5. Congruence of Triangles and Inequalities in a Triangle

Exercise 5A

1. Question

In a ΔABC, if AB=AC and ∠A=70°, find ∠B and ∠C.

Answer

Given that

AB = AC and ∠A = 70°

To find: ∠B and ∠C

AB = AC and also ∠A = 70°

As two sides of triangle are equal, we say that ΔABC is isosceles triangle.

Hence by the property of isosceles triangle, we know that base angles are also equal.

ie. we state that ∠B = ∠C. …(1)

Now,

Sum of all angles in any triangle = 180°

∴ ∠A + ∠B + ∠C = 180°

Hence,

70° + ∠B + ∠C = 180°

2 ∠B = 180° - 70° …from (1)

∴ 2∠B= 110°

∠B = 55°

Therefore, our base angles, ∠B and ∠C, are 55° each.

2. Question

The vertical angle of an isosceles triangle is 100°. Find its base angles.

Answer

Given: The given triangle is isosceles triangle. Also vertex angle is 100°

To find: Measure of base angles.

It is given that triangle is isosceles.

So let our given triangle be ΔABC.

And let ∠A be the vertex angle, which is given as ∠A= 100°

By the property of isosceles triangle, we know that base angles are equal.

So,

∠B = ∠C ...(1)

We know that,

Sum of all angles in any triangle = 180°

∴ ∠A + ∠B + ∠C = 180°

100° + 2∠B = 180° ...from (1)

∴ 2∠B = 180° - 100°

2∠B = 80°

∴∠B = 40°

Therefore, our base angles, ∠B and ∠C, are 40° each.

3. Question

In a ΔABC, if AB=AC and ∠B=65°, find ∠C and ∠A.

Answer

Given: In ΔABC,

AB=AC and ∠B=65°

To find : ∠A and ∠C

It is given that AB=AC and ∠B=65°

As two sides of the triangle are equal, we say that triangle is isosceles triangle, with vertex angle A.

Hence by the property of isosceles triangle we know that base angles are equal.

∴ ∠B = ∠C

∴∠C = ∠B =65°

Also, We know that,

Sum of all angles in any triangle = 180°

∴∠A + ∠B + ∠C = 180°

∠A + 65° + 65° = 180°

∠A + 130° = 180°

∴∠A = 180° - 130°

∠A = 50°

Hence, ∠C = 65° and ∠A = 50°

4. Question

In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Answer

Given: Our given triangle is isosceles triangle. Also, the vertex angle is twice the sum of the base angles

To find: Measures of angles of triangle.

It is given that that given triangle is isosceles triangle.

Let vertex angle be y and base angles be x each.

So by given condition,

$y = 2(x + x)$

$\therefore y = 4x$

Also, We know that,

Sum of all angles in any triangle = 180°

$\therefore y + x + x = 180°$

$y + 2x = 180°$

$4x + 2x = 180°$

$\therefore 6x = 180°$

$x = 30°$

$\therefore y = 4 \times 30°$

$y = 120°$

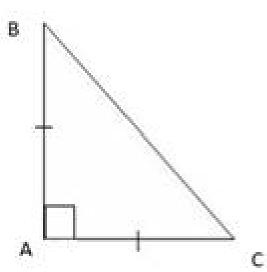
Hence, vertex angle is 120° and base angles are 30° each.

5. Question

What is the measure of each of the equal angles of a right-angled isosceles triangle?

Answer

Here given triangle is isosceles right angled triangle.



So let our triangle be ΔABC, right angled at A.

$\therefore \angle A = 90°$

Here, AB = AC, as our given triangle is isosceles triangle.

Hence, base angles, $\angle B$ and $\angle C$ are equal.

Also, We know that,

Sum of all angles in any triangle = 180°

$\therefore \angle A + \angle B + \angle C = 180°$

$90° + 2 \angle B = 180°$

$2\angle B = 90°$

$\angle B = 45°$

Hence the measure of each of the equal angles of a right-angled isosceles triangle is 45°

6. Question

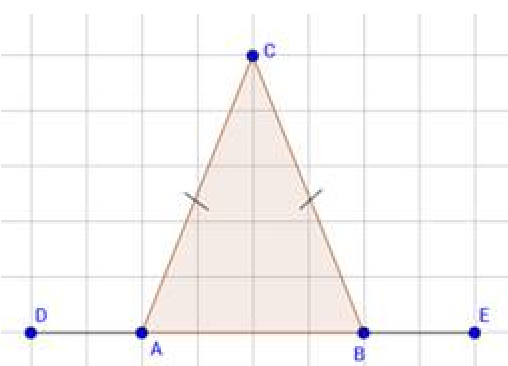
If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Answer

Given: ΔABC is isosceles triangle.

To prove: $\angle CAD = \angle CBE$

Let ΔABC be our isosceles triangle as shown in the figure.



We know that base angles of the isosceles triangle are equal.

Here, $\angle CAB = \angle CBA$ ....(1)

Also here, $\angle CAD$ and $\angle CBE$ are exterior angles of the triangle.

So, we know that,

$\angle CAB + \angle CAD = 180°$... exterior angle theorem

And $\angle CBA + \angle CBE = 180°$ ... exterior angle theorem

So from (1) and above statement, we conclude that,

∠CAB +∠CAD = 180°

And ∠CAB +∠CBE = 180°

Which implies that,

∠CAD = 180° - ∠CAB

And ∠CBE = 180° - ∠CAB

Hence we say that ∠CAD = ∠CBE

∴For the isosceles triangle, the exterior angles so formed are equal to each other.

7. Question

Find the measure of each exterior angle of an equilateral triangle.

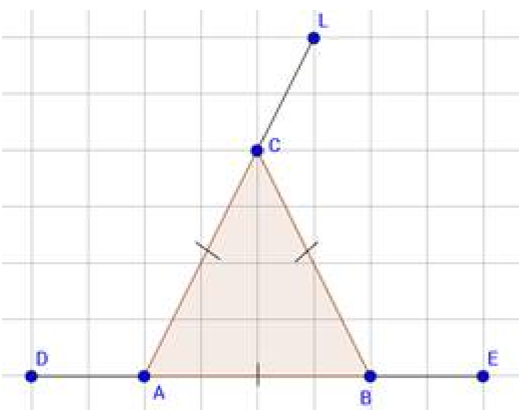
Answer

Given: ΔABC is equilateral triangle.

To prove: ∠CAD = ∠CBE = ∠BCL

Proof:

Let our triangle be ΔABC, which is equilateral triangle as shown in the figure.



Hence all angles are equal and measure 60° each.

∴ ∠CAB = ∠CBA = ∠BCA = 60° ...(1)

Also here, ∠CAD and ∠CBE are exterior angles of the triangle.

So, we know that,

∠CAB +∠CAD = 180° ... exterior angle theorem

∠CBA + ∠CBE = 180° ... exterior angle theorem

∠BCA + ∠BCL = 180° ... exterior angle theorem

From (1) and above statements, we state that,

60° +∠CAD = 180°

60° + ∠CBE = 180°

60° + ∠BCL = 180°

Simplifying above statements,

∠CAD = 180° - 60° = 120°

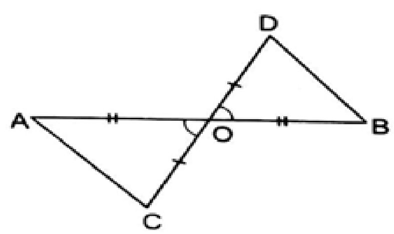
∠CBE = 180° - 60° = 120°

∠BCL = 180° - 60° = 120°

Hence, the measure of each exterior angle of an equilateral triangle is 120°

8. Question

In the given figure, O is the midpoint of each of the line segments AB and CD. Prove that AC=BD and AC||BD.



Answer

Given: AO = OB , DO = OC

To prove: AC=BD and AC||BD

Proof:

It is given that, O is the midpoint of each of the line segments AB and CD.

This implies that AO = OB and DO = OC

Here line segments AB and CD are concurrent.

So,

∠AOC = ∠BOD …. As they are vertically opposite angles.

Now in ΔAOC and ΔBOD,

AO = OB,

OC = OD

Also, ∠AOC = ∠BOD

Hence, ΔAOC ≅ ΔBOD … by SAS property of congruency

So,

AC = BD … by cpct

∴ ∠ACO = ∠BDO … by cpct

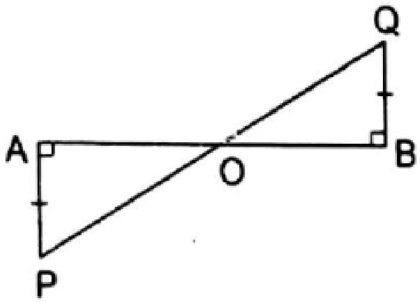
But ∠ACO and ∠BDO are alternate angles.

∴ We conclude that AC is parallel to BD.

Hence we proved that AC=BD and AC||BD

9. Question

In the adjoining figure, PA ⊥ AB, QB ⊥ AB and PA=QB. If PQ intersects AB at O, show that O is the midpoint of AB as well as that of PQ.



Answer

Given: PA ⊥ AB, QB ⊥ AB and PA=QB

To prove: AO = OB and PO = OQ

It is given that PA ⊥ AB and QB ⊥ AB.

This means that ΔPAO and ΔQBO are right angled triangles.

It is also given that PA=QB

Now in ΔPAO and ΔQBO,

∠OAP = ∠OBQ = 90°

PO = OQ

Hence by hypotenuse-leg congruency,

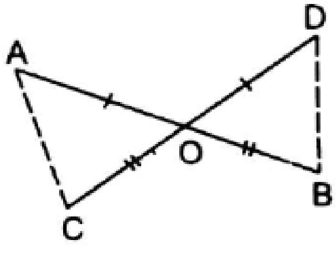
ΔPAO ≅ ΔQBO

∴AO = OB and PO = OQ ….by cpct

Hence proved that AO = OB and PO = OQ

10. Question

Let the line segments AB and CD intersect at O in such a way that OA=OD and OB=OC. Prove that AC=BD but AC may not be parallel to BD.



Answer

Given: AO = OD and CO = OB

To prove: AC = BD

Proof :

It is given that AO = OD and CO = OB

Here line segments AB and CD are concurrent.

So,

∠AOC = ∠BOD …. As they are vertically opposite angles.

Now in ΔAOC and ΔDOB,

AO = OD,

CO = OD

Also, ∠AOC = ∠BOD

Hence, ΔAOC ≅ ΔBOD … by SAS property of congruency

So,

AC = BD … by cpct

Here,

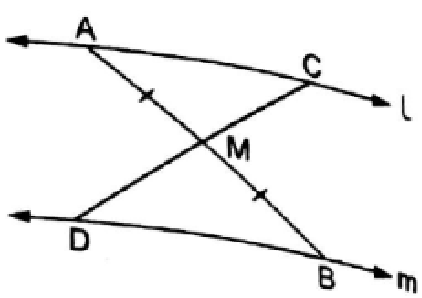
∠ACO ≠ ∠BDO or ∠OAC ≠ ∠OBD

Hence there are no alternate angles, unless both triangles are isosceles triangle.

Hence proved that AC=BD but AC may not be parallel to BD.

11. Question

In the given figure, l∥m and M is the midpoint of AB. Prove that M is also the midpoint of any line segment CD having its end points at l and m respectively.



Answer

Here it is given that l ∥ m ie. AC ||DB.

Also given that AM = MB

Now in ΔAMC and ΔBMD,

∠CAM = ∠DBM … Alternate angles

AM = MB

∠AMC = ∠BMD … vertically opposite angles

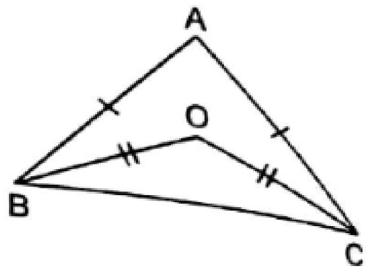
Hence, ΔAMC ≅ ΔBMD … by ASA property of congruency

∴ CM = MD …cpct

Hence proved that M is also the midpoint of any line segment CD having its end points at l and m respectively.

12. Question

In the given figure, AB=AC and OB=OC. Prove that ∠ABO=∠ACO. Give that AB=AC and OB=OC.



Answer

ΔABC and ΔOBC are isosceles triangle.

∴ ∠ABC = ∠ACB and ∠OBC = ∠OCB ….(1)

Also,

∠ABC = ∠ABO + ∠OBC

And ∠ACB = ∠ACO + ∠OCB

From 1 and above equations, we state that,

∠ABC = ∠ABO + ∠OBC

And ∠ABC = ∠ACO + ∠OBC

This implies that,

∠ABO = ∠ABC - ∠OBC

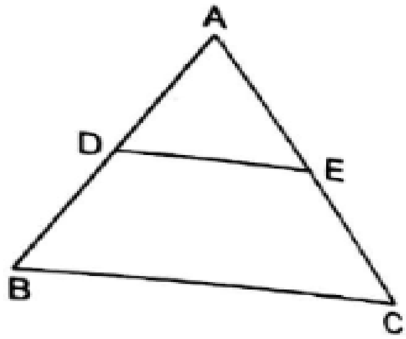
And ∠ACO = ∠ABC - ∠OBC

Hence,

∠ABO = ∠ACO = ∠ABC - ∠OBC

13. Question

In the given figure, ABC is a triangle in which AB=AC and D is a point on AB. Through D, a line DE is drawn parallel to BC and meeting AC at E. Prove that AD=AE.



Answer

Given that AB = AC and also DE || BC.

So by Basic proportionality theorem or Thales theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \frac{DB}{AD} = \frac{EC}{AE}$

Now adding 1 on both sides,

$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

$\frac{DB+AD}{AD} = \frac{EC+AE}{AE}$

$\frac{AB}{AD} = \frac{AC}{AE}$ ... as AB = AD + DE and AC = AE + EC

But is given that AB = AC,

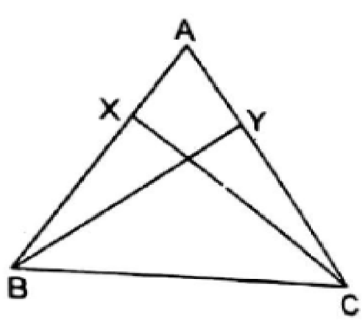
$\therefore \frac{AB}{AD} = \frac{AB}{AE}$

Hence,

AD = AE.

14. Question

In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of ΔABC such that AX=AY. Prove that CX=BY.



Answer

Here it is given that AX = AY.

Now in ΔCXA and ΔBYA,

AX = AY

∠XAC = ∠YAB … Same angle or common angle.

AC = AB … given condition Hence by SAS property of congruency,

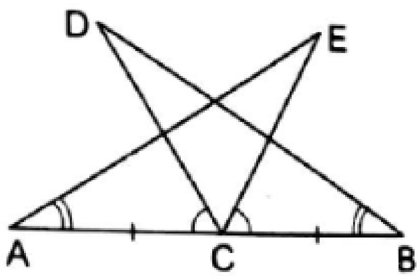
ΔCXA ≅ ΔBYA

Hence by cpct, we conclude that,

CX = BY

15. Question

In the given figure, C is the midpoint of AB. If ∠DCA=∠ECB and ∠DBC=∠EAC, prove that DC=EC.



Answer

It is given that AC = BC , ∠DCA = ∠ECB and ∠DBC = ∠EAC.

Adding angle ∠ECD both sides in ∠DCA = ∠ECB, we get,

∠DCA + ∠ECD = ∠ECB + ∠ECD

∴∠ECA = ∠DCB …addition property

Now in ΔDBC and ΔEAC,

∠ECA = ∠DCB

BC = AC

∠DBC = ∠EAC

Hence by ASA postulate, we conclude,

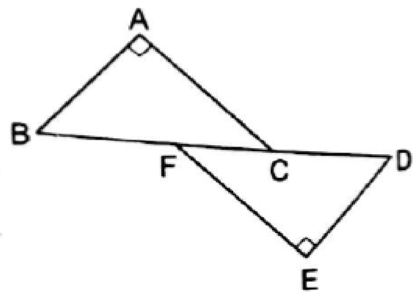
ΔDBC ≅ ΔEAC

Hence, by cpct, we get,

DC = EC

16. Question

In the given figure, BA ⊥ AC and DE ⊥ EF such that BA=DE and BF=DC. Prove that AC=EF.



Answer

Given : BA ⊥ AC and DE ⊥ EF such that BA=DE and BF=DC

To prove: AC = EF

Proof:

In ΔABC, we have,

BC = BF + FC

And , in ΔDEF,

FD = FC + CD

But, BF = CD

So, BC = BF + FC

And, FD = FC +BF

∴ BC = FD

So, in ΔABC and ΔDEF, we have,

∠BAC = ∠DEF … given

BC = FD

AB = DE …given

Thus by Right angle - Hypotenuse- Side property of congruence, we have,

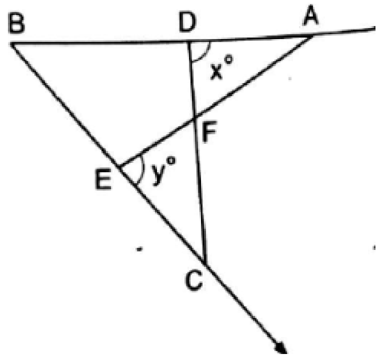
ΔABC ≅ ΔDEF

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ AC = EF

17. Question

In the given figure, if x=y and AB=CB, then prove that AE=CD.



Answer

Given: x=y and AB=CB

To prove: AE = CD

Proof:

In ΔABE, we have,

∠AEC = ∠EBA + ∠BAE …Exterior angle theorem

y° = ∠EBA + ∠BAE

Now in ΔBCD, we have,

x° = ∠CBA + ∠BCD

Since, given that,

x = y ,

∠CBA + ∠BCD = ∠EBA + ∠BAE

∴ ∠BCD = ∠BAE … as ∠CBA and ∠EBA and same angles.

Hence in ΔBCD and ΔBAE,

∠B = ∠B

BC = AB …given

∠BCD = ∠BAE

Thus by ASA property of congruence, we have,

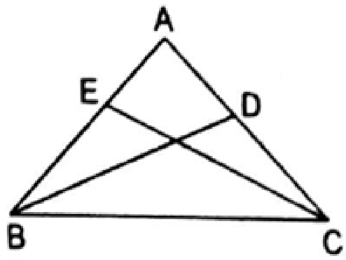
ΔBCD ≅ ΔBAE

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ CD = AE

18. Question

ABC is a triangle in which AB=AC. If the bisectors of ∠B and ∠C meet AC and AB in D and E respectively, prove that BD=CE.



Answer

Given: AB=AC and BD and AB are angle bisectors of ∠B and ∠C

To prove: BD = CE

Proof:

In ΔABD and ΔACE,

∠ABD = $\frac{1}{2}$ ∠B

And ∠ACE = $\frac{1}{2}$ ∠C

But ∠B = ∠C as AB = AC … As in isosceles triangle, base angles are equal

∠ABD = ∠ACE

AB = AC

∠A = ∠A

Thus by ASA property of congruence,

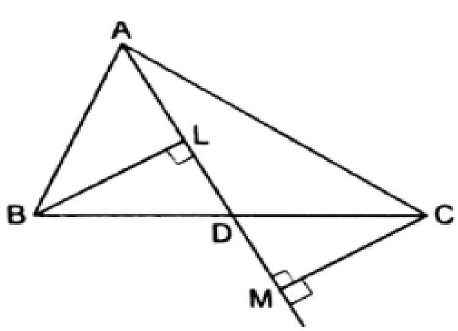
ΔABD ≅ ΔACE

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ BD = CE

19. Question

In the adjoining figure, AD is a median of ΔABC. If BL and CM are drawn perpendiculars on AD and AD produced, prove that BL=CM



Answer

Given: BC = DC and BL ⊥ AD and DM ⊥ CM

To prove: BL=CM

Proof:

In ΔBLD and ΔCMD,

∠BLD = ∠CMD = 90° … given

∠BLD = ∠MDC … vertically opposite angles

BD = DC … given

Thus by AAS property of congruence,

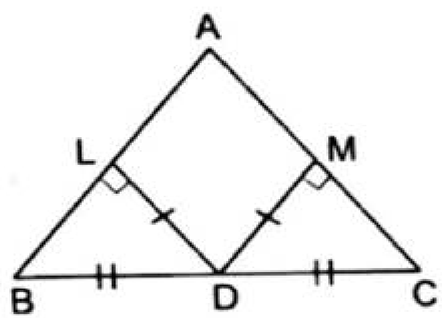
ΔBLD ≅ ΔCMD

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ BL = CM

20. Question

In ΔABC, D is the midpoint of BC. If DL ⊥ AB and DM ⊥ AC such that DL=DM, prove that AB=AC.



Answer

Given: BD = DC and DL ⊥ AB and DM ⊥ AC such that DL=DM

To prove: AB = AC

Proof:

In right angled triangles ΔBLD and ΔCMD,

∠BLD = ∠CMD = 90°

BD = CD … given

DL = DM … given

Thus by right angled hypotenuse side property of congruence,

ΔBLD ≅ ΔCMD

Hence, we know that, corresponding parts of the congruent triangles are equal

∠ABD = ∠ACD

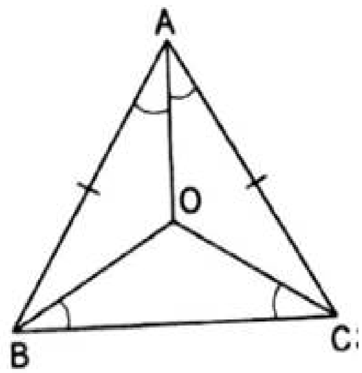
In ΔABC, we have,

∠ABD = ∠ACD

∴ AB= AC …. Sides opposite to equal angles are equal

21. Question

In ΔABC, AB=AC and the bisectors of ∠B and ∠C meet at a point O. prove that BO=CO and the ray AO is the bisector of ∠A.



Answer

Given: In ΔABC, AB=AC and the bisectors of ∠B and ∠C meet at a point O.

To prove: BO=CO and ∠BAO = ∠CAO

Proof:

In , ΔABC we have,

∠OBC = $\frac{1}{2}$ ∠B

∠OCB = $\frac{1}{2}$ ∠C

But ∠B = ∠C … given

So, ∠OBC = ∠OCB

Since the base angles are equal, sides are equal

∴ OC = OB …(1)

Since OB and OC are bisectors of angles ∠B and ∠C respectively, we have

∠ABO = $\frac{1}{2}$ ∠B

∠ACO = $\frac{1}{2}$ ∠C

∴∠ABO = ∠ACO …(2)

Now in ΔABO and ΔACO

AB = AC … given

∠ABO = ∠ACO … from 2

BO = OC … from 1

Thus by SAS property of congruence,

ΔABO ≅ ΔACO

Hence, we know that, corresponding parts of the congruent triangles are equal

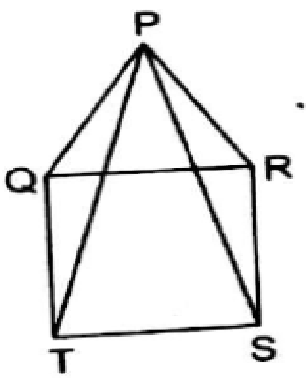
∠BAO = ∠CAO

ie. AO bisects ∠A

22. Question

In the given figure, PQR is an equilateral triangle and QRST is a square. Prove that

(i) PT=PS, (ii) ∠PSR=15°.



Answer

Given: PQR is an equilateral triangle and QRST is a square

To prove: PT=PS and ∠PSR=15°.

Proof:

Since ΔPQR is equilateral triangle,

∠PQR = ∠PRQ = 60°

Since QRTS is a square,

∠RQT = ∠QRS = 90°

In ΔPQT,

∠PQT = ∠PQR + ∠RQT

= 60° + 90°

= 150°

In ΔPRS,

∠PRS = ∠PRQ + ∠QRS

= 60° + 90°

= 150°

∴ ∠PQT = ∠PRS

Thus in ΔPQT and ΔPRS,

PQ = PR … sides of equilateral triangle

∠PQT = ∠PRS

QT = RS … side of square

Thus by SAS property of congruence,

ΔPQT ≅ ΔPRS

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ PT = PS

Now in ΔPRS, we have,

PR = RS

∴ ∠PRS = ∠PSR

But ∠PRS = 150°

SO, by angle sum property,

∠PRS + ∠PSR + ∠SPR = 180°

150° + ∠PSR + ∠SPR = 180°

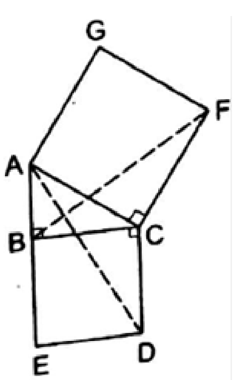
2∠PSR = 180° - 150°

2∠PSR = 30°

∠PSR = 15°

23. Question

In the given figure, ABC is a triangle, right angled at B. If BCDE is a square on side BC and ACFG is a square on AC, prove that AD=BF.



Answer

Given: ∠ABC = 90° , BCDE is a square on side BC and ACFG is a square on AC

To prove: AD = EF

Proof:

Since BCDE is square,

∠BCD = 90° …(1)

In ΔACD,

∠ACD = ∠ACB + ∠BCD

= ∠ACB + 90° …(2)

In ΔBCF,

∠BCF = ∠BCA + ∠ACF

Since ACFG is square,

∠ACF = 90° …(3)

From 2 and 3, we have,

∠ACD = ∠BCF ….(4)

Thus in ΔACD and ΔBCF, we have,

AC = CF ...sides of square

∠ACD = ∠BCF …from 4

CD = BC … sides of square

Thus by SAS property of congruence,

ΔACD ≅ ΔBCF

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ AD = BF

24. Question

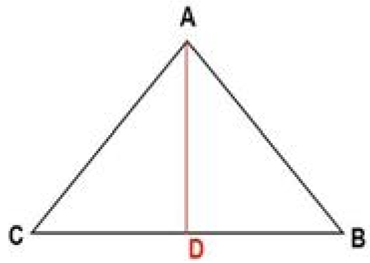
Prove that median from the vertex of an isosceles triangle is the bisector of the vertical angle.

Answer

Given: ΔABC is isosceles triangle where AB = AC and BD = DC

To prove: ∠BAD = ∠DAC

Proof:



In ΔABD and ΔADC

AB = AC …given

BD = DC …given

AD = AD … common side

Thus by SSS property of congruence,

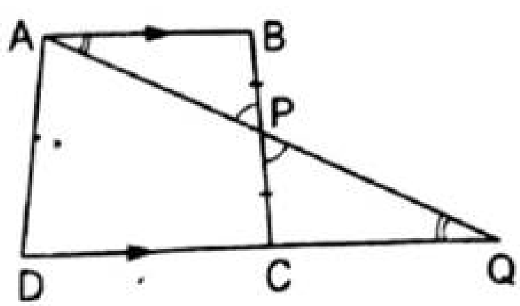
ΔABD ≅ ΔADC

Hence, we know that, corresponding parts of the congruent triangles are equal

∠BAD = ∠DAC

25. Question

In the given figure, ABCD is a quadrilateral in which AB∥DC and P is the midpoint of BC. On producing, AP and DC meet at Q. prove that (i) AB=CQ, (ii) DQ=DC+AB.



Answer

Given: ABCD is a quadrilateral in which AB∥DC and BP = PC

To prove: AB=CQ and DQ=DC+AB

Proof:

In ΔABP and ΔPCQ we have,

∠PAB = ∠PQC …alternate angles

∠APB = ∠CPQ … vertically opposite angles

BP = PC … given

Thus by AAS property of congruence,

ΔABP ≅ ΔPCQ

Hence, we know that, corresponding parts of the congruent triangles are equal

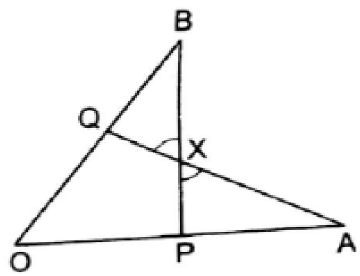
∴ AB = CQ …(1)

But, DQ = DC + CQ

= DC + AB …from 1

26. Question

In the given figure, OA=OB and OP=OQ. Prove that (i) PX=QX, (ii) AX=BX.



Answer

Given: OA=OB and OP=OQ

To prove: PX=QX and AX=BX

Proof:

In ΔOAQ and ΔOPB, we have

OA = OB …given

∠O = ∠O …common angle

OQ = OP … given

Thus by SAS property of congruence,

ΔOAP ≅ ΔOPB

Hence, we know that, corresponding parts of the congruent triangles are equal

∠OBP = ∠OAQ …(1)

Thus, in ΔBXQ and ΔPXA, we have,

BQ = OB – OQ

And PA = OA − OP

But OP = OQ

And OA = OB …given

Hence, we have, BQ = PA …(2)

Now consider ΔBXQ and ΔPXA,

∠BXQ = ∠PXA … vertically opposite angles

∠OBP = ∠OAQ …from 1

BQ = PA … from 2

Thus by AAS property of congruence,

ΔBXQ ≅ ΔPXA

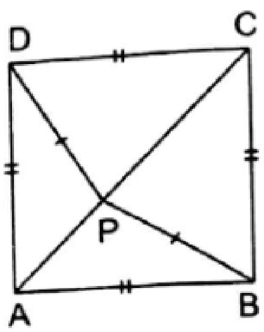
Hence, we know that, corresponding parts of the congruent triangles are equal

∴ PX = QX

And AX = BX

27. Question

In the given figure, ABCD is a square and P is a point inside it such that PB=PD. Prove that CPA is a straight line.



Answer

Given: ABCD is a square and PB=PD

To prove: CPA is a straight line

Proof:

ΔAPD and ΔAPB,

DA = AB …as ABCD is square

AP = AP … common side

PB = PD … given

Thus by SSS property of congruence,

ΔAPD ≅ ΔAPB

Hence, we know that, corresponding parts of the congruent triangles are equal

∠APD = ∠APB …(1)

Now consider ΔCPD and ΔCPB,

CD = CB … ABCD is square

CP = CP … common side

PB = PD … given

Thus by SSS property of congruence,

$\Delta CPD \cong \Delta CPB$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle CPD = \angle CPB$ … (2)

Now,

Adding both sides of 1 and 2,

$\angle CPD + \angle APD = \angle APB + \angle CPB$ …(3)

Angels around the point P add upto 360°

$\therefore \angle CPD + \angle APD + \angle APB + \angle CPB = 360°$

From 4,

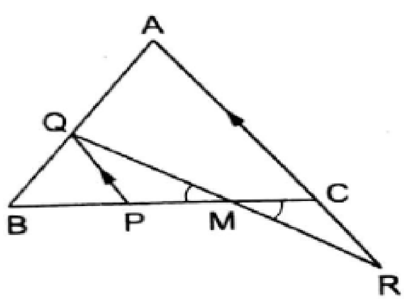
$2(\angle CPD + \angle APD) = 360°$

$\angle CPD + \angle APD = \frac{360°}{2} = 180°$

This proves that CPA is a straight line.

28. Question

In the given figure, ABC is an equilateral triangle, PQ ∥AC and AC is produced to R such that CR=BP. Prove that QR bisects PC.



Answer

Given: ABC is an equilateral triangle, PQ ∥AC and CR=BP

To prove: QR bisects PC or PM = MC

Proof:

Since, ΔABC is equilateral triangle,

$\angle A = \angle ACB = 60°$

Since, PQ ∥AC and corresponding angles are equal,

$\angle BPQ = \angle ACB = 60°$

In ΔBPQ,

∠B= ∠ACB = 60°

∠BPQ = 60°

Hence, ΔBPQ is an equilateral triangle.

∴ PQ = BP = BQ

Since we have BP = CR,

We say that PQ = CR …(1)

Consider the triangles ΔPMQ and ΔCMR,

∠PQM = ∠CRM …alternate angles

∠PMQ = ∠CMR … vertically opposite angles

PQ = CR … from 1

Thus by AAS property of congruence,

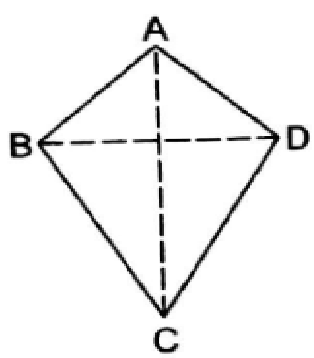
ΔPMQ ≅ ΔCMR

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ PM = MC

29. Question

In the given figure, ABCD is a quadrilateral in which AB=AD and BC=DC. Prove that (i) AC bisects ∠A and ∠C, (ii) AC is the perpendicular bisector of BD.



Answer

Given: ABCD is a quadrilateral in which AB=AD and BC=DC

To prove: AC bisects ∠A and ∠C, and AC is the perpendicular bisector of BD

Proof:

In ΔABC and ΔADC, we have

AB = AD …given

BC = DC … given

AC = AC … common side

Thus by SSS property of congruence,

ΔABC ≅ ΔADC

Hence, we know that, corresponding parts of the congruent triangles are equal

∠BAC = ∠DAC

∴ ∠BAO = ∠DAO …(1)

It means that AC bisects ∠BAD ie ∠A

Also, ∠BCA = ∠DCA … cpct

It means that AC bisects ∠BCD, ie ∠C

Now in ΔABO and ΔADO

AB = AD …given

∠BAO = ∠DAO … from 1

AO = AO … common side

Thus by SAS property of congruence,

ΔABO ≅ ΔADO

Hence, we know that, corresponding parts of the congruent triangles are equal

∠BOA = ∠DAO

But ∠BOA + ∠DAO = 180°

2∠BOA = 180°

∴ ∠BOA = $\frac{180^\circ}{2}$ = 90°

Also ΔABO ≅ ΔADO

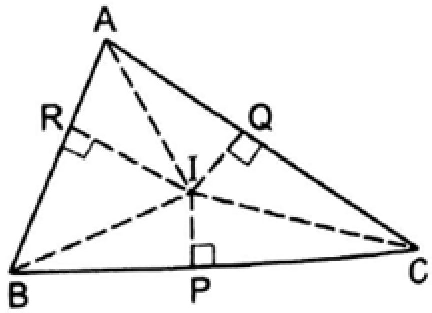
So, BO = OD

Which means that AC = BD

30. Question

In the given figure, the bisectors of ∠B and ∠C of ΔABC meet at I If IP ⊥BC, IQ ⊥CA and IR ⊥ AB, prove that

(i) IP=IQ=IR, (ii) IA bisects ∠A.



Answer

Given: IP ⊥BC, IQ ⊥CA and IR ⊥ AB and the bisectors of ∠B and ∠C of ΔABC meet at I

To prove: IP=IQ=IR and IA bisects ∠A

Proof:

In ΔBIP and ΔBIR we have,

∠PBI = ∠RBI …given

∠IRB = ∠IPB = 90° …Given

IB = IB …common side

Thus by AAS property of congruence,

ΔBIP ≅ ΔBIR

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ IP = IR

Similarly,

IP = IQ

Hence, IP = IQ = IR

Now in ΔAIR and ΔAIQ

IR = IQ …proved above

IA = IA … Common side

∠IRA = ∠IQA = 90°

Thus by SAS property of congruence,

ΔAIR ≅ ΔAIQ

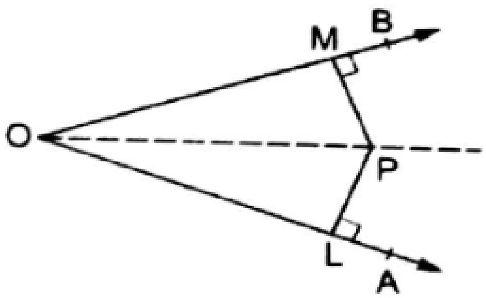
Hence, we know that, corresponding parts of the congruent triangles are equal

∴ ∠IAR = ∠IAQ

This means that IA bisects ∠A

31. Question

In the adjoining figure, P is a point in the interior of ∠AOB. If PL ⊥ OA and PM ⊥ OB such that PL=PM, show that OP is the bisector of ∠AOB



Answer

Given: P is a point in the interior of ∠AOB and PL ⊥ OA and PM ⊥ OB such that PL=PM

To prove: ∠POL = ∠POM

Proof:

In ΔOPL and ΔOPM, we have

∠OPM = ∠OPL = 90° …given

OP = OP …common side

PL = PM … given

Thus by Right angle hypotenuse side property of congruence,

ΔOPL ≅ ΔOPM

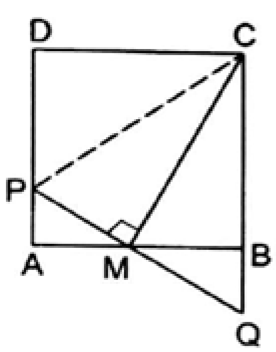
Hence, we know that, corresponding parts of the congruent triangles are equal

∴ ∠POL = ∠POM

Ie. OP is the bisector of ∠AOB

32. Question

In the given figure, ABCD is a square, M is the midpoint of AB and PQ ⊥ CM meets AD at P and CB produced at Q. prove that (i) PA=BQ, (ii) CP=AB+PA.



Answer

Given: ABCD is a square, AM = MB and PQ ⊥ CM

To prove: PA=BQ and CP=AB+PA

Proof:

In ΔAMP and ΔBMQ, we have

∠AMP = BMQ …vertically opposite angle

∠PAM = ∠MBQ = 90° …as ABCD is square

AM = MB …given

Thus by AAS property of congruence,

ΔAMP ≅ ΔBMQ

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ PA = BQ and MP = MQ …(1)

Now in ΔPCM and ΔQCM

PM = QM … from 1

∠PMC = ∠QMC … given

CM = CM … common side

Thus by AAS property of congruence,

ΔPCM ≅ ΔQCM

Hence, we know that, corresponding parts of the congruent triangles are equal

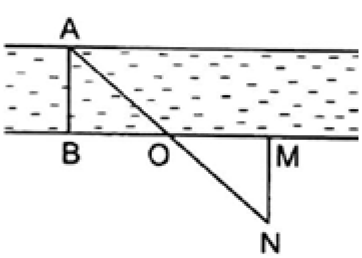
∴ PC = QC

PC = QB + CB

PC = AB + PA …as AB = CB and PA = QB

33. Question

In the adjoining figure, explain how one can find the breadth of the river without crossing it.



Answer

Given: AB ⊥ BO and NM ⊥ OM

In ΔABO and ΔNMO,

∠OBA = ∠OMN

OB = OM …O is mid point of BM

∠BOA = ∠MON …vertically opposite angles

Thus by AAS property of congruence,

ΔABO ≅ ΔNMO

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ AB = MN

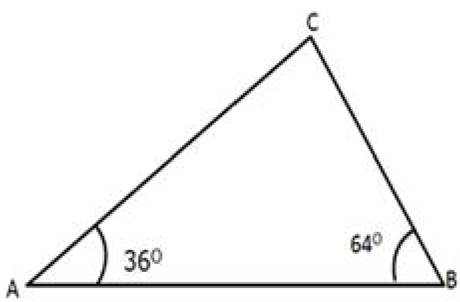
Hence, we can calculate the width of the river by calculating MN

34. Question

In ΔABC, if ∠A=36° and ∠B=64°, name the longest and shortest sides of the triangle.

Answer

Given: ∠A=36° and ∠B=64°



To find: The longest and shortest sides of the triangle

Given that ∠A=36° and ∠B=64°

Hence, by the angle sum property in ΔABC, we have

∠A + ∠B + ∠C = 180°

36° + 64°+ ∠C = 180°

100° + ∠C = 180°

∠C = 80°

So, we have ∠A=36°, ∠B=64° and ∠C = 80°

∴∠C is largest and ∠A is shortest

Hence,

Side opposite to ∠C is longest.

∴AB is longest

Side opposite to ∠A is shortest.

∴ BC is shortest

35. Question

In ΔABC, if ∠A=90°, which is the longest side?

Answer

It is given that ∠A=90°.

In right angled triangle at 90°

Sum of all angles in triangle is 180°, so other two angles must be less that 90°

So, other angles are smaller than ∠A.

Hence ∠A is largest angle.

We know that side opposite to largest angle is largest.

∴BC is longest side, which is opposite to ∠A.

36. Question

In ΔABC, if ∠A=∠B=45°, name the longest side.

Answer

In ΔABC given that ∠A=∠B=45°

So, by the angle sum property in ΔABC, we have

∠A + ∠B + ∠C = 180°

45° + 45° + ∠C = 180°

90° + ∠C = 180°

∴ ∠C = 180° - 90°

∠C = 90°

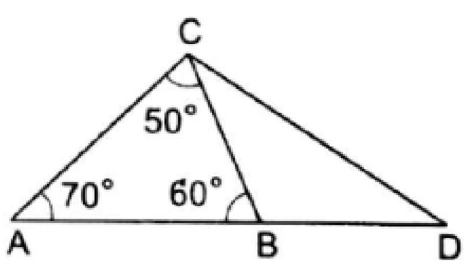
Hence, largest angle is ∠C

We know that side opposite to largest angle is longest, which is AB

Hence our longest side is AB

37. Question

In ΔABC, side AB is produced to D such that BD=BC. If ∠B=60° and ∠A=70°, prove that (i) AD>CD and (ii) AD>AC.



Answer

Given: In ΔABC, BD=BC and ∠B=60° and ∠A=70°

To prove: AD>CD and AD>AC

Proof:

In ΔABC, by the angle sum property, we have

∠A + ∠B + ∠C = 180°

70° + 60° + ∠C = 180°

130° + ∠C = 180°

∠C = 50°

Now in ΔBCD we have,

∠CBD = ∠DAC + ∠ACB … as ∠CBD is the exterior angle of ∠ABC

= 70° + 50°

Since BC = BD …given

So, ∠BCD = ∠BDC

∴ ∠BCD + ∠BDC = 180° - ∠CBD

= 180° - 120° = 60°

2∠BCD = 60°

∠BCD = ∠BDC = 30°

Now in ΔACD we have

∠A = 70°, ∠D = 30°

And ∠ACD = ∠ACB + ∠BCD

= 50° + 30° = 80°

∴∠ACD is greatest angle

So, the side opposite to largest angle is longest, ie AD is longest side.

∴AD > CD

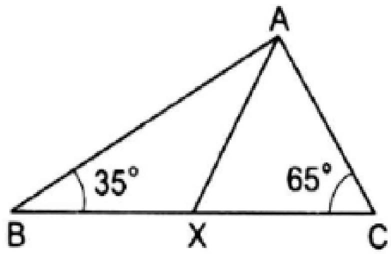
Since, ∠BDC is smallest angle,

The side opposite to ∠BDC, ie AC, is the shortest side in ΔACD.

∴AD > AC

38. Question

In ΔABC, ∠B=35°,∠C=65° and the bisector of ∠BAC meets BC in X. Arrange AX, BX and CX in descending order.



Answer

Given: In ΔABC, ∠B=35°,∠C=65° and ∠BAX = ∠XAC

To find: Relation between AX, BX and CX in descending order.

In ΔABC, by the angle sum property, we have

∠A + ∠B + ∠C = 180°

∠A + 35° + 65° = 180°

∠A + 100° = 180°

∴ ∠A = 80°

But ∠BAX = $\frac{1}{2}$ ∠A

= $\frac{1}{2}$ × 80° = 40°

Now in ΔABX,

∠B = 35°

∠BAX = 40

And ∠BXA = 180° - 35° - 40°

= 105°

So, in ΔABX,

∠B is smallest, so the side opposite is smallest, ie AX is smallest side.

∴ AX < BX ...(1)

Now consider ΔAXC,

∠CAX = $\frac{1}{2}$ × ∠A

=$\frac{1}{2}$ × 80° = 40°

∠AXC = 180° - 40° - 65°

= 180° - 105° = 75°

Hence, in ΔAXC we have,

∠CAX = 40°, ∠C = 65°, ∠AXC =75°

∴∠CAX is smallest in ΔAXC

So the side opposite to ∠CAX is shortest

Ie CX is shortest

∴ CX <AX …. (2)

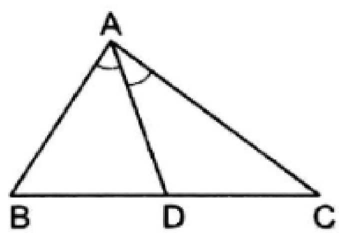
From 1 and 2 ,

BX > AX > CX

This is required descending order

39. Question

In ΔABC, if AD is the bisector of ∠A, show that AB>BD and AC>DC



Answer

Given: ∠BAD = ∠DAC

To prove: AB>BD and AC>DC

Proof:

In ΔACD,

∠ADB = ∠DAC + ∠ACD … exterior angle theorem

= ∠BAD + ∠ACD … given that ∠BAD = ∠DAC

∠ADB > ∠BAD

The side opposite to angle ∠ADB is the longest side in ΔADB

So, AB > BD

Similarly in ΔABD

∠ADC = ∠ABD + ∠BAD … exterior angle theorem

= ∠ABD + ∠CAD … given that ∠BAD = ∠DAC

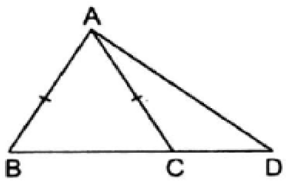
∠ADC > ∠CAD

The side opposite to angle ∠ADC is the longest side in ΔACD

So, AC > DC

40. Question

In the given figure, ABC is a triangle in which AB=AC. If D be a point on BC produced, prove that AD>AC.



Answer

Given: AB=AC

To prove: AD>AC

Proof:

In ΔABC,

∠ACD = ∠B + ∠BAC

= ∠ACB + ∠BAC …as ∠C = ∠B as AB = AC

= ∠CAD + ∠CDA +∠BAC …as ∠ACB = ∠CAD + ∠CDA

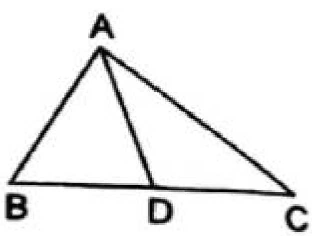
∴∠ACD > ∠CDA

So the side opposite to ∠ACD is the longest

∴AD > AC

41. Question

In the adjoining figure, AC>AB and AD is the bisector of ∠A. show that ∠ADC>∠ADB.



Answer

Given: AC>AB and ∠BAD = ∠DAC

To prove: ∠ADC>∠ADB

Proof:

Since AC > AB

∠ABC > ∠ACB

Adding $\frac{1}{2}$ ∠A on both sides

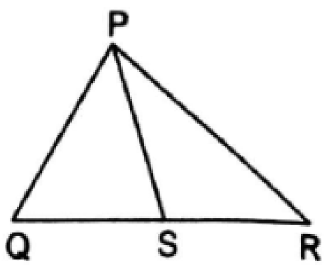
$\angle ABC + \frac{1}{2}\angle A > \angle ACB + \frac{1}{2}\angle A$

∠ABC + ∠BAD > ∠ACB + ∠DAC … As AD is a bisector of ∠A

∴ ∠ADC > ∠ADB

42. Question

In ΔPQR, if S is any point on the side QR, show that PQ+QR+RP>2PS.



Answer

Given: S is any point on the side QR

To prove: PQ+QR+RP>2PS.

Proof:

Since in a triangle, sum of any two sides is always greater than the third side.

So in ΔPQS, we have,

PQ + QS > PS …(1)

Similarly, ΔPSR, we have,

PR + SR > PS …(2)

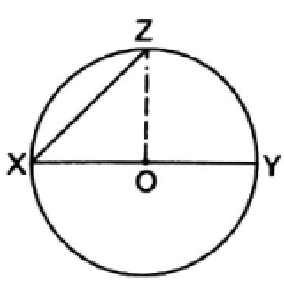
Adding 1 and 2

PQ + QS + PR + SR > 2PS

PQ + PR + QR > 2PS …as PR = QS +SR

43. Question

In the given figure, O is the center of the circle and XOY is a diameter. If XZ is any other chord of the circle, show that XY>XZ.



Answer

Given: XOY is a diameter and XZ is any chord of the circle.

To prove: XY>XZ

Proof:

In ΔXOZ,

OX + OZ > XZ … sum of any sides in a triangle is a greater than its third side

∴ OX + OY > XZ … As OZ = OY, radius of circle

Hence, XY > XZ …As OX + OY = XY

44. Question

If O is a point within ΔABC, show that:

(i) AB+AC>OB+OC

(ii) AB+BC+CA>OA+OB+OC

(iii) OA+OB+OC> $\frac{1}{2}$ (AB+BC+CA)

Answer

Given: O is a point within ΔABC

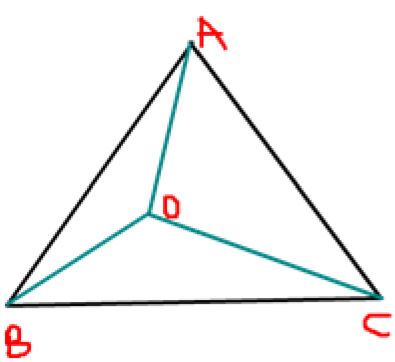
To prove:

(i) AB+AC>OB+OC

(ii) AB+BC+CA>OA+OB+OC

(iii) OA+OB+OC> $\frac{1}{2}$ (AB+BC+CA)

Proof:



In ΔABC,

AB +AC >BC ….(1)

And in ΔOBC,

OB + OC > BC …(2)

Subtracting 1 from 2 we get,

(AB + AC) − (OB + OC) > (BC − BC)

Ie AB + AC > OB + OC

From I, AB + AC > OB + OC

Similarly, AB + BC > OA + OC

And AC + BC > OA + OB

Adding both sides of these three inequalities, we get,

(AB + AC) + (AB + BC) + (AC + BC) > (OB + OC) + (OA + OC) + (OA + OB)

Ie. 2(AB + BC + AC) > 2(OA + OB + OC)

∴ AB + BC + OA > OA + OB + OC

In ΔOAB,

OA + OB > AB ...(1)

In ΔOBC,

OB + OC > BC ...(2)

In ΔOCA

OC + OA > CA ...(3)

Adding 1,2 and 3,

(OA + OB) + (OB + OC) + (OC+ OA) >AB + BC +CA

Ie. 2(OA + OB + OC) > AB + BC + CA ∴ OA + OB + OC > $\frac{1}{2}$ (AB + BC + CA)

45. Question

Can we draw a triangle ABC with AB=3cm, BC=3.5cm and CA=6.5cm? Why?

Answer

Our given lengths are AB=3cm, BC=3.5cm and CA=6.5cm.

∴ AB + BC = 3 + 3.5 = 6.5 cm

But CA = 6.5 cm

So, AB + BC = CA

A triangle can be drawn only when the sum of two sides is greater than the third side

So, a triangle cannot be drawn with such lengths

CCE Questions

1. Question

Which of the following is not a criterion for congruence of triangles?

A. SSA

B. SAS

C. ASA

D. SSS

Answer

From the above given four options, SSA is not a criterion for the congruence of triangles

∴ Option (A) is correct

2. Question

If AB=QR, BC=RP and CA=PQ, then which of the following holds?

A. ΔABC ≅ ΔPQR

B. ΔCBA ≅ ΔPQR

B. ΔCAB ≅ ΔPQR

D. ΔBCA ≅ ΔPQR

Answer

It is given in the question that,

AB = QR

BC = RP

And, CA = PQ

∴ By SSS congruence criterion

$\Delta CBA \cong \Delta PQR$

Hence, option (B) is correct

3. Question

If ΔABC≅ΔPQR AND ΔABC is not congruent to ΔRPQ, then which of the following is not true?

A. BC=PQ

B. AC=PR

C. BC=QR

D. AB=PQ

Answer

According to the condition given in the question, If $\Delta ABC \cong \Delta PQR$ and ΔABC is not congruent to ΔRPQ

Then, clearly BC $\neq$ PQ

∴ It is false

Hence, option (A) is correct

4. Question

It is given that ΔABC≅ΔFDE in which AB=5cm, ∠B=40°, ∠A=80° and FD=5cm. Then, which of the following is true?

A. ∠D=60°

B. ∠E=60°

C. ∠F=60°

D. ∠D=80°

Answer

It is given in the question that,

$\Delta ABC \cong \Delta FDE$ where,

AB = 5 cm

FD = 5 cm

∠ B = 40°

∠ A = 80°

We know that sum of all angles of a triangle is equal to 180^0

∴ ∠ A + ∠ B + ∠ C = 180^o

$80^o + 40^o$ + ∠ C = 180^o

∠ C = $180^o - 120^0$

= 60^o

As, Angle C = Angle E

∴ Angle E = 60^o

Hence, option (B) is correct

5. Question

In ΔABC, AB=2.5cm and BC=6cm. Then, the length of AC cannot be

A. 3.4

B. 4 cm

C. 3.8 cm

D. 3.6 cm

Answer

It is given in the question that,

In ΔABC

AB = 2.5 cm

BC = 6 cm

We know that, the length of a side must be less than the sum of the other two sides

Let us assume the side of AC be x cm

$\therefore x < 2.5 + 6$

$x < 8.5$

Also, we know that the length of a side must be greater then the difference between the other two sides

$\therefore x > 6 - 2.5$

$x > 3.5$

Hence, the limits of the value of x is

$3.5 < x < 8.5$

∴ It is clear the length of AC cannot be 3.4 cm

Hence, option (A) is correct

6. Question

In ΔABC, ∠A=40° and ∠B=60°, Then, the longest side of ΔABC is

A. BC

B. AC

C. AB

D. cannot be determined

Answer

It is given in the question that,

In ΔABC, $\angle A = 40^{0}$

$\angle B = 60^{0}$

We know that, sum of all angles of a triangle is equal to 180^{0}

∴ ∠A + ∠B + ∠C = 180^{o}

60^{o} + 40^{o} + ∠C = 180^{o}

∠C = 180^{o} – 100^{o}

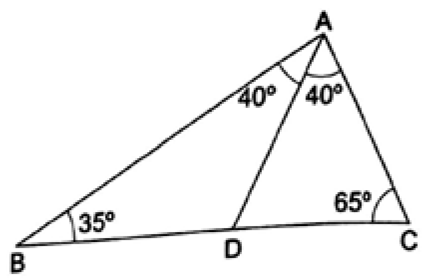
∠C = 80^{o}

Hence, the side which is opposite to ∠C is the longest side of the triangle

∴ Option (C) is correct

7. Question

In ΔABC, ∠B=35°, ∠C=65° and the bisector AD of ∠BAC meets BC at D. Then, which of the following is true?



A. AD>BD>CD

B. BD>AD>CD

C. AD>CD>BD

D. None of these

Answer

It is given in the question that,

In ΔABC, we have

∠B = 35^{o}

∠C = 65^{o}

Also the bisector AD of ∠BAC meets at D

∴ ∠A + ∠B + ∠C = 180^{o}

∠A + 35^{o} + 65^{o} = 180^{o}

∠A = 180^{o} – 100^{o}

∠A = 80^{o}

As, AD is the bisector of ∠BAC

∴ ∠BAD = ∠CAD = 40^{o}

In ΔABD, we have

∠BAD > ∠ABD

BD > AD

Also, in ΔACD

∠ACD > ∠CAD

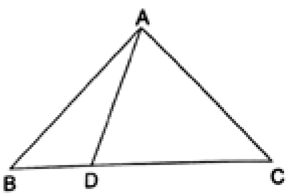
AD > CD

Hence, BD > AD > CD

∴ Option (B) is correct

8. Question

In the given figure, AB>AC. Then, which of the following is true?



A. AB<AD

B. AB=AD

C. AB>AD

D. cannot be determined

Answer

From the given figure, we have

AB > AC

∴ ∠ACB > ∠ABC

Also, ∠ADB > ACD

∠ADB > ACB > ∠ABC

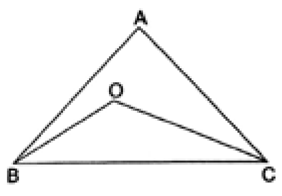
∠ADB > ∠ABD

∴ AB > AD

Hence, option (C) is correct

9. Question

In the given figure, AB>AC. If BO and CO are the bisectors of ∠B and ∠C respectively, then



A. OB=OC

B. OB>OC

C. OB<OC

Answer

From the given figure, we have

AB > AC

Also, ∠C > ∠B

$\frac{1}{2}\angle C > \frac{1}{2}\angle B$

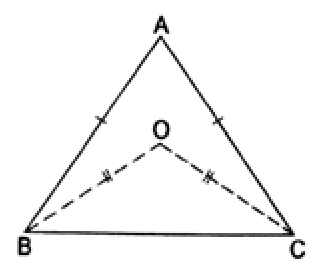
∠OCB > ∠OBC (Given)

∴ OB > OC

Hence, option (C) is correct

10. Question

In the given figure, AB=AC and OB=OC. Then, ∠ABO: ∠ACO=?



A. 1:1

B. 2:1

C. 1:2

D. None of these

Answer

It is given in the question that,

In ΔOAB and ΔOAC, we have

AB = AC

OB = OC

OA = OA (Common)

∴ By SSS congruence criterion

$\Delta OAB \cong \Delta OAC$

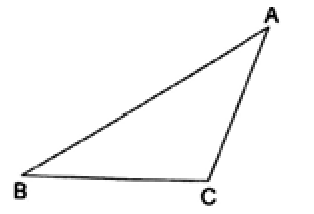
∴ ∠ABO = ∠ACO

So, ∠ABO: ∠ACO = 1: 1

Hence, option (A) is correct

11. Question

In ΔABC, IF ∠C>∠B, then



A. BC>AC

B. AB>AC

C. AB<AC

D. BC<A

Answer

It is given in the question that,

In ΔABC, we have

∠C > ∠B

We know that, side opposite to the greater angle is larger

∴ AB >AC

Hence, option (B) is correct

12. Question

O is any point in the interior of ΔABC. Then, which of the following is true?

A. (OA+OB+OC)> (AB+BC+CA)

B. (OA+OB+OC) $>\frac{1}{2}$ (AB+BC+CA)

C. (OA+OB+OC) $<\frac{1}{2}$ (AB+BC+CA)

D. None of these

Answer

From the given question, we have

In ΔOAB, ΔOBC and ΔOCA we have:

OA + OB > AB

OB + OC > BC

And, OC + OA > AC

Adding all these, we get:

2 (OA + OB + OC) > (AB + BC + CA)

(OA + OB + OC > $\frac{1}{2}(AB + BC + CA)$

∴ Option (C) is correct

13. Question

If the altitudes from two vertices of a triangle to the opposite sides are equal, then the triangle is

A. Equilateral

B. isosceles

C. Scalene

D. right-angled

Answer

It is given in the question that,

In ΔABC, BL is parallel to AC

Also, CM is parallel AB such that BL = CM

We have to prove that: AB = AC

Now, in ΔABL and ΔACM we have:

BL = CM (Given)

∠BAL = ∠CAM (Common)

∠ALB = ∠AMC (Each angle equal to 90^o)

∴ By AAS congruence criterion

ΔABL ≅ ΔACM

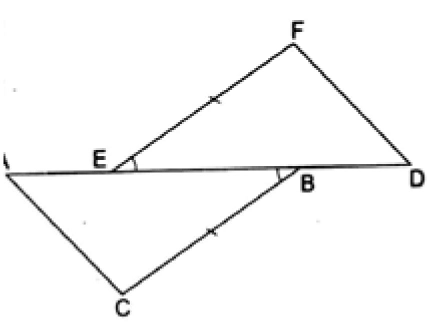
AB = AC (By Congruent parts of congruent triangles)

As opposite sides of the triangle are equal, so it is an isosceles triangle

Hence, option (B) is correct

14. Question

In the given figure, AE=DB, CB=EF And ∠ABC=∠FED. Then, which of the following is true?



A. ΔABC ≅ ΔDEF

B. ΔABC ≅ ΔEFD

C. ΔABC ≅ ΔFED

D. ΔABC ≅ ΔEDF

Answer

From the given figure, we have

AE = DB

And, CB = EF

Now, AB = (AD − DB)

= (AD − AE)

DE = (AD − AE)

Now, in ΔABC and ΔDEF we have:

AB = DE

CB = EF

∠ABC = ∠FED

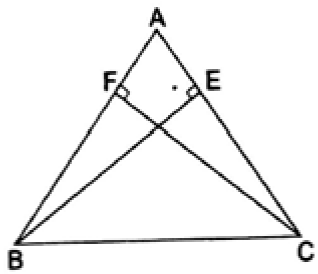
∴ By SAS congruence criterion

ΔABC ≅ ΔDEF

Hence, option (A) is correct

15. Question

In the given figure, BE ⊥ CA and CF ⊥ BA such that BE=CF. Then, which of the following is true?



A. ΔABE ≅ ΔACF

B. ΔABE ≅ ΔAFC

C. ΔABE ≅ ΔCAF

D. ΔABE ≅ ΔFAC

Answer

From the given figure, we have

BE is perpendicular to CA

Also, CF is perpendicular to BA

And, BE = CF

Now, in ΔABE and ΔACF we have:

BE = CF (Given)

∠BEA = ∠CFA = 90^{0}

∠A = ∠A (Common)

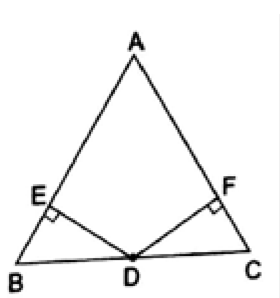
∴ By AAS congruence criterion

ΔABE ≅ ΔACF

Hence, option (A) is correct

16. Question

In the given figure, D is the midpoint of BC, DE ⊥ AB and DF ⊥ AC such that DE=DF. Then, which of the following is true?



A. AB=AC

B. AC=BC

C. AB=BC

D. None of these

Answer

From the given figure, we have

D is the mid-point of BC

Also, DE is perpendicular to AB

DF is perpendicular to AC

And, DE = DF

Now, in ΔBED and ΔCFD we have:

DE = DF

BD = CD

∠E = ∠F = 90^{o}

∴ By RHS congruence rule

ΔBED ≅ ΔCFD

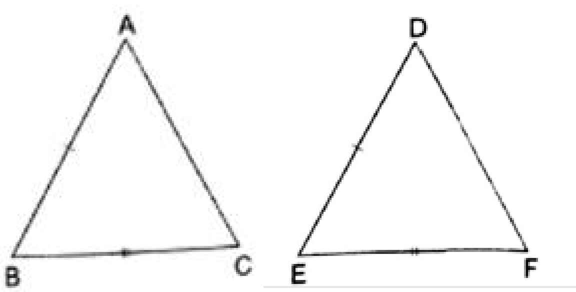
Thus, ∠B = ∠C

AC = AB

Hence, option (A) is correct

17. Question

In ΔABC and ΔDEF, it is given that AB=DE and BC=EF. In order that ΔABC≅ΔDEF, we must have



A. ∠A=∠D

B. ∠B=∠E

C. ∠C=∠F

D. none of these

Answer

From the question, we have:

In ΔABC and ΔDEF

AB = DE (Given)

BC = EF (Given)

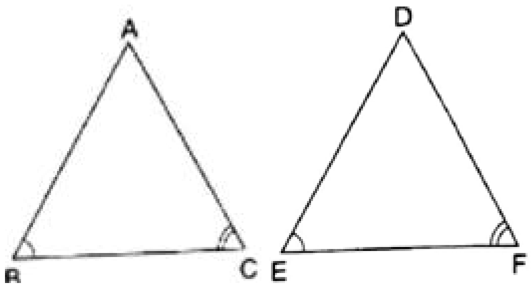
So, in order to have ΔABC ≅ ΔDEF

∠B must be equal to ∠E

∴ Option (B) is correct

18. Question

In ΔABC and ΔDEF, it is given that ∠B=∠E and ∠C=∠F. In order that ΔABC ≅ DEF, we must have



A. AB=DF

B. AC=DE

C. BC=EF

D. ∠A=∠ D

Answer

From the question, we have:

In ΔABC and ΔDEF

∠B = ∠E (Given)

∠C = ∠F (Given)

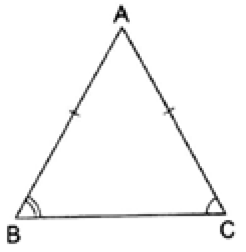
So, in order to have ΔABC ≅ ΔDEF

BE must be equal to EF

∴ Option (C) is correct

19. Question

In ΔABC and ΔPQR, it is given that AB=AC, ∠C=∠P and ∠P=∠Q. Then, the two triangles are



A. Isosceles but not congruent

B. Isosceles and congruent

C. Congruent but not isosceles

D. Neither congruent not isosceles

Answer

It is given in the question that,

In ΔABC and ΔPQR, we have

AB = AC

Also, ∠C = ∠B

As, ∠C = ∠P and, ∠B = ∠Q

∴ ∠P = ∠Q

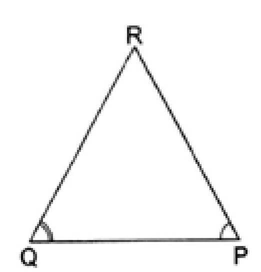
So, both triangles are isosceles but not congruent

Hence, option (A) is correct

20. Question

Which is true?

A. A triangle can have two right angles.



B. A triangle can have two obtuse angles.

C. A triangle can have two acute angles.

D. An exterior angle of a triangle is less than either of the interior opposite angles.

Answer

We know that,

Sum of all angles of a triangle is equal to 180^o

∴ A triangle can have two acute angles because sum of two acute angles of a triangle is always less than 180^o

Thus, it satisfies the angle sum property of a triangle

Hence, option (C) is correct

21. Question

Three statements are given below:

(I) In a ΔABC in which AB=AC, the altitude AD bisects BC.

(II) If the altitudes AD, BE and CF of ΔABC are equal, then ΔABC is equilateral.

(III) If D is the midpoint of the hypotenuse AC of a right ΔABC, then BD=AC.

Which is true?

A. I only

B. II only

C. I and II

D. II and III

Answer

Here we can clearly see that the true statements are as follows:

(I) In a ΔABC in which AB=AC, the altitude AD bisects BC.

(II) If the altitudes AD, BE and CF of ΔABC are equal, then ΔABC is equilateral.

∴ Option C is correct

22. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	**Reason (R)**
If AD is a median of ΔABC, then AB+AC>2AD.	The angles opposite to equal sides of a triangle are equal.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

According to the question,

In ΔABD and ΔACD,

Since, sum of any two sides of a triangle is greater than the third side.

AB + DB > AD (i)

AC + DC > AD (ii)

Adding (i) and (ii)

AB + AC + DB + DC > 2AD

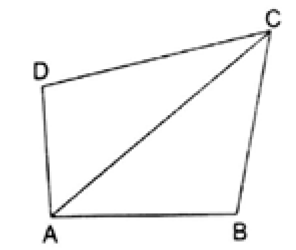
AB + AC + BC > 2AD

Hence, the assertion and the reason are both true, but Reason does not explain the assertion.

∴ Option B is correct

23. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
In a quadrilateral ABCD, we have (AB+BC+CD+DA)>2AC. 	The sum of any two sides of a triangle is greater than the third side.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Since, sum of two sides is greater than the third side

∴AB + BC > AC (i)

CD + DA > AC (ii)

Adding (i) and (ii),

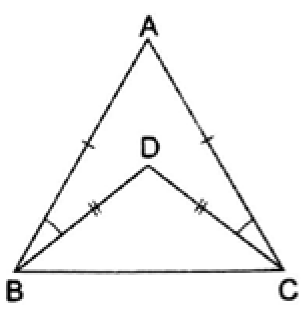
AB + BC +CD +DA > 2AC

Hence, the assertion is true and also the reason gives the right explanation of the assertion.

∴ Option A is correct

24. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
ΔABC and ΔDBC are two isosceles triangles on the same base BC. Then, ∠ABD=∠ACD. 	The angles opposite to equal sides of a triangle are equal.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Since, angles opposite to equal sides are equal

AB = AC

∠ABC = ∠ACB (i)

DB = DC

∠DBC = ∠DCB (ii)

Subtracting (ii) from (i),

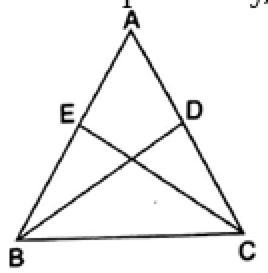
∠ABC - ∠DBC = ∠ACB - ∠DCB

Hence, the assertion is true and also the reason gives the right explanation of the assertion.

∴ Option A is correct

25. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
It is always possible to draw a triangle whose sides measure 4 cm, 5cmand 10cm respectively.	In an isosceles ΔABC with AB=AC, if BD and CE are bisectors of ∠B and ∠C respectively, then BD=CE. 

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

In ΔBDC and ΔCEB,

∠DCB = ∠EBC (Given)

BC = CB (Common)

∠B = ∠C (AC = AB)

$\frac{1}{2}\angle B = \frac{1}{2}\angle C$

∠CEB = ∠BCE

∴ ΔBDC ≅ ΔCEB

BD = CE (By c.p.c.t.)

And, we know that the sum of two sides is always greater than the third side in any triangle.

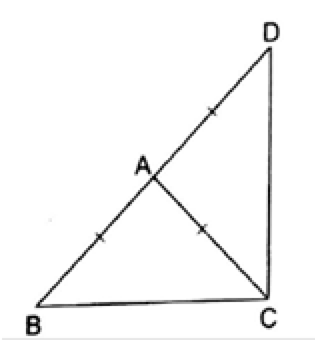
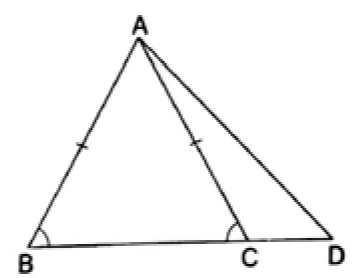
But, (5 + 4) < 10

Hence, the reason is true, but the assertion is false.

∴ Option D is true

26. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	**Reason (R)**
In the given figure, ΔABC is given with AB=AC and BA is produced to D, such that AB=AD. Then, ∠BCD=90°.  	In the given figure AB=AC and D is a point on BC produced. Then, AB>AD. 

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

According to the question,

AB = AC

∠ACB = ∠ABC (i)

Now, ∠ACD >∠ACB =∠ABC (Side BC is produced to D)

And, In ΔADC, side DC is produced to B

∠ACB>∠ADC (ii)

∠ABC>∠ADC

Now, using (i) and (ii),

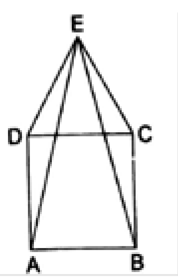
AD >AB

Hence, the reason is wrong but the assertion is true.

∴ Option C is correct

27. Question

Match the following columns.

Column I	Column II
(a)In ΔABC, if AB=AC and ∠A=50°, then ∠C=................	(p) its perimeter
(b) The vertical angle of an isosceles triangle is 130°. Then, each base angle is.......	(q) 15°
(c) The sum of three altitudes of a ΔABC is less than............	(r) 65°
(d) In the given figure, ABCD is a square and ΔEDC is an equilateral triangle. Then, ∠EBC is............. 	(s) 25°

The correct answer is:

(A)-........., (B)-........., (C)-........., (D)-.........,

Answer

The parts of the question are solved below:

a. Given: In △ABC, AB = AC and ∠A=50°

Thus, ∠B = ∠C

Now, ∠A + ∠B + ∠C = 180° (The angle sum property of triangle)

50 + 2∠B = 180°

2∠B = 130°

∠C = ∠B = 65°

b. As per the question,

Let the vertical angle be A and ∠ B = ∠ C

Now, ∠A + ∠B + ∠C = 180° (The angle sum property of triangle)

130 + 2∠B = 180°

2∠B = 50°

∠C = ∠B = 25°

c. We know that, the sum of three altitudes of a triangle ABC is less than its perimeter.

d. Here, ABCD is a square and EDC is a equilateral triangle.

∴ ED = CD = AB = BC = AD = EC

In ΔECB,

EC = BC

∠C = ∠B = x

∠ECD = 60° and ∠DCB = 90°

∠ECB = 60° + 90°

= 150°

Now, x + x + 150° = 180°

2x = 30°

x = 15°

∴∠EBC = 15°

∴ a = r, b = s, c = p, d = q

28. Question

Fill in the blanks with < or >.

(A) (Sum of any two sides of a triangle)………… (the third side)

(B) (Difference of any two sides of a triangle)………… (the third side)

(C) (Sum of three altitudes of a triangle)……….. (sum of its three sides)

(D) (Sum of any two sides of a triangle)……. (twice the median to the 3rd side)

(E) (Perimeter of a triangle)………… (Sum of its three medians)

Answer

a) Sum of any two sides of a triangle > the third side

b) Difference of any two sides of a triangle < the third side

c) Sum of three altitudes of a triangle < sum of its three side

d) Sum of any two sides of a triangle > twice the median to the 3rd side

e) Perimeter of a triangle > sum of its three medians

29. Question

Fill in the blanks:

(A)Each angle of an equilateral triangle measures……………..

(B) Medians of an equilateral triangle are………..

(C) In a right triangle the hypotenuse is the ……….side.

(D) Drawing a ΔABC with AB=3cm, BC=4cm and CA=7cm is………..

Answer

a) Each angle of an equilateral triangle measures **<u>60°</u>**

b) Medians of an equilateral triangle are **<u>equal</u>**

c) In a right triangle, the hypotenuse is the **<u>longest</u>** side

d) Drawing a △ABC with AB = 3cm, BC = 4cm and CA = 7cm is **<u>not possible</u>**.

Formative Assessment (Unit Test)

1. Question

In an equilateral ΔABC, find ∠A.

Answer

We know that,

In any equilateral triangle all the angles are equal

Let the three angles of the triangle ∠A, ∠B and ∠C be x

∴ x + x + x = 180°

3x = 180°

$x = \frac{180}{3}$

x = 60

Hence, ∠A = 60^{o}

2. Question

In a ΔABC, if AB=AC and ∠B=65°, find ∠A.

Answer

It is given in the question that,

In triangle ABC, AB = AC

∠B = 65°

As ABC is an isosceles triangle

∴ ∠C = ∠B

∠C = 65°

Now, we now that sum of all angles of a triangle is 180^{o}

∴ ∠A + ∠B + ∠C = 180°

∠A + 65° + 65° = 180°

∠A + 130° = 180°

∠A = 180° – 130°

∠A = 50°

3. Question

In a right ΔABC, ∠B=90°. Find the longest side.

Answer

It is given in the question that,

In right triangle ABC,

∠B = 90^{o}

So, ∠A + ∠C = 90^{o}

∴ ∠A, ∠C < ∠B

Hence, the side opposite to ∠B is longest

Thus, AC is the longest side

4. Question

In a ΔABC, ∠B > ∠C. Which of AC and AB is longer?

Answer

It is given in the question that,

In triangle ABC, ∠B > ∠C

We know that, in a triangle side opposite to greater angle is longer

∴ AC is longer than AB

5. Question

Can we construct a ΔABC in which AB=5cm, BC=4cm and AC=9cm? Why?

Answer

We know that,

The sum of two sides must be greater than the third side

In this case, we have

AB + BC = 5 + 4 = 9 cm

AC = 9 cm

∴ AC must be greater than the sum of AB and BC

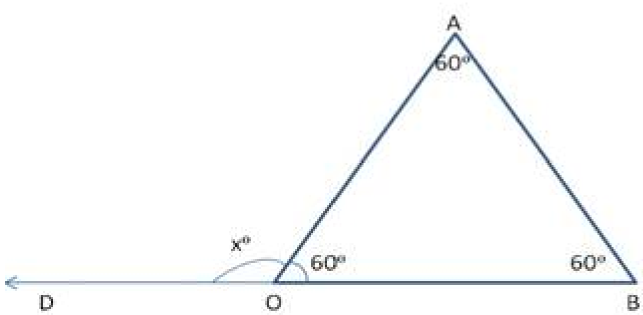
Hence, the sum of two sides is not greater than the third side. So, ΔABC cannot be constructed

6. Question

Find the measure of each exterior angle of an equilateral triangle.

Answer

From the figure, we have



∠AOD is the exterior angle

∴ ∠AOD + ∠AOB = 180^o

60^o + ∠AOB = 180^o

∠AOB = 180^o – 60^o

∠AOB = 120^o

Hence, the measure of each of the exterior angle of an equilateral triangle is 120^o

7. Question

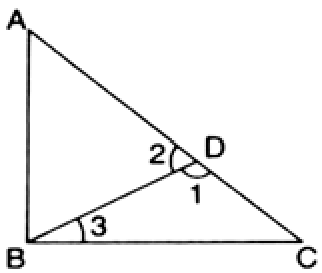
Show that the difference of any two sides of a triangle is less than the third side

Answer

In a triangle let AC > AB

Then, along AC draw AD = AB and join BD

Proof: In Δ ABD,



∠ ABD = ∠ ADB (AB = AD) ….(i)

∠ ABD = ∠ 2 (angles opposite to equal sides) ….(ii)

Now, we know that the exterior angle of a triangle is greater than either of its opposite interior angles.

∴∠ 1 >∠ABD

∠1 > ∠2 ….(iii)

Now, from (ii)

∠2 > ∠3 ….(iv) (∠2 is an exterior angle)

Using (iii) and (iv),

∠1 > ∠3

BC > DC (side opposite to greater angle is longer)

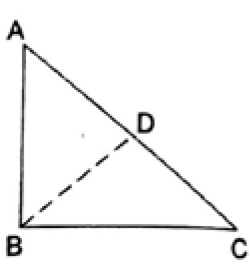
BC > AC – AD

BC > AC – AB (since, AB = AD)

Hence, the difference of two sides is less than the third side of a triangle

8. Question

In a right ΔABC, ∠B=90° and D is the mid-point of AC. Prove that BD= $\frac{1}{2}$ AC.



Answer

It is given in the question that,

In right triangle ABC, ∠B = 90^o

Also D is the mid-point of AC

∴ AD = DC

∠ADB = ∠BDC (BD is the altitude)

BD = BD (Common)

So, by SAS congruence criterion

∴ ΔADB ≅ ΔCDB

∠A = ∠C (CPCT)

As, ∠B = 90^o

So, by using angle sum property

∠A = ∠ABD = 45^o

Similarly, ∠BDC = 90^o (BD is the altitude)

∠C = 45^o

∠DBC = 45^o

∠ABD = 45^o

Now, by isosceles triangle property we have:

BD = CD and

BD = AD

AS, AD + DC = AC

BD + BD = AC

2BD = AC

BD = $\frac{1}{2}AC$

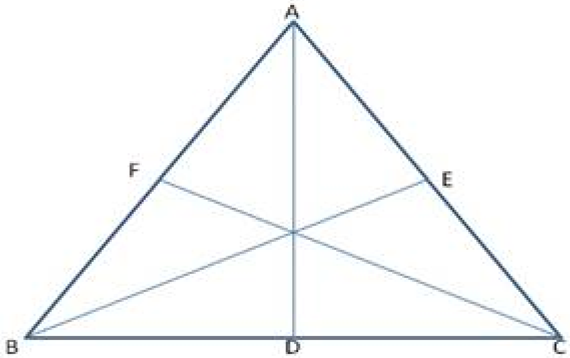
Hence, proved

9. Question

Prove that the perimeter of a triangle is greater than the sum of its three medians

Answer

Let ABC be the triangle where D, E and F are the mid-points of BC, CA and AB respectively



As, we know that the sum of two sides of the triangle is greater than twice the median bisecting the third side

∴ AB + AC > 2AD

Similarly, BC + AC > 2CF

Also, BC + AB > 2BE

Now, by adding all these we get:

(AB + BC) + (BC + AC) + (BC + AB) > 2AD + 2CD + 2BE

2 (AB + BC + AC) > 2(AD + BE + CF)

∴ AB + BC + AC > AD + BE + CF

Hence, the perimeter of the triangle is greater than the sum of its medians

10. Question

Which is true?

(A) A triangle can have two acute angles.

(B) A triangle can have two right angles.

(C) A triangle can have two obtuse angles.

(D) An exterior angles of a triangle is always less than either of the interior opposite angles.

Answer

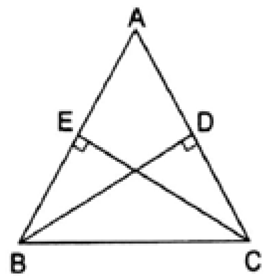
We know that,

A triangle can have two acute angles because the sum of two acute angles is always less than 180^{o} which satisfies the angle sum property of a triangle

Hence, option (A) is correct

11. Question

In ΔABC, BD ⊥ AC and CE ⊥ AB such that BE=CD. Prove that BD=CE.



Answer

It is given that,

BD is perpendicular to AC and CE is perpendicular to AB

Now, in ΔBDC and ΔCEB we have:

BE = CD (Given)

∠BEC = ∠CDB = 90^{o}

And, BC = BC (Common)

∴ By RHS congruence rule

ΔBDC ≅ ΔCEB

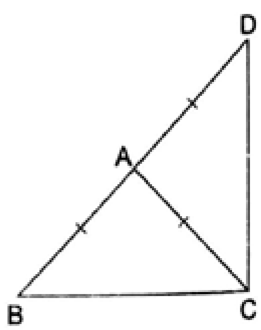
BD = CE (By CPCT)

Hence, proved

12. Question

In ΔABC, AB=AC. Side BA is produced to D such that AD=AB.

Prove that ∠BCD=90°.



Answer

It is given in the question that,

In ΔABC,

AB = AC

We know that, angles opposite to equal sides are equal

∴ ∠ACB = ∠ABC

Now, in ΔACD we have:

AC = AD

∠ADC = ∠ACD (The Angles opposite to equal sides are equal)

By using angle sum property in triangle BCD, we get:

∠ABC + ∠BCD + ∠ADC = 180^o

∠ACB + ∠ACB + ∠ACD + ∠ACD = 180^o

2 (∠ACB + ∠ACD) = 180^o

2 (∠BCD) = 180^o

∠BCD = $\frac{180}{2}$

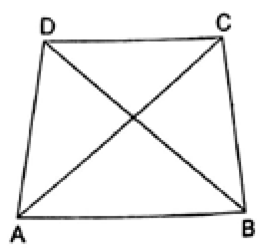
∠BCD = 90^o

Hence, proved

13. Question

In the given figure, it is given that AD=BC and AC=BD.

Prove that ∠CAD=∠CBD and ∠ADC=∠BCD.



Answer

From the given figure,

In triangles DAC and CBD, we have:

AD = BC

AC = BD

DC = DC

So, by SSS congruence rule

ΔADC ≅ ΔBCD

∴ By Congruent parts of congruent triangles we have:

∠CAD = ∠CBD

∠ADC = ∠BCD

∠ACD = ∠BDC

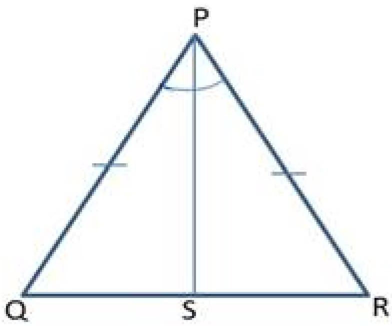
Hence, proved

14. Question

Prove that the angles opposite to equal sides of a triangle are equal

Answer

We have a triangle PQR where PS is the bisector of ∠ P



Now in ΔPQS and ΔPSR, we have:

PQ = PR (Given)

PS = PS (Common)

∠ QPS = ∠ PRS (As PS is the bisector of ∠ P)

∴ By SAS congruence rule

ΔPQS ≅ ΔPSR

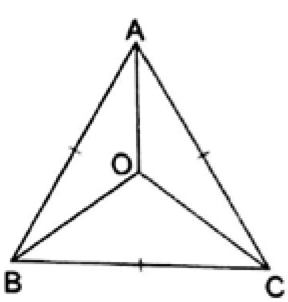
∠ Q = ∠ R (By Congruent parts of congruent triangles)

Hence, it is proved that the angles opposite to equal sides of a triangle are equal

15. Question

In an isosceles ΔABC, AB=AC and the bisectors of ∠B and ∠C intersect each other at O. Also, O and A are joined.

Prove that: (i) OB=OC (ii) ∠OAB=∠OAC



Answer

From the given figure, we have:

(i) In ΔABO and ΔACO

AB = AC (Given)

AO = AO (Common)

∠ ABO = ∠ ACO

∴ By SAS congruence rule

$\Delta ABO \cong \Delta ACO$

OB = OB (By CPCT)

(ii) As, By SAS congruence rule

ΔABO ≅ ΔACO

∴ ∠ OAB = ∠ OAC (By Congruent parts of congruent triangles)

Hence, proved

16. Question

Prove that, of all line segments that can be drawn to a given line, from a point, not lying on it, the perpendicular line segment is the shortest

Answer

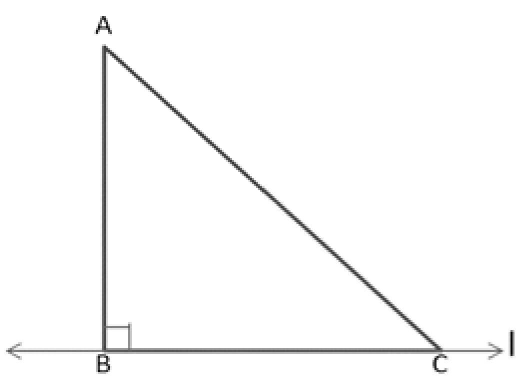
It is given in the question that,

l is the straight line and A is a point that is not lying on l

AB is perpendicular to line l and C is the point on l

As, ∠ B = 90^o

So in ΔABC, we have:



∠ A + ∠ B + ∠ C = 180^o

∠ A + ∠ B = 90^o

∴ ∠ C < 90^o

∠ C < ∠ B

AB < AC

As C is that point which can lie anywhere on l

∴ AB is the shortest line segment drawn from A to l

Hence, proved

1. Question

Each question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	**Reason (R)**
Each angle of an equilateral triangle is 60°.	Angles opposite to equal sides of a triangle are equal.

A. Both Assertion (A) and Reason (R) are true but Reason (R) is a correct explanation of Assertion (A)

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A)

C. Assertion (A) is true and Reason (R) is false

D. Assertion (A) is false and Reason (R) is true

Answer

We know that,

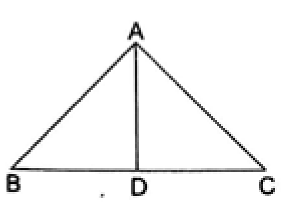
Each angle of an equilateral triangle is equal to 60^o also angles opposite to equal sides of a triangle are equal to each other

∴ Both assertion and reason are true and reason is the correct explanation of the assertion

Hence, option (A) is correct

18. Question

Each question consists of two statements, namely, Assertion (A) and Reason (R). Please select the correct answer.

Assertion (A)	Reason (R)
If AD is a median of ΔABC, then AB+AC>2AD. 	In a triangle the sum of two sides is greater than the third side.

A. Both Assertion (A) and Reason (R) are true but Reason (R) is a correct explanation of Assertion (A)

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A)

C. Assertion (A) is true and Reason (R) is false

D. Assertion (A) is false and Reason (R) is true

Answer

From the given figure in the question, we have

In ΔABD, we have:

AB + BD > AD

Similarly, in ΔADC

AC + CD > AD

Adding both expressions, we get:

AB + AC + BD + CD > AD + AD

AB + AC + BD + DC > 2AD

AB + AC + BC > 2AD

∴ Assertion and reason both are true and reason is the correct explanation of the assertion

Hence, option (A) is correct

19. Question

Math the following columns:

Column I	Column II
(a)In ΔABC, if AB=AC and ∠A=70°, then ∠C=...........	(p) less
(b) The vertical angle of an isosceles triangle is 120°. Each base angle is.........	(q) greater
(c) The sum of three medians of a triangle is .... than the perimeter.	(r) 30°
(d) In a triangle, the sum of any two sides is always .... than the third side.	(s) 55°

The correct answer is:

(a)-……, (b)-……, (c)-……., (d)-……

Answer

a) In ΔABC, ∠ A=70°

As AB = AC and we know that angles opposite to equal sides are equal

∴ In triangle ABC,

$\angle A + \angle B + \angle C = 180^o$

$70^o + 2\angle C = 180^o$

$2\angle C = 180^o - 70^o$

$\angle C = \frac{110}{2}$

$\therefore \angle C = 55^o$

(b) We know that,

Angles opposite to equal sides are equal

It is given that, vertical angle of the isosceles triangle = 120^{o}

Let the base angle be x

∴ 120° + x + x = 180°

120° + 2x = 180°

2x = 180° – 120°

2x = 60°

$x = \frac{60}{2}$

x = 30°

Hence, each base angle of the isosceles triangle is equal to 30^{o}

(c) We know that,

The sum of the three medians of the triangle is always less than the perimeter

(d) We know that,

In a triangle the sum of any two sides is always greater than the third side

Hence, the correct match is as follows:

(a) – (s)

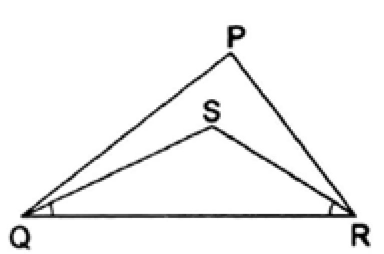
(b) – (r)

(c) – (p)

(d) – (q)

20. Question

In the given figure, PQ>PR and QS and RS are the bisectors of ∠Q and ∠R respectively. Show that SQ>SR



Answer

It is given in the question that,

PQ > PR

And, QS and RS are the bisectors of ∠ Q and ∠ R

We have, angle opposite to the longer side is greater

∴ PQ > PR

∠ R > ∠ Q

$\frac{1}{2}\angle R > \frac{1}{2}\angle Q$

∠ SRQ > ∠ RQS

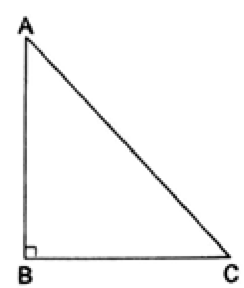
SQ > SR

Hence, proved

21. Question

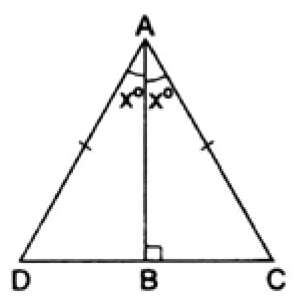
In the given figure, ABC is a triangle right-angled at B such that ∠BCA=2∠BAC.

Show that AC=2BC.



Answer

We will have to make the following construction in the given figure:



Produce CB to D in such a way that BD=BC and join AD.

Now, in ΔABC and ΔABD,

BC=BD (constructed)

AB=AB (common)

∠ABC=∠ABD (each 90°)

∴ by S.A.S.

ΔABC ≅ ΔABD

∠CAB=∠DAB and AC=AD (by c.p.c.t.)

∴∠CAD=∠CAB+∠BAD

=x°+x°

=2x°

But, AC=AD

∠ACD=∠ADB=2x°

∴ ΔACD is equilateral triangle.

AC=CD

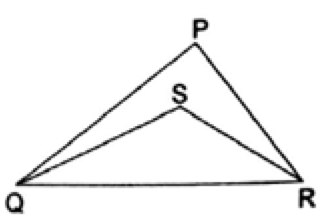
AC=2BC

Hence, proved

22. Question

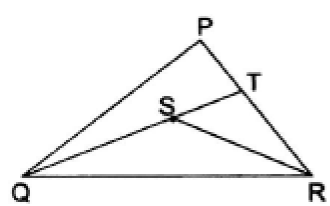
S is any point in the interior of ΔPQR.

Show that (SQ+SR)<(PQ+PR).



Answer

Following construction is to be made in the given figure.



Extend QS to meet PR at T.

Now, in Δ PQT,

PQ+PT>QT (sum of two sides is greater than the third side in a triangle)

PQ+PT>SQ+ST (i)

Now, In Δ STR,

ST+TR>SR (ii)(sum of two sides is greater than the third side in a triangle)

Now, adding (i) and (ii),

PQ+PT+ST+TR>SQ+ST+SR

PQ+PT+TR>SQ+SR

PQ+PR>SQ+SR

SQ+SR<PQ+PR

Hence, proved

23. Question

Show that in a quadrilateral ABCD

AB+BC+CD+DA>AC+BD.

Answer

Here, ABCD is a quadrilateral and AC and BD are its diagonals.

Now, As we that, sum of two sides of a triangle is greater than the third side.

∴ In Δ ACB,

AB + BC > AC (i)

In Δ BDC,

CD + BC > BD (ii)

In Δ BAD,

AB + AD>BD (iii)

In Δ ACD,

AD + DC > AC (iv)

Now, adding (i), (ii), (iii) and (iv):

AB + BC + CD + BC + AB + AD + AD + DC> AC + BD + BD + AC

2AB + 2BC + 2CD + 2AD > 2AC + 2BD

Thus, AB + BC + CD + AD > AC + BD

Hence, proved