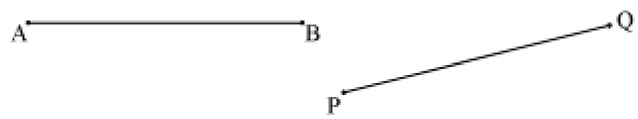
Similarity

Difference Between Similarity and Congruence

Congruency of line segments:

"***Two line segments are congruent to each other if their lengths are equal".***

Consider the following line segments.



Here, the line segments AB and PQ will be congruent to each other, if they are of equal length.

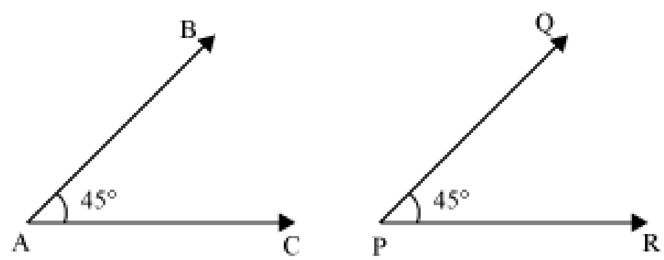
Conversely, we can say that, ***"Two line segments are of equal length if they are congruent to each other".***

i.e. if $\overline{AB} \cong \overline{PQ}$, then AB = PQ.

Congruency of angles:

"***Two angles are said to be congruent to each other if they have the same measure".***

The angles shown in the following figures are congruent to each other as both the angles are of the same measure 45°.



Thus, we can write ∠BAC ≅ ∠QPR.

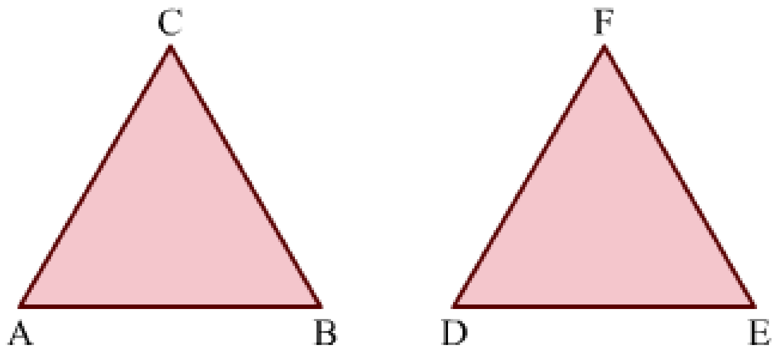
Its converse is also true.

"***If two angles are congruent to each other, then their measures are also equal".***

There is one special thing about congruent figures that their corresponding parts are always equal.

For example, if two triangles are congruent then their corresponding sides will be equal. Also, their corresponding angles will be equal.

Look at the following triangles.



Here, ΔABC ≅ ΔDEF under the correspondence ΔABC ↔ ΔDEF. This correspondence rule represents that in given triangles, AB ↔ DE (AB corresponds to DE), BC ↔ EF, CA ↔ FD, ∠A ↔ ∠D, ∠B ↔ ∠E, ∠C ↔ ∠F. These are **corresponding parts of congruent triangles** (CPCT), ΔABC and ΔDEF.

Since ΔABC and ΔDEF are congruent, their corresponding parts are equal.
Therefore, AB = DE, BC = EF, CA = FD

And, ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

Similarly, we can apply the method of CPCT on other congruent triangles also.

Let us now try and apply what we have just learnt in some examples.

Example 1:

Find which of the pairs of line segments are congruent.

(i) **(ii)**

Solution:

(i) Lengths of the two line segments are not same. Therefore, they are not congruent.

(ii) Each of the line segments is of length 3.1 cm, i.e. they are equal. Therefore, they are congruent.

Example 2:

If $\overline{AB} \cong \overline{PQ}$ and $\overline{PQ}$ = 9 cm, then find the length of $\overline{AB}$.

Solution:

Since $\overline{AB} \cong \overline{PQ}$, i.e. line segment AB is congruent to line segment PQ, therefore, $\overline{AB}$ and $\overline{PQ}$ are of equal length.

∴ $\overline{AB}$ = 9 cm

Example 3:

If ∠ABC ≅∠PQR and ∠PQR = 75º, then find the measure of ∠ABC.

Solution:

If two angles are congruent, then their measures are equal.

Since ∠ABC ≅ ∠PQR,

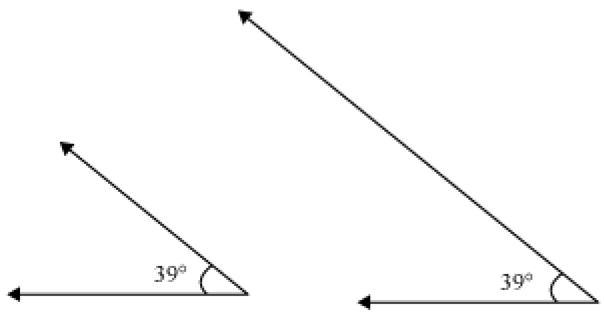
∴ ∠ABC = ∠PQR

Therefore, ∠ABC = 75º

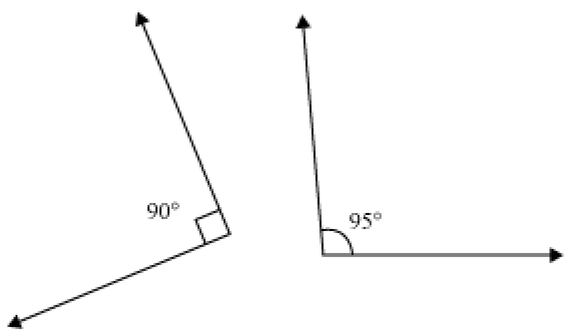
Example 4:

Which of the following pairs of angles are congruent?

(i)



(ii)



Solution:

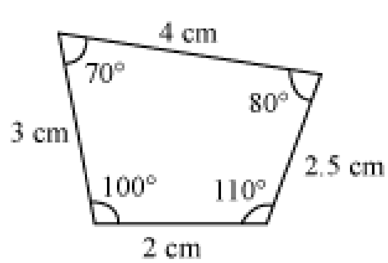
(i) The measure of both the angles is the same. Therefore, they are congruent**.**

(ii) The measures of the two angles are different. Therefore, they are not congruent.

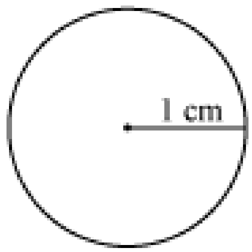
Example 5:

Identify the pairs of similar and congruent figures from the following.

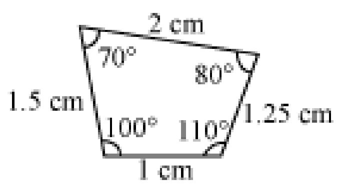
(i)



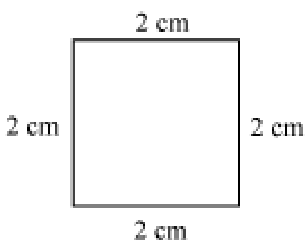
(ii)


1 cm

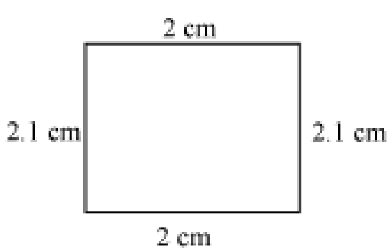
(iii)


2 cm
70°
80°
1.5 cm
1.25 cm
100°
110°
1 cm

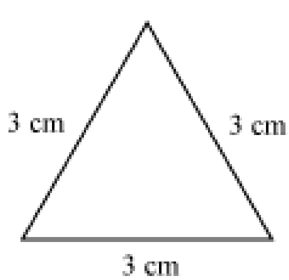
(iv)


2 cm
2 cm
2 cm
2 cm

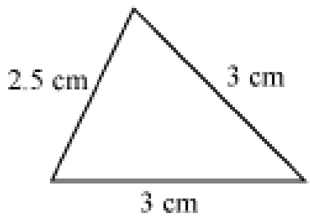
(v)


2 cm
2.1 cm
2.1 cm
2 cm

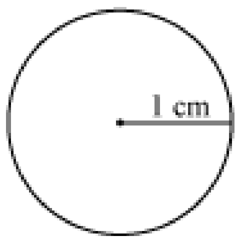
(vi)


3 cm
3 cm
3 cm

(vii)



(viii)



Solution:

Figures **(i)** and **(iii)** are similar because their corresponding angles are equal and their corresponding sides are in the same ratio. However, these figures are not congruent as they are of different sizes.

Figures **(ii)** and **(viii)** are congruent as they are of the same shape and size (circles with radius 1 cm each).

Example 6:

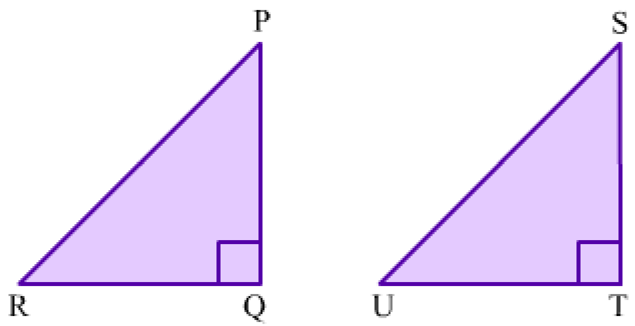
Are the following figures similar or congruent?

Solution:

The two given figures show two one-rupee coins. As both the figures represent the same coin in two different sizes, they are similar to each other. However, the pictures are not congruent because of their different sizes.

Example 7:

In the following figure, ΔPQR and ΔSTU are congruent.



If PQ = 8 cm, QR = 6 cm then find the perimeter of ΔSTU.

Solution:

In ΔPQR, we have

PQ = 8 cm, QR = 6 cm and ∠Q = 90°

Applying Pythagoras theorem in ΔPQR, we obtain

$RP^2 = PQ^2 + QR^2$

$\Rightarrow RP^2 = 8^2 + 6^2$

$\Rightarrow RP^2 = 64 + 36$

$\Rightarrow RP^2 = 100$

⇒ RP = 10 cm

Since ΔPQR and ΔSTU are congruent, their corresponding parts will be equal.

Therefore,

PQ = 8 cm = ST (CPCT)

QR = 6 cm = TU and (CPCT)

RP = 10 cm = US (CPCT)

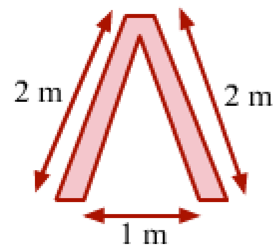
∴ Perimeter of ΔSTU = ST + TU + US = 8 cm + 6 cm + 10 cm = 24 cm

Mid Point Theorem and Its Converse

Midpoint Theorem

The given image is that of a food outlet named Adam's Diner. The logo of the outlet, as you can see on top of the building, is the letter 'A'.

The carpenter who made the logo fixed the two legs of the logo, each of length 2 m, such that the distance between the two legs at the base was 1 m.

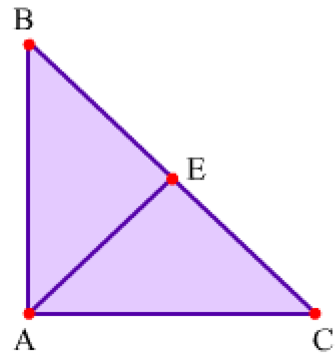


The carpenter then put the horizontal bar of the logo at the **mid-point** of the two legs so that it fitted exactly in the middle of the legs. The midpoint theorem came to his aid while doing so. In this lesson, we will study this theorem, its proof and its converse, and solve some related examples.

Whiz Kid

Right triangle median theorem

The length of the median on the hypotenuse of a right triangle is half that of the hypotenuse.



Here, AE = 1/2 × BC

Activity

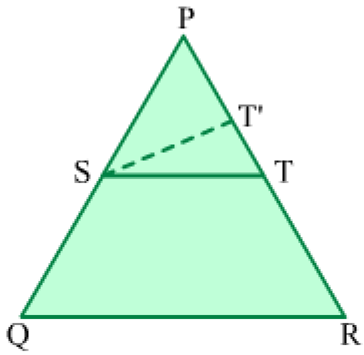
Perform the following activity to verify the midpoint theorem.

- Draw a large scalene triangle on a sheet of paper.
- Name its vertices A, B and C. Find the midpoints of sides AB and AC and name them D and E respectively. Draw a line joining the two midpoints.
- Cut out ΔABC and then cut it along line segment DE.
- Draw a quadrilateral BDEC. Place ΔADE on it such that vertices E and D of the triangle are respectively on vertex C and line segment BC of the quadrilateral. Mark the point where vertex D touches line segment BC.
- Shift ΔADE such that its vertices D and E are respectively on vertex B and line segment BC of the quadrilateral. Mark the point where vertex E touches line segment BC.
- What do you notice about the lengths of DE and BC? Do the marked points coincide?

Proof of the Converse of the Midpoint Theorem

Statement: The line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side.

Given: ΔPQR in which S is the midpoint of PQ and ST||QR



To prove: T is the midpoint of PR

Construction: Take a point T′ on PR and join S and T′.

Proof: Let us say that T is not the midpoint of PR. Let T′ be the midpoint of PR.

In ΔPQR, S is the midpoint of PQ and T′ is the midpoint of PR.

∴ By the midpoint theorem, ST′||QR … (1)

It is given that ST||QR. … (2)

From 1 and 2, we conclude that ST||ST′, which is not possible.

So, our assumption was wrong.

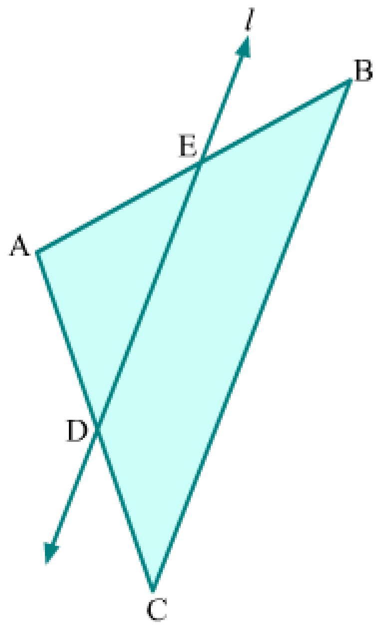
Hence, T is the midpoint of PR.

Solved Examples

Easy

Example 1:

Line *l* cuts the isosceles ΔABC, with AB = AC = 10 cm and BC = 15 cm, in such a way that ΔADE is also isosceles with AD = AE = 5 cm. Find the length of DE.



Solution:

It is given that ΔABC is isosceles with AB = AC = 10 cm, BC = 15 cm and AD = AE = 5 cm.

AC = AD + DC

⇒ DC = AC − AD = (10 − 5) cm = 5 cm

Since AD = DC, D is the midpoint of AC.

Similarly, E is the midpoint of AB.

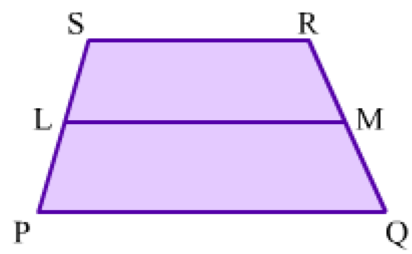
Using the midpoint theorem, we get:

DE = $\frac{BC}{2} = \frac{15}{2}$ cm = 7.5 cm

Medium

Example 1:

In the given trapezium PQRS, PQ||SR and L is the midpoint of line segment PS. If LM||PQ, then prove that RM = QM.



Solution:

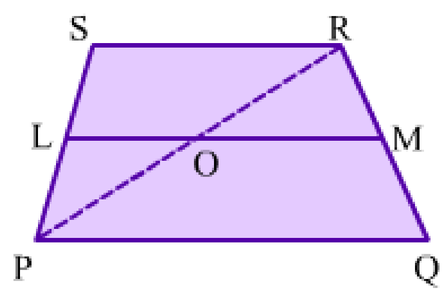
It is given that:

LM||PQ

PQ||SR

⇒ LM||SR(∵ Lines parallel to the same line are also parallel)

Construction: Join P to R such that PR intersects LM at point O.



In ΔPSR, L is the midpoint of PS and LO||RS.

So, by the converse of the midpoint theorem, we get O as the midpoint of PR.

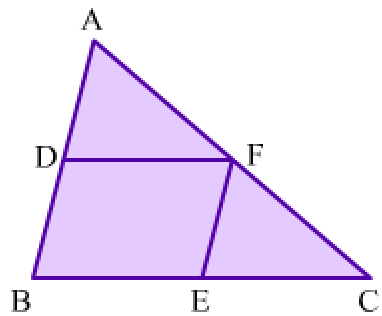
Now, in ΔPQR, O is the midpoint of PR and OM||PQ.

Again, by the converse of the midpoint theorem, we get M as the midpoint of QR.

⇒ RM = QM

Example 2:

In the shown ΔABC, D, E and F are the midpoints of sides AB, BC and AC respectively. Prove that BEFD is a parallelogram.



Solution:

In ΔABC, D and F are the midpoints of AB and AC respectively.

So, by the midpoint theorem, we obtain DF||BC

⇒ DF||BE … (1)

Again, in ΔABC, E and F are the midpoints of BC and AC respectively.

So, by the midpoint theorem, we obtain EF||BA

⇒ EF||BD … (2)

Now in quadrilateral BEFD, we have:

DF||BE and EF||BD (From 1 and 2)

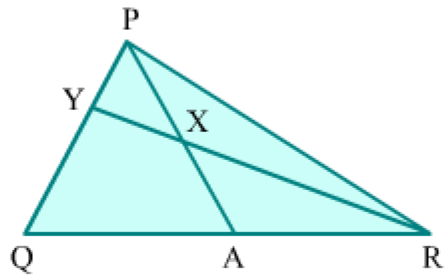
⇒ BEFD is a parallelogram. (∵ There are two pairs of parallel opposite sides)

Hard

Example 1:

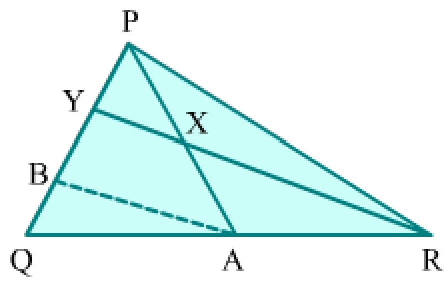
In the given ΔPQR, PA is the median to side QR and X is the midpoint of PA.

When produced, RX meets PQ at Y. Prove that $PY = \frac{1}{3}PQ$.



Solution:

Construction: Draw a line AB such that it is parallel to RY.



In ΔPAB, X is the midpoint of AP and XY||AB.

So, by the converse of the midpoint theorem, we obtain Y as the midpoint of PB.

⇒ PY = YB … (1)

In ΔQYR, A is the midpoint of QR. (∵ AP is the median to side QR)

Also, RY||AB

So, by the converse of the midpoint theorem, we obtain B as the midpoint of YQ.

⇒ YB = BQ … (2)

From equations 1 and 2, we obtain:

PY = YB = BQ … (3)

From the figure, we have:

PQ = PY + YB + BQ

⇒ PQ = PY + PY + PY

$\Rightarrow$ PQ = 3PY

$$\Rightarrow \therefore PY = \frac{1}{3} PQ$$

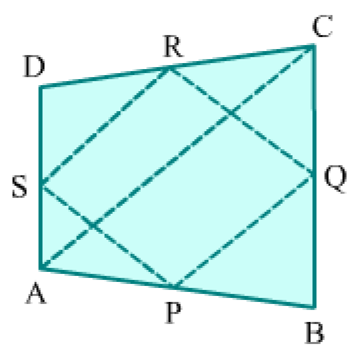
Example 2:

Prove that the quadrilateral formed by joining the midpoints of all sides of a quadrilateral is a parallelogram.

Solution:

Let ABCD be a quadrilateral and P, Q, R and S the respective midpoints of sides AB, BC, CD and DA.

Construction: Draw diagonal AC of quadrilateral ABCD.



To prove that PQRS is a parallelogram, we have to prove that it has one pair of parallel and equal opposite sides.

In ΔACD, R and S are the midpoints of sides CD and DA respectively.

So, by the midpoint theorem, we obtain SR||AC and $SR = \frac{1}{2} AC$ … (1)

Now, in ΔABC, P and Q are the midpoints of sides AB and BC respectively.

So, by midpoint theorem, we obtain PQ||AC and $PQ = \frac{1}{2} AC$ … (2)

From 1 and 2, we obtain:

PQ||RS(∵ Lines parallel to the same line are also parallel)

PQ = RS

In quadrilateral PQRS, we have one pair of parallel and equal opposite sides, i.e., PQ||RS and PQ = RS. Thus, PQRS is a parallelogram.

Properties of Parallel Lines and Their Transversals with Respect to Intercepts

Aline that intersects two (or more) distinct lines at different points is known as transversal the portion of the transversal lying between these two distinct lines is known as **intercept.**

To understand intercept better, let us draw two lines and their transversal in a few different ways.

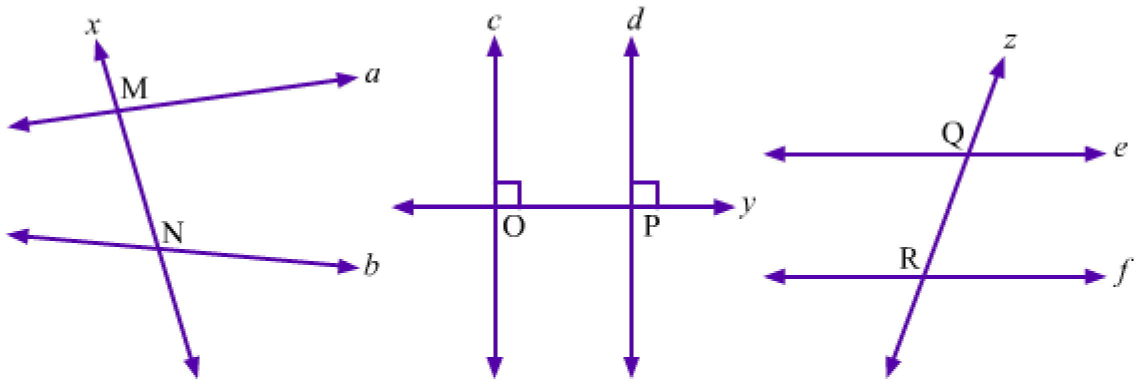


Fig. (1) Fig. (2) Fig. (3)

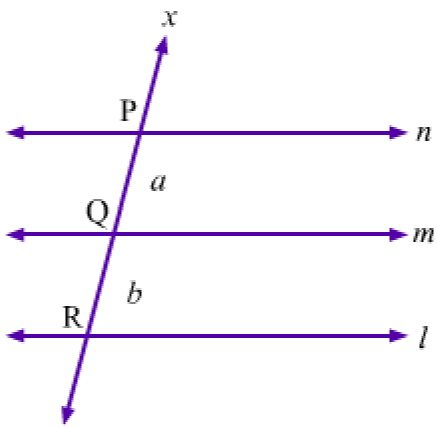
In fig. (1), *x* is the transversal which intersects lines *a* and *b* at points M and N respectively. So, line segment MN is the intercept made by lines *a* and *b* on transversal *x*.

Similarly, in fig. (2), OP is the intercept made by lines *c* and *d* on transversal *y*.

In fig. (3), QR is the intercept made by lines *e* and *f* on transversal *z*.

If two lines are parallel, then the transversal exhibit a few interesting properties related to the intercepts formed on it.

Let us consider the following figure.

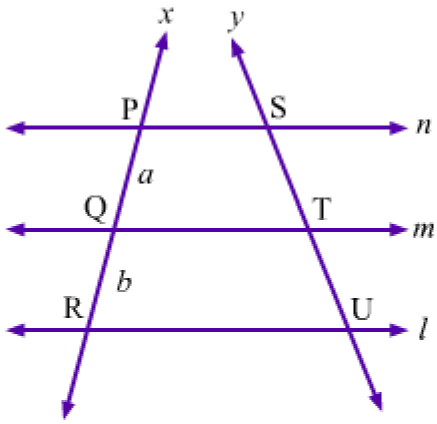


Here, lines *l*, *m* and *n* are parallel to each other, and line *x* is the transversal making intercepts PQ and QR.

Let the length of the intercept PQ be *a* and that of intercept QR be *b*.

$$\therefore \frac{PQ}{QR} = \frac{a}{b}$$

Now, let us draw another transversal *y* with respect to lines *l*, *m* and *n* making intercept ST and TU.



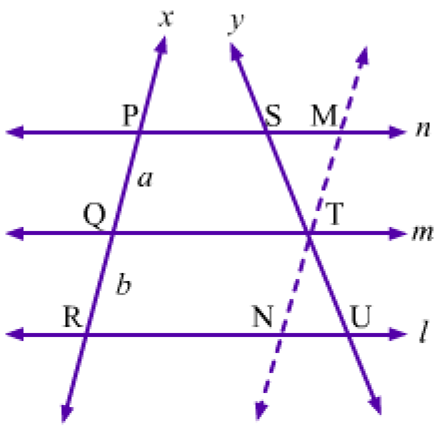
There is a property about intercepts made by three parallel lines on two different transversals. It is stated as follows:

The intercepts made by three parallel lines on one transversal are in the same ratio as the corresponding intercepts made by the same lines on any other transversal.

$$\Rightarrow \frac{PQ}{QR} = \frac{ST}{TU} = \frac{a}{b}$$

Let us prove this property.

For this, let us consider the same figure and draw a line parallel to *x* through T, which intersects *n* and *l* at points M and N respectively.



We have, PM || QT(As *n* || *m*)

PQ|| MT(By construction)

Thus, PQTM is a parallelogram.

⇒PQ = MT = *a*…(1) (Opposite sides of a parallelogram are equal.)

Similarly, QRNT is a parallelogram.

⇒QR = TN = *b*…(2) (Opposite sides of a parallelogram are equal.)

Now, in ΔTSMand ΔTUN:

∠STM= ∠UTN(Vertically opposite angles)

∠TSM= ∠TUN(Alternate angles)

∠TMS= ∠TNU(Alternate angles)

∴ ΔTSM~ΔTUN (By AAA criterion of similarity)

$$\Rightarrow \frac{TS}{TU} = \frac{MT}{NT} \quad \text{(Property of similar triangles)}$$

$$\Rightarrow \frac{TS}{TU} = \frac{a}{b} \quad \text{[Using (1) and (2)]}$$

$$\Rightarrow \frac{TS}{TU} = \frac{PQ}{QR}$$

Hence proved.

There may be a case when the intercepts made by three parallel lines on first transversal, i.e. *x*, are congruent.

∴ PQ = QR

$$\Rightarrow \frac{PQ}{QR} = 1$$

Since, $\frac{PQ}{QR} = \frac{TS}{TU}$,

$$\frac{TS}{TU} = 1$$

⇒ ST = TU

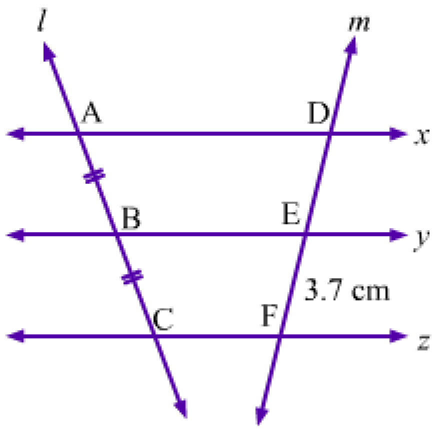
This property can be stated as follows:

If three parallel lines form congruent intercepts on one transversal, then the intercepts formed by them on other transversals are also congruent.

Let us go through a few examples to understand the concept better.

Example1:

In the given figure, *x* || *y* || *z* and AB = BC. Find DF, if EF = 3.7 cm.



Solution:

It is given that *x* || *y* || *z*, AB = BC and EF = 3.7 cm.

In the given figure, *l*and *m*are two transversals with respect to parallel lines *x*, *y* and *z*. Also, AB and BC are intercepts formed by three parallel lines *x*, *y* and *z* on transversal *l*, such that AB = BC.

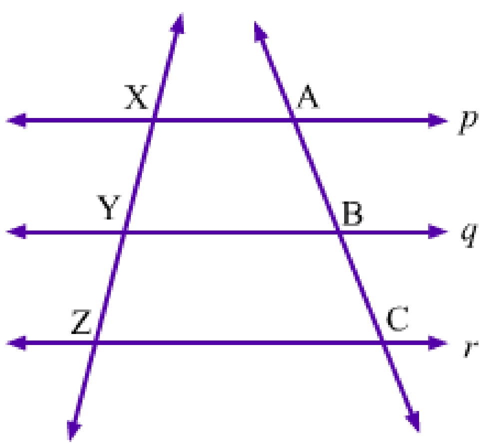
We know that if three parallel lines form congruent intercepts on one transversal, then intercepts formed by them on other transversals are also congruent.

∴DE = EF = 3.7cm(Since AB = BC)

Thus, DF = DE + EF = 3.7 cm + 3.7 cm = 7.4 cm.

Example 2:

In the given figure, *p* || *q*|| *r*.If XY = 2.2 cm and BC = 5 cm, then what is the numerical value of AB× YZ?



Solution:

It is given that *p* || *q*|| *r*, XY = 2.2 cm and BC = 5 cm.

Now, we know that the intercepts made by three parallel lines on one transversal are in the same ratio as the corresponding intercepts made by the same lines on any other transversal.

$\therefore \frac{XY}{YZ} = \frac{AB}{BC}$

$\Rightarrow AB \times YZ = XY \times BC$

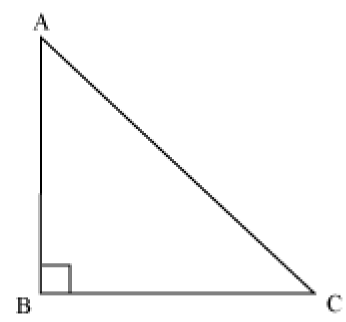
$\Rightarrow AB \times YZ = 2.2 \times 5$

$\Rightarrow AB \times YZ = 11$

Hence, the required value is 11.

Pythagoras Theorem and Its Applications

Look at the following right triangle ABC.



We have the following relationship between the sides of a right-angled triangle ABC.

$(AC)^2 = (AB)^2 + (BC)^2$

This relation between the sides of a right-angled triangle is known as **Pythagoras Theorem.**

Let us understand the proof of the theorem with the help of the given video.

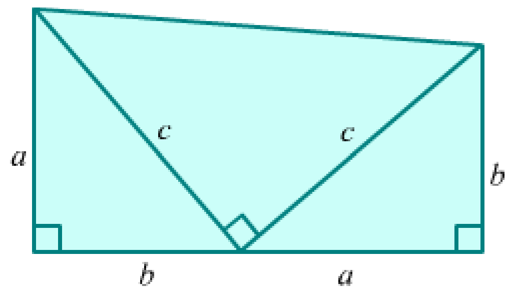
This theorem has several other proofs also. Let us discuss two of them here.

Proof by American President:

The 20th president of United States, James Garfield proved this theorem taking two right angled triangles having sides as *a*, *b* and *c* and an other right angled triangle with side *c*.

The proof given by him is as follows:

All three triangles are combined as shown in the following figure.



Thus, we got a trapezium.

$$\text{Area of trapezium} = \frac{1}{2} \times (\text{Sum of lengths of parallel sides}) \times \text{Height}$$

$$\Rightarrow \text{Area of trapezium} = \frac{1}{2} \times (a+b)(a+b)$$

$$\Rightarrow \text{Area of trapezium} = \frac{a^2+2ab+b^2}{2} \qquad ...(1)$$

And,

$$\text{Sum of areas of 3 triangles} = \frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2} \qquad ...(2)$$

$$\frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2} = \frac{a^2+2ab+b^2}{2} \qquad [\text{Using (1) and (2)}]$$

$$\Rightarrow \frac{2ab+c^2}{2} = \frac{a^2+2ab+b^2}{2}$$

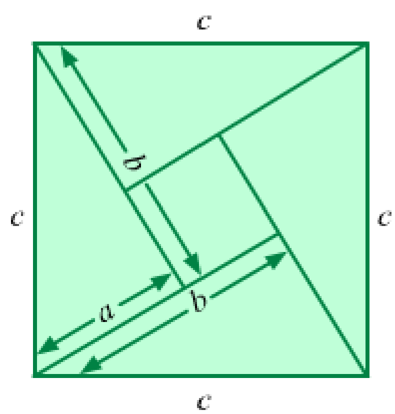
$\Rightarrow 2ab + c^2 = a^2 + 2ab + b^2$

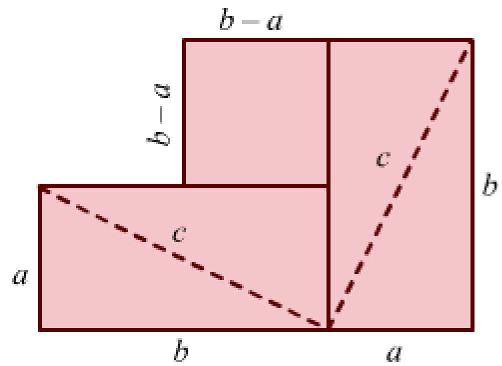
$\Rightarrow c^2 = a^2 + b^2$

This theorem of right angled triangles was also known to Indian, Chinese, Greek and Babylonian mathematician long before Pythagoras lived. Thus, it was proved by the mathematicians of that time differently.

Proof by Indian Mathematician:

Bhaskaracharya, the great Indian mathematician of 2nd century AD, used the below given diagrams to prove this theorem.





Area of 4 triangles $= 4(\frac{1}{2}ab) = 2ab$

Area of small square = $(b – a)^2 = b^2 – 2ab + a^2$

Area of big square = Area of 4 triangles + Area of small square

$\Rightarrow c^2 = 2ab + (b^2 – 2ab + a^2)$

$\Rightarrow c^2 = 2ab + b^2 – 2ab + a^2$

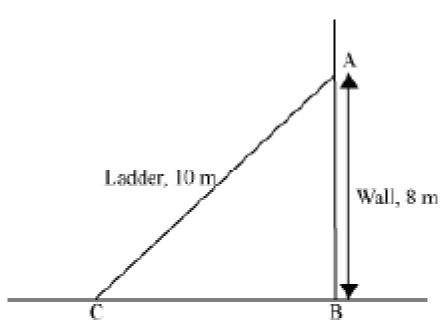
$\Rightarrow c^2 = b^2 + a^2$

Bhaskaracharya proved this theorem by using property of similarity also, but we will not discuss it here.

Note: Conventionally in ΔABC, we consider the lengths of sides opposite to vertices A, B and C as *a*, *b* and *c* respectively.

In real life, we come across many situations where a right angle is formed. Let us consider such a situation.

A 10m long ladder is placed on a wall such that the ladder touches the wall at 8m above the ground. This situation can be shown geometrically as follows.



In the above figure, AB is the wall of height 8 m and AC is the ladder of length 10 m. We know that a wall is perpendicular to the floor, i.e. AB is perpendicular to BC. Thus, ∠ABC is a right angle.

Now, can we calculate the distance of the foot of the ladder from the base of the wall?

We can calculate the distance of the foot of the ladder from the base of the wall by using Pythagoras theorem. To understand how Pythagoras theorem is helpful in this case, look at the following video.

In this way, we can use Pythagoras theorem in many situations where right-angled triangle is formed.

Is the converse of Pythagoras theorem also true?

Yes, the converse of Pythagoras theorem is also true.

Its converse can be stated as follows:

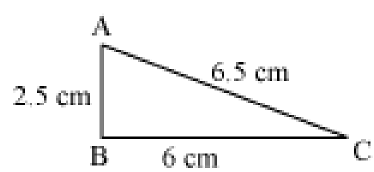
"**In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle".**

But how will we prove it?

Thus, in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Using the converse, we can check whether the given triangle is a right triangle or not.

Let ABC be a triangle with sides AB = 2.5 cm, BC = 6 cm and CA = 6.5 cm. Can we say that the triangle ABC is a right-angled triangle?



Here, $(AB)^2 = (2.5)^2 = 6.25$

$(BC)^2 = 6^2 = 36$

And, $(CA)^2 = (6.5)^2 = 42.25$

Therefore, we obtain

$(AB)^2 + (BC)^2 = 6.25 + 36 = 42.25 = (CA)^2$

Thus, using the converse of Pythagoras theorem, we can say that the angle opposite to the side CA, i.e. ∠B, is a right angle.

Pythagorean triplet: Any three natural numbers *a*, *b*, *c* form a Pythagorean triplet if it satisfies $a^2 = b^2 + c^2$ irrespective of order.

General form to find the Pythagorean Triplets:

For natural number: $2n, (n^2 - 1), (n^2 + 1)$ where *n* may be even or odd.

For odd natural numbers: $n, 1/2(n^2 - 1), 1/2(n^2 + 1)$ where *n* is odd, $n \in N$.

Any number of pythagorean triplets can be generated by giving values to *n*.

For example, the number 39, 80, and 89 forms a Pythagorean triplet.

$39^2 = 1521$

$80^2 = 6400$

$89^2 = 7921$

Now, 1521 + 6400 = 7921

$\therefore 39^2 + 80^2 = 89^2$

Here, the square of a number is equal to the sum of the squares of the other two numbers. Therefore, we can say that 39, 80, and 89 forms a Pythagorean triplet.

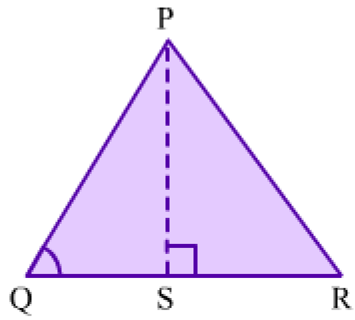
Applications of Pythagoras theorem:

Though Pythagoras theorem holds true for only right angled triangle, it can be applied to obtuse and acute angled triangles as well. The obtained results are discussed below:

(1) In acute angled ΔPQR , if PS ⊥ QR, QSR is a line and ∠Q < 90° then

$PR^2 = PQ^2 + QR^2 – 2QR . QS$

Proof: Observe the acute angled triangle ΔPQR.



Here, PS ⊥ QR, QSR is a line and ∠Q < 90°.

In ΔPSQ, we have

$PQ^2 = PS^2 + QS^2$ ...(1) (By Pythagoras theorem)

In ΔPSR, we have

$PR^2 = PS^2 + SR^2$ (By Pythagoras theorem)

$\Rightarrow PR^2 = PS^2 + (QR - QS)^2$

$\Rightarrow PR^2 = PS^2 + QR^2 + QS^2 - 2QR \cdot QS$

$\Rightarrow PR^2 = (PS^2 + QS^2) + QR^2 - 2QR \cdot QS$

$\Rightarrow PR^2 = PQ^2 + QR^2 - 2QR \cdot QS$ [Using (1)]

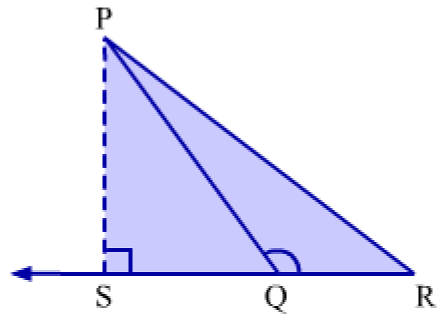
Hence proved.

(2) In obtuse angled ΔPQR , if PS ⊥ QR, SQR is a line and ∠Q > 90° then

$PR^2 = PQ^2 + QR^2 + 2QR \cdot QS$

Proof:

Observe the acute angled triangle ΔPQR.



Here, PS ⊥ QR, SQR is a line and ∠Q > 90°.

In ΔPSQ, we have

$PQ^2 = PS^2 + QS^2$ ...(1) (By Pythagoras theorem)

In ΔPSR, we have

$PR^2 = PS^2 + SR^2$ (By Pythagoras theorem)

$\Rightarrow PR^2 = PS^2 + (QR + QS)^2$

$\Rightarrow PR^2 = PS^2 + QR^2 + QS^2 + 2QR \cdot QS$

$\Rightarrow PR^2 = (PS^2 + QS^2) + QR^2 + 2QR \cdot QS$

$\Rightarrow PR^2 = PQ^2 + QR^2 + 2QR \cdot QS$ [Using (1)]

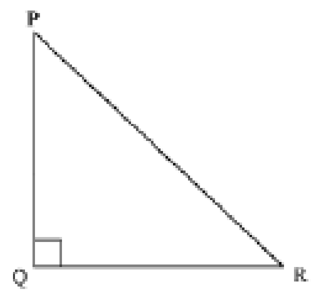
Hence proved.

Now, let us discuss some more examples based on Pythagoras theorem and its converse.

Example 1:

ΔPQR is an isosceles triangle, right angled at Q. Prove that $PR^2 = 2PQ^2$.

Solution:



Here, PQR is an isosceles triangle, right angled at Q. Therefore,

PQ = QR … (1)

Now, using Pythagoras theorem, we obtain

$(PR)^2 = (PQ)^2 + (QR)^2$

Using equation (1), we obtain

$(PR)^2 = (PQ)^2 + (PQ)^2$

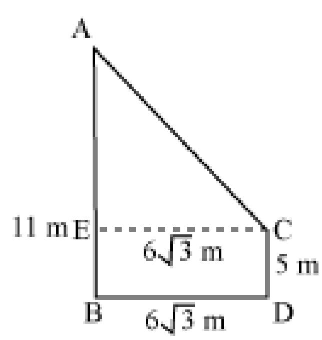
$(PR)^2 = 2(PQ)^2$

Example 2:

Two poles are of length 5m and 11m. The distance between the feet of the poles is $6\sqrt{3}$ m. Find the distance between the tops of the poles.

Solution:

The figure for the given situation can be drawn as follows.



In the above figure, the poles are denoted by AB and CD, where AB = 11 m and CD = 5 m. The distance between the feet of the poles, i.e. BD, is $6\sqrt{3}$ m.

Let us draw a perpendicular CE from C on AB.

AC is the distance between the top of the poles.

Here, BD = CE = $6\sqrt{3}$ m

And AE = AB− BE

= AB − CD [since BE = CD]

= (11 − 5) m

= 6 m

Using Pythagoras theorem in ΔACE, we obtain

$(AC)^2 = (AE)^2 + (EC)^2$

$(AC)^2 = (6)^2 m^2 + \left(6\sqrt{3}\right)^2 m^2$

$= (36 + 108) m^2$

$= 144 m^2$

⇒ AC = $\sqrt{144 m^2}$

⇒ AC = 12 m

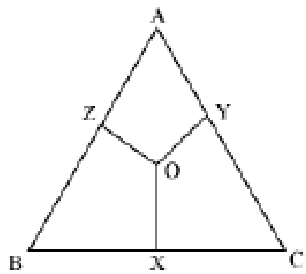
Thus, the distance between the tops of the poles is 12 m.

Example 3:

O is any point in the interior of ΔABC and OX, OY, and OZ are the perpendiculars drawn from O to BC, CA, and AB respectively.

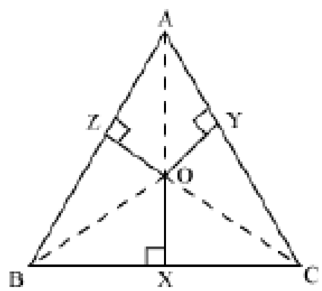
Prove that

$AZ^2 + BX^2 + CY^2 = AY^2 + CX^2 + BZ^2$



Solution:

Let us join OA, OB, and OC in the given figure.



Using Pythagoras theorem in ΔOZA, we obtain

$OA^2 = OZ^2 + AZ^2$

or $AZ^2 = OA^2 - OZ^2$ … (i)

Similarly, in ΔBOX and ΔCOY respectively, we obtain

$BX^2 = OB^2 - OX^2$ … (ii)

and $CY^2 = OC^2 - OY^2$ … (iii)

On adding equations (i), (ii) and (iii), we obtain

$AZ^2 + BX^2 + CY^2 = OA^2 + OB^2 + OC^2 - OZ^2 - OX^2 - OY^2$

$AZ^2 + BX^2 + CY^2 = (OA^2 - OY^2) + (OB^2 - OZ^2) + (OC^2 - OX^2)$ … (iv)

Using Pythagoras theorem in ΔOYA, we obtain

$OA^2 = OY^2 + AY^2$

Or $AY^2 = OA^2 - OY^2$

Similarly, $BZ^2 = OB^2 - OZ^2$

and $CX^2 = OC^2 - OX^2$

Using these in equation (iv), we obtain

$AZ^2 + BX^2 + CY^2 = AY^2 + BZ^2 + CX^2$

Hence, proved.