CBSE Test Paper 03
CH-08 Binomial Theorem

Section A

1. If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^n$ are in A.P. then
   a. $n^2-9n+18=0$
   b. $n^2-6n+12=0$
   c. $n^2-9n+14=0$
   d. $n^2-5n+14=0$
2. Coefficient of a^2b^5 in the expansion of $(a+b)^3(a-2b)^4$ is
   a. 48
   b. 24
   c. 96
   d. -24
3. In Pascal's triangle, each row begins with 1 and ends in
   a. -1
   b. 0
   c. 2
   d. 1
4. If the 21 st and 22nd terms in the expansion of $(1+x)^{44}$ are equal, find x
   a. $\frac{7}{6}$

b. $\frac{5}{8}$

c. $\frac{7}{8}$

d. $\frac{6}{8}$

5. $\sqrt{5}\left\{(\sqrt{5}+1)^4-(\sqrt{5}-1)^4\right\}$is

a. 0

b. 212

c. 240

d. 224

6. Fill in the blanks: An approximation of $(0.99)^5$, using the first three terms of its binomial expansion is ________.
7. Fill in the blanks: In the objective function Z = ax + by, x and y are called ________ variables.
8. Find the middle term in the expansion of $\left(\frac{2}{3}x^2-\frac{3}{2x}\right)^{20}$.
9. Find the constant term in the expansion of $\left(x-\frac{1}{x}\right)^{10}$.
10. Find the coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$.
11. Using binomial theorem, evaluate each of the following: $(102)^5$
12. Show that the middle term in the expansion of $(1 + x)^{2n}$ is $\frac{1\cdot3\cdot5\ldots(2n-1)}{n!}$ 2^n x^n, where n is a positive integer.
13. Expand the given expression $\left(x+\frac{1}{x}\right)^6$
14. Find the middle terms in the expansion of : $\left(3x-\frac{x^3}{6}\right)^7$
15. The 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x + a)^n$ are respectively 84, 280 and 560. Find the values of x, a and n.

CBSE Test Paper 03

CH-08 Binomial Theorem

Solution

Section A

1. (c) $n^2 - 9n + 14 = 0$

   Explanation: Given that $^nC_1$, $^nC_2$ and $^nC_3$ are in A.P

   Hence we have $^nC_1 + {^nC_3} = 2.{^nC_2}$
   $\Rightarrow n + \frac{n(n-1)(n-2)}{6} = 2 \cdot \frac{n(n-1)}{2}$
   $\Rightarrow n\left[\frac{6+(n-1)(n-2)}{6}\right] = n(n-1)$
   $\Rightarrow 6 + (n-1)(n-2) = 6(n-1)$
   $\Rightarrow n^2 - 3n + 8 = 6n - 6$
   $\Rightarrow n^2 - 9n + 14 = 0$

2. (d) -24

   Explanation: We have $(a+b)^3(a-2b)^4 = [{^3C_0}\, a^3 + {^3C_1}\, a^2b + {^3C_2}\, a^1b^2 + {^3C_3}\, b^3]$

   $[{^4C_0}\,(a)^4 + {^4C_1}\,(a)^3\,(-2b) + {^4C_2}\,(a)^2(-2b)^2 + {^4C_3}\,(a)^1(-2b)^3 + {^4C_4}\,(-2b)^4]$
   $= \left(a^3 + 3a^2b + 3a^1b^2 + b^3\right)\left(a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4\right)$
   Hence we have the sum of the terms containing a^2b^5 are
   $3a^2b \times 16b^4 - 32ab^3 \times 3a^1b^2 + 24a^2b^2 \times b^3 = a^2b^5(48 - 96 + 24) = -24a^2b^5$

3. (d) 1

   Explanation: The pascal's triangle is given by

   1 1

   1 2 1

   1 3 3 1

4. (c) $\frac{7}{8}$

   Explanation: We have the general terms of $(x+a)^n$ is $T_{r+1} = {^nC_r}\,(x)^{n-r}a^r$

Now consider $(1+x)^{44}$

Here $T_{r+1} = ^{44}C_r \quad (1)^{44-r}(x)^r$

So, $T_{21} = T_{20+1} = ^{44}C_{20}(x)^{20}$ and $T_{22} = T_{21+1} = ^{44}C_{21}(x)^{21}$

Given $T_{21} = T_{22} \Rightarrow ^{44}C_{20} \quad (x)^{20} = ^{44}C_{21} \quad (x)^{21}$

$\Rightarrow x = \frac{44}{44C_{21}} = \frac{(44)!}{(20)!\cdot 24!}\frac{(21)!\cdot 23!}{(44)!} = \frac{21}{24} = \frac{7}{8}$

5. (c) 240

Explanation: $\sqrt{5}\left\{(\sqrt{5}+1)^4 - (\sqrt{5}-1)^4\right\}$

Let $a = \sqrt{5} \quad and \quad b = 1$

Then $(\sqrt{5}+1)^4 - (\sqrt{5}-1)^4 = (a+b)^4 - (a-b)^4$

= $[^nC_0\, a^4 + {}^4C_1\, a^3b + {}^4C_2\, a^2b^2\, {}^4C_3\, a^1b^3 + {}^4C_4\, b^4] - [{}^4C_0\, a^4 - {}^4C_1\, a^3b + {}^4C_2\, a^2b^2 - {}^4C_3\, a^1b^3 + {}^4C_4\, b^4]$

= $2\, [{}^4C_1\, a^3b + {}^4C_3\, a^1b^3]$

$= 2\left[4a^3b + 4a^1b^3\right] = 8ab\left[a^2+b^2\right]$

$\therefore (\sqrt{5}+1)^4 - (\sqrt{5}-1)^4 = 8\cdot\sqrt{5}\cdot 1\left[(\sqrt{5})^2 + 1^2\right] = 48\sqrt{5}$

Hence $\sqrt{5}\left\{(\sqrt{5}+1)^4 - (\sqrt{5}-1)^4\right\} = \sqrt{5}.48\sqrt{5} = 240$

6. 0.951

7. decision

8. Here n = 20, which is an even number. So, $\left(\frac{20}{2}+1\right)^{\text{th}}$ term i.e. 11^{th} term is the middle term.

Hence, the middle term = $T_{11} = T_{10+1} = {}^{20}C_{10}\left(\frac{2}{3}x^2\right)^{20-10}\left(-\frac{3}{2x}\right)^{10} = {}^{20}C_{10}x^{10}$.

9. We have, $\left(x - \frac{1}{x}\right)^{10}$

General term is

$T_{r+1} = {}^{10}C_r(x)^{10-r}\left(\frac{-1}{x}\right)^r = ^{10}C_r x^{10-r}\frac{(-1)^r}{x^r}$ = (- 1)r ${}^{10}C_r x^{10-2r}$.......(i)

We have to find constant term i.e., the term independent of x. For this, put 10 - 2r = 0

$\Rightarrow$ r = 5

On putting r = 5 in Eq. (i), we get

$T_{5+1} = (-1)^5\, {}^{10}C_5 x^{10-2(5)}$

$= (-1) \times \frac{10!}{5!5!} = (-1) \times \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1} \times 1$

= - 252

Hence, constant term in the expansion of $\left(x - \frac{1}{x}\right)^{10}$ is - 252.

10. Coefficient of x^n in (1 + x) $(1 - x)^n$

    = Coefficient of x^n in $(1 - x)^n$ + Coefficient of x^{n-1} in $(1 - x)^n$

    $= (-1)^n \, {}^nC_n + (-1)^{n-1} \, {}^nC_{n-1}$

    $= (-1)^n (1 - n)$

11. $(102)^5 = (100 + 2)^5$

    Using binomial theorem, we have

    $(100+2)^5 = {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2$
    $+ {}^5C_3(100)^2(2)^3 + {}^5C_4(100)(2)^4 + {}^5C_5(2)^5$

    $= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5$

    = 10000000000 + 1000000000+ 40000000 + 800000 + 8000 + 32

    = 11040808032

12. As 2n is even, the middle term in the expansion of $(1 + x)^{2n}$ is $\left(\frac{2n}{2} + 1\right)$ th i.e., (n + 1)th term, which is given by

    $T_{n+1} = {}^{2n}C_n(1)^{2n-n}(x)^n = {}^{2n}C_n x^n = \frac{(2n)!}{n!(2n-n)!} \cdot x^n$

    $= \frac{(2n)!}{n!n!} \cdot x^n$

    $= \frac{2n(2n-1)(2n-2)\cdots4\cdot3\cdot2\cdot1}{n!n!} \cdot x^n$

    $= \frac{1\cdot2\cdot3\cdot4\cdots(2n-2)(2n-1)(2n)}{n!n!} \cdot x^n$

    $= \frac{[1\cdot3\cdot5\ldots(2n-1)][2\cdot4\cdot6\cdots(2n)]}{n!n!} \cdot x^n$

    $= \frac{[1\cdot3\cdot5\cdots(2n-1)][2\cdot(2\cdot2)\cdot(2\cdot3)\cdots(2\cdot n)]\cdot x^n}{n!n!}$

    $= \frac{[1\cdot3\cdot5\cdots(2n-1)]2^n[1\cdot2\cdot3\cdots n]}{n!n!} \cdot x^n$

    $= \frac{[1\cdot3\cdot5\cdots(2n-1)]2^n \cdot n!}{n!n!} \cdot x^n$

    $= \frac{[1\cdot3\cdot5\cdots(2n-1)]\cdot2^n}{n!} \cdot x^n$

13. Using binomial theorem for the expansion of $\left(x + \frac{1}{x}\right)^6$ we have

    $\left(x + \frac{1}{x}\right)^6 = = {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2 + {}^6C_3(x)^3\left(\frac{1}{x}\right)^3$

$+{}^6C_4(x)^2\left(\frac{1}{x}\right)^4+{}^6C_5(x)\left(\frac{1}{x}\right)^5+{}^6C_6\left(\frac{1}{6}\right)^6$

$= x^6+6\cdot x^5\cdot\frac{1}{x}+15\cdot 4x^4\cdot\frac{1}{x^2}+20\cdot x^3\cdot\frac{1}{x^3}+15\cdot x^2\cdot\frac{1}{x^4}+6\cdot x\cdot\frac{1}{x^5}+\frac{1}{x^6}$

$= x^6+6x^4+15x^2+20+\frac{15}{x^2}+\frac{6}{x^4}+\frac{1}{x^6}$

14. Hence n = n7, which is odd number.

So, $\left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+1}{2}+1\right)^{th}$ i.e 4th and 5th terms are two middle terms

$\therefore T_4=T_{3+1}={}^7C_3(3x)^{7-3}\left(-\frac{x^3}{6}\right)^3=(-1)^3 7C_3(3x)^4\left(\frac{x^3}{6}\right)^3$

font-family : Tahoma font - size: 8px $\Rightarrow T_4=-35\times 81x^4\times\frac{x^9}{216}=-\frac{105x^{13}}{8}$

and $T_5=T_{4+1}={}^7C_4(3x)^{7-4}\left(-\frac{x^3}{6}\right)^4={}^7C_4(3x)^3\left(-\frac{x^3}{6}\right)^4$

$\Rightarrow T_5=35\times 27x^3\times\frac{x^{12}}{1296}=\frac{35x^{15}}{48}$

Hence, the middle terms are $-\frac{105x^{13}}{8}$ and $\frac{35x^{15}}{48}$

15. It is given that: T_3 = 84, T_4 = 280 and T_5 = 560

We have, $\frac{T_{r+1}}{T_r}=\frac{{}^nC_r x^{n-r}a^r}{{}^nC_{r-1}x^{n-r+1}a^{r-1}}=\frac{n-r+1}{r}\cdot\frac{a}{x}$

$\therefore \quad \frac{T_4}{T_3}=\frac{n-2}{3}\cdot\frac{a}{x}$ and $\frac{T_5}{T_4}=\frac{n-3}{4}\cdot\frac{a}{x}$

$\Rightarrow \quad \frac{280}{84}=\frac{n-2}{3}\cdot\frac{a}{x}$ and $\frac{560}{280}=\frac{n-3}{4}\cdot\frac{a}{x}$ [T_3 = 84, T_4 = 280 and T_5 = 560]

$\Rightarrow \quad \frac{10}{3}=\frac{n-2}{3}\cdot\frac{a}{x}$ and $\frac{2}{1}=\frac{n-3}{4}\cdot\frac{a}{x}$

$\Rightarrow \quad \frac{a}{x}=\frac{10}{n-2}$ and $\frac{a}{x}=\frac{8}{n-3}$

$\Rightarrow \quad \frac{10}{n-2}=\frac{8}{n-3}$ $\Rightarrow$10n - 30 = 8n - 16 $\Rightarrow$ n = 7

Putting n = 7 in $\frac{a}{x}=\frac{10}{n-2}$, we get

$\frac{a}{x}=\frac{10}{5}\Rightarrow 2x=a$

Now, T_3 = 84

$\Rightarrow {}^nC_2\, x^{n-2}\, a^2 = 84$

$\Rightarrow {}^7C_2\, x^5\, (2x)^2 = 84$ [$\because$ a = 2x and n = 7]

$\Rightarrow 21\times 2^4\times x^7 = 84 \Rightarrow x^7 = 1 \Rightarrow x = 1$

$\therefore$ a = 2x = 2 $\times$ 1 = 2

Hence, n = 7, a = 2 and x = 1.