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Electrostatic Potential and Capacitance

TOPIC 1 Electrostatic Potential and Equipotential Surfaces

1. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge $(+q)$ each, while 2, 4, 6, 8, 10 have charge $(-q)$ each. The potential V and the electric field E at the centre of the circle are respectively :
(Take $V = 0$ at infinity) **[Sep. 05, 2020 (II)]**

(a) $V = \frac{10q}{4\pi\varepsilon_0 R}$; $E = 0$

(b) $V = 0$; $E = \frac{10q}{4\pi\varepsilon_0 R^2}$

(c) $V = 0$; $E = 0$

(d) $V = \frac{10q}{4\pi\varepsilon_0 R}$; $E = \frac{10q}{4\pi\varepsilon_0 R^2}$

2. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively :
[Sep. 03, 2020 (I)]

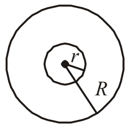
(a) 4.5 μC on both (b) +4.5 μC and –4.5 μC
(c) 3 μC and 6 μC (d) 6 μC and 3 μC

3. Concentric metallic hollow spheres of radii R and $4R$ hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference $V(R) - V(4R)$ is : **[Sep. 03, 2020 (II)]**

(a) $\frac{3Q_1}{16\pi\varepsilon_0 R}$ (b) $\frac{3Q_2}{4\pi\varepsilon_0 R}$

(c) $\frac{Q_2}{4\pi\varepsilon_0 R}$ (d) $\frac{3Q_1}{4\pi\varepsilon_0 R}$

4. A charge Q is distributed over two concentric conducting thin spherical shells radii r and R ($R > r$). If the surface charge densities on the two shells are equal, the electric potential at the common centre is : **[Sep. 02, 2020 (II)]**



(a) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$

5. A point dipole $= \vec{p} - p_o\hat{x}$ kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively : (Take V = 0 at infinity)
[12 April 2019 I]

(a) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$ (b) $0, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

(c) $0, \frac{|\vec{p}|}{4\pi\varepsilon_0 d^3}$ (d) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

6. A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed v at z = 4a. The minimum value of v such that it crosses the origin is : **[10 April 2019 I]**

(a) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (b) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

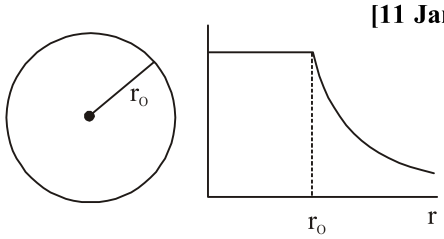
(c) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (d) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

7. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of – 4 Q, the new potential difference between the same two surfaces is : **[8 April 2019 I]**

(a) –2V (b) 2V (c) 4V (d) V

8. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is : **[8 Jan. 2019 II]**
(a) 320 V (b) −48V (c) 180 V (d) −520 V

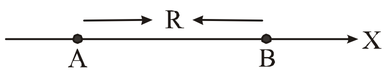
9. The given graph shows variation (with distance r from centre) of : **[11 Jan. 2019 I]**



(a) Electric field of a uniformly charged sphere
(b) Potential of a uniformly charged spherical shell
(c) Potential of a uniformly charged sphere
(d) Electric field of a uniformly charged spherical shell

10. A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another.
The total potential at a point at distance r from their common centre, where r < a, would be: **[10 Jan. 2019 I]**
(a) $\frac{Q}{12\pi\epsilon_0}\frac{ab+bc+ca}{abc}$ (b) $\frac{Q(a^2+b^2+c^2)}{4\pi\epsilon_0(a^3+b^3+c^3)}$
(c) $\frac{Q}{4\pi\epsilon_0(a+b+c)}$ (d) $\frac{Q(a+b+c)}{4\pi\epsilon_0(a^2+b^2+c^2)}$

11. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ are placed on the x-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is: **[10 Jan. 2019 I]**
(a) $\frac{R}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}R}{\sqrt{2}+1}$
(c) $\frac{R}{\sqrt{2}-1}$ (d) $\frac{\sqrt{2}R}{\sqrt{2}-1}$

12. Consider two charged metallic spheres S_1 and S_2 of radii R_1 and R_2, respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1/E_2 = R_1/R_2$. Then the ratio V_1(on S_1)/V_2(on S_2) of the electrostatic potentials on each sphere is: **[8 Jan. 2019 II]**
(a) R_1/R_2 (b) $(R_1/R_2)^2$
(c) (R_2/R_1) (d) $\left(\frac{R_1}{R_2}\right)^3$

13. Three concentric metal shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is: **[2018]**
(a) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (b) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$
(c) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (d) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

14. There is a uniform electrostatic field in a region. The potential at various points on a small sphere centred at P, in the region, is found to vary between in the limits 589.0 V to 589.8 V. What is the potential at a point on the sphere whose radius vector makes an angle of 60° with the direction of the field ? **[Online April 8, 2017]**
(a) 589.5 V (b) 589.2 V (c) 589.4 V (d) 589.6 V

15. Within a spherical charge distribution of charge density ρ(r), N equipotential surfaces of potential V_0, $V_0 + \Delta V$, $V_0 + 2\Delta V$, ...$V_0 + N\Delta V$ ($\Delta V > 0$), are drawn and have increasing radii r_0, r_1, r_2,......... r_N, respectively. If the difference in the radii of the surfaces is constant for all values of V_0 and ΔV then : **[Online April 10, 2016]**
(a) ρ(r) = constant (b) $\rho(r) \propto \frac{1}{r^2}$
(c) $\rho(r) \propto \frac{1}{r}$ (d) $\rho(r) \propto r$

16. The potential (in volts) of a charge distribution is given by
$V(z) = 30 - 5z^2$ for $|z| \le 1$m
$V(z) = 35 - 10|z|$ for $|z| \ge 1$ m.
V(z) does not depend on x and y. If this potential is generated by a constant charge per unit volume ρ_0 (in units of ε_0) which is spread over a certain region, then choose the correct statement. **[Online April 9, 2016]**
(a) $\rho_0 = 20\,\varepsilon_0$ in the entire region
(b) $\rho_0 = 10\,\varepsilon_0$ for $|z| \le 1$ m and $p_0 = 0$ elsewhere
(c) $\rho_0 = 20\,\varepsilon_0$ for $|z| \le 1$ m and $p_0 = 0$ elsewhere
(d) $\rho_0 = 40\,\varepsilon_0$ in the entire region

17. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[2015]**
(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(b) $2R = R_4$
(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(d) $R_1 \ne 0$ and $(R_2 - R_1) > (R_4 - R_3)$

18. An electric field $\vec{E} = (25\hat{i} + 30\hat{j})NC^{-1}$ exists in a region of space. If the potential at the origin is taken to be zero then the potential at $x = 2$ m, $y = 2$ m is : **[Online April 11, 2015]**
(a) −110 J (b) −140 J (c) −120 J (d) −130 J

19. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is:
(a) 120 J/C (b) −120 J/C **[2014]**
(c) −80 J/C (d) 80 J/C

20. Consider a finite insulated, uncharged conductor placed near a finite positively charged conductor. The uncharged body must have a potential : **[Online April 23, 2013]**
(a) less than the charged conductor and more than at infinity.
(b) more than the charged conductor and less than at infinity.
(c) more than the charged conductor and more than at infinity.
(d) less than the charged conductor and less than at infinity.

21. Two small equal point charges of magnitude q are suspended from a common point on the ceiling by insulating mass less strings of equal lengths. They come to equilibrium with each string making angle θ from the vertical. If the mass of each charge is m, then the electrostatic potential at the centre of line joining them will be $\left(\frac{1}{4\pi \in_0} = k\right)$. **[Online April 22, 2013]**
(a) $2\sqrt{k\,mg\tan\theta}$ (b) $\sqrt{k\,mg\tan\theta}$
(c) $4\sqrt{k\,mg/\tan\theta}$ (d) $\sqrt{k\,mg/\tan\theta}$

22. A point charge of magnitude $+1\ \mu C$ is fixed at (0, 0, 0). An isolated uncharged spherical conductor, is fixed with its center at (4, 0, 0). The potential and the induced electric field at the centre of the sphere is : **[Online April 22, 2013]**
(a) 1.8×10^5 V and -5.625×10^6 V/m
(b) 0 V and 0 V/m
(c) 2.25×10^5 V and -5.625×10^6 V/m
(d) 2.25×10^5 V and 0 V/m

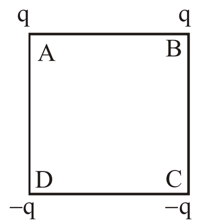
23. A charge of total amount Q is distributed over two concentric hollow spheres of radii r and R $(R > r)$ such that the surface charge densities on the two spheres are equal. The electric potential at the common centre is **[Online May 19, 2012]**
(a) $\frac{1}{4\pi\varepsilon_0}\frac{(R-r)Q}{(R^2+r^2)}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)Q}{2(R^2+r^2)}$
(c) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)Q}{(R^2+r^2)}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{(R-r)Q}{2(R^2+r^2)}$

24. The electric potential $V(x)$ in a region around the origin is given by $V(x) = 4x^2$ volts. The electric charge enclosed in a cube of 1 m side with its centre at the origin is (in coulomb) **[Online May 7, 2012]**
(a) $8\varepsilon_0$ (b) $-4\varepsilon_0$ (c) 0 (d) $-8\varepsilon_0$

25. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre and a, b are constants. Then the charge density inside the ball is: **[2011]**
(a) $-6a\varepsilon_0 r$ (b) $-24\pi a\varepsilon_0$
(c) $-6a\varepsilon_0$ (d) $-24\pi a\varepsilon_0 r$

26. An electric charge $10^{-3}\ \mu$ C is placed at the origin (0, 0) of $X-Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be **[2007]**
(a) 4.5 volts (b) 9 volts
(c) Zero (d) 2 volt

27. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then **[2007]**



(a) $\vec{E}$ changes, V remains unchanged
(b) $\vec{E}$ remains unchanged, V changes
(c) both $\vec{E}$ and V change
(d) $\vec{E}$ and V remain unchanged

28. The potential at a point x (measured in μ m) due to some charges situated on the x-axis is given by $V(x) = 20/(x^2 - 4)$ volt. The electric field E at $x = 4\ \mu$ m is given by **[2007]**
(a) (10/9) volt/ μ m and in the +ve x direction
(b) (5/3) volt/ μ m and in the –ve x direction
(c) (5/3) volt/ μ m and in the +ve x direction
(d) (10/9) volt/ μ m and in the –ve x direction

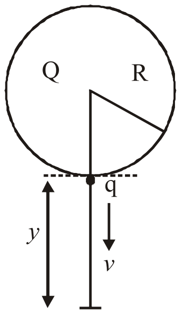
29. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is **[2005]**
(a) $\frac{q}{2\pi \in_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ (b) $\frac{qR}{4\pi \in_0 d^2}$
(c) $\frac{q}{4\pi \in_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ (d) zero

30. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P, a distance $\frac{R}{2}$ from the centre of the shell is **[2003]**
(a) $\frac{2Q}{4\pi\varepsilon_o R}$ (b) $\frac{2Q}{4\pi\varepsilon_o R} - \frac{2q}{4\pi\varepsilon_o R}$
(c) $\frac{2Q}{4\pi\varepsilon_o R} + \frac{q}{4\pi\varepsilon_o R}$ (d) $\frac{(q+Q)2}{4\pi\varepsilon_o R}$

TOPIC 2 Electric Potential Energy and Work Done in Carrying a Charge

31. A solid sphere of radius R carries a charge $Q+q$ distributed uniformaly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then : (assume the remaining portion to be spherical).

[Sep. 05, 2020 (I)]



(a) $v^2 = y\left[\dfrac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$

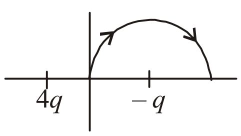
(b) $v^2 = y\left[\dfrac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(c) $v^2 = 2y\left[\dfrac{Qq\,R}{4\pi\varepsilon_0 (R+y)^3 m} + g\right]$

(d) $v^2 = 2y\left[\dfrac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

32. A two point charges $4q$ and $-q$ are fixed on the x-axis at $x = -\dfrac{d}{2}$ and $x = \dfrac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to $x = d$ along the semicircle as shown in the figure, the energy of the charge will :

[Sep. 04, 2020 (I)]



(a) increase by $\dfrac{3q^2}{4\pi\varepsilon_0 d}$

(b) increase by $\dfrac{2q^2}{3\pi\varepsilon_0 d}$

(c) decrease by $\dfrac{q^2}{4\pi\varepsilon_0 d}$

(d) decrease by $\dfrac{4q^2}{3\pi\varepsilon_0 d}$

33. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to : **[Sep. 03, 2020 (II)]**

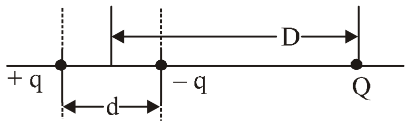
(a) 1 : 2 (b) 10 : 7
(c) 2 : 1 (d) 5 : 7

34. In free space, a particle A of charge 1 μC is held fixed at a point P. Another particle B of the same charge and mass 4 μg is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is :

$\left[Take\ \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2}\right]$ **[10 April 2019 II]**

(a) 1.0m/s (b) 3.0×10^4 m/s
(c) 2.0×10^3 m/s (d) 1.5×10^2 m/s

35. A system of three charges are placed as shown in the figure:

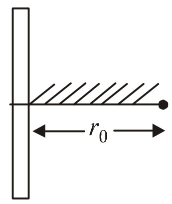


If D >> d, the potential energy of the system is best given by **[9 April 2019 I]**

(a) $\dfrac{1}{4\pi\in_0}\left[\dfrac{-q^2}{d} \dfrac{-qQd}{2D^2}\right]$ (b) $\dfrac{1}{4\pi\in_0}\left[\dfrac{-q^2}{d} + \dfrac{2qQd}{D^2}\right]$

(c) $\dfrac{1}{4\pi\in_0}\left[+\dfrac{q^2}{d} + \dfrac{qQd}{D^2}\right]$ (d) $\dfrac{1}{4\pi\in_0}\left[-\dfrac{q^2}{d} - \dfrac{qQd}{D^2}\right]$

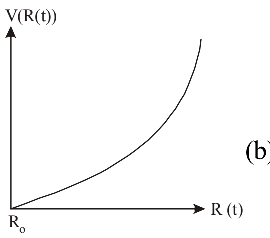
36. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to : **[8 April 2019 II]**

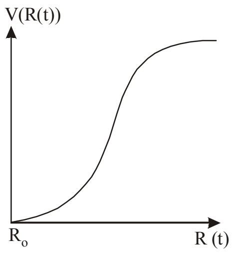


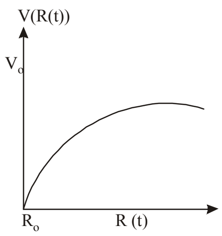
(a) $v \propto e^{+r/r_0}$ (b) $v \propto \sqrt{ln\left(\dfrac{r}{r_0}\right)}$

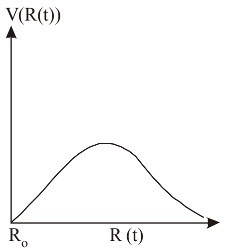
(c) $v \propto ln\left(\dfrac{r}{r_0}\right)$ (d) $v \propto \left(\dfrac{r}{r_0}\right)$

37. There is a uniform spherically symmetric surface charge density at a distance R_o from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed V (R(t)) of the distribution as a function of its instantaneous radius R(t) is: **[12 Jan. 2019 I]**

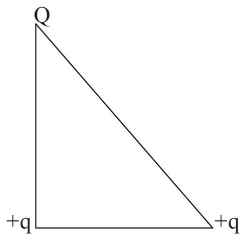
(a) 

(b) 

(c) 

(d) 

38. Three charges Q, + q and + q are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is : **[11 Jan. 2019 I]**



(a) + q (b) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

(c) $\frac{-q}{1+\sqrt{2}}$ (d) –2q

39. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, – 2) and (0, – 2). The work required to put a fifth charge Q at the origin of the coordinate system will be: **[10 Jan. 2019 II]**

(a) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (b) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{Q^2}{2\sqrt{2}\,\pi\varepsilon_0}$ (d) $\frac{Q^2}{4\pi\varepsilon_0}$

40. **Statement 1 :** No work is required to be done to move a test charge between any two points on an equipotential surface.

Statement 2 : Electric lines of force at the equipotential surfaces are mutually perpendicular to each other. **[Online April 25, 2013]**

(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is false, Statement 2 is true.

41. An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero. **[2012]**

Statement -1 When a charge q is taken from the centre to the surface of the sphere its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement -2 The electric field at a distance r ($r<R$) from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of statement 1.
(b) Statement 1 is true Statement 2 is false.
(c) Statement 1 is false Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1

42. Two positive charges of magnitude 'q' are placed, at the ends of a side (side 1) of a square of side '$2a$'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is **[2011 RS]**

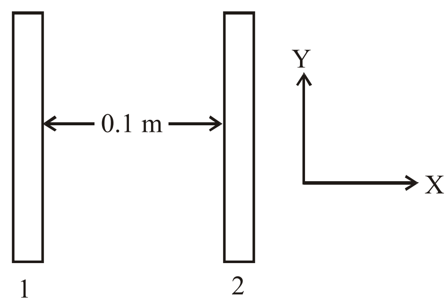
(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$

43. Two points P and Q are maintained at the potentials of 10 V and – 4 V, respectively. The work done in moving 100 electrons from P to Q is: **[2009]**

(a) 9.60×10^{-17}J (b) -2.24×10^{-16} J
(c) 2.24×10^{-16} J (d) -9.60×10^{-17} J

44. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (i.e., plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2? ($e = 1.6\times10^{-19}$ C, $m_e = 9.11\times10^{-31}$ kg) **[2006]**



(a) 2.65×10^6 m/s (b) 7.02×10^{12} m/s
(c) 1.87×10^6 m/s (d) 32×10^{-19} m/s

45. A charged particle 'q' is shot towards another charged particle 'Q' which is fixed, with a speed 'v'. It approaches 'Q' upto a closest distance r and then returns. If q were given a speed of '$2v$' the closest distances of approach would be **[2004]**

(a) $r/2$ (b) $2r$ (c) r (d) $r/4$

46. On moving a charge of 20 coulomb by 2 cm, 2 J of work is done, then the potential difference between the points is **[2002]**

(a) 0.1 V (b) 8 V (c) 2 V (d) 0.5 V

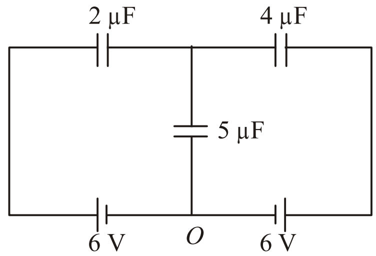
TOPIC 3 Capacitors, Grouping of Capacitors and Energy Stored in a Capacitor

47. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is : **[Sep. 05, 2020 (I)]**

(a) $\frac{25}{6}CV^2$ (b) $\frac{3}{2}CV^2$

(c) zero (d) $\frac{9}{2}CV^2$

48. In the circuit shown, charge on the 5 μF capacitor is : **[Sep. 05, 2020 (II)]**



(a) 18.00 μC (b) 10.90 μC

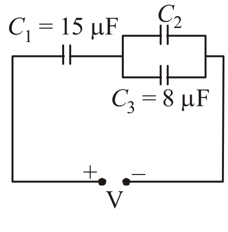
(c) 16.36 μC (d) 5.45 μC

49. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is : **[Sep. 04, 2020 (II)]**

(a) $\frac{1}{2}CV_0^2$ (b) $\frac{1}{3}CV_0^2$

(c) $\frac{1}{4}CV_0^2$ (d) $\frac{1}{6}CV_0^2$

50. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is : **[Sep. 03, 2020 (I)]**



(a) 450 μC (b) 590 μC

(c) 160 μC (d) 650 μC

51. A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J then value of X to the nearest integer is ________. **[NA Sep. 02, 2020 (I)]**

52. A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is : **[Sep. 02, 2020 (II)]**

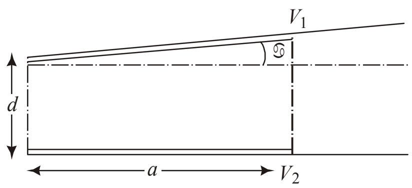
(a) 15 μF (b) 30 μF

(c) 20 μF (d) 10 μF

53. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1 V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be: **[8 Jan. 2020 I]**

(a) 4.2 μF (b) 3.2 μF (c) 1.6 μF (d) 8.4 μF

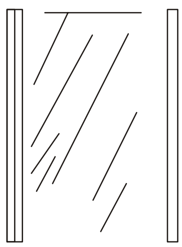
54. A capacitor is made of two square plates each of side 'a' making a very small angle a between them, as shown in figure. The capacitance will be close to: **[8 Jan. 2020 II]**



(a) $\frac{\epsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$ (b) $\frac{\epsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$

(c) $\frac{\epsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$ (d) $\frac{\epsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$

55. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression: **[7 Jan. 2020 I]**



(a) $\frac{AK\epsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$

(b) $\frac{A\epsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

(c) $\frac{A\epsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$

(d) $\frac{AK\epsilon_0}{d}(1+\alpha d)$

56. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) **[NA 7 Jan. 2020 II]**

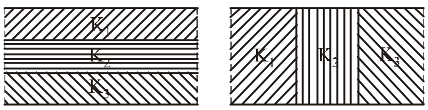
57. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is: **[9 April 2020 II]**

(a) $\frac{nV}{K+n}$ (b) V

(c) $\frac{V}{K+n}$ (d) $\frac{(n+1)V}{(K+n)}$

58. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitors is filled as shown in Fig. I, and the second one is filled as shown in Fig. II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitors (I) and E_2 to capacitors (II) : **[12 April 2019 I]**



(I) (II)

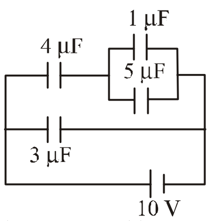
(a) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2}$

(b) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2}{K_1K_2K_3}$

(c) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

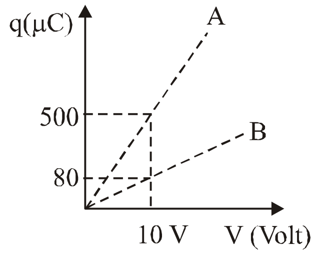
(d) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2}{9K_1K_2K_3}$

59. In the given circuit, the charge on 4 μF capacitor will be : **[12 April 2019 II]**



(a) 5.4 μC (b) 9.6 μC (c) 13.4 μC (d) 24 μC

60. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are : **[10 April 2019 I]**



(a) 40 μF and 10 μF (b) 60 μF and 40 μF
(c) 50 μF and 30 μF (d) 20 μF and 30 μF

61. A capacitor with capacitance 5μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 ¼F, how much work is done? **[9 April 2019 I]**

(a) 6.25×10^{-6} J (b) 3.75×10^{-6} J
(c) 2.16×10^{-6} J (d) 2.55×10^{-6} J

62. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2. What is the dielectric constant if the capacitance is 15 pF ? **[8 April 2019 I]**

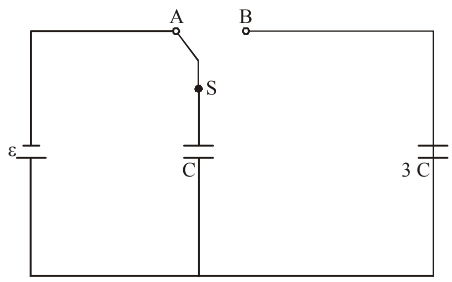
(given “$_0 = 8.86 \times 10^{-12}$ C^2/Nm^2)

(a) 3.8 (b) 8.5 (c) 4.5 (d) 6.2

63. A parallel plate capacitor has 1μF capacitance. One of its two plates is given + 2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is : **[8 April 2019 II]**

(a) 3V (b) 1V (c) 5V (d) 2V

64. In the figure shown, after the switch ‘S’ is turned from position ‘A’ to position ‘B’, the energy dissipated in the circuit in terms of capacitance ‘C’ and total charge ‘Q’ is: **[12 Jan. 2019 I]**



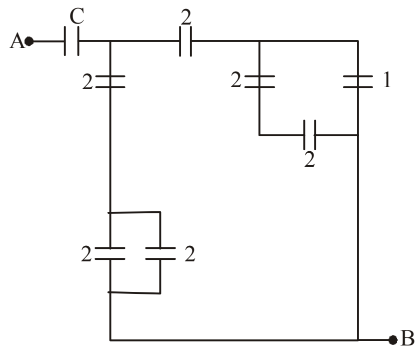
(a) $\frac{1}{8}\frac{Q^2}{C}$ (b) $\frac{3}{8}\frac{Q^2}{C}$ (c) $\frac{5}{8}\frac{Q^2}{C}$ (d) $\frac{3}{4}\frac{Q^2}{C}$

65. A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is :

(Take $\in_0 = 8.85 \times 10^{-12} \frac{C^2}{N-M^2}$) **[12 Jan. 2019 II]**

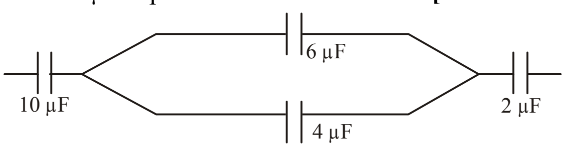
(a) 7.85×10^{-10} C (b) 6.85×10^{-10} C
(c) 8.85×10^{-10} C (d) 9.85×10^{-10} C

66. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF. **[12 Jan. 2019 II]**



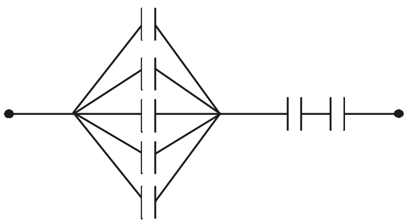
(a) $\frac{7}{11}$ μF (b) $\frac{6}{5}$ μF (c) 4 μF (d) $\frac{7}{10}$ μF

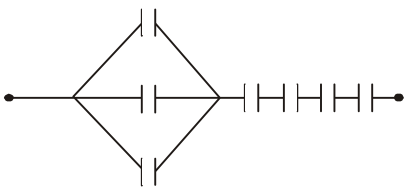
67. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30μC. The charge on the right plate of the 6μF capacitor is : **[11 Jan. 2019 I]**

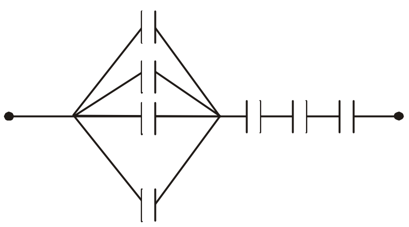


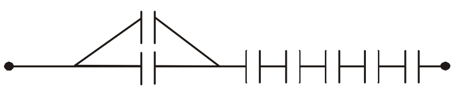
(a) –12 μC (b) +12 μC
(c) –18 μC (d) +18 μC

68. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)$ μF. Which of the combinations, shown in figures below, will achieve the desired value? **[11 Jan. 2019 II]**

(a) 

(b) 

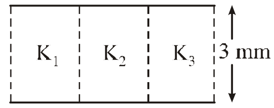
(c) 

(d) 

69. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is: **[10 Jan. 2019 II]**

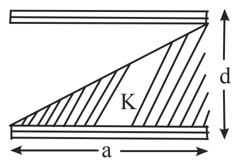
(a) 692 pJ (b) 508 pJ
(c) 560 pJ (d) 600 pJ

70. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 1(4)$ The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be: **[10 Jan. 2019 I]**



(a) 4 (b) 14 (c) 12 (d) 36

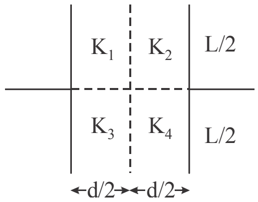
71. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is: **[9 Jan. 2019 I]**



(a) $\frac{K \epsilon_0 a^2}{2d(K+1)}$ (b) $\frac{K \epsilon_0 a^2}{d(K-1)} \ln K$

(c) $\frac{K \epsilon_0 a^2}{d} \ln K$ (d) $\frac{1}{2}\frac{K \epsilon_0 a^2}{d}$

72. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be: **[9 Jan. 2019 II]**



(a) $K = \frac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(b) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

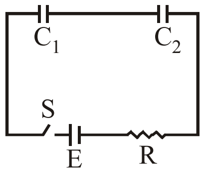
(c) $K = \frac{(K_1 + K_2)\,(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(d) $K = \frac{(K_1 + K_4)\,(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$

73. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $k = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be: **[2018]**

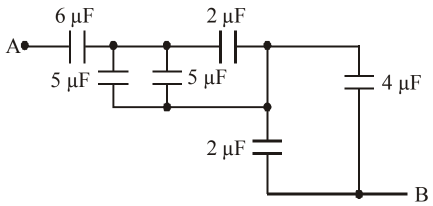
(a) 1.2 n C (b) 0.3 n C (c) 2.4 n C (d) 0.9 n C

74. In the following circuit, the switch S is closed at t = 0. The charge on the capacitor C_1 as a function of time will be given by $\left(C_{eq} = \frac{C_1C_2}{C_1 + C_2}\right)$. **[Online April 16, 2018]**



(a) $C_{eq}E[1 - \exp(-t/RC_{eq})]$
(b) $C_1E[1 - \exp(-tR/C_1)]$
(c) $C_2E[1 - \exp(-t/RC_2)]$
(d) $C_{eq}E\exp(-t/RC_{eq})$

75. The equivalent capacitance between *A* and *B* in the circuit given below is:



[Online April 15, 2018]

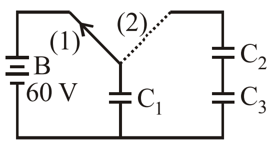
(a) 4.9 μF (b) 3.6 μF (c) 5.4 μF (d) 2.4 μF

76. A parallel plate capacitor with area $200 cm^2$ and separation between the plates 1.5cm, is connected across a battery of emf V. If the force of attraction between the plates is 25×10^{-6}N, the value of V is approximately: **[Online April 15, 2018]**

$$\left(\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N.m^2}\right)$$

(a) 150V (b) 100V (c) 250V (d) 300V

77. A capacitor C_1 is charged up to a voltage V = 60V by connecting it to battery B through switch (1), Now C_1 is disconnected from battery and connected to a circuit consisting of two uncharged capacitors $C_2 = 3.0$μF and $C_3 = 6.0$μF through a switch (2) as shown in the figure. The sum of final charges on C_2 and C_3 is: **[Online April 15, 2018]**

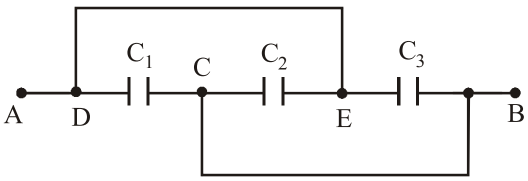


(a) 36μC (b) 20μC (c) 54μC (d) 40μC

78. A capacitance of $2\mu F$ is required in an electrical circuit across a potential difference of 1.0 kV. A large number of $1\mu F$ capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is **[2017]**

(a) 24 (b) 32 (c) 2 (d) 16

79. A combination of parallel plate capacitors is maintained at a certain potential difference.



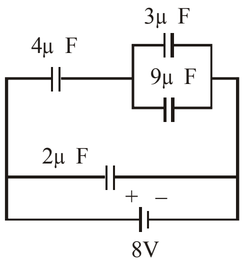
When a 3 mm thick slab is introduced between all the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab. **[Online April 9, 2017]**

(a) 3 (b) 4 (c) 5 (d) 6

80. The energy stored in the electric field produced by a metal sphere is 4.5 J. If the sphere contains 4 μC charge, its radius will be : [Take : $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N - m^2/C^2$] **[Online April 8, 2017]**

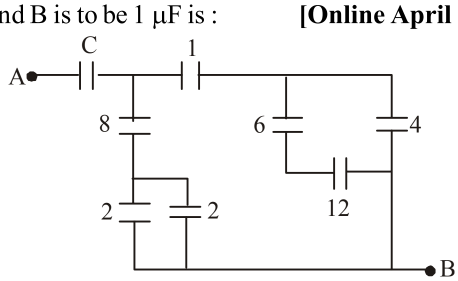
(a) 20mm (b) 32mm (c) 28mm (d) 16mm

81. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distance 30 m from it, would equal : **[2016]**



(a) 420 N/C (b) 480 N/C
(c) 240 N/C (d) 360 N/C

82. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is : **[Online April 10, 2016]**

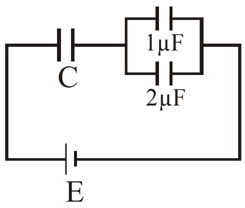


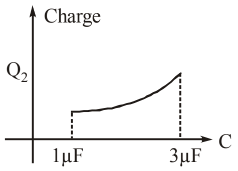
(a) $\frac{32}{23}\mu F$ (b) $\frac{31}{23}\mu F$ (c) $\frac{33}{23}\mu F$ (d) $\frac{34}{23}\mu F$

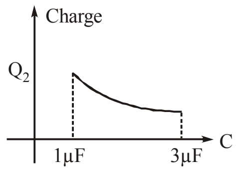
83. Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6μF. This can be done by connecting them : **[Online April 9, 2016]**

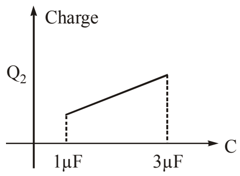
(a) all in series
(b) all in parallel
(c) two in parallel and one in series
(d) two in series and one in parallel

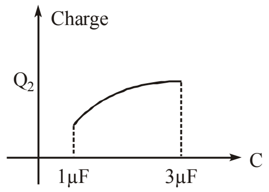
84. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by: (*figures are drawn schematically and are not to scale*) **[2015]**



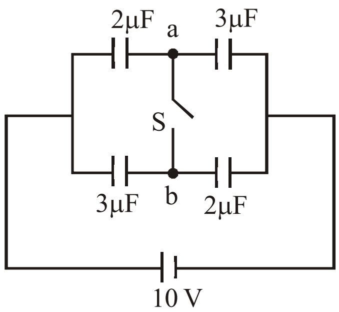
(a)


(b)


(c)


(d)


85. In figure a system of four capacitors connected across a 10 V battery is shown. Charge that will flow from switch S when it is closed is : **[Online April 11, 2015]**



(a) 5 μC from b to a (b) 20 μC from a to b
(c) zero (d) 5 μC from a to b

86. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dialectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m the charge density of the positive plate will be close to: **[2014]**

(a) $6\times10^{-7}\ C/m^2$ (b) $3\times10^{-7}\ C/m^2$
(c) $3\times10^{4}\ C/m^2$ (d) $6\times10^{4}\ C/m^2$

87. The gap between the plates of a parallel plate capacitor of area A and distance between plates d, is filled with a dielectric whose permittivity varies linearly from $\in_1$ at one plate to $\in_2$ at the other. The capacitance of capacitor is: **[Online April 19, 2014]**

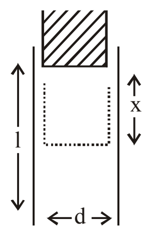
(a) $\in_0(\in_1+\in_2)A/d$
(b) $\in_0(\in_2+\in_1)A/2d$
(c) $\in_0 A/[d\,ln(\in_2/\in_1)]$
(d) $\in_0(\in_2-\in_1)A/[d\,ln(\in_2/\in_1)]$

88. The space between the plates of a parallel plate capacitor is filled with a 'dielectric' whose 'dielectric constant' varies with distance as per the relation:
$K(x) = K_o + \lambda x$ (λ = a constant)
The capacitance C, of the capacitor, would be related to its vacuum capacitance C_o for the relation : **[Online April 12, 2014]**

(a) $C=\frac{\lambda d}{ln(1+K_o\lambda d)}C_o$ (b) $C=\frac{\lambda}{d.ln(1+K_o\lambda d)}C_o$

(c) $C=\frac{\lambda d}{ln(1+\lambda d/K_o)}C_o$ (d) $C=\frac{\lambda}{d.ln(1+K_o/\lambda d)}C_o$

89. A parallel plate capacitor is made of two plates of length l, width w and separated by distance d. A dielectric slab (dielectric constant K) that fits exactly between the plates is held near the edge of the plates. It is pulled into the capacitor by a force $F=-\frac{\partial U}{\partial x}$ where U is the energy of the capacitor when dielectric is inside the capacitor up to distance x (See figure). If the charge on the capacitor is Q then the force on the dielectric when it is near the edge is: **[Online April 11, 2014]**



(a) $\frac{Q^2 d}{2\omega l^2\varepsilon_o}K$ (b) $\frac{Q^2\omega}{2dl^2\varepsilon_0}(K-1)$

(c) $\frac{Q^2 d}{2wl^2\varepsilon_o}(K-1)$ (d) $\frac{Q^2 w}{2dl^2\varepsilon_o}K$

90. Three capacitors, each of 3 μF, are provided. These cannot be combined to provide the resultant capacitance of: **[Online April 9, 2014]**

(a) 1 μF (b) 2 μF (c) 4.5 μF (d) 6 μF

91. A parallel plate capacitor having a separation between the plates d, plate area A and material with dielectric constant K has capacitance C_0. Now one-third of the material is replaced by another material with dielectric constant 2K, so that effectively there are two capacitors one with area

$\frac{1}{3}$ A, dielectric constant 2K and another with area $\frac{2}{3}$ A and dielectric constant K. If the capacitance of this new capacitor is C then $\frac{C}{C_0}$ is **[Online April 25, 2013]**

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

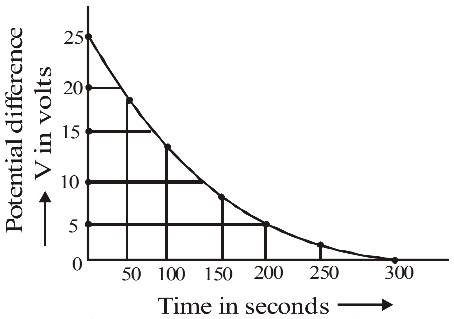
92. To establish an instantaneous current of 2 A through a 1 μF capacitor ; the potential difference across the capacitor plates should be changed at the rate of : **[Online April 22, 2013]**

(a) 2×10^4 V/s (b) 4×10^6 V/s
(c) 2×10^6 V/s (d) 4×10^4 V/s

93. A uniform electric field $\vec{E}$ exists between the plates of a charged condenser. A charged particle enters the space between the plates and perpendicular to $\vec{E}$. The path of the particle between the plates is a : **[Online April 9, 2013]**

(a) straight line (b) hyperbola
(c) parabola (d) circle

94. The figure shows an experimental plot discharging of a capacitor in an RC circuit. The time constant τ of this circuit lies between : **[2012]**



(a) 150 sec and 200 sec (b) 0 sec and 50 sec
(c) 50 sec and 100 sec (d) 100 sec and 150 sec

95. The capacitor of an oscillatory circuit is enclosed in a container. When the container is evacuated, the resonance frequency of the circuit is 10 kHz. When the container is filled with a gas, the resonance frequency changes by 50 Hz. The dielectric constant of the gas is **[Online May 26, 2012]**

(a) 1.001 (b) 2.001 (c) 1.01 (d) 3.01

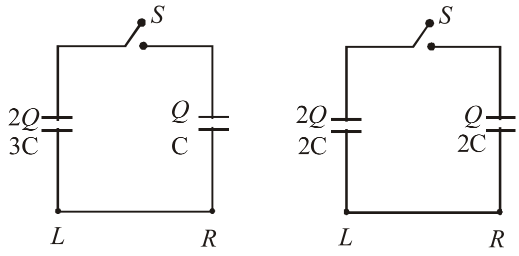
96. Statement 1: It is not possible to make a sphere of capacity 1 farad using a conducting material.
Statement 2: It is possible for earth as its radius is 6.4×10^6 m. **[Online May 26, 2012]**

(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(d) Statement 1 is true, Statement 2 is false.

97. A series combination of n_1 capacitors, each of capacity C_1 is charged by source of potential difference 4 V. When another parallel combination of n_2 capacitors each of capacity C_2 is charged by a source of potential difference V, it has the same total energy stored in it as the first combination has. The value of C_2 in terms of C_1 is then **[Online May 12, 2012]**

(a) $16\frac{n_2}{n_1}C_1$ (b) $\frac{2C_1}{n_1 n_2}$
(c) $2\frac{n_2}{n_1}C_1$ (d) $\frac{16C_1}{n_1 n_2}$

98. Two circuits (a) and (b) have charged capacitors of capacitance C, 2C and 3C with open switches. Charges on each of the capacitor are as shown in the figures. On closing the switches **[Online May 7, 2012]**



(a) No charge flows in (a) but charge flows from R to L in (b)
(b) Charges flow from L to R in both (a) and (b)
(c) Charges flow from R to L in (a) and from L to R in (b)
(d) No charge flows in (a) but charge flows from L to R in (b)

99. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be **[2010]**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2

100. A parallel plate capacitor with air between the plates has capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now **[2008]**

(a) 1.8 pF (b) 45 pF (c) 40.5 pF (d) 20.25 pF

101. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **[2007]**

(a) zero (b) $\frac{1}{2}(K-1)\,CV^2$

(c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)\,CV^2$

102. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is 'C' then the resultant capacitance is **[2005]**

(a) $(n+1)\,C$ (b) $(n-1)\,C$

(c) nC (d) C

103. A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass 'm'. If the temperature of the block is raised by 'ΔT', the potential difference 'V' across the capacitance is **[2005]**

(a) $\frac{mC\Delta T}{s}$ (b) $\sqrt{\frac{2mC\Delta T}{s}}$

(c) $\sqrt{\frac{2ms\Delta T}{C}}$ (d) $\frac{ms\Delta T}{C}$

104. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor **[2003]**

(a) decreases (b) remains unchanged

(c) becomes infinite (d) increases

105. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is **[2003]**

(a) 16×10^{-32} joule (b) 3.1×10^{-26} joule

(c) 4×10^{-10} joule (d) 32×10^{-32} joule

106. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to **[2002]**

(a) CV (b) $\frac{1}{2}nCV^2$ (c) CV^2 (d) $\frac{1}{2n}CV^2$

107. Capacitance (in F) of a spherical conductor with radius 1 m is **[2002]**

(a) 1.1×10^{-10} (b) 10^{-6}

(c) 9×10^{-9} (d) 10^{-3}

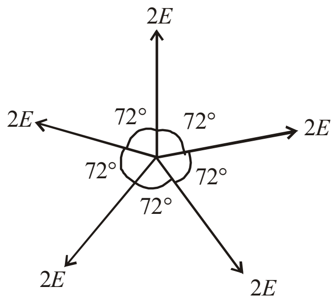
Hints & Solutions

1. **(c)** Potential at the centre, $V_C = \frac{KQ_{net}}{R}$

$\because Q_{net} = 0$

$\therefore V_C = 0$

Let E be electric field produced by each charge at the centre, then resultant electric field will be $E_C = 0$, since equal electric field vectors are acting at equal angle so their resultant is equal to zero.



2. **(d)** Total charge $Q_1 + Q_2 = Q'_1 + Q'_2$

$= 12\mu C - 3\mu C = 9\mu C$

Two isolated conducting sphres S_1 and S_2 are now connected by a conducting wire.

$\therefore V_1 = V_2 = \frac{KQ'_1}{\frac{2}{3}R} = \frac{KQ'_2}{\frac{R}{3}} = 12 - 3 = 9\ \mu\text{C}$

$Q'_1 = 2Q'_2 \Rightarrow 2Q'_2 + Q'_2 = 9\mu C$

$\therefore Q'_1 = 6\mu C$ and $Q'_2 = 3\mu C$

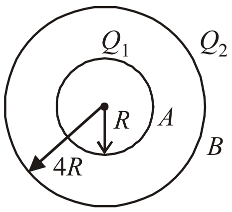
3. **(a)** We have given two metallic hollow spheres of radii R and $4R$ having charges Q_1 and Q_2 respectively.

Potential on the surface of inner sphere (at A)

$V_A = \frac{kQ_1}{R} + \frac{kQ_2}{4R}$

Potential on the surface of outer sphere (at B)

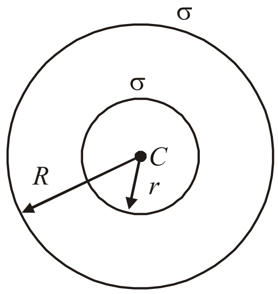
$V_B = \frac{kQ_1}{4R} + \frac{kQ_2}{4R}$ $\quad\left(\text{Here, k} = \frac{1}{4\pi\varepsilon_0}\right)$



Potential difference,

$\Delta V = V_A - V_B = \frac{3}{4}\cdot\frac{kQ_1}{R} = \frac{3}{16\pi\epsilon_0}\cdot\frac{Q_1}{R}$

4. **(d)** Let σ be the surface charge density of the shells.



Charge on the inner shell, $Q_1 = \sigma 4\pi r^2$

Charge on the outer shell, $Q_2 = \sigma 4\pi R^2$

$\therefore$ Total charge, $Q = \sigma 4\pi(r^2 + R^2)$

$\Rightarrow \sigma = \frac{Q}{4\pi(r^2 + R^2)}$

Potential at the common centre,

$V_C = \frac{KQ_1}{r} + \frac{KQ_2}{R} \quad \left(\text{where } K = \frac{1}{4\pi\varepsilon_0}\right)$

$= \frac{K\sigma 4\pi r^2}{r} + \frac{K\sigma 4\pi R^2}{R}$

$= K\sigma 4\pi(r + R)$

$= \frac{KQ4\pi(r + R)}{4\pi(r^2 + R^2)}$

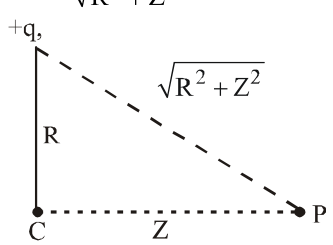
$= \frac{1}{4\pi\varepsilon_0}\frac{(r + R)Q}{(r^2 + R^2)}$

5. **(b)** The electric potential at the bisector is zero and electric field is antiparallel to the dipole moment.

$\therefore\ V = 0$ and $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-\vec{P}}{d^3}\right)$

6. **(c)** Potential at any point of the charged ring

$V_p = \frac{Kq}{\sqrt{R^2 + Z^2}}$



R = 3a

Z = 4a

$\ell = \sqrt{R^2 + Z^2} = 5a$

The minimum velocity (v_0) should just sufficient to reach the point charge at the center, therefore

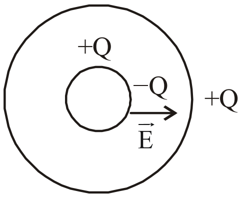
$$\frac{1}{2}mv_0^2 = q\,[V_C - V_P]$$

$$= q\left[\frac{Kq}{3a} - \frac{Kq}{5a}\right]$$

$$v_0^2 = \frac{4Kq^2}{15ma} = \frac{4}{15}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{ma}$$

$$\Rightarrow v_0 = \sqrt{\frac{2}{m}}\left(\frac{2q^2}{15 \times 4\pi\varepsilon_0 a}\right)^{\frac{1}{2}}$$

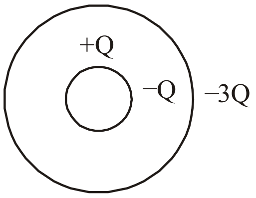
7. **(d)** When charge Q is on inner solid conducting sphere



Electric field between spherical surface

$$\vec{E} = \frac{KQ}{r^2} \quad So \int \vec{E}.d\vec{r} = V \ \ given$$

Now when a charge –4Q is given to hollow shell



Electric field between surface remain unchanged.

$$\vec{E} = \frac{KQ}{r^2}$$

as, field inside the hollow spherical shell = 0

$\therefore$ Potential difference between them remain unchanged

i.e. $\int \vec{E}.d\vec{r} = V$

8. **(c)** Given, $\vec{E} = (Ax + B)\hat{i}$

or $E = 20x + 10$

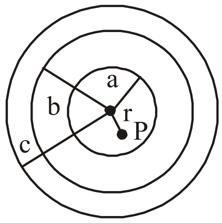
Using $V = \int E dx$, we have

$$V_2 - V_1 = \int_{-5}^{1} (20x + 10)dx = -180 \text{ V}$$

or $V_1 - V_2 = 180$ V

9. **(b)** Electric potential is constant inside a charged spherical shell.

10. **(d)**



Potential at point P, $V = \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$

Since surface charge densities are equal to one another i.e., $\sigma_a = \sigma_b = \sigma_c$

$$\therefore \ Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$$

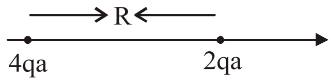
$$\therefore Q_a = \left[\frac{a^2}{a^2 + b^2 + c^2}\right]Q$$

$$Q_b = \left[\frac{b^2}{a^2 + b^2 + c^2}\right]Q$$

$$Q_c = \left[\frac{c^2}{a^2 + b^2 + c^2}\right]Q$$

$$\therefore \ V = \frac{Q}{4\pi \epsilon_0}\left[\frac{(a + b + c)}{a^2 + b^2 + c^2}\right]$$

11. **(d)** Let at a distance 'x' from point B, both the dipoles produce same potential



$$\therefore \frac{4qa}{(R + x)} = \frac{2qa}{(x^2)}$$

$$\Rightarrow \ \sqrt{2}x = R + x \Rightarrow x = \frac{R}{\sqrt{2} - 1}$$

Therefore distance from A at which both of them produce the same potential

$$= \frac{R}{\sqrt{2} - 1} + R = \frac{\sqrt{2}R}{\sqrt{2} - 1}$$

12. **(b)** Electric field at a point outside the sphere is given by

$$E = \frac{1Q}{4\pi \epsilon_0 r^2} \quad \text{But } \rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\therefore E = \frac{\rho R^3}{3 \epsilon_0 r^2}$$

At surface $r = R$

$$\therefore E = \frac{\rho R^3}{3 \epsilon_0}$$

Let ρ_1 and ρ_2 are the charge densities of two sphere.

$$E_1 = \frac{\rho R_1}{3\varepsilon_0} \text{ and } E_2 = \frac{\rho_2 R_2}{3\varepsilon_0}$$

$\because \frac{E_1}{E_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{R_1}{R_2}$

This gives $\rho_1 = \rho_2 = \rho$

Potential at a point outside the sphere

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{\rho R^3}{3\varepsilon_0 r}\left(\because \rho = \frac{Q}{\frac{4}{3}\pi R^3}\right)$$

At surface, $r = R$

$$V = \frac{\rho R^2}{3\varepsilon_0} \text{ so, } V_1 = \frac{\rho R_1^2}{3\varepsilon_0} \text{ and } V_2 = \frac{\rho R_2^2}{3\varepsilon_0}$$

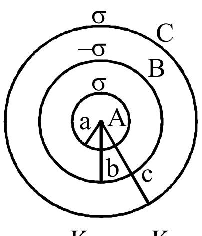
$$\therefore \frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^2$$

13. **(b)** Potential outside the shell, $V_{outside} = \frac{KQ}{r}$

where r is distance of point from the centre of shell

Potential inside the shell, $V_{inside} = \frac{KQ}{R}$

where 'R" is radius of the shell



$$V_B = \frac{Kq_A}{r_b} + \frac{Kq_B}{r_b} + \frac{Kq_C}{r_c}$$

$$V_B = \frac{1}{4\pi\epsilon_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

14. **(c)** Potential gradient is given by,

$\Delta V = E.d$

$0.8 = Ed\,(\max)$

$\Delta V = Ed\cos\theta = 0.8 \times \cos 60 = 0.4$

Hence, maximum potential at a point on the sphere $= 589.4$ V

15. **(c)** As we know electric field, $E = \frac{-dv}{dr}$

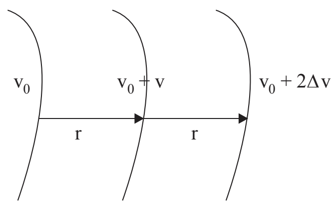
E = constant $\therefore$ dv and dr same

$$E = \frac{K\phi}{r^2} = c$$

$$\Rightarrow \phi \propto r^2$$

$$\therefore \phi = \int_0^r \rho 4\pi r^2 dr$$

$$\Rightarrow \rho \propto \frac{1}{r}$$



16. **(b)** $\Sigma_1 = \frac{-dv}{dr} = 10\,|z|$

$\Sigma_2 = \frac{-dv}{dr} = 10$ (constant : E)

$\therefore$ The source is an infinity large non conducting thick plate of thickness 2 m.

$$\therefore 10Z \cdot 10A = \frac{\rho \cdot A \propto Z}{\varepsilon_0}$$

$r_0 = 10e_0$ for $|z| \le 1$ m.

17. **(a)** We know, $V_0 = \frac{Kq}{R} = V$ surface

Now, $V_i = \frac{Kq}{2R^3}(3R^2 - r^2)$ [For r < R]

At the centre of sphere r = 0. Here

$$V = \frac{3}{2}V_0$$

Now, $\frac{5}{4}\frac{Kq}{R} = \frac{Kq}{2R^3}(3R^2 - r^2)$

$$\Rightarrow R_2 = \frac{R}{\sqrt{2}}$$

$$\frac{3}{4}\frac{Kq}{R} = \frac{Kq}{R^3}$$

$$\frac{1}{4}\frac{Kq}{R} = \frac{Kq}{R_4}$$

$R_4 = 4R$

Also, $R_1 = 0$ and $R_2 < (R_4 - R_3)$

18. **(a)** As we know, $E = -\frac{dv}{dx}$

Potential at the point x = 2m, y = 2m is given by :

$$\int_0^V dV = -\int_0^{2,2}(25dx + 30dy)$$

on solving we get,

V = – 110 volt.

19. **(c)** Potential difference between any two points in an electric field is given by,

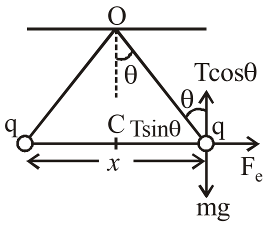
$dV = -\vec{E}\cdot\vec{dx}$

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_O = -[10x^3]_0^2 = -80 \text{ J/C}$$

20. **(a)** The potential of uncharged body is less than that of the charged conductor and more than at infinity.

21. (c)



In equilibrium, $F_e = T\sin\theta$

$mg = T\cos\theta$

$$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg}$$

$$\therefore\ x = \sqrt{\frac{q^2}{4\pi\epsilon_0 \tan\theta\ mg}}$$

Electric potential at the centre of the line

$$V = \frac{kq}{x/2} + \frac{kq}{x/2} = 4\sqrt{kmg/\tan\theta}$$

22. (c) $q = 1\mu C = 1\times10^{-6} C$

$r = 4\text{ cm} = 4\times10^{-2}\text{ m}$

Potential $V = \frac{kq}{r} = \frac{9\times10^9\times10^{-6}}{4\times10^{-2}} = 2.25\times10^5$ V.

Induced electric field $E = -\frac{kq}{r^2}$

$$= \frac{9\times10^9\times1\times10^{-6}}{16\times10^{-4}} = -5.625\times10^6 \text{ V/m}$$

23. (c) Let q_1 and q_2 be charge on two spheres of radius 'r' and 'R' respectively

As, $q_1 + q_2 = Q$

and $\sigma_1 = \sigma_2$ [Surface charge density are equal]

$$\therefore\ \frac{q_1}{r\pi r^2} = \frac{q_2}{4\pi R^2}$$

So, $q_1 = \frac{Qr^2}{R^2+r^2}$ and $q_2 = \frac{QR^2}{R^2+r^2}$

Now, potential, $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{R}\right]$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{R^2+r^2} + \frac{QR}{R^2+r^2}\right]$$

$$= \frac{Q(R+r)}{R^2+r^2}\frac{1}{4\pi\varepsilon_0}$$

24. (c) Charges reside only on the outer surface of a conductor with cavity.

25. (c) Electric field

$$E = -\frac{d\phi}{dr} = -2ar \quad ....(i)$$

By Gauss's theorem

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad ....(ii)$$

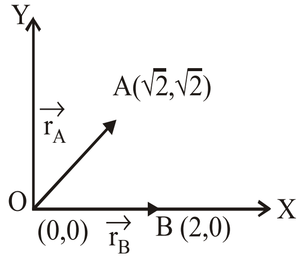
From (i) and (ii),

$Q = -8\pi\varepsilon_0 ar^3$

$\Rightarrow dq = -24\pi\varepsilon_0 ar^2\, dr$

Charge density, $\rho = \frac{dq}{4\pi r^2 dr} = -6\varepsilon_0 a$

26. (c)



The distance of point $A\left(\sqrt{2},\sqrt{2}\right)$ from the origin,

$$r_A = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4} = 2 \text{ units.}$$

The distance of point $B(2, 0)$ from the origin,

$$r_B = \sqrt{(2)^2 + (0)^2} = 2 \text{ units.}$$

Now, potential at A, due to charge $\theta = 10^{-3}\mu C$

$$V_A = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{(r_A)}$$

Potential at B, due to charge $Q = 10^{-3}$ QC $V_B = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{(r_B)}$

$\therefore$ Potential difference between the points A and B is given by

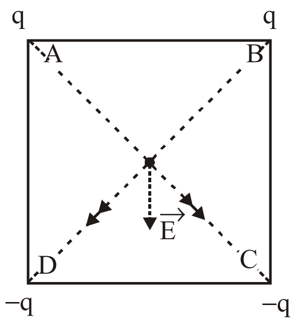
$$V_A - V_B = \frac{1}{4\pi\epsilon_0}\cdot\frac{10^{-3}}{r_A} - \frac{1}{4\pi\epsilon_0}\cdot\frac{10^{-3}}{r_B}$$

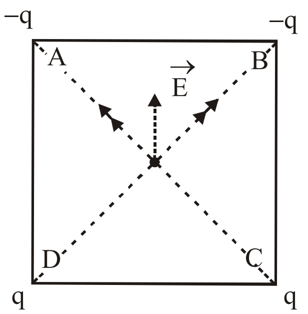
$$= \frac{10^{-3}}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = \frac{10^{-3}}{4\pi\epsilon_0}\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\times 0 = 0.$$

27. (a) As shown in the figure, the resultant electric fields before and after interchanging the charges will have the same magnitude, but opposite directions.

As potential is a scalar quantity, So the potential will be same in both cases.





28. **(a)** Given, potential $V(x) = \dfrac{20}{x^2 - 4}$ volt

Electric field $E = -\dfrac{dV}{dx} = -\dfrac{d}{dx}\left(\dfrac{20}{x^2 - 4}\right)$

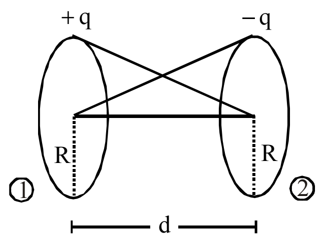
$\Rightarrow E = +\dfrac{40x}{(x^2 - 4)^2}$

At $x = 4\ \mu m$,

$E = +\dfrac{40 \times 4}{(4^2 - 4)^2} = +\dfrac{160}{144} = +\dfrac{10}{9}$ volt / μm.

Positive sign indicates that $\vec{E}$ is in +ve x-direction.

29. **(a)**



Potential at the center of ring of charge $+q$ = potential due to iteself + potential due to other ring of charge $-q$.

$$\Rightarrow V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{\sqrt{R^2 + d^2}}\right]$$

Potential at the centre of ring of charge $-q$ = potential due to itself + potential due to other ring of charge $+q$.

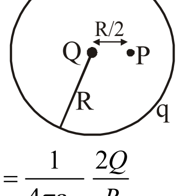
$$\Rightarrow V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{R} + \frac{q}{\sqrt{R^2 + d^2}}\right]$$

$\Delta V = V_1 - V_2$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} + \frac{q}{R} - \frac{q}{\sqrt{R^2 + d^2}} - \frac{q}{\sqrt{R^2 + d^2}}\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{\sqrt{R^2 + d^2}}\right]$$

30. **(c)** Electric potential due to charge Q at point P is



$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R/2} = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R}$$

Electric potential due to charge q inside the shell is

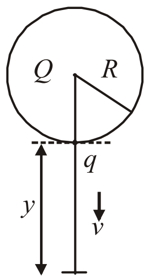
$$V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

∴ The net electric potential at point P is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R} + \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

31. **(d)** By using energy conservation,

$$\Delta KE + (\Delta PE)_{\text{Electro}} + (\Delta PE)_{\text{gravitational}} = 0$$



$$\frac{1}{2}mV^2 + \left(k\frac{Qq}{R+y} - k\frac{Qq}{R}\right) + (-mgy) = 0$$

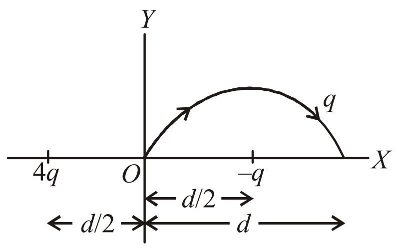
$$\Rightarrow \frac{1}{2}mV^2 = mgy + kQq\left(\frac{1}{R} - \frac{1}{R+y}\right)$$

$$\Rightarrow V^2 = 2gy + \frac{2kQq}{m}\frac{y}{R(R+y)}$$

$$\text{or, } V^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$$

32. **(d)** Change in potential energy, $\Delta u = q(V_f - V_i)$

Potential of $-q$ is same as initial and final point of the path.



$$\Delta u = q\left(\frac{k4q}{3d/2} - \frac{k4q}{d/2}\right) = -\frac{4q^2}{3\pi\varepsilon_0 d}$$

–ve sign shows the energy of the charge is decreasing.

33. **(c)** According to work energy theorem, gain in kinetic energy is equal to work done in displacement of charge.

$$\therefore \quad \frac{1}{2}mv^2 = q\Delta V$$

Here, ΔV = potential difference between two positions of charge q.

For same q and ΔV.

$$v \propto \frac{1}{\sqrt{m}}$$

Mass of hydrogen ion $m_H = 1$
Mass of helium ion $m_{He} = 4$

$$\therefore \quad \frac{v_H}{v_{He}} = \sqrt{\frac{4}{1}} = 2:1.$$

34. **(c)** Using conservation of energy

$$U_i = U_F + \frac{1}{2}mv^2$$

$$\frac{kq_1q_2}{r_1} = \frac{kq_1q_2}{r_2} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}mv^2 = kq_1q_2\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$v^2 = \frac{2kq_1q_2}{m}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2\times 9\times 10^9 \times 10^{-12}}{4\times 10^{-6}\times 10^{-3}}\left[1 - \frac{1}{9}\right] = 4\times 10^{+6}$$

$v = 2 \times 10^3$ m/s

35. **(d)** $$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q(-q)}{d} + \frac{qQ}{\left(D + \frac{d}{2}\right)} + \frac{(-q)Q}{\left(D - \frac{d}{2}\right)}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right], \text{Ignoring } \frac{d^2}{4}$$

36. **(b)** Using, $[K + U]_i = [K + U]_f$

or $\quad 0 + Vq = mv^2 + v'q$

or $\quad mv^2 = (V - V')q$

$$= -q\int_{r_0}^{r} Edr = q\int_{r_0}^{r}\frac{\lambda}{2\pi\epsilon_0 r}dr = \frac{\lambda q}{2\pi\epsilon_0}\left(\frac{\ln^3}{r_0}\right)$$

$$\Rightarrow v \propto \sqrt{l\,n\frac{r}{r_0}}$$

37. **(c)** Total energy of charge distribution is constant at any instant t.

$U_f + K_f = U_i + K_i$

$$\text{i.e., } \frac{1}{2}mV^2 + \frac{KQ^2}{2R} = 0 + \frac{KQ^2}{2R_0}$$

$$\therefore \frac{1}{2}mV^2 = \frac{KQ^2}{2R_0} - \frac{KQ^2}{2R}$$

$$\therefore V = \sqrt{\frac{2}{m}\frac{KQ^2}{2}\left(\frac{1}{R_0} - \frac{1}{R}\right)}$$

$$\therefore V = \sqrt{\frac{KQ^2}{m}\left(\frac{1}{R_0} - \frac{1}{R}\right)} = C\sqrt{\frac{1}{R_0} - \frac{1}{R}}$$

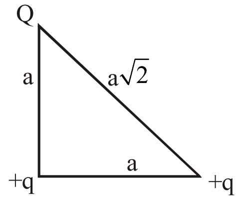
Also the slope of V – R curve will go on decreasing.

38. **(b)** Net electrostatic energy for the system

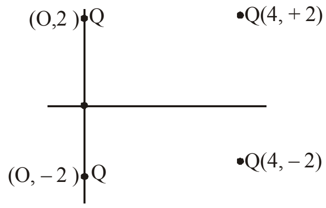
$$U = K\left[\frac{q^2}{a} + \frac{Qq}{a} + \frac{Qq}{a\sqrt{2}}\right] = 0$$

$$\Rightarrow q = -Q\left[1 + \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow Q = \frac{-q\sqrt{2}}{\sqrt{2}+1}$$



39. **(b)**



Potential at origin

$$v = \frac{KQ}{2} + \frac{KQ}{2} + \frac{KQ}{\sqrt{20}} + \frac{KQ}{\sqrt{20}}$$

and potential at $\infty = 0 = KQ\left(1 + \frac{1}{\sqrt{5}}\right)$

$\therefore$ Work required to put a fifth charge Q at origin W =

$$VQ = \frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{5}}\right)$$

40. **(c)** The work done in moving a charge along an equipotential surface is always zero.
The direction of electric field is perpendicular to the equipotential surface or lines.

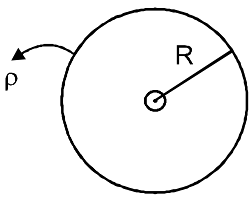
41. **(c)** The potential energy at the centre of the sphere

$$U_c = \frac{3}{2}\frac{KQq}{R}$$

The potential energy at the surface of the sphere

$$U_s = \frac{KqQ}{R}$$

Now change in the energy



$$\Delta U = U_c - U_s$$

$$= \frac{KQq}{R}\left[\frac{3}{2} - 1\right] = \frac{KQq}{2R}$$

Where $Q = \rho.V = \rho.\frac{4}{3}\pi R^3$

$$\Delta U = \frac{2K}{3}\frac{\pi R^3\rho q}{R}$$

$$\Delta U = \frac{2}{3}\times\frac{1}{4\pi\epsilon_0}\frac{\pi R^3\rho q}{R}$$

$$\Delta U = \frac{R^2\rho q}{6\epsilon_0}$$

Using Gauss's law

$$\int \vec{E}\cdot\overrightarrow{dA} = \frac{q_{en}}{E_0} = \frac{\beta\times\frac{4}{3}\pi R^3}{E_0}$$

$$\Rightarrow \int EdA(\cos\theta) = \frac{\beta\times 4\pi R^3}{3E_0}$$

$$\Rightarrow E(4\pi R^2) = \beta\times\frac{4}{3}\pi R^3\times\frac{1}{E_0}$$

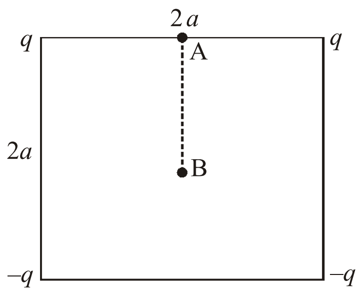
$$\Rightarrow E = \frac{\beta r}{3E_0}(r<R)$$

42. **(d)** Initial potential of the charge,

$$V_A = \frac{2kq}{a} - \frac{2kq}{a\sqrt{5}}$$

$$\Rightarrow V_A = \frac{1}{4\pi E}\frac{2q}{a}\left(1-\frac{1}{\sqrt{5}}\right)$$

(Here potential due to each $q = \frac{kq}{a}$ and potential due to each $-q = \frac{-kq}{a\sqrt{5}}$)



Final potential of the charge

$V_B = 0$

($\because$ Point B is equidistant from all the four charges)

$\therefore$ Using work energy theorem,

$(W_{AB})_{\text{electric}} = Q(V_A - V_B)$

$$= \frac{2qQ}{4\pi E_0 a}\left[1-\frac{1}{\sqrt{5}}\right]$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2Qq}{a}\left[1-\frac{1}{\sqrt{5}}\right]$$

43. **(c)** Work done, $W_{PQ} = q(V_Q - V_P)$

$= (-100\times 1.6\times 10^{-19})(-4-10)$

$= +2.24\times 10^{-16}\text{J}$

44. **(a)** Gain in kinetic energy = work done by potential difference

$$eV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2\times 1.6\times 10^{-19}\times 20}{9.1\times 10^{-31}}} = 2.65\times 10^6\text{ m/s}$$

45. **(d)** $\frac{1}{2}mv^2 = \frac{kQq}{r}$

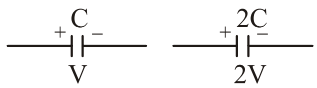
$$\Rightarrow \frac{1}{2}m(2v)^2 = \frac{kqQ}{r'} \Rightarrow r' = \frac{r}{4}$$

46. **(a)** By using

$W = q(V_B - V_A)$

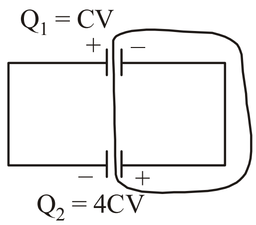
$$\therefore V_B - V_A = \frac{2\text{J}}{20\text{C}} = 0.1\text{J/C} = 0.1\text{V}$$

47. **(b)** When capacitors C and $2C$ capacitance are charged to V and 2V respectively.



$Q_1 = CV \quad Q_2 = 2C\times 2V = 4CV$

When connected in parallel



By conservation of charge

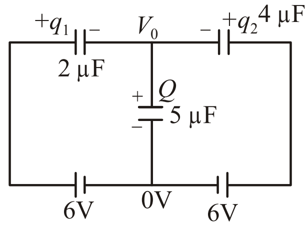
$$4CV - CV = (C+2C)V_{\text{common}}$$

$$V_{\text{common}} = \frac{3CV}{3C} = V$$

Therefore final energy of this configuration,

$$U_f = \left(\frac{1}{2}CV^2 + \frac{1}{2}\times 2CV^2\right) = \frac{3}{2}CV^2$$

48. **(a)**



Let q_1 and q_2 be the charge on the capacitors of 2μF and 4μF. Then charge on capacitor of 5μF

$$Q = q_1 + q_2$$

$$\Rightarrow 5V_0 = 2(6-V_0) + 4(6-V_0)$$

$$\Rightarrow 5V_0 = 12 - 2V_0 + 24 - 4V_0$$

$$\Rightarrow 11V_0 = 36 \Rightarrow V_0 = \frac{36}{11}V$$

$$\Rightarrow Q = 5V_0 = \frac{180}{11}\mu C$$

49. **(d)** When two capacitors with capacitance C_1 and C_2 at potential V_1 and V_2 connected to each other by wire, charge begins to flow from higher to lower potential till they acquire common potential. Here, some loss of energy takes place which is given by.

Heat loss, $H = \frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$

In the equation, put $V_2 = 0$, $V_1 = V_0$

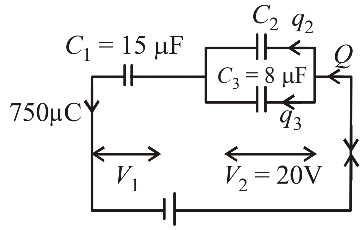
$C_1 = C,\ C_2 = \frac{C}{2}$

Loss of heat $= \frac{C\times\frac{C}{2}}{2\left(C+\frac{C}{2}\right)}(V_0-0)^2 = \frac{C}{6}V_0^2$

$H = \frac{1}{6}CV_0^2$

50. **(b)** According to question,

$Q = 750\mu C = q_2 + q_3$



Capacitors C_2 and C_3 are in parallel hence,
Voltage across C_2 = voltage across C_3 = 20 V
Change on capacitor C_3,

$q_3 = C_3 \times V_3 = 8\times 20 = 160\mu C$

$\therefore q_2 = 750\mu C - 160\mu C = 590\mu C$

51. **(4)**

Given, $C_1 = 5\ \mu F$ and $V_1 = 220$ Volt
When capacitor C_1 fully charged it is disconnected from the supply and connected to uncharged capacitor C_2.
$C_2 = 2.5\ \mu F$, $V_2 = 0$
Energy change during the charge redistribution,

$\Delta U = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$

$= \frac{1}{2}\times\frac{5\times 2.5}{(5+2.5)}(220-0)^2\ \mu J$

$= \frac{5}{2\times 3}\times 22\times 22\times 100\times 10^{-6}$ J

$= \frac{5\times 11\times 22}{3}\times 10^{-4}\text{ J} = \frac{55\times 22}{3}\times 10^{-4}$ J

$= \frac{1210}{3}\times 10^{-4}\text{ J} = \frac{1210}{3}\times 10^{-3}\text{ J} \simeq 4\times 10^{-2}$ J

According to questions, $\frac{x}{100} = 4\times 10^{-2}$

$\therefore x = 4$

52. **(a)** Given,

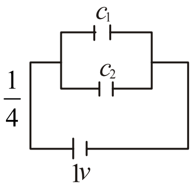
Capacitance of capacitor, $C_1 = 10\ \mu F$
Potential difference before removing the source voltage, $V_1 = 50$ V
If C_2 be the capacitance of uncharged capacitor, then common potential is

$V = \frac{C_1V_1 + C_2V_2}{C_1+C_2}$

$\Rightarrow 20 = \frac{10\times 50 + 0}{20 + C} \Rightarrow C = 15\ \mu F$

53. **(c)** In parallel combination, $C_{eq} = C_1 + C_2 = 10\ \mu F$
When connected across 1 V battery, then

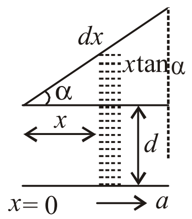
$\frac{U_1}{U_2} = \frac{\left(\frac{1}{2}C_1V^2\right)}{\left(\frac{1}{2}C_2V^2\right)} = \frac{1}{4} \Rightarrow \frac{C_1}{C_2} = \frac{1}{4}$



$\therefore C_2 = 8\ \mu F$ and $C_1 = 2\ \mu F$
Now C_1 and C_2 are connected in series combination,

$\therefore C_{\text{equivalent}} = \frac{C_1C_2}{C_1+C_2} = \frac{2\times 8}{2+8} = \frac{16}{10} = 1.6\mu F$

54. **(a)**



Consider an infinitesimal strip of capacitor of thickness dx at a distance x as shown.
Capacitance of parallel plate capacitor of area A is given by $C = \frac{\varepsilon_0 A}{t}$
[Here t = seperation between plates]
So, capacitance of thickness dx will be

$\therefore dC = \frac{\varepsilon_0 a dx}{d + x\tan\alpha}$

Total capacitance of system can be obtained by integrating with limits x = 0 to x = a

$\therefore C_{eq} = \int dC = a\varepsilon_0 \int_{x=0}^{x=a} \frac{dx}{x\tan\alpha + d}$

[By Binomial expansion]

$\Rightarrow C_{eq} = \frac{a\varepsilon_0}{d}\int_0^a\left(1 - \frac{x\tan\alpha}{d}\right)dx = \frac{a\varepsilon_0}{d}\left(x - \frac{x^2\tan\alpha}{2d}\right)_0^a$

$\Rightarrow C_{eq} = \frac{a^2\varepsilon_0}{d} = \left(1 - \frac{a\tan\alpha}{2d}\right) = \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$

55. **(a)** Given, $K(x) = K(1+\alpha x)$

Capacitance of element, $C_{el} = \frac{K\varepsilon_0 A}{dx}$

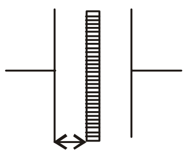
$$\Rightarrow C_{el} = \frac{\varepsilon_0 K(1+\alpha x)A}{dx}$$

$$\therefore \int d\left(\frac{1}{C}\right) = \frac{1}{C_{el}} = \int_0^d \left(\frac{dx}{\varepsilon_0 KA(1+\alpha x)}\right)$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 KA\alpha}[ln(1+\alpha x)]_0^d$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 KA\alpha} ln(1+\alpha d)[\alpha \text{ d} << 1]$$

$$= \frac{1}{\varepsilon_0 KA\alpha}\left[\alpha d - \frac{\alpha^2 d^2}{2}\right]$$



$$= \frac{1}{\varepsilon_0 KA}\left[1 - \frac{\alpha d}{2}\right]$$

$$\therefore C = \frac{\varepsilon_0 KA}{d\left(1 - \frac{\alpha d}{2}\right)} \Rightarrow C = \frac{\varepsilon_0 KA}{d}\left(1 + \frac{\alpha d}{2}\right)$$

56. **(6)** In the first condition, electrostatic energy is

$$U_i = \frac{1}{2}CV_0^2 = \frac{1}{2} \times 60 \times 10^{-12} \times 400 = 12 \times 10^{-9} J$$

In the second condition $U_F = \frac{1}{2}C'V'^2$

$$U_f = \frac{1}{2}2C.\left(\frac{V_0}{2}\right)^2 \qquad \left(\because C' = 2C, V' = \frac{V_0}{2}\right)$$

$$= \frac{1}{4} \times 60 \times 10^{-12} \times (20)^2 = 6 \times 10^{-9} J$$

Energy lost $= U_i - U_f = 12\times10^{-9}J - 6\times10^{-9}J = 6\ nJ$

57. **(d)** $V' = \frac{CV + (nC)V}{kC + nC}$

$$\frac{(n+1)V}{k+n}$$

58. **(c)** $\frac{1}{C_1} = \frac{d/3}{k_1\varepsilon_0 A} + \frac{d/3}{k_2\varepsilon_0 A} + \frac{d/3}{k_3\varepsilon_0 A}$

or $C_1 = \frac{3k_1k_2k_3\varepsilon_0 A}{d(k_1k_2 + k_2k_3 + k_3k_1)}$

C_2

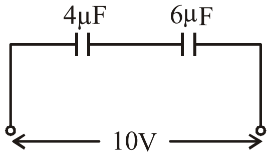
$$= \frac{k_1\varepsilon_0(A/3)}{d} + \frac{k_2\varepsilon_0(A/3)}{d} + \frac{k_3\varepsilon_0(A/3)}{d}$$

$$= \frac{(k_1+k_2+k_3)\varepsilon_0 A}{3d}$$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2}C_1V^2}{\frac{1}{2}C_2V^2}$$

$$= \frac{E_1}{E_2} = \frac{9k_1k_2k_3}{(k_1+k_2+k_3)(k_1k_2+k_2k_3+k_3k_1)}$$

59. **(d)** $V_1 + V_2 = 10$



and $4V_1 = 6V_2$

On solving above equations, we get

$V_1 = 6\text{ V}$

Charge on 4 μf,

$q = CV_1 = 4 \times 6 = 24\ \mu\text{C}$.

60. **(a)** Equivalent capacitance in series combination (C') is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C' = \frac{C_1C_2}{C_1 + C_2}$$

For parallel combination equivalent capacitance

$C'' = C_1 + C_2$

For parallel combination

$q = 10(C_1 + C_2)$

$q_1 = 500\ \mu C$

$500 = 10(C_1 + C_2)$

$C_1 + C_2 = 50\mu F$ ....(i)

For Series Combination–

$$q_2 = 10\frac{C_1C_2}{(C_1 + C_2)}$$

$80 = 10\frac{C_1C_2}{50}$ From equation ....(ii)

$C_1C_2 = 400$ ....(iii)

From equation (i) and (ii)

$C_1 = 10\mu F \quad C_2 = 40\mu F$

61. **(b)** $\omega = \omega_f - v_i = \frac{q}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right)$

$$= \frac{(5\times10)^2}{2}\left(\frac{1}{2} - \frac{1}{5}\right) \times 10^6$$

$= 3.75 \times 10^{-6}$J

62. **(b)** Capacitance of a capacitor with a dielectric of dielectric constant k is given by

$$C = \frac{k \in_0 A}{d}$$

$$\because E = \frac{V}{d} \qquad \therefore C = \frac{k \in_0 AE}{V}$$

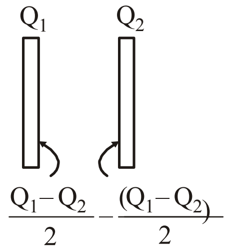
$$15 \times 10^{-12} = \frac{k \times 8.86 \times 10^{-12} \times 10^{-4} \times 10^6}{500}$$

$k = 8.5$

63. **(b)** $V = \frac{Q}{C}$

$= \left(\frac{Q_1 - Q_2}{2C}\right)$

$= \left(\frac{4-2}{2\times 1}\right) = 1\,V$



64. **(b)** Energy stored in the system initially

$U_i = \frac{1}{2}CE^2$

$U_f = \frac{1}{2}\frac{Q^2}{C_{eq}} = \frac{(CE)^2}{2\times 4C} = \frac{1}{2}\frac{CE^2}{4}$

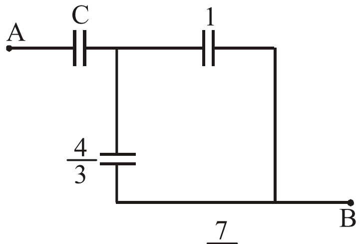
[As $Q = CE$, and $C_{eq} = 4C$]

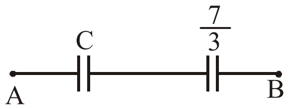
$\Delta U = \frac{1}{2}CE^2 \times \frac{3}{4} = \frac{3}{8}CE^2 = \frac{3}{8}\frac{Q^2}{C}$

65. **(c)** $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

$\therefore Q = \varepsilon_0 . E . A = 8.85 \times 10^{-12} \times 100 \times 1$
$= 8.85 \times 10^{-10} C$

66. **(a)**





For series combination

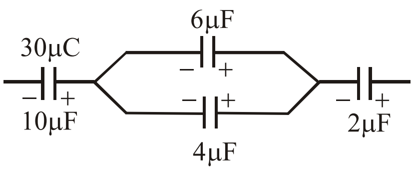
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$\Rightarrow \frac{\frac{7C}{3}}{\frac{7}{3}+C} = \frac{1}{2}$

$\Rightarrow 14\,C = 7 + 3\,C$

$\Rightarrow C = \frac{7}{11}\mu F$

67. **(d)**



As given in the figure, 6μF and 4μF are in parallel. Now using charge conservation

Charge on 6μF capacitor $= \frac{6}{6+4} \times 30 = 18\mu C$

Since charge is asked on right plate therefore is $+18\mu C$

68. **(b)** As required equivalent capacitance should be

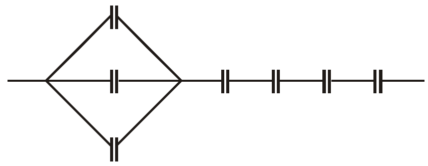
$C_{eq} = \frac{6}{13}\mu F$

Therefore three capacitors must be in parallel and 4 must be in series with it.

$\frac{1}{C_{eq}} = \left[\frac{1}{3C}\right] + \left[\frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}\right]$

$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F \quad [\text{as } C = 2\mu F]$

So, desired combination will be as below:



69. **(b)** $W = -\Delta u$

$= (-1)\left|\frac{(c\varepsilon)^2}{2kc} - \frac{(c\varepsilon)^2}{2c}\right|$

$= \frac{\varepsilon^2 c}{2}\,\frac{k-1}{k}$

$= 508\,J$

70. **(c)** Let dielectric constant of material used be K.

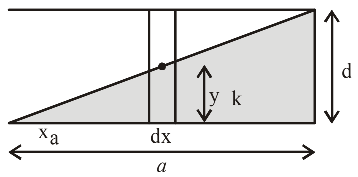
$\frac{k_1 \epsilon_0 A_1}{d} + \frac{k_2 \epsilon_0 A_2}{d} + \frac{k_3 \epsilon_0 A_3}{d} = \frac{k \epsilon_0 A}{d}$

or $\frac{10 \epsilon_0 A/3}{d} + \frac{12 \epsilon_0 A/3}{d} + \frac{14 \epsilon_0 A/3}{d} = \frac{K \epsilon_0 A}{d}$

$\frac{\epsilon_0 A}{d}\left(\frac{10}{3} + \frac{12}{3} + \frac{14}{3}\right) = \frac{K \epsilon_0 A}{d}$

$\therefore K = 12$

71. **(b)**



From figure, $\frac{y}{x} = \frac{d}{a} \Rightarrow y = \frac{d}{a}x$

$dy = \frac{d}{a}(dx) \;\Rightarrow \frac{1}{dc} = \frac{y}{K\varepsilon_0 adx} + \frac{(d-y)}{\varepsilon_0 adx}$

$\frac{1}{dc} = \frac{y}{\varepsilon_0 abx}\left(\frac{y}{k} + d - y\right)$

$\int dc = \int \frac{\varepsilon_0 adx}{\frac{y}{k} + d - y}$

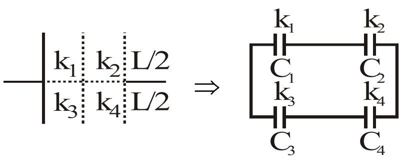
or, $c = \varepsilon_0 a . \frac{a}{d}\int_0^d \frac{dy}{d + y\left(\frac{1}{k} - 1\right)}$

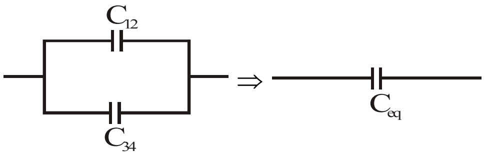
$$= \frac{\varepsilon_0 a^2}{\left(\frac{1}{k}-1\right)d}\left[\ell n\left(d + y\left(\frac{1}{k}-1\right)\right)\right]_0^d$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{d + d\left(\frac{1}{k}-1\right)}{d}\right)$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k \in_0 a^2 \ell nk}{(k-1)d}$$

72. (Bonus)





$$C_{12} = \frac{C_1C_2}{C_1+C_2} = \frac{\frac{k_1 \in_0 \frac{L}{2}\times L}{d/2}\cdot\frac{k_2\left[\in_0 \frac{L}{2}\times L\right]}{d/2}}{(k_1+k_2)\left[\frac{\in_0 \frac{L}{2}\times L}{d/2}\right]}$$

$$C_{12} = \frac{k_1k_2}{k_1+k_2}\frac{\in_0 L^2}{d}$$

in the same way we get,

$$C_{34} = \frac{k_3k_4}{k_3+k_4}\frac{\in_0 L^2}{d}$$

$$\therefore C_{eq} = C_{12} + C_{34} = \left[\frac{k_1k_2}{k_1+k_2} + \frac{k_3k_4}{k_3+k_4}\right]\frac{\in_0 L^2}{d} \quad ..(i)$$

Now if $k_{eq} = K$, $C_{eq} = \frac{k \in_0 L^2}{d}$ ...(ii)

on comparing equation (i) to equation (ii), we get

$$k_{eq} = \frac{k_1k_2(k_3+k_4) + k_3k_4(k_1+k_2)}{(k_1+k_2)(k_3+k_4)}$$

This does not match with any of the options so this must be a bonus.

73. **(a)** Charge on Capacitor, $Q_i = CV$

After inserting dielectric of dielectric constant

$= K\ Q_f = (kC)V$

Induced charges on dielectric

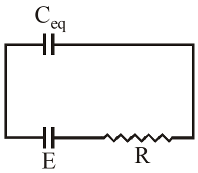
$Q_{ind} = Q_f - Q_i = KCV - CV$

$$(K-1)CV = \left(\frac{5}{3}-1\right) \times 90\,pF \times 2V = 1.2nc$$

74. **(a)** During charging charge on the capacitor increases with time. Charge on the capacitor C_1 as a function of time, $Q = Q_0(1 - e^{-t/RC})$

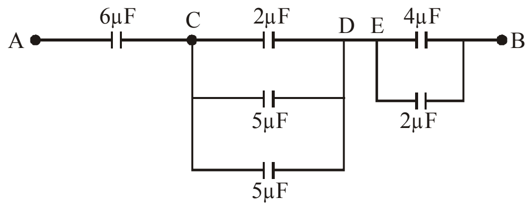
$$Q = C_{eq}E\left[1 - e^{-t/RC_{eq}}\right]$$

$(\because\ Q_0 = C_{eq}E)$



Both capacitor will have charge as they are connected in series

75. **(d)** The simplified circuit of the circuit given in question as follows:



The equivalent capacitance between C & D capacitors of 2μF, 5μF and 5 μF are in parallel.

$\therefore$ C_{CD} = 2 + 5 + 5 = 12 μF ($\because$ In parallel grouping $C_{eq} = C_1 + C_2 + ... + C_n$)

Similarly equivalent capacitance between E & B C_{EB} = 4 + 2 = 6μF

Now equivalent capacitance between A & B

$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$$

$\Rightarrow C_{eq} = \frac{12}{5} = 2.4\ \mu F$ ($\because$ In series grouping, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$)

76. **(c)** Given area of Parallel plate capacitor, $A = 200\ cm^2$

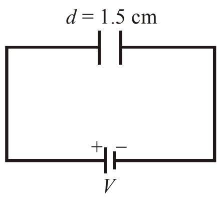
Separation between the plates, $d = 1.5$ cm

Force of attraction between the plates, $F = 25 \times 10^{-6}$N

$F = QE$

$$F = \frac{Q^2}{2A \in_0} \quad \left(\text{E due to parallel plate } = \frac{\sigma}{2\in_0} = \frac{Q}{A2\in_0}\right)$$

But $Q = CV = \frac{\in_0 A(V)}{d}$



$$\therefore \quad F = \frac{(\in_0 AV)^2}{d^2 \times 2A\in_0}$$

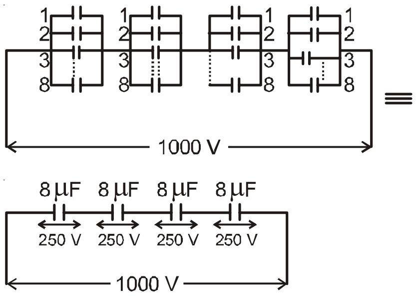
$$= \frac{(\in_0 A)^2 \times V^2}{d^2 \times 2 \times (A\in_0)} = \frac{(\in_0 A)\times V^2}{d^2 \times 2}$$

$$\text{or,} \quad 25\times10^{-6} = \frac{(8.85\times10^{-12})\times(200\times10^{-4})\times V^2}{2.25\times10^{-4}\times 2}$$

$$\Rightarrow \quad V = \sqrt{\frac{25\times10^{-6}\times2.25\times10^{-4}\times2}{8.85\times10^{-12}\times200\times10^{-4}}} \approx 250\ V$$

77. **(a)** The sum of final charges on C_2 and C_3 is 36 μC.

78. **(b)** To get a capacitance of 2μF arrangement of capacitors of capacitance 1μF as shown in figure 8 capacitors of 1μF in parallel with four such branches in series i.e., 32 such capacitors are required.



$$\frac{1}{C_{eq}}=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8} \quad \therefore\ C_{eq}=2\ \mu F$$

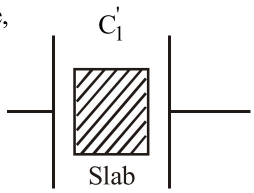
79. **(c)** Before introducing a slab capacitance of plates

$$C_1=\frac{\varepsilon_0 A}{3}$$

If a slab of dielectric constant K is introduced between plates then

$$C=\frac{K\varepsilon_0 A}{d} \text{ then } C_1'=\frac{\varepsilon_0 A}{2.4}$$

C_1 and C_1' are in series hence,



$$\frac{\varepsilon_0 A}{3}=\frac{k\frac{\varepsilon_0 A}{3}\cdot\frac{\varepsilon_0 A}{2.4}}{k\frac{\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}}$$

$3k=2.4k+3$

$0.6k=3$

Hence, the dielectric constant of slap is given by,

$$k=\frac{30}{6}=5$$

80. **(d)** Energy of sphere $=\dfrac{Q^2}{2C}$

$$4.5=\frac{16\times10^{-12}}{2C}$$

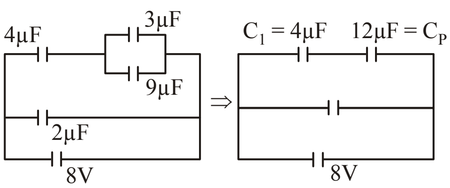
$$C=\frac{16\times10^{-12}}{9}=4\pi\varepsilon_0 R$$

(capacity of spherical conductor)

$$R=\frac{16\times10^{-12}}{9}\times\frac{1}{4\pi\varepsilon_0} \qquad \because\ \frac{1}{4\pi\varepsilon_0}=9\times10^9$$

$$=9\times10^9\times\frac{16}{9}\times10^{-12}=16\text{ mm}$$

81. **(a)**



Charge on C_1 is $q_1=\left[\left(\frac{12}{4+12}\right)\times8\right]\times4=24\mu C$

The voltage across C_P is $V_P=\frac{4}{4+12}\times8=2V$

∴ Voltage across 9μF is also 2V

∴ Charge on 9μF capacitor = 9 × 2 = 18μC

∴ Total charge on 4 μF and 9μF = 42μC

$$\therefore E=\frac{KQ}{r^2}=9\times10^9\times\frac{42\times10^{-6}}{30\times30}=420\,NC^{-1}$$

82. **(a)** Capacitors 2μF and 2μF are parallel, their equivalent = 4 μF

6μF and 12 μF are in series, their equivalent = 4 μF

Now 4μF (2 and 2 μF) and 8μF in series $=\frac{3}{8}\mu F$

And 4μF (12 & 6 μF) and 4μF in parallel = 4 + 4 = 8μF

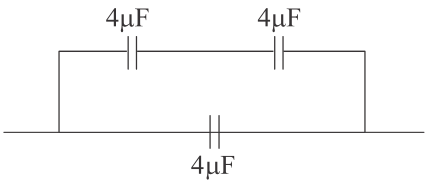
8μF in series with 1μF $=\frac{1}{8}+1\Rightarrow\frac{8}{9}\mu F$

$$\text{Now } C_{eq}=\frac{8}{9}+\frac{8}{3}=\frac{32}{9}$$

$$C_{eq}\text{ of circuit}=\frac{32}{9}$$

$$\text{With } C-\frac{1}{C_{eq}}=\frac{1}{C}+\frac{9}{32}=1\Rightarrow C=\frac{32}{23}$$

83. **(d)** To get effective capacitance of 6 μF two capacitors of 4 μF each connected in sereies and one of 4 μF capacitor in parallel with them.



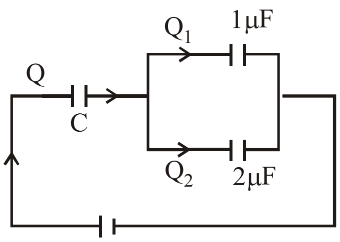
Two capacitances in series

$$\therefore\ \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

1 capacitor in parallel

$$\therefore\ C_{eq}=C_3+C=4+2=6\ \mu F$$

84. **(d)**

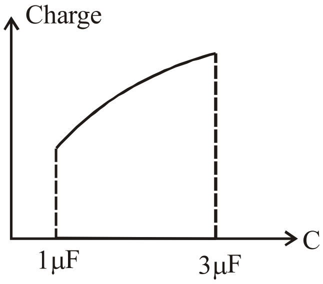


$$\text{From figure, } Q_2=\frac{2}{2+1}Q=\frac{2}{3}Q$$

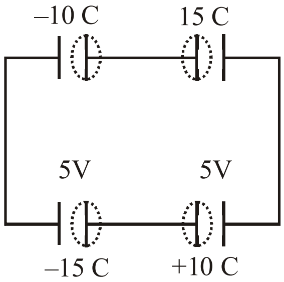
$Q = E\left(\dfrac{C \times 3}{C+3}\right)$

$\therefore Q_2 = \dfrac{2}{3}\left(\dfrac{3CE}{C+3}\right) = \dfrac{2CE}{C+3}$

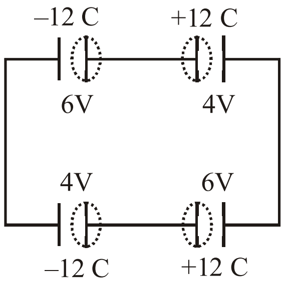
Therefore graph *d* correctly dipicts.



85. **(a)** when switch is closed



When switch is open



Charge of 5μc flows from *b* to *a*

86. **(a)** Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by,

$E = \dfrac{\sigma}{K\varepsilon_0}$

Then, charge density

$\sigma = K\varepsilon_0 E$

$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$

$\approx 6 \times 10^{-7}\ \text{C/m}^2$

87. **(d)**

88. **(c)** The value of dielectric constant is given as,

$K = K_0 + \lambda x$

And, $V = \int_0^d E\,dr \Rightarrow V = \int_0^d \dfrac{\sigma}{K} dx$

$= \sigma\int_0^d \dfrac{1}{(K_0 + \lambda x)} dx = \dfrac{\sigma}{\lambda}\left[\ln(K_0 + \lambda d) - \ln K_0\right]$

$= \dfrac{\sigma}{\lambda}\ln\left(1 + \dfrac{\lambda d}{K_0}\right)$

Now it is given that capacitance of vacuum = C_0.

Thus, $C = \dfrac{Q}{V}$

$= \dfrac{\sigma.s}{v}$ (Let surface area of plates = s)

$= \dfrac{\sigma.s}{\dfrac{\sigma}{\lambda}\ln\left(1+\dfrac{\lambda d}{K_0}\right)}$

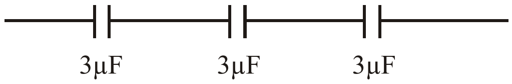
$= s\lambda.\dfrac{d}{d}\dfrac{1}{\ln\left(1+\dfrac{\lambda d}{K_0}\right)}$ ($\because$ in vacuum $\varepsilon_0 = 1$)

$c = \dfrac{\lambda d}{\ln\left(1+\dfrac{\lambda d}{K_0}\right)} \cdot C_0$ $\left(\text{here, } C_0 = \dfrac{s}{d}\right)$

89. **(c)**

90. **(d)** Possible combination of capacitors

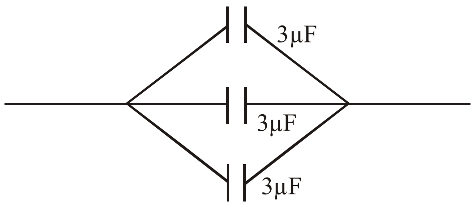
(i) Three capacitors in series combination



$\dfrac{1}{C_{eq}} = \dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{3}$

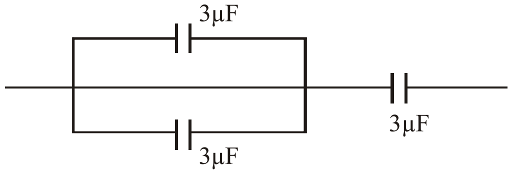
$\therefore \dfrac{1}{C_{eq}} = 1\mu F$

(ii) Three capacitors in parallel combination



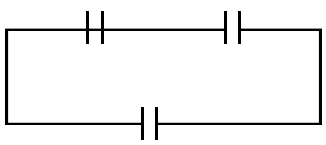
$C_{eq} = 3 + 3 + 3 = 9\ \mu F$

(iii) Two capacitors in parallel and one is in series



$C_{eq} = 2\mu F$

(iv) Two capacitors in series and one is in parallel



$C_{eq} = 4.5\ \mu F$

91. **(b)** $C_0 = \dfrac{k \in_0 A}{d}$

$$C = \frac{k \in_0 2}{3d} + \frac{2k \in_0 A}{3d} = \frac{4}{3}\frac{k \in_0 A}{d}$$

$$\therefore \frac{C}{C_0} = \frac{\frac{4}{3}\frac{k \in_0 A}{d}}{\frac{k \in_0 A}{d}} = \frac{4}{3}$$

92. **(c)** As, $C = \frac{Q}{V} = \frac{It}{V}$

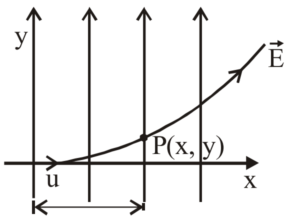
$\Rightarrow \frac{V}{t} = \frac{I}{C} = \frac{2}{1 \times 10^{-6}}$

$= 2 \times 10^6$ V/s

93. **(c)** When charged particle enters perpendicularly in an electric field, it describes a parabolic path

$$y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{4}\right)^2$$

This is the equation of parabola.



94. **(d)** The discharging of a capacitor is given as

$q = q_0 \, exp[-t/RC]$

RC = time constant = τ

$q = q_0 e^{-t/\tau}$

If e is the capacitance of the capacitor

$q = CV$ and $q = CV_0$

Thus, $CV = CV_0 e^{t/\tau}$

$V = V_0 e^{-t/\tau}$ ...(i)

From the graph (given in the problem when $t = 0.5$, $V = 25$ i.e.,

$V_0 = 25$ volt.

and when $t = 200$, $V = 5$ volt

Thus equation (i) becomes

$5 = 25e^{-200/\tau}$

$\Rightarrow \; 1/5 = e^{-200/\tau}$

Taking $\log_e$ on both sides

$\log_e \frac{1}{5} = -200/\tau \Rightarrow -\frac{200}{\tau} = \log e^5$

$$\tau = \frac{200}{\log_e 5}$$

or $\tau = \frac{200}{\log_e\left(\frac{10}{2}\right)} = \frac{200}{\log_e 10 - \log_e 2}$

$$\tau = \frac{200}{2.302 - 0.693} = \frac{200}{1.609} = 124.300$$

Which lies between 100 s and 150 s

95. **(c)** The dielectric constant of the gas is 1.01

96. **(d)** Capacitance of sphere is given by :

$C = 4\pi \in_0 r$

If, C = 1F then radius of sphere needed:

$$r = \frac{C}{4\pi \in_0} = \frac{1}{4\pi \times 8.85 \times 10^{-12}}$$

or, $r = \frac{10^{12}}{4\pi \times 8.85} = 9 \times 10^9$ m

9×10^9 m is very large, it is not possible to obtain such a large sphere. Infact earth has radius 6.4×10^6 m only and capacitance of earth is 711μF.

97. **(d)** Equivalent capacitance of n_2 number of capacitors each of capacitance C_2 in parallel $= n_2 C_2$

Equivalent capacitance of n_1 number of capacitors each of capacitances C_1 in series.

Capacitance of each is $C_1 = \frac{C_1}{n_1}$

According to question, total energy stored in both the combinations are same

i.e., $\frac{1}{2}\left(\frac{C_1}{n_1}\right)(4V)^2 = \frac{1}{2}(n_2 C_2)V^2$

$$\therefore C_2 = \frac{16C_1}{n_1 n_2}$$

98. **(c)** Charge (or current) always flows from higher potential to lower potential.

$$\text{Potential} = \frac{\text{Charge}}{\text{Capacitance}}$$

99. **(c)** Initial energy of capacitor, $E_1 = \frac{q_1^2}{2C}$

Final energy of capacitor,

$$E_2 = \frac{1}{2}E_1 = \frac{q_1^2}{4C} = \frac{\left(\frac{q_1}{\sqrt{2}}\right)^2}{2C}$$

$\therefore t_1$ = time for the charge to reduce to $\frac{1}{\sqrt{2}}$ of its initial value

and t_2 = time for the charge to reduce to $\frac{1}{4}$ of its initial value

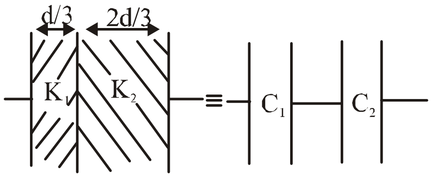
We have, $q_2 = q_1 e^{-t/CR} \Rightarrow \ln\left(\frac{q_2}{q_1}\right) = -\frac{t}{CR}$

$\therefore \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{-t_1}{CR}$ ...(1)

and $\ln\left(\frac{1}{4}\right) = \frac{-t_2}{CR}$ ...(2)

By (1) and (2), $\frac{t_1}{t_2} = \frac{\ln\left(\frac{1}{\sqrt{2}}\right)}{\ln\left(\frac{1}{4}\right)} = \frac{1}{2}\frac{\ln\left(\frac{1}{2}\right)}{2\ln\left(\frac{1}{2}\right)} = \frac{1}{4}$

100. (c)



The capacitance with air between the plates

$$C = \frac{\varepsilon_0 A}{d} = 9\text{pF}$$

On introducing two dielectric between the plates, the given capacitance is equal to two capacitances connected in series where

$$C_1 = \frac{k_1 \in_0 A}{d/3} = \frac{3 \in_0 A}{d/3}$$

$$= \frac{3 \times 3 \in_0 A}{d} = \frac{9 \in_0 A}{d}$$

and

$$C_2 = \frac{k_2 \in_0 A}{2d/3} = \frac{3k_2 \in_0 A}{2d}$$

$$= \frac{3 \times 6 \in_0 A}{2d} = \frac{9 \in_0 A}{d}$$

The equivalent capacitance C_{eq} is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{d}{9 \in_0 A} + \frac{d}{9 \in_0 A} = \frac{2d}{9 \in_0 A}$$

$$\therefore C_{eq} = \frac{9}{2} \frac{\in_0 A}{d} = \frac{9}{2} \times 9\ pF = 40.5 pF$$

101. (a) The potential energy of a charged capacitor is given by $U = \frac{Q^2}{2C}$.

When a dielectric slab is introduced between the plates the energy is given by $\frac{Q^2}{2KC}$, where K is the dielectric constant.

Again, when the dielectric slab is removed slowly its energy increases to initial potential energy. Thus, work done is zero.

102. (b) As n plates are joined alternately positive plate of all $(n-1)$ capacitor are connected to one point and negative plate of all $(n-1)$ capacitors are connected to other point. It means $(n-1)$ capacitors joined in parallel.

$\therefore$ Resultant capacitance $= (n-1)C$

103. (c) Applying conservation of energy,

Electric potential energy of capacitor = heat absorbed

$$\frac{1}{2}CV^2 = m.s\,\Delta t; \quad V = \sqrt{\frac{2m.s.\Delta t}{C}}$$

104. (b) The capacitance without aluminium foil is

$$C = \frac{\varepsilon_0 A}{d}$$

Here, d is distance between the plates of a capacitor

A = Area of plates of capacitor

When an aluminium foil of thickness t is introduced between the plates.

Capacitance, $C' = \frac{\varepsilon_0 A}{d - t}$

If thickness of foil is negligible 50 $d - t \sim d$. Hence, $C = C'$.

105. (d) The work done is stored in the form of potential energy which is given by

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$\therefore U = \frac{1}{2} \times \frac{\left(8 \times 10^{-18}\right)^2}{100 \times 10^{-6}} = 32 \times 10^{-32}\text{ J}$$

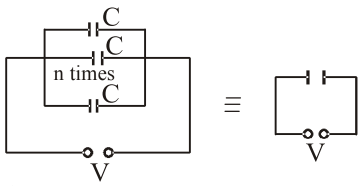
106. (b) In parallel, equivalent capacitance of n capacitor of capacitance C

$C' = nC$

Energy stored in this capacitor

$$E = \frac{1}{2}C^1V^2$$

$$\Rightarrow E = \frac{1}{2}(nC)V^2 = \frac{1}{2}nCV^2$$



Alternatively

Each capacitor has a potential difference of V between the plates.

So, energy stored in each capacitor

$$= \frac{1}{2}CV^2.$$

$\therefore$ Energy stored in n capacitor

$$= \left[\frac{1}{2}CV^2\right] \times n$$

107. (a) Capacitance of spherical conductor $= 4\pi E_0 R$

Here, R is radius of conductor

$$\therefore C = 4\pi \in_0 R = \frac{1}{9 \times 10^9} \times 1 = 1.1 \times 10^{-10} F$$