t} dt$
$= \log|1+t| - \log|t+2| + c$
$= \log\left|\frac{t+1}{t+2}\right| + c$
So,
$I = \int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx = \log\left|\frac{\sin x+1}{\sin x+2}\right| + c$
12. According to the question ,
Given parabola is y^2 = x......(i)
vertex of parabola is (0, 0)
axis of parabola lies along X-axis.
Given equation of line is x + y = 2.......(ii)
For, x + y = 2

x	2	0

y	0	2

So, line passes through the points (2, 0) and (0, 2).

Now, let us sketch the graph of given curve and line as shown below:



On putting x = 2 - y from Eq. (ii) in Eq. (i), we get

$y^2 = 2 - y$

$\Rightarrow y^2 + y - 2 = 0$

$\Rightarrow y^2 + 2y - y - 2 = 0$

$\Rightarrow y(y+2) - 1(y+2) = 0$

$\Rightarrow (y-1)(y+2) = 0$

$\therefore$ y = 1 or -2

When y = 1, then x = 2 - y = 1

When y = -2, then x = 2 - y = 2 - (- 2) = 4

So, points of intersection are (1, 1) and (4, - 2).

Now, required area = $\int_{-2}^{1} \left[x_{(line)} - x_{\text{ (parabola)}}\right] dy$

$= \int_{-2}^{1} \left(2 - y - y^2\right) dy$

$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$

$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)$

$= 2 - \frac{5}{6} + 6 - \frac{8}{3}$

$= 8 - \frac{5}{6} - \frac{8}{3}$

$= \frac{48-5-16}{6}$

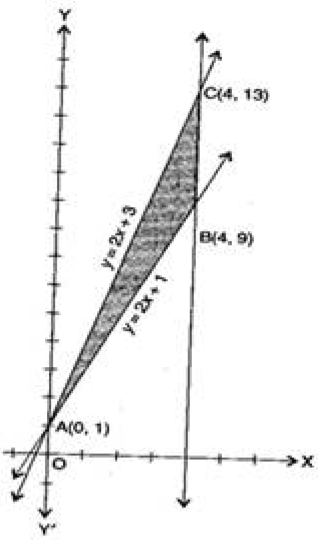
$= \frac{48-21}{6}$

$= \frac{27}{6}$

$= \frac{9}{2}$ sq units.

OR

Equations of one side of triangle is



y = 2x + 1 ...(i)

second line of triangle is y = 3x + 1 ...(ii)

third line of triangle is x = 4 ...(iii)
Solving eq. (i) and (ii), we get x = 0 and y = 1
∴ Point of intersection of lines (i) and (ii) is A (0, 1)
Putting x = 4 in eq. (i), we get y = 9
∴ Point of intersection of lines (i) and (iii) is B (4, 9)
Putting x = 4 in eq. (i), we get y = 13
∴ Point of intersection of lines (ii) and (iii) is C (4, 13)
∴ Area between line (ii) i.e., AC and x - axis

$$= \left|\int_0^4 y dx\right| = \left|\int_0^4 (3x+1)\, dx\right| = \left(\frac{3x^2}{2} + x\right)_0^4$$

= 24 + 4 = 28 sq. units ...(iv)
Again Area between line (i) i.e., AB and x - axis

$$= \left|\int_0^4 y dx\right| = \left|\int_0^4 (2x+1)\, dx\right| = \left(x^2 + x\right)_0^4$$

= 16 + 4 = 20 sq. units ...(v)
Therefore, Required area of $\triangle ABC$
= Area given by (iv) – Area given by (v)
= 28 – 20 = 8 sq. units

13. We know that any plane through the line of intersection of the two given plane is

$$[\vec{r}. \quad (2\hat{i} - 3\hat{j} + 4\hat{k}) - 1] + \lambda[\vec{r}.(i - \hat{j}) + 4] = 0$$

=$\vec{r}.[(2+\lambda)\hat{i} - (3+\lambda)\hat{j} + 4\hat{k}] = 1 - 4\lambda$ ....(i)
If this plane is perpendicular to the plane

$$\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$$

Then

$$2(2+\lambda) + (3+\lambda) + 4 = 0$$

$$3\lambda + 11 = 0 \Rightarrow \lambda = -\frac{11}{3}$$

Put $\lambda = -\frac{11}{3}$ in Equation.(i) we get the required equation of the plane is

$$\vec{r} \cdot (-5\hat{i} + 2\hat{j} + 12\hat{k}) = 47$$

and given that equation of line is
x - 1= 2y - 4= 3z -12
=$\frac{x-1}{1} = \frac{y-2}{1/2} = \frac{z-4}{1/3}$
In vector form, equation of line is

$$\vec{r} = \hat{i} + 2\hat{j} + 4\hat{k} + \lambda\left(\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{3}\hat{k}\right)$$

This line $\vec{r} = \hat{i} + 2\hat{j} + 4\hat{k} + \lambda\left(\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{3}\hat{k}\right)$
passes though a point with position vector
$\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$ and parallel to the vector
$\vec{b} = \hat{i} + \frac{1}{2}\hat{j} + \frac{1}{3}\hat{k}$
The plane $\vec{r} \cdot (-5\hat{i} + 2\hat{j} + 12\hat{k}) = 47$ contains the given line if

i. it passes through $\hat{i} + 2\hat{j} + 4\hat{k}$
ii. it is parallel to the line

We have, $(\hat{i} + 2\hat{j} + 4\hat{k}).(-5\hat{i} + 2\hat{j} + 12\hat{k})$
= - 5 + 4 + 48 = 47
So, the plane passes through the point $\hat{i} + 2\hat{j} + 4\hat{k}$ and $\left(\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{3}\hat{k}\right).(-5\hat{i} + 2\hat{j} + 12\hat{k})$.
= - 5 + 1 + 4 = 0
Therefore, the plane is parallel to the line.
Hence, the plane contains the given line.

CASE-BASED/DATA-BASED

14. Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore \quad P(E) = \frac{50}{100} = \frac{1}{2}, P(M) = \frac{35}{100} = \frac{7}{20}$ and $P(E \cap M) = \frac{25}{100} = \frac{1}{4}$

i. Required probability = P(E|M)

$$= \frac{P(E \cap M)}{P(M)} = \frac{\frac{1}{4}}{\frac{7}{20}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$$

ii. Required probability = P(M|E)

$$= \frac{P(M \cap E)}{P(E)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$