CHAPTER 15

Surface Area and Volume of Solids

Introduction

We live in a three dimensional world. If we can see and touch a 3-D figure then we can measure its length, breadth and height. Many times we need to measure some other aspects of these figures such as volume, area etc. For example, while buying-selling land we need to know the area; to know how much material is needed to make a statue we need to know the volume etc.

Before we learn how to find the area and volume of 3-D shapes, let us open them up.

Surface Net for Making 3-D Shapes

Kamli and Mangi had a cubical box made of cardboard. They cut the edges of the box using a pair of scissors as shown in figure –1 and spread it out (figure–2). They discussed with each-other about the shape of the open box. After some time, they joined the edges of the box again using cello-tape and were very happy. They showed the box to Mangi's father and talked about what they had done.

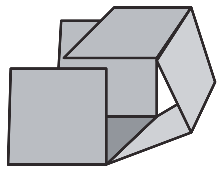


Figure - 1

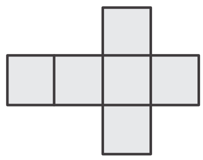


Figure - 2

Mangi's father was very pleased with their work. He asked if the box could be opened in any other way. Kamli and Mangi opened up the box in a different way and got the shape shown in figure–3. They immediately tried to make the box again using the new shape.

Mangi's father asked the two children to cut one corner of the cubical box to make other open shapes. The children made the shapes shown below.

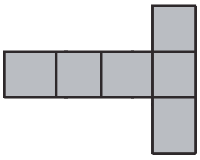


Figure - 3

Mangi's father discussed different ways of getting various shapes. In this activity, we saw that when we cut along the edges of a cubic cardboard box and spread it out, we can many different shapes. These flat figures are known as surface net of cubes. The surface

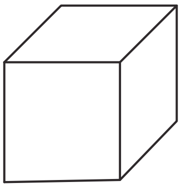


Figure - 4

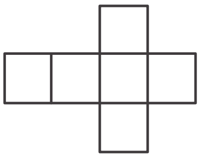


Figure - 5

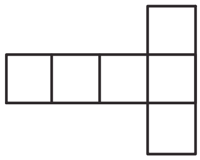


Figure - 6

net of a three dimensional figure is two dimensional. Figures-5 and 6 represent two surface nets of the same cube. Can we obtain more surface nets for this cube?

Try These

1. Take several cubical cardboard boxes and open them in various ways by cutting their edges. How many different open flat figures did you get?
2. Draw the surface net for a cube. We can get 11 different surface nets for a single cube.

   How many surface nets are possible for a cuboid?
3. Using cardboard, prepare a cubical box with sides equal to 4 cm.
4. Using cardboard, prepare a cuboidal box with sides equal to 12 cm, 6 cm and 8 cm.

Parts of a Cube and Cuboid

(i) Face, Edge and Vertex

We have already learnt about cube, cuboid and cylinder in previous classes. In this chapter we will learn about different parts of cubes and cuboids.

Explore

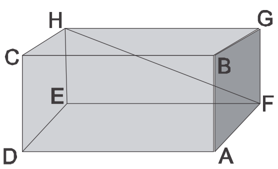


Figure - 7

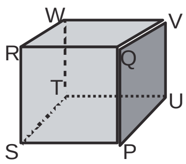


Figure - 8

See the figures of cube and cuboid given below. Here ABCDEFGH is a cuboid and PQRSTUVW is a cube. The nomenclature cube and cuboid is based on the vertices of the figure.

Can you count the faces, edges and vertices of the cube and cuboid and name them?

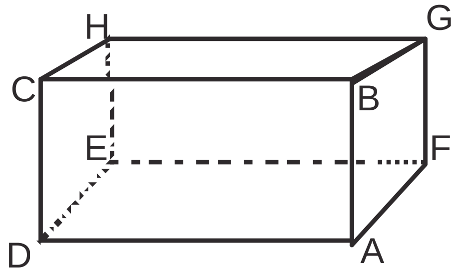
What is the relation between the faces and vertices of a cube and a cuboid? Look carefully at figures-7 and 8 and discuss with your friends. Write your answers in your notebook. Some points related to ABCDEFGH are given below:

- The cuboid has 8 vertices and these are A, B, C, D, E, F, G and H
- A cuboid has 12 edges. Opposite edges in a cuboid are equal. For example, in the cuboid given to us edges AB and CD and EF and HG are equal.
- A cuboid has six faces. In the given figure, the six faces are ABCD, EFGH, AFGB, DEHC, AFED and BGHC. ABCD and EFGH are equal to each other. Similarly, AFGB and DEHC are equal.

Look at figures-7 and 8 and tell which of the edges are equal and which of the faces are equal.

(ii) Diagonal of a Cube and Cuboid

A teacher asked her students: What is the shape of a chalk box or a classroom? All the students answered that they are like cuboids. Then the teacher called five students and asked them to place a pencil inside the chalk box. The students found out that the pencil did not fit in the box if it was laid flat on any of the faces.



Suppose ABCDEFGH is a chalk box. If we try to place the pencil flat on face ADEF (the bottom of the box), it will not fit because the length of the pencil is more than the length of the box. So, does this mean that the pencil cannot be put inside the box? What if we put the pencil in such a way that its ends are towards A and E or D and F? If we put the pencil on ADEF in such a way that it lies along AE or DF then it is possible that the pencil may fit in the box because the length of AE and DF is more than the lengths AD or DE.

(Take another cuboidal box.) If the pencil still does not fit after placing it along AE then is there any other way to place the pencil in the box so that it fits?

If we put the pencil in such a way that its ends are on H and A or G and D then it more probable that the pencil would fit in the box. This means that this distance is the longest length inside the box. These lengths (AH and GD) are the space diagonals of the given cuboid. In cuboid ABCDEFGH, the apace diagonals are AH, GD, FC and EB. The distance between two opposite vertices of any face of the cuboid is called face diagonal.

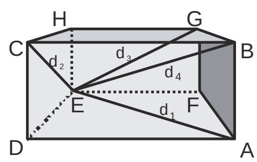


Figure - 9

Now the teacher asked the students to look at figure-9 and then take a cuboidal box and measure the lengths corresponding to d_1, d_2, d_3 and d_4 in the figure using a thread.

Which of the lengths is longest? What will we call these distances?

d_1, d_2, and d_3 are the diagonals but what is d_4? Can we call it a diagonal? It is also a diagonal but different from other three because it does not lie along any of the faces of the cuboid.

Thus, d_1, d_2, d_3 and d_4 are all diagonals but of two different types. d_1, d_2, and d_3 lie along the face of the cuboid and are known as face diagonals while d_4 is present inside the space and is called diagonal of cuboid or space diagonal.

Face and Space Diagonal

If we look at the cardboard box we will find two types of diagonals – one type along the faces of the cuboid and the other type which covers the entire cuboid. The diagonals on the face of the cuboid are called face cuboids and the one which is present inside the three dimensional space of the box is called space diagonal.

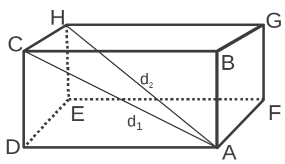


Figure - 10

In geometry, the face diagonals of a cube or cuboid are line segments which connect the vertices on the same face while space diagonal connects vertices of different faces. In the given diagonal (figure-10), AH is space diagonal of the cuboid while AC is a face diagonal.

We can obtain 16 diagonals in a cube or cuboid of which 12 are face diagonals and 4 are diagonals (or space diagonal) of the cuboid.

Try These

Draw a cube and cuboid and name their diagonals. Separately count the diagonals for the cube and cuboid and see how many are face and how many are space diagonals.

Finding out the Diagonal of Cube and Cuboid

Cuboid

Given a room, we have to place a bamboo which is longer than its height, breadth or length inside it. Suppose we know the length, width and height of the room and also the length of the bamboo then can we find out whether the bamboos would fit in the room or not? How can we find the maximum length of any object so that it can fit in the room? That is, we need to know the relation between the space diagonal and length, width and height of a box to know the maximum length of any pencil, twig, or piece of paper which can be fitted inside it.

We have already studied about cube, cuboid and their diagonals. Now we need to find out how to calculate the length of diagonals given the sides of a cube or cuboid.

Face Diagonal

How will we find out the length of a face diagonal?

We know that ΔADC is a right angled triangle in which AD = *a* unit or DC = *c* unit.

Therefore by Bodhayan-Pythagoras theorem:

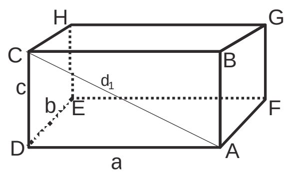


Figure - 11

$$AC = \sqrt{AD^2 + DC^2}$$

$$d_1 = \sqrt{a^2 + c^2}$$

Thus, length of face diagonal = AC = $\sqrt{a^2 + c^2}$ unit

Similarly we can find out lengths of face diagonals AE or AG

$$AE = d_2 = \sqrt{a^2 + b^2} \text{ unit}$$

$$AG = d_3 = \sqrt{b^2 + c^2} \text{ unit}$$

Hence, the cuboid in which all the sides are of different lengths have diagonals of three different lengths.

Diagonals of a Cuboid

In the given cuboid (Figure-11) the lengths of sides are a units, b units and units respectively. AH is one of the diagonals of the cuboid.

In the figure-11, AE is a face diagonal and its length is $\sqrt{a^2 + b^2}$ unit

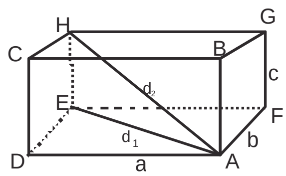


Figure - 12

In a right angled triangle (Figure-12). We can calculate the length of the diagonal AH by Bodhaya-Pythagoras theorem.

$$AH = \sqrt{AE^2 + EH^2}$$

$$= \sqrt{(a^2 + b^2) + c^2}$$

$$= \sqrt{a^2 + b^2 + c^2}$$

$\therefore$ the length of the diagonal, AH =

$$= \sqrt{a^2 + b^2 + c^2} \text{ unit}$$

Hence, length of the space diagonal $= \sqrt{(\text{Length})^2 + (\text{Width})^2 + (\text{Height})^2}$ Unit

Are the lengths of the other 3 space diagonals same? Identify these diagonals.

Diagonals are ...., ..., and ....

Find their lengths by using the Bodhayan-Pythagoras theorem.

Cube

If a side of given cube is 'a' units then length of the face diagonal $= \sqrt{a^2 + a^2}$

$= \sqrt{2a^2}$

$= a\sqrt{2}$ Units

All the face diagonals of the cube are of same length.

Space diagonal of the cube $= \sqrt{a^2 + a^2 + a^2}$

$= \sqrt{3a^2}$

$= a\sqrt{3}$ Units

Example-1. In a cuboid, the length is 10 *cm*, breadth is 4 *cm* and height is 5 *cm*. Find the length of the space diagonal of the cube.

Solution : Length, breadth and height of the cuboid is given, we have to calculate the length of the space diagonal of the cuboid.

We know that

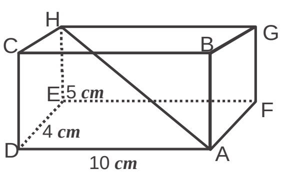


Figure - 13

Diagonal of the cuboid $= \sqrt{(\text{Length})^2 + (\text{Width})^2 + (\text{Height})^2}$

$= \sqrt{(10)^2 + (4)^2 + (5)^2}$

$= \sqrt{100 + 16 + 25}$

$= \sqrt{141}$

$= 11.87$ *cm*

Hence, the length of the space diagonal of given cuboid is 11.87 *cm*.

Example-2. Calculate the lengths of face diagonal and space diagonal of a cube having each side equal to 6 cm.

Solution : Side of the cube is 6 cm (given). We have to calculate the length of space diagonal of the cuboid.

We know that face diagonal of the cuboid = $a\sqrt{2}$ *unit* where *a* is the side of the cube.

Hence, face diagonal of the cube $= 6\sqrt{2}\ cm$

Since space diagonal of the cube $= a\sqrt{3}\ unit$

Hence, diagonal of the cube $= 6\sqrt{3}\ cm$

Exercise-1

1. A cuboid is of length 8 *m*, breadth 4 *m* and height 2 *m*. Calculate the lengths of all the diagonals.
2. Calculate the length of face diagonal of the cube whose side is $12\sqrt{3}$ *cm* long. What is the length of the space diagonal of that cube?
3. Calculate the length of the possible longest pole which can be placed in a room of length 10 *m*, width 10 *m* and height 5 *m*.

Cylinder

Cylinder is a 3-dimensional figure in which 2 similar and congruent circular surfaces are joined to each other with the help of a curved surface.

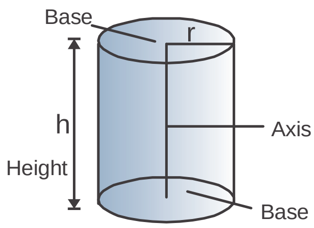


Figure - 14

Pipes, tubelight, etc. are some examples of cylinders.

The perpendicular distance between the circular surfaces is called the height of the cylinder and the circular surface is called base of the cylinder. The line segment which joins the centers of the two circular surfaces (base) is called axis of the cylinder.

Types of Cylinder

Right Circular or Oblique Cylinder

When the two bases lie exactly over each other and the axis is perpendicular to the base then the cylinder is called a right circular cylinder. If the base of the right circular cylinder is slightly shifted so that axis is no longer perpendicular to the base then it becomes an oblique cylinder (Figure-16).

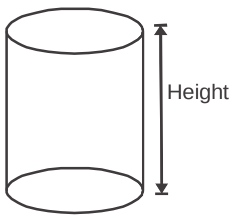


Figure - 15

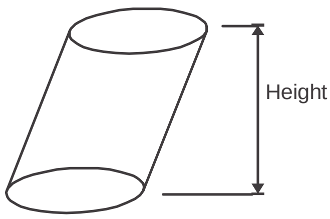


Figure - 16

Net of Cylinder

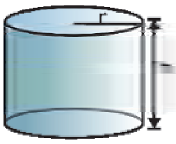


Figure - 17

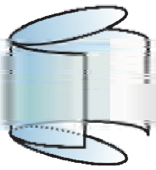


Figure - 18

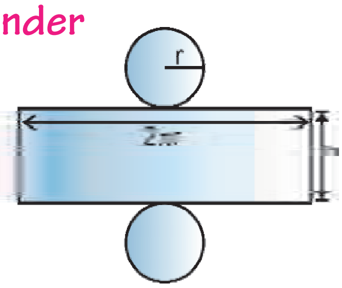


Figure - 19

Take one cylinder where both ends are closed. Let the radius of this cylinder be 'r' and height be 'h' units. When we cut the **curved** surface of the cylinder and spread it out (figures- 18, 19) then we obtain the surface net of the cylinder.

In the net obtained of cylinder, the length of rectangle (curved surface of cylinder) is $2\pi r$ unit and breadth is (height of cylinder) h units. The radii of the two circles is 'r' unit.

Try These

1. Make the net of a cylinder whose height is 7 cm and radius of base is 2 cm.
2. Take a sheet of drawing paper and make a cylinder of height 7*cm* and radius 2 *cm*.

Surface Area of Right Circular Cylinder

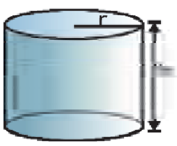


Figure - 20 (i)

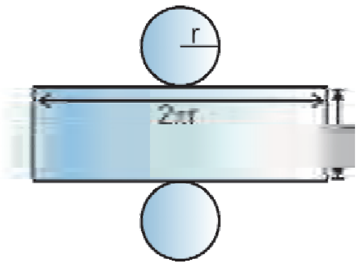


Figure - 20(ii)

The net diagram of a cylinder of radius 'r' and height 'h' will look similar to the one given in figure 20(ii). In this example, the width of rectangle is equal to the height of cylinder, h and length of rectangle is equal to circumference of the circle, $2\pi r$.

Hence, area of the curved surface of cylinder = Area of rectangle

$$= 2\pi rh \text{ unit}$$

And total surface area of the cylinder = Area of curved Surface + Area of both bases

$$= 2\pi rh + \pi r^2 + \pi r^2$$

$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r(h + r) \text{ Square unit}$$

Where r is radius of circular base and h is height of cylinder.

Volume of a Right Circular Cylinder

We know that volume of a cuboid is can be obtained by multiplying its height with the area of its base. The shape given in figure-21 is a cuboid. If we increase the number of sides from 4 to 5 and so on we can see a **gradual** change in shape from figure-21 to figure-22. You will observe that it will change slowly into a right circular cylinder because the base is gradually becoming circular. When the number of sides becomes infinite then this base changes into a circle and the whole figure becomes a right circular cylinder.

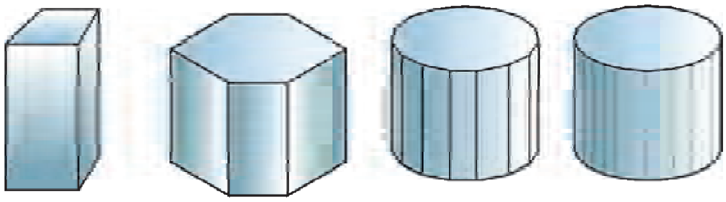


Figure - 21

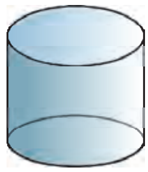


Figure - 22

Therefore, we can say that we can obtain the formula of volume of a cylinder from that of a cuboid. Volume of cylinder is the product of the area of its base and its height.

Let '*r*' be the radius of the base and '*h*' be the height, then

Volume of cylinder = Area of its base × height

$= \pi r^2 \times h$

$= \pi r^2 h$ *unit cube*

Try These

1. Take a sheet of paper. Make a cylinder by folding it along its length. Find the area and volume of the figure obtained. Now fold the same sheet along its breadth and find the area and volume. What can you say about the obtained volume and area?
2. Take some Rs. 5/- coins and form stacks of different heights by arranging them one above the other. Now, calculate the area and volume of the obtained figures. How many techniques can be used to calculate the area and volume?

Calculation of Curved Surface and Volume

Often cylindrical vessels are used to measure volumes. At other times we have to calculate how much metal is needed to make a cylinder or what is the amount of paint needed to colour a cylindrical surface? Or, how much paper is needed to cover it completely? For all of this we have to calculate the curved surface area and volume of cylinders. Let us see how this can be done.

Example-3. The circumference of the base of a right circular cylinder is 44 *cm*. If the height of the cylinder is 10 *cm*, calculate the curved surface area and volume of the cylinder.

Solution : Lets '*r*' be the radius of the base of a cylinder and '*h*' its height.

Given, height of cylinder, $h = 10$ *cm*.

Circumference of the base of cylinder $2\pi r = 44$ *cm*

$$r = \frac{44}{2\pi}$$

$$r = \frac{44}{2} \times \frac{7}{22}$$

$$r = 7 \quad cm$$

Area of curved surface of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 10$$

Area of curved surface of cylinder = 440 *sq.cm.*

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 10$$

Volume of cylinder = 1540 cm^3.

Example-4. Two right circular cylinders of same height have their radii in the ratio 3:4. Find the ratio of the volumes of the cylinders.

Solution : Let the radius of the cylinders be r_1 and r_2 respectively and height be h (Why?)

Because, the ratio of the radii of cylinders is 3:4,

$$\therefore \frac{r_1}{r_2} = \frac{3}{4}$$

Or $r_1 = 3r, r_2 = 4r$ (Why?)

Hence, volume of first cylinder $= \pi r_1^2 h$

And, volume of second cylinder $= \pi r_2^2 h$

$\therefore$ Ratio of volumes of both cylinders, $\dfrac{\pi r_1^2 h}{\pi r_2^2 h} = \dfrac{r_1^2}{r_2^2}$

$$\frac{\pi r_1^2 h}{\pi r_2^2 h} = \frac{(3r)^2}{(4r)^2}$$

$$= \frac{9r^2}{16r^2}$$

$$= \frac{9}{16}$$

Example-5. For a science project, Aisha has to make kaleidoscope with chart paper so that its surface is cylindrical. What would be the area of chart paper required by her if radius of kaleidoscope is 2.1 *cm* and height is 20*cm*?

Solution : Given

Radius of Kaleidoscope, $r = 2.1$ *cm*.

Height of Kaleidoscope, $h = 20$ *cm*.

Required Area of chart paper = Area of Kaleidoscope

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 2.1 \times 20$$

$$= 264 \ sq.cm$$

Exercise - 2

1. Radius of the base of a cylinder is 14 *cm* and its height is 10 *cm*. Find the area of the curved surface and total surface area of the cylinder.
2. The area of curved surface of cylinder is 3696 sq.cm. If the radius of the base of cylinder is 14 *cm* then find the height of the cylinder.
3. Area of curved surface of a cylinder, whose height is 14 cm, is 88 sq.cm. Find the diameter of the cylinder.
4. Diameter and height of a cylindrical pillar are 50 *cm* and 3.5 *m* respectively. Find the cost of painting the curved surface of pillar at a rate of Rs. 12.50 per m^2.
5. The diameter of a roller is 84 *cm* and its length is 120 *cm*. It takes 500 complete revolutions of the roller to level a playground once. Find the area of the playground in m^2.
6. Find the volume of cylinder if its radius is 3 *cm* and height is 14 *cm*.
7. The area of the base of a cylinder is 154 *sq.cm.* and height is 10 *cm*. Find the volume of the cylinder.
8. The circumference of the base of a cylinder is 88 *cm* and height is 10 *cm*. Find the volume of the cylinder.

9. Volume of a cylinder is 3080 cm^3 and its height 20 cm. Find the radius of the cylinder.

10. 11 l juice is filled in a cylindrical vessel of height 35 cm. Find the diameter of the jar. (1 l = 1000 cm^3)

11. A thin cylindrical tin contains 1 litre paint. If diameter of the tin is 14 cm then what is the height of the tin?

12. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve to 50 patients?

13. A 18 m long thin wire is drawn by melting a copper rod of diameter 1 m and length 8 m. Calculate the thickness of the wire.

14. A well has been dug whose radius is 7m and which is 20 m deep. A 22m × 14 m platform has been constructed from the soil excavated when digging the well. Find the height of the platform.

15. How many coins of diameter 1.75 cm and thickness 2 cm can be made by melting down a cuboid of sides 5.5 cm, 10 cm and 3.5 cm?

16. Volume and curved surface of a cylinder are 24750 cm^3 and 3300 cm^2 respectively. Find the height and radius of the base of cylinder.

Cone

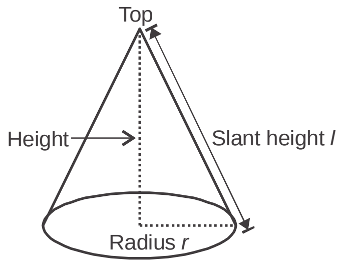


Figure - 23

Cone is a 3-dimensional shape with a circular base and a pointed top. The top and base are connected by two line segments. One joins the top to the circumference of the base and this is the slant height (l) of the cone. The line segment which joins the top of the cone to the center of the base and which is perpendicular to the base is called altitude or height of the right circular cone.

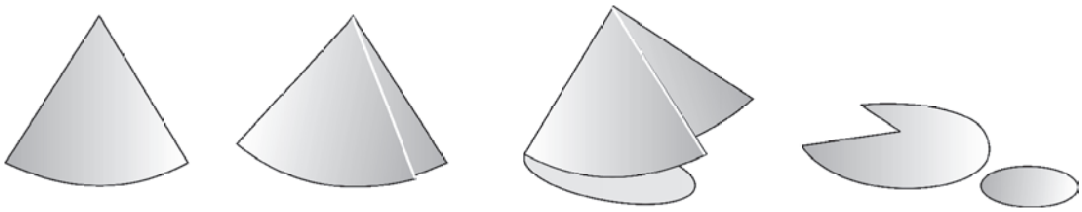


Figure - 24

1. Net of a Cone

See the pictures of the cone given below. If we cut the cone along its slant height and the edges of its base, then we will get a figure similar to the one shown in figure-25.

In the net diagram of a cone includes a sector of radius '*l*' units and a circle of radius '*r*' units.

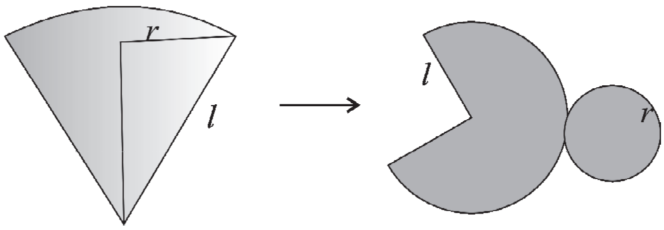


Figure - 25

2. Surface Area of Cone

If '*r*' is the radius of base of a cone and its slant height is '*l*' units then to find the surface area, we have to calculate the curved surface area and surface area of the base.

We have discussed that if we cut the cone open then we obtain a curved surface.

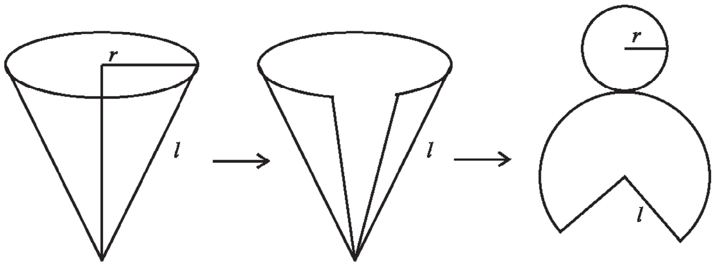


Figure - 26 (i) *Figure - 26 (ii)* *Figure - 26 (iii)*

To calculate the area of curved surface of the cone, we have to calculate the area of the sector obtained in the net diagram.

Curved surface area of cone = Area of the sector of a circle of radius '*l*' unit

$$= \frac{1}{2}(2\pi r)l$$

$$= \pi rl \ sq.unit$$

Area of the base of the cone = Area of a circle having radius '*r*'.

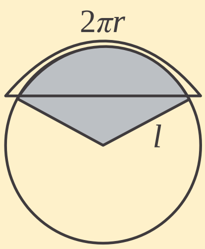
$$= \pi r^2 \ sq.unit$$

Total surface area of a cone = Curved surface area + Area of the base of the cone

$$= \pi rl + \pi r^2$$

$$= \pi r(r+l)$$

Hence, the total surface area of the cone where radius of base is '*r*' and slant height is *l* is $\pi r(r+l)$.



NOTE:

In the cone given above, the circumference of the circular base is $2\pi r$ This is the sector of a circle in which radius is l unit. We know that ratio of the area of the shaded region and area of the circle will be same as the ratio of the length of arc of sector to the circumference of the circle.

This means that,

$$\frac{\text{Area of the Sector of a Circle}}{\text{Area of Circle}} = \frac{\text{Length of Arc of Sector}}{\text{Circumference of Circle}}$$

$$\frac{\text{Area of the Sector of a Circle}}{\pi l^2} = \frac{\text{Length of Arc of Sector}}{2\pi l}$$

$$\text{Area of the Sector of a Circle} = \frac{\text{Length of Arc of Sector} \times \pi l^2}{2\pi l}$$

$$\text{Area of the Sector of a Circle} = \frac{\text{Length of Arc of Sector} \times l}{2}$$

$$\text{Area of the Sector of a Circle} = \frac{2\pi r \times l}{2}$$

$$\text{Area of the Sector of a Circle} = \pi rl$$

Where, $2\pi r$ is length of the arc of sector of a circle and l is radius of the circle.

3. Volume of Cone

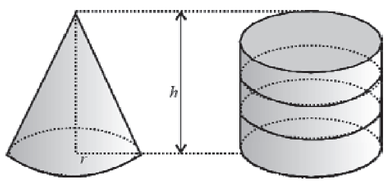


Figure - 28

Let us do an activity to understand the relation between the volumes of a cone and a cylinder.

Make a cylinder and a cone having the same base and of same height.

Fill the cone with sand and then put it sand in the cylinder. Is the cylinder completely filled with sand?

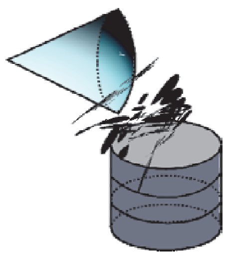


Figure - 29

To completely fill the cylinder with the sand, how many times did you repeat this process?

In this way you will find out that if the area of the base of cone and cylinder is same and they are of the same height then volume of the cylinder is thrice the volume of the cone.

$\therefore$ 3 × volume of cone = volume of cylinder

$$\text{Volume of cone} = \frac{1}{3}(\text{Volume of cylinder}) = \frac{1}{3}(\text{Height} \times \text{Area of base})$$

Since volume of cone is the product of its height and the area of its base, therefore.

Volume of cone $= \frac{1}{3} \times A \times h$ where A is the area of base and h is height of the cylinder.

Area of base $A = \pi r^2$

Hence, volume of cone = $\frac{1}{3} \times \pi r^2 h$ *Cube unit*

Volume of cylinder, in which r is the radius of the base and h is the height is $\pi r^2 h$.

Hence, volume of cone is one third of volume of cylinder given that radius of the base and height are same for both the figures.

Example-6. Diameter of a cone is 12 *cm* and its height 8 *cm*. Find the curved surface area and volume of cone.

Solution : Let r be the radius and h be height and l be the slant height of the cone.

Given: Height of cone = h = 8 *cm*.

Diameter of cone = $2r$ = 12 *cm*.

Radius of cone r = 6 *cm*.

Slant height of cone, $l = \sqrt{h^2 + r^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$

$= \sqrt{100} = 10$ *cm*

Curved surface area of cone $= \pi r l$

$= \pi \times 6 \times 10 = 60\pi\ cm^2$

Volume of Cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8$

$= 96\pi\ cm^2$

Example-7. 65π square meter cloth is used to make a tent in the shape of a cone. Slant height of tent is 13*m*, find its height and radius.

Solution : Let radius be r, h be height and l be slant height of the cone.

Given: Slant height of cone, l = 13 *m*.

Area of the cloth which is used in cone shaped tent is equal to the curved surface area of cone (Why?). Therefore,

Curved surface area of cone $= 65\pi$

$\pi r l = 65\pi$

$$r = \frac{65\pi}{\pi l}$$

$$r = \frac{65}{13}$$

$$r = 5 \text{ meter}$$

Slant height, $l = \sqrt{h^2 + r^2}$

$$l^2 = h^2 + r^2$$

$$h^2 = l^2 - r^2$$

$$= (13)^2 - (5)^2$$

$$= 169 - 25$$

$$h^2 = 144$$

$$h = 12 \text{ meter}$$

Radius of the base of the cone is 5*m* and its height 12*m*.

Exercise - 3

1. Find the curved surface area of the right circular cone in which slant height is 10 *cm* and radius of the base is 7 *cm*.
2. If the curved surface area of a cone 77π *sq.cm* and radius of its base is 14 *cm* then find the height of the cone.
3. If the slant height of the cone is 21 *cm* and diameter of its base is 14 *cm* then find the total surface area of the cone.
4. If the radius of the base of a cone shaped hat (like the ones worn by Jokers) is 7 *cm* and its height is 24 *cm* then find the area of the sheet needed to make 10 such hats.
5. Height of a cone shaped tent is 5 *m* and its radius is 12 *m*. Find the slant height of the cone and the cost of the canvas which is used to make the tent. Cost of canvas is 70 rupees per square meter.
6. Find the volume of a cone whose base area is 300 *sq.cm* and height is 15 *cm*.
7. Find the height of the cone if its volume is 550 cm^3 and its diameter is 10 *cm*.
8. The circumference of the base of a cone shaped cup is 22 *cm* and height is 6 *cm*. Then what is the maximum volume of water which can be kept in it.
9. If the radius of a 1 *m* long metallic rod (in cylindrical shape) is 3.5 cm then how many cones of radius 1 *cm* and height 2.1 *cm* can be formed by melting down the rod?

10. We have a right angled triangle having sides 21 *cm*, 28 *cm* and 35 *cm*. If we take the side of 28 *cm* as axis and rotate the triangle around it then write the name of the figure obtained and calculate its volume.

11. If the radius of the base of a cone and a cylinder and their height is similar then calculate the ratio of their volumes.

Sphere

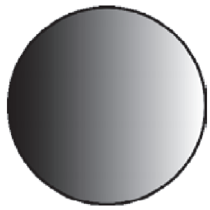


Figure - 31

The figure obtained by rotating a circle around it diameter is called a sphere. A sphere is a solid figure such that each point on it is equidistant from its center.

Surface area of sphere

Activity-1

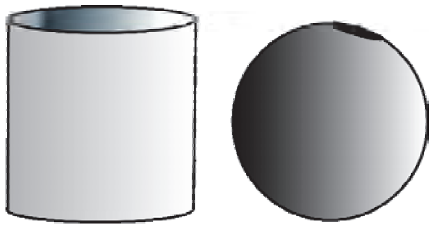


Figure - 32

We know that surface area represents the outer surface of any solid figure. Let us compare the surface area of a cylinder with that of a sphere.

Take a cylinder or sphere in which the radius of the base of the cylinder and radius of sphere are same and the height of the cylinder is twice the radius of the sphere. Take a piece of rope as well

Mark a point halfway along the height of the cylinder. Now, start winding the rope starting from the bottom or the top of the cylinder and end at this point. Cut this rope and wind it around the sphere.

You will find that half of the sphere is covered by this string. Now, through this activity we can say that area of the curved surface of a cylinder is similar to the area of the sphere when radius of the base of cylinder and radius of sphere are equal and height of cylinder is equal to the diameter of sphere.

We can say that,

Surface Area of sphere = Surface area of curved part of cylinder.

$= 2\pi rh$

$= 2\pi r\,(2r)$

$= 4\pi r^2$ *sq.units.*

Therefore, surface area of sphere = $4pr^2$ *sq.unit*, where *r* is the radius of sphere.

Activity-2

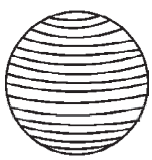


Figure - 33

Take a string and wind it completely around a ball making sure that no space is left in between and that there are no overlaps (see figure-33). If we make circles using this string where the radius of the circle is equal to the radius of the sphere then we will be able to make 4 circles (figure-34). Area of each circle is πr^2.

So, surface area of sphere $= 4 \times$ Area of circle

$= 4\pi r^2$

Therefore, surface area of sphere is $4\pi r^2$ Sq.unit, where r is the radius of the sphere. Then, the surface area of hemispheres can be obtained in the following ways.

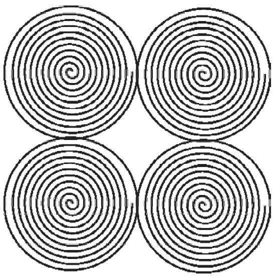


Figure - 34

Surface Area of Hemisphere $= \frac{1}{2} \times$ (Surface Area of sphere)

$= \frac{1}{2}\left(4\pi r^2\right)$

$= 2\pi r^2$

Total area of the hemisphere $= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$ *sq.unit*

Hence,

Surface Area of Sphere	$= 4\pi r^2$ *sq.unit*
Surface Area of Hemisphere	$= 2\pi r^2$ *sq.unit*
Total surface area of Hemisphere	$= 3\pi r^2$ *sq.unit*

Volume of Sphere

Volume of sphere is proportional to the cube of its radius. When radius is increased the volume increases correspondingly. Value of volume is represented by $\frac{4}{3}\pi r^3$.

Example-8. If the radius of a solid sphere is 7 *cm*, then find its surface area and volume.

Solution : Given, radius of sphere, $r = 7$ *cm*.

Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (7)^2$

$= 4 \times \frac{22}{7} \times 7 \times 7$

$= 616 \ cm^3$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (7)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= 1437.33 \ cm^3$$

Example-9. Find the total surface area of a hemisphere of diameter 14 *cm*.

Solution : Let *r* be the radius of hemisphere.

Given, diameter of hemisphere = 14 *cm*

$\therefore \qquad 2r = 14 \ cm$

Or $r = 7 \ cm$

$\because$ Total surface area of hemisphere $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times (7)^2$$

$$= 462 \ sq.cm$$

Example-10. A big sphere is made by melting down 64 small spheres each of radius 2 cm. Find the radius of the big sphere.

Solution : Let *r cm* is the radius of small sphere.

Given = $r = 2 \ cm$

Volume of each small sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (2)^3 = \frac{32}{3}\pi \ cm^3$

$\because$ Volume of 64 small spheres = $64 \times \frac{32}{3}\pi$

$$= \frac{2048\pi}{3}$$

By melting down the small 64 spheres a big sphere is made. So the volume of the big sphere is equal to the combined volume of 64 small spheres. Let 'R' be the radius of the big sphere.

Volume of big sphere = Volume of 64 small sphere

$$\frac{4}{3}\pi R^3 = \frac{2048\pi}{3}$$

$$R^3 = \frac{2048\pi \times 3}{3 \times 4\pi}$$

$$R^3 = 512$$

$$R = 8 \text{ } cm$$

Hence, radius of big sphere = 8 *cm*.

Example-11. Radius of a solid metallic sphere is 3 *cm*. If the density of the metal is 8 *gm*/*cm*^{3} then find the mass of the metal.

Solution : We know that product of density and volume is equal to mass. Therefore, we first calculate the volume of the sphere.

Let *r cm* be the radius of the sphere.

$$r = 3 \text{ } cm$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (3)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$$

$$= 113.14 \text{ } cm^3$$

Because density of metal is 8*gm*/*cm*^{3}, hence mass of 1 *cm*^{3} is 8 *g*.

$\therefore$ Mass of sphere $= \text{Volume} \times \text{Density}$

$$= 113.14 \times 8$$

$$= 905.12 \text{ gram}$$

$$= 0.9051 \text{ } kg \text{ (approx.)}$$

Exercise - 4

1. Find the surface area of a sphere having radius equal to 21 *cm*.
2. Diameter of a globe is 14 *cm*. Find its surface area.
3. Surface area of a sphere is 154 *sq. cm*^{2}. Find the diameter of the sphere.
4. Find the volume of sphere if its radius is 3 *cm*.
5. By melting down 21 small metallic balls, each of radius 2 *cm*, one big sphere is made. Calculate the volume of this sphere.
6. By melting down a sphere of radius 10.5 *cm* some small cones are made, each of height 3 *cm* and radius 3.5 *cm*. Find the number of such cones.

7. After filling the air in a spherical balloon of radius 7 *cm* it becomes 14 *cm*. Calculate the ratio of the surface area of the balloon in both conditions.

8. Calculate the volume of a sphere having surface area equal to 154 $sq.cm^2$.

9. Ratio of the volume of 2 spheres is 64 : 27. Calculate the ratio of their surface areas.

10. Radius of a solid sphere is 12 *cm*. How many spheres of radius 6 cm can be made by melting this sphere.

11. If the volume and surface area of a sphere are equal then calculate its radius.

12. A child converts a cone of height 24 *cm* and base radius 6 *cm* into a sphere. Calculate the radius of the sphere.

13. By melting down 3 spherical balls of radius 6 *cm*, 8 *cm* and 10 *cm* one big solid sphere is made. Calculate the radius of the new solid sphere.

Surface Area and Volume of a Combination of Solids Shapes

In our daily life we see many figures that are a combination of different shapes. For example, a capsule is a combination of a cylinder and 2 hemispheres stuck at the ends of the cylinder. Similarly, a spinning top has a hemispherical part and another that is cone shaped. Therefore, we have to think of methods to calculate the surface area and volume of these figures.

Figure - 35

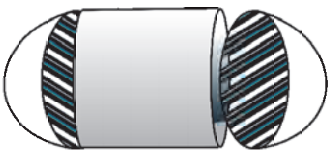


Figure - 36

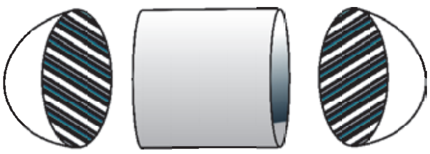


Figure - 37

Let us discuss the container which is shown in figure-35. We have to calculate the area and volume of an iron sheet needed to make this container but the container is not of a shape for which we already have some formula. If we have some solid figure which is similar to the one shown in figure 36, then what we do?

In such situations, we divide the figure into smaller parts so that we can calculate their area and volume easily and obtain the solution of the problem. We see that this capsule is made by joining the hemispheres at the ends of the solid cylinder. If we cut the container, then this figure will look as shown in figures – 36 and 37.

So to make the required container, the area of iron sheet needed = curved surface area of first hemisphere + curved surface area of cylinder + curved surface area of second hemisphere.

Volume of container = volume of first hemisphere + Volume of cylinder + Volume of second hemisphere.

Example-12. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 *cm* and total height of the vessel is 13 *cm*. Find the area of the iron sheet needed to make this container and volume of the vessel. (Thickness of iron sheet is negligible).

Solution : Diameter of hemisphere $= 14\ cm$

$\therefore$ Radius of hemisphere $= 7\ cm$

Height of the cylindrical portion of vessel $= 13-7$

$= 6\ cm$

Curved Surface area of the cylindrical part $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 6$$

$$= 264\ cm^2$$

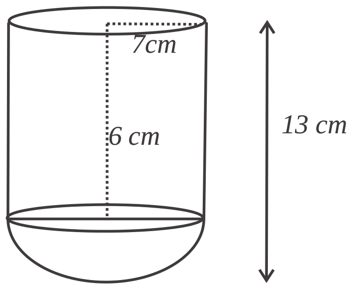


Figure - 38

Surface area of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 7$$

$$= 308\ sq.cm$$

Therefore, to make the container

Required area of sheet = Curved surface area of cylinder + Area of Hemisphere

= 264 *sq.cm.* + 308 *sq.cm.*

= 572 *sq.cm.*

Volume of cylindrical part $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 6$$

$$= 924\ cm^3$$

And, Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= 718.6 \quad cm^3$$

Hence, capacity of vessel = Volume of cylinder + Volume of hemisphere

$$= 924\, cm^3 + 718.6 \quad cm^3$$

$$= 1642.6 \quad cm^3$$

Exercise - 5

1. In figure-39, the object is made of two solids - a cube and a hemisphere. The base of the figure is a cube with edges of 5 *cm* and the hemisphere fixed on the top has a diameter of 4.2 *cm*. Find the total surface area of the object (take $\pi = 22/7$).

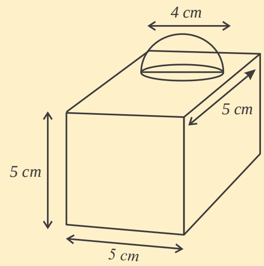


Figure - 39

2. A toy has two parts. One part is cone shaped and has radius of 5 *cm*. and it is placed on the top of a hemisphere having similar radius. Total height of toy is 17 *cm*. Find the total surface area of the toy?

3. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 *cm*. The height of the cone is equal to its radius. Find the volume of the solid in terms of π.

4. A spherical glass vessel has a cylindrical neck which is 4 *cm* long and 2 *cm* in diameter. The diameter of the spherical part is 6 *cm*. Find the amount of water it can hold?

5. The upper portion of the greenhouse shown in figure-40 is semicircular. This greenhouse is made up of cloth. It has a wooden door of size 1.2 *m*. × 0.5 *m*. Find the area of the cloth required to cover the green house completely.

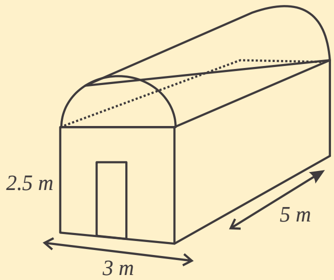


Figure - 40

What We Have Learnt

1. We have learnt to make the net diagrams of 3-dimensional figures like cuboid, cylinder, cone etc. and to draw and understand the net diagram as well.
2. To identify and understand the top, base, surface and edge of a cube and a cuboid.
3. To identify and understand the different kinds of diagonals in cube and cuboid.
4. Calculate the area and volume of a 3-D figure like cone, cylinder, sphere etc.
5. To calculate the area and volume of objects which are a combination of different figures.

ANSWER KEY

Exercise - 1

1. $2\sqrt{21}$ m 2. 36 cm 3. 15 m

Exercise - 2

1. 880 $cm.sq.$ and 2112 $cm.sq$ 2. 42 cm 3. 2 cm 4. ₹68.75
5. 1584 m^2 6. 396 cm^3 7. 1540 cm^3 8. 6160 cm^3
9. 7 cm 10. 20 cm 11. 6.49 cm 12. 7700 cm^3
13. 2 / 3   14. 2.5 m 15. 35   16. 15 cm and 35 cm

Exercise - 3

1. 220 cm^2 2. $6\sqrt{2}$ cm 3. 616 cm^2 4. 5500 cm^2
5. ₹34320   6. 1500 cm^3 7. 21 cm 8. 77 cm^3
9. 1750   10. 12936 cm^3 11 1 : 3

Exercise - 4

1. 5544 cm^2 2. 196π cm^2 3. 7 cm 4. 36π cm^3
5. 224π cm^3 6. 126   7. 1 : 4   8. 179.66 cm^3 9. 16 : 9
10. 8   11. 3 $unit$ 12. 6 cm 13. 12 cm^3

Exercise - 5

1. 163.86 cm^2 2. 115π cm^2 3. ₹22000   4. π cm^3
5. 44π cm^3 6. 62.9 cm^2