CBSE Test Paper 04

Chapter 14 Oscillations

1. The time period of a simple pendulum is given by (l is length of pendulum and g the acceleration due to gravity). **1**
   a. $T = 2\pi\sqrt{\frac{l}{g^2}}$
   b. $T = 2\pi\sqrt{\frac{l^2}{g}}$
   c. $T = 2\pi\sqrt{\frac{l}{g}}$
   d. $T = 2\pi\sqrt{\frac{g}{l}}$

2. A spring has a certain mass suspended from it and its period for vertical oscillations is T_1 . The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillations is now T_2 .The ratio of T_1/ T_2 is **1**
   a. $\frac{1}{2}$
   b. $2\sqrt{2}$
   c. $\sqrt{2}$
   d. 1.0

3. The damped system differential equation is **1**
   a. $m\frac{d^2x}{d^2t} + kx = 0$
   b. $m\frac{d^2x}{d^2t} + b\frac{dx}{dt} + kx = 0$
   c. $m\frac{d^2x}{d^2t} + b + kx = 0$
   d. $m\frac{d^2x}{d^2t} + b\frac{dx}{dt} = 0$
4. The length of the second’s pendulum on the surface of earth is 1m. The length of second’s pendulum on the surface of moon, where g is 1/6th the value on earth is **1**
   a. 36 m
   b. 6 m
   c. 1/36 m

d. 1/6 m

5. What is the ratio between the height H of a mountain and the depth h of a mine if a pendulum swings with the same period at the top of the mountain and at the bottom of the mine? **1**

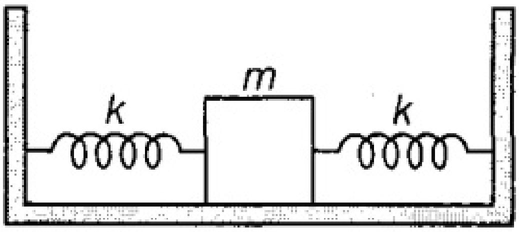
   a. $\frac{1}{2}$
   b. 1
   c. 4
   d. 6.0

6. If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob density of the fluid), the time period of the pendulum increased or decrease. **1**

7. The soldiers marching on a suspended bridge are advised to go out of steps. Why? **1**

8. What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion? **1**

9. Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown in figure. Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations. **2**



10. What are the two basic characteristics of a simple harmonic motion? **2**

11. What is ratio of frequencies of the vertical oscillations when two springs of spring constant K are connected in series and then in parallel? **2**

12. At what distance from the mean position, is the kinetic energy in a simple harmonic oscillator equal to potential energy? **3**

13. A particle executes SHM with a time period of 2 s and amplitude 5 cm. Find **3**

    i. displacement
    ii. velocity and
    iii. acceleration after 1/3 s starting from the mean position.

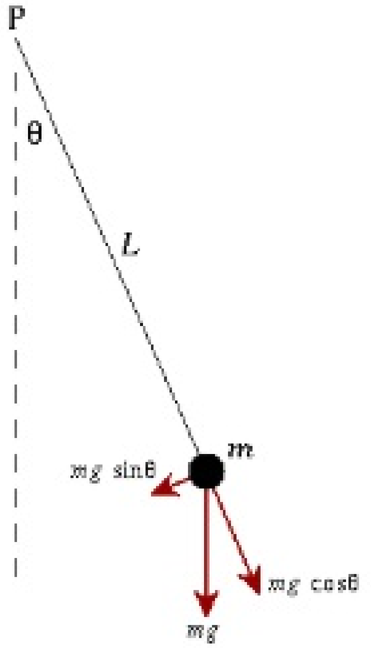
14. Show that the total energy of a body executing SHM is independent of time? **3**

15. A cylindrical piece of cork of base area A, density ρ and height L floats in a liquid of density ρ_{L}. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically and find its time period of oscillations. **5**

Answer

1. c. $T = 2\pi\sqrt{\frac{l}{g}}$

   Explanation: The figure shows the motion of a simple pendulum



   The restoring force is $F = -mg\, sin\, \theta$ as the angle θ is small so $sin\, \theta \simeq \theta$
   we have $F = -mg\ \theta$ from S H M and also $F = -kx$
   $\therefore mg\theta = kx$
   $\frac{m}{k} = \frac{g\theta}{x}$
   as $l = \frac{\theta}{x}$
   $\frac{m}{k} = \frac{g}{l}$
   As $\omega = \sqrt{\frac{k}{m}}$
   So $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$
   $= 2\pi\sqrt{\frac{l}{g}}$

2. d. 1.0

   Explanation: As the spring constant k=F/x i.e. restoring force per unit displacement remains same thus time period in both cases will be same. Thus their ratio will be equal to 1.

3. b. $m\frac{d^2x}{d^2t}+b\frac{dx}{dt}+kx=0$

Explanation: The forces on the system is restoring force $F=-kx$ and the damping force is $F=-bv=-b\frac{dx}{dt}$ where 'k' is the force constant and 'b' is the damping constant

The net force is F= m*a which is equal to the sum of restoring force $F=-kx$ and the damping force is $F=-bv=-b\frac{dx}{dt}$

$m\frac{d^2x}{dt^2}=-kx-b\frac{dx}{dt}$

$\therefore m\frac{d^2x}{dt^2}+kx+b\frac{dx}{dt}=0$

4. d. 1/ 6m

Explanation: Time period of a second pendulum is 1 s. Relation for time period is T= $2\pi\sqrt{\frac{L}{g}}$

if T remain constant $L\ \alpha\ g$

$\frac{L_{earth}}{L_{moon}}=\frac{g_{earth}}{g_{moon}}$

given L_{earth} = 1m

$\frac{1}{L_{moon}}=\frac{g}{\frac{g}{6}}L_{moon}=\frac{1}{6}m$

5. a. $\frac{1}{2}$

Explanation: Gravity above the earth surface is given by

$g=g_0(1-\frac{2h}{R})$

And below the earth surface is given by

$g=g_0(1-\frac{d}{R})$

Where h and d are hight and depth below the earth surface. As given time period at both location is same

T1=T2

$2\pi\sqrt{\frac{L}{g_{up}}}=2\pi\sqrt{\frac{L}{g_{down}}}$

$\Rightarrow g_{up}=g_{down}$

$g_0(1-\frac{2h}{R})=g_0(1-\frac{d}{R})$

$\Rightarrow\ 2h=d$

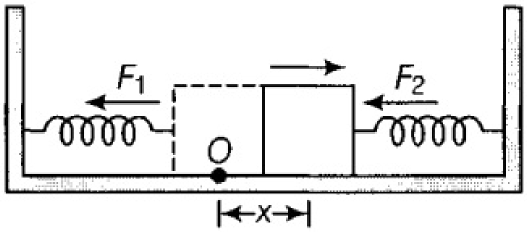
$\frac{h}{d}=\frac{1}{2}$

6. Increased.

7. The soldiers marching on a suspended bridge are advised to go out of steps because in such a case the frequency of marching steps matches with natural frequency of the suspended bridge and hence resonance takes place, as a result amplitude of oscillation increases enormously which may lead to the collapsing of bridge.

8. A periodic motion repeats after a definite time interval T. So, y(t) = y(t + T) = y(t + 2T) etc.

9. Force exerted by left spring, trying to pull the mass towards the mean position.
   i.e. Restoring force of the left spring, $F_1 = -kx$

   Similarly, force exerted by the right spring, trying to push the mass towards the mean position,
   Restoring force of the right spring, $F_2 = -kx$. k is the spring constant of both the springs.
   The net force acting on the mass due to both the springs is
   $F = F_1 + F_2 = -2kx$
   Thus, the force acting on the mass is proportional to its displacement x and is directed towards its mean position. Which are the two main features of an SHM. Hence, the motion of the mass m is simple harmonic. Comparing the equation, F = -2kx with general equation of restoring force, F =- k'x.
   We have, equivalent force constant k' = 2k.
   The time period of oscillation is T $=2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$

10. Two basic characteristics of a simple harmonic motion are:

    i. Acceleration is directly proportional to displacement from mean position, and the direction of acceleration is towards mean position.
    ii. Restoring force is directly proportional to displacement, the direction of force and

displacement are opposite i.e., $F=-kx$.

11. If two spring of spring constant K are connected in parallel, then effective resistance in parallel = K_P = K + K = 2K

Let f_P = frequency in parallel combination.

$f_P=\frac{1}{2\pi}\sqrt{\frac{K_p}{M}}$

Put the value of K_P

$f_p=\frac{1}{2\pi}\sqrt{\frac{2K}{M}}\rightarrow(1)$

In Series combination, effective spring constant for 2 sprigs of spring constant K is :-

$\frac{1}{K_S}=\frac{1}{K}+\frac{1}{K}$

$\frac{1}{K_S}=\frac{K+K}{K\times K}=\frac{2K}{K^2}$

$\frac{1}{K_S}=\frac{K}{2}$ or $\mathrm{K_s}=\frac{K}{2}$

Let f_S = frequency in series combination

$f_S=\frac{1}{2\pi}\sqrt{\frac{K_s}{M}}$

$f_S=\frac{1}{2\pi}\sqrt{\frac{K}{M}}$

$f_S=\frac{1}{2\pi}\sqrt{\frac{K}{2M}}\rightarrow(2)$

Divide equation (2) by (1)

$\frac{f_S}{f_P}=\frac{\frac{1}{2\pi}\sqrt{\frac{K}{2M}}}{\frac{1}{2\pi}\sqrt{\frac{2K}{M}}}$

$\frac{f_S}{f_P}=\frac{1}{2\pi}\sqrt{\frac{K}{2M}}\times\frac{2\pi\times\sqrt{M}}{\times\sqrt{2K}}$

$\frac{f_S}{f_P}=\sqrt{\frac{K\times M}{2M\times 2K}}$

$\frac{f_S}{f_P}=\sqrt{\frac{1}{4}}$

$\frac{f_S}{f_P}=\frac{1}{2}$

$f_S:f_P=1:2$

12. Let the displacement of particle executing S.H.M = Y

Amplitude of particle executing S.H.M = a

Mass of particle = m

Angular velocity = ω

The kinetic energy= $\frac{1}{2}m\,\omega^2\left(a^2-y^2\right)$

$Potential\ energy = \frac{1}{2}m\ \omega^2 y^2$

If kinetic energy = Potential energy

$\frac{1}{2}m\ \omega^2\ \left(a^2 - y^2\right) = \frac{1}{2}m\ \omega^2 y^2$

$a^2 - y^2 = y^2$

$a^2 = 2y^2$

$a = \sqrt{2y} \rightarrow$ Square root on both sides

$Y = \frac{a}{\sqrt{2}}$

13. Here, T = 2 s, A = 5 cm, t = $\frac{1}{3}$ s

    i. For the particle starting from mean position, (i.e.ϕ = 0) displacement,
    x = A sinωt = $A\sin\frac{2\pi}{T}t$
    $= 5\sin\frac{2\pi}{2} \times \frac{1}{3} = 5\sin\frac{\pi}{3}$ (since t = 1/3 s)
    $= 5 \times \frac{\sqrt{3}}{2}$ = 4.33 cm
    ii. Velocity of the particle executing SHM, $v = \frac{dx}{dt} = \frac{d(A\sin\omega t)}{dt} = A\omega\cos\omega t$
    $= \frac{2\pi A}{T}\cos\frac{2\pi}{T}t = \frac{2\pi\times5}{2}\cos\frac{\pi}{3}$
    $= 5 \times 3.14 \times 0.5$
    = 7.85 cm s^{-1}
    iii. Acceleration of the particle executing SHM, a = $\frac{dv}{dt} = \frac{d(A\omega\cos\omega t)}{dt} = -A\omega^2\sin\omega t$
    $= -\frac{4\pi^2 A}{T^2}\sin\frac{2\pi}{T}t$
    $= -\frac{4\times9.87\times5}{4}\sin\frac{\pi}{3}$
    $= -9.87 \times 5 \times \frac{\sqrt{3}}{2}$ = -42.73 cm s^{-2}
    $\therefore |a| = 42.73\ \text{cms}^{-2}$

14. $\Rightarrow$ Let y = displacement at any time‘t’
    $\Rightarrow$ a = amplitude
    $\Rightarrow$ ω = Angular frequency
    $\Rightarrow$ v = velocity,
    $\Rightarrow$ y = a Sin ωt
    $\Rightarrow$ So, $v = \frac{dy}{dt} = \frac{d}{dt}(\text{a}\sin\omega\text{t})$
    $\Rightarrow$ v = a ω cos wt
    $\Rightarrow$ Now, kinetic energy = K. E. = $\frac{1}{2}mv^2$
    $\Rightarrow K.E. = \frac{1}{2}m\omega^2 a^2\cos^2\omega t \rightarrow$ 1)

$\Rightarrow$ Potential energy $= \frac{1}{2}ky^2$

$\Rightarrow P.E = \frac{1}{2}ka^2\sin^2\omega t \rightarrow 2)$

Adding equation (1) & (2)

$\Rightarrow$ Total energy = K.E. + P.E.

$= \frac{1}{2}m\omega^2a^2\cos^2\omega t + \frac{1}{2}ka^2\sin^2\omega t$

$\Rightarrow$ Since $\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega^2 m = k^2$

$\Rightarrow$ Total energy $= \frac{1}{2}m\omega^2a^2\cos^2\omega t + \frac{1}{2}ka^2\sin^2\omega t$

$= \frac{1}{2}ka^2\cos^2\omega t + \frac{1}{2}ka^2\sin_2\omega t$

$= \frac{1}{2}ka^2\left(\cos^2\omega t + \sin^2\omega t\right)$

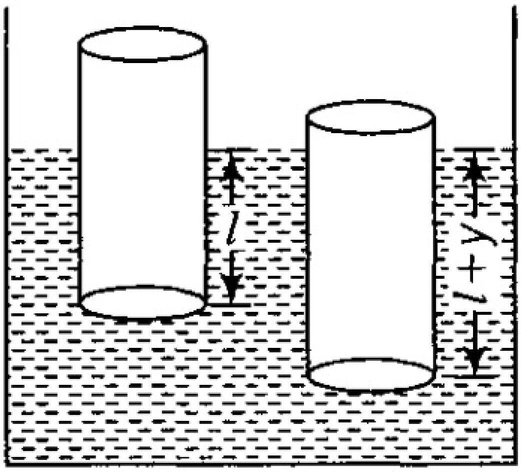
$\Rightarrow$ Total energy $\frac{1}{2}ka^2$

$\Rightarrow$ Thus total mechanical energy is always constant is equal to $\frac{1}{2}ka^2$. The total energy is independent to time.

15. Consider a cylinder of mass m, length L, density of material ρ and uniform area of cross-section A.

Therefore, mass of the cylinder(m) = A Lρ

Let the cylinder is floating in the liquid of density ρ_1



In equilibrium, let l be the length of cylinder dipping in liquid.

In equilibrium, weight of cylinder = Weight of liquid displaced

$\Rightarrow$ mg = A l ρ_1 g

$\Rightarrow$ m = A lρ_1 ...(ii)

Now say the cylinder is pushed down by y into the liquid, then

Total upward thrust, F_2 = A (l + y) ρ_1 g (since effective depth = l+y)

Restoring force, F = - (F_2 - mg)

$\Rightarrow$F = -[A(l + y)ρ_1g - Alρ_1 g] = -A$\rho_1$gy........(iii)

We know that In SHM, F $\propto$ - y

$\Rightarrow$ F = - k y ...(iv)

Comparing equation (iii) with equation (iv) we get,

Spring factor, k = Aρ_1g

Inertia factor = mass of the cylinder(m) = ALρ

Now, we know the formula of time period, T = $2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

Hence, T = $2\pi\sqrt{\frac{AL\rho}{A\rho_1 g}} = 2\pi\sqrt{\frac{L\rho}{\rho_1 g}}$.............(v)

Using, m = Alρ_1 = ALρ

So, $l\rho_1 = L\rho$

Using the above value we get time period,

T = $2\pi\sqrt{\frac{l\rho_1}{g\rho_1}} = 2\pi\sqrt{\frac{l}{g}}$