Motion

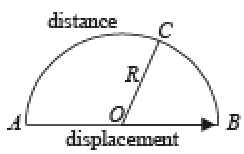
Fill in the Blanks

Q.1. A particle moves in a circle of radius R. In half the period of revolution its displacement is ____________and distance covered is ________.   (1983 - 2 Marks)

Ans. 2R, πR

Solution. Displacement = AOB = 2 R

Displacement = AOB = 2 R



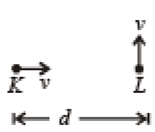
Q.2. Four persons K, L, M, N are initially at the four corners of a square of side d. Each person now moves with a uniform speed v in such a way that K always moves directly towards L, L directly towards M, M directly towards N, and N directly towards K. The four persons will meet at a time ....   (1984- 2 Marks)

Ans. d/v

Solution. The relative velocity of K w.r.t L along the line KL is

$$\vec{v}_{KL} = \vec{v}_K - \vec{v}_L = \vec{v}_K + (-\vec{v}_L)$$
$$= v$$

(∴ the component of velocity of L along KL is zero)

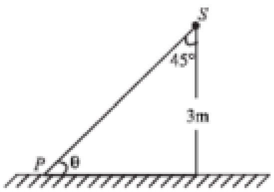


The displacement of K till K and L meet is d.

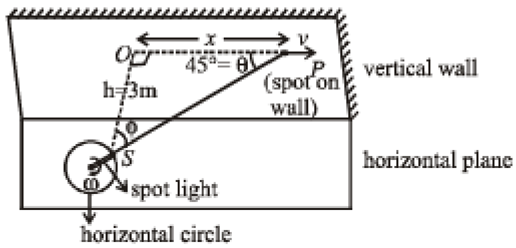
∴ Time taken for K and L to meet will be = d/v

Q.3. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 radian/second. The spot of light P moves along the wall at a distance of 3 m.

The velocity of the spot P when q = 45° (see fig.) is .... m/s (1987 - 2 Marks)



Ans. 0.6 m/s



Solution.

The velocity (v) of spot = dx / dt

and the angular speed (w) of spot light =dØ/dt

FromΔSOP,

$$\tan\phi = \frac{x}{h} \qquad \therefore \quad x = h\tan\phi$$

$$\therefore \quad \frac{dx}{dt} = h\sec^2\phi\,\frac{d\phi}{dt} \qquad \therefore \quad v = (h\sec^2\phi)\,\omega$$

$\therefore$ v = 3 sec^2 45° × 0.1 [$\therefore$ q + φ= 90°]

$\therefore$ v = 3 × 2 × 0.1 = 0.6 m/s

B True/False

Q.1. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed (Neglect air resistance). (1983 - 2 Marks)

Ans. T

Solution. When the two balls are thrown vertically upwards with the same speed u then their final speed v at the point of projection is $v^2 - u^2 = 2 \times g \times s$

Here, $s = 0$

$\therefore v = u$ for both the cases

Q.2. A projectile fired from the ground follows a parabolic path.

The speed of the projectile is minimum at the top of its path. (1984 - 2 Marks)

Ans. T

Solution. T.E. = P.E. + K.E.

T.E. = Constant

At P, K.E. is minimum and P.E. is maximum. Since K.E. is minimum speed is also minimum.

Q.3. Two iden tical trains are moving on r ails along the equator on the earth in opposite directions with the same speed.
They will exert the same pressure on the rails. (1985 - 3 Marks)

Ans. F

Solution. The pressure exerted will be different because one train is moving in the direction of earth's rotation and other in the opposite direction.

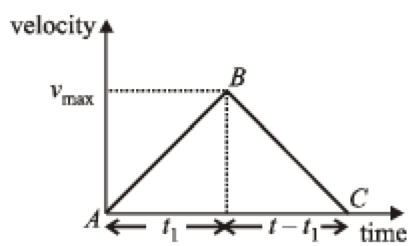
Subjective Question

Q.1. A car accelerates from rest at a constant rate a for some time after which it decelerates at a constant rate b to come to rest. If the total time lapse is t seconds, evaluate. (1978) (i) maximum velocity reached, and (ii) the total distance travelled.

Ans.

$$\frac{\alpha\beta}{\alpha+\beta}t;\ \frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}t^2$$

Solution.



Distance travelled = area of Δ ABC

$$= \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times t \times v_{max}$$

$$= \frac{1}{2} \times t \times \frac{\alpha\beta}{\alpha+\beta} t = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right) t^2$$

Q.2.The displacement x of particle moving in one dimension, under the action of a constant force is related to the time t by the equation t =√x+ 3 where x is in meters and t in seconds. Find

(i) The displacement of the particle when its velocity is zero, and

(ii) The work done by the force in the first 6 seconds.

Ans.(i) 0 ; (ii) 0

Solution. $\sqrt{x} = t - 3 \Rightarrow x = t^2 + 9 - 6t \quad \therefore \quad v = \frac{dx}{dt} = 2t - 6$

(i) For velocity to be zero, 2 t – 6 = 0 ⇨ t = 3 sec.

The displacement is x = 9 + 9 – 6 × 3 = 0

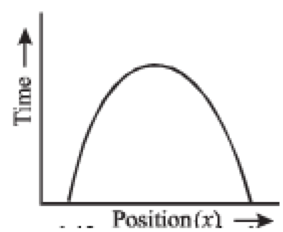
$a = \frac{dv}{dt} = 2 \quad \therefore \quad \text{At } t = 0, v = -6 \text{ ms}^{-1}$

At t = 6 sec, v = 6 ms^{-1}

$\therefore$ Work done = Change in K.E. = [$K.E_f$ – $K.E_i$]

$$= \frac{1}{2}m(6)^2 - \frac{1}{2}m(6)^2 = 0$$

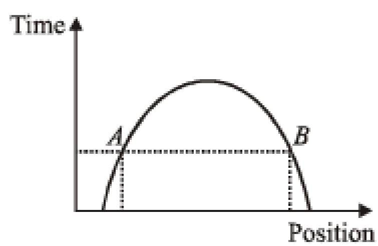
Q.3. Answer the following giving reasons in brief :



Is the time variation of position, shown in the figure observed in nature? (1979)

Ans. No

Solution.

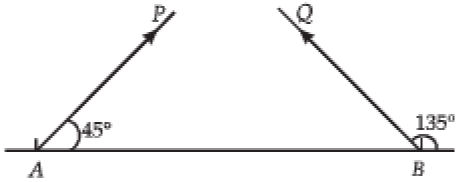


As shown, at a given instant of time, the body is at two different positions A and B which is not possible.

Q.4. Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from points A and B on the ground.

The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal AB as shown in the figure.

Each particle has an initial speed of 49 m/s. The separation AB is 245 m.



Both particle travel in the same vertical plane and undergo a collision After the collision, P retraces its path, Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground? Take g = 9.8 m/s^2.

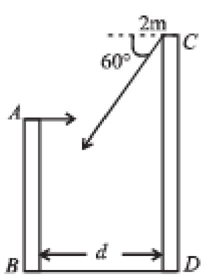
Ans. mid point of AB, 3.53 sec.

Solution. If a body dr ops fr om a h eigh t H above the ground then the time taken by it to reach the ground

$$t=\sqrt{\frac{2H}{g}} \qquad \therefore \qquad t=\sqrt{\frac{2\times 61.25}{9.8}}=3.53\,s$$

Q.5. Two towers AB and CD are situated a distance d apart as shown in figure.

AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD. (1994 - 6 Marks) Simultaneously another object of mass 2 m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each

other. 

(i) Calculate the distance ‘d’ between the towers and,

(ii) Find the position where the objects hit the ground.

Ans. 17.32, 11.547 m from B

Solution.

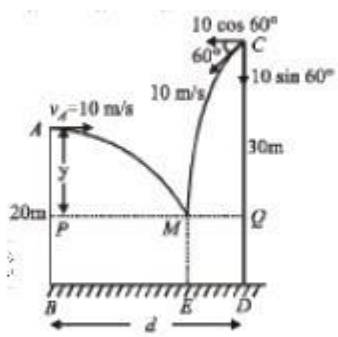
(i) Let t be the time taken for collision.

For mass m thrown horizontally from A.

For horizontal motion PM = 10 t ... (i) For vertical motion $u_y = 0$; $s_y = y$; $a_y = g$; $t_y = t$

$\therefore y = 1/2\, gt^2$ ... (ii)

$v_y = u_y + a_y t = gt$ ... (iii)



For mass 2m thrown from C

For horizontal motion QM = [10 cos 60°] t ... (iv)

For vertical motion $v_y = 10 \sin 60° = 5\sqrt{3}$; $a_y = g$ $s_y = y + 10$; $t_y = t$

Now, $v_y = 5\sqrt{3} + gt$ ... (v)

and $(s_y) = u_y t + 1/2\, a_y t^2$

$\Rightarrow y + 10 = 5\sqrt{3}\ 1/2\, gt^2$ ... (vi)

From (ii) and (vi)

$$\frac{1}{2}gt^2 + 10 = 5\sqrt{3}\,t + \frac{1}{2}gt^2 \Rightarrow t = \frac{2}{\sqrt{3}}\text{ sec}$$

$$\therefore \quad BD = PM + MQ = 10\,t + 5\,t = 15\,t = 15 \times \frac{2}{\sqrt{3}}$$

$= 10\sqrt{3} = 17.32$ m

(ii) Applying conservation of linear momentum (during collision of the masses at M) in the horizontal direction

$m \times 10 - 2\,m\,10 \cos 60° = 3\,m \times v_x$

$\Rightarrow 10\,m - 10\,m = 3\,m \times v_X \Rightarrow v_X = 0$

Since, the horizontal momentum comes out to be zero, the combination of masses will drop vertically downwards and fall at E.

$$BE = PM = 10\,t = 10 \times \frac{2}{\sqrt{3}} = 11.547\text{ m}$$

Q.6.Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed 5 √3 m s–1 at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find (i) the time-interval between the firings, and (ii) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane. (1996 - 5 Marks)

Ans.1 sec, (5√3 , 5) in metres

Solution.

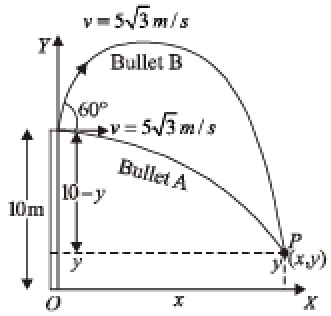
For Bullet A. Let t be the time taken by bullet A to reach P.

Vertical motion

$$u_y = 0;\ s_y = 10 - y;\ a_y = 10\text{ m/s}^2;\ t_y = t$$

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$10 - y = 5\ t^2$



Horizontal motion $x = 5\sqrt{3}\ t$ ... (ii)

For bullet B.

Let (t + t') be the time taken by bullet B to reach P Vertical Motion Let us consider upward direction negative and downward positive. Then

$$u_y = -5\sqrt{3} \sin 60° = -7.5 \text{ m/s}, a_y = +10 \text{ m/s}^2$$

$$s_y = +(10 - y);\ t_y = t + t',\ s_y = u_y t + \frac{1}{2} a_y t^2$$

$$10 - y = -7.5\,(t + t') + 5\,(t + t')^2 \qquad \text{... (iii)}$$

Horizontal motion

$x = (5\ \sqrt{3} \cos 60°)\ (t + t\ ')$

$$\Rightarrow 5\sqrt{3}\,t + 5\sqrt{3}\,t' = 2x \qquad \text{... (iv)}$$

Substituting the value of x from (ii) in (iv), we get

$5\sqrt{3}t + 5\sqrt{3}t\ ' = 10\sqrt{3}t$

$\Rightarrow t = t\ '$

Putting t = t ' in eq. (iii) $y - 10 = 15\ t - 20\ t^2$ ... (v)

Adding (i) and (v)

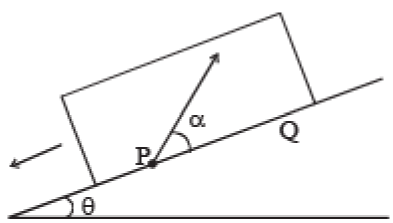
$0 = 15\,t - 15\,t^2 \Rightarrow t = 1$ sec.

(ii) Putting t = 1 in eq. (ii), we get $x = 5\sqrt{3}$

Putting t = 1 in eq. (i), we get y = 5

Therefore, the coordinates of point P are ($5\sqrt{3}$, 5) in metres.

Q.7.A large, heavy box is sliding without friction down a smooth plane of inclination q. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u, and the direction of projection makes an angle a with the bottom as shown in Figure. (1998 - 8 Marks)



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when particle was projected.

Ans.

(a) $\dfrac{u^2 \sin 2\alpha}{g\cos\theta}$ **(b)** $\dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

Solution.

(a) u is the relative velocity of the particle with respect to the box. Resolve u. u_x is the relative velocity of particle with respect to the box in x-direction. u_y is the relative velocity with respect to the box in y-direction.

Since, there is no velocity of the box in the y-direction, therefore this is the vertical velocity of the particle with respect to ground also.

Y-direction motion (Taking relative terms w.r.t. box)

$u_y = + u \sin\alpha$

$a_y = - g \cos\emptyset$

$s_y = 0$ (activity is taken till the time the particle comes back to the box.)

$t_y = t$

$$s_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 0 = (u \sin \alpha)\, t - \frac{1}{2} g \cos\theta \times t^2$$

$$\Rightarrow \quad t = 0 \text{ or } t = \frac{2u \sin\alpha}{g \cos\theta}$$

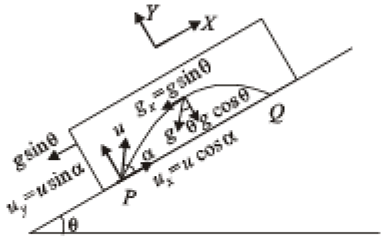
X - direction motion (Taking relative terms w.r.t. box)

$$u_x = + u \cos \alpha \text{ ; } a_x = 0, t_x = t, s_x = s_x$$

$$s_x = u_x t + \frac{1}{2} a_x t^2 \Rightarrow s_x = u \cos \alpha \times \frac{2u \sin\alpha}{g \cos\theta} = \frac{u^2 \sin 2\alpha}{g \cos\theta}$$

(b) For the observer (on ground) to see the horizontal displacement to be zero, the distance travelled by the box in time $\left(\frac{2u \sin\alpha}{g \cos\theta}\right)$ should be equal to the range of the particle.

Let the speed of the box at the time of projection of particle be U. Then for the motion of box with respect to ground.



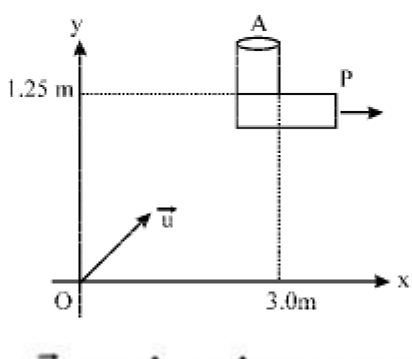
$$u_x = -U;\ a_x = -g\sin\theta;\ t_y = \frac{2u\sin\alpha}{g\cos\theta};\ s_x = \frac{-u^2\sin 2\alpha}{g\cos\theta}$$

$$s_x = u_x t + \frac{1}{2}a_x t^2$$

$$\frac{-u^2\sin 2\alpha}{g\cos\theta} = -U\left(\frac{2u\sin\alpha}{g\cos\theta}\right) - \frac{1}{2}g\sin\theta\left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$$

On solving we get $U = \frac{u\cos(\alpha+\theta)}{\cos\theta}$

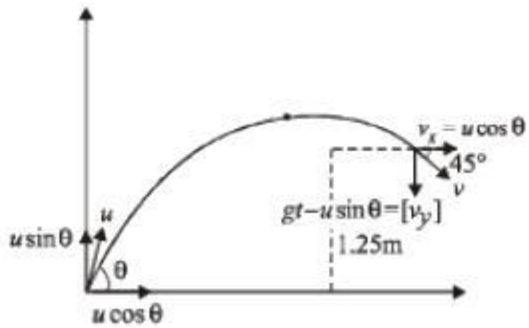
Q.8. An object A is kept fixed at the point x = 3 m and y = 1.25 m on a plank P raised above the ground. At time t = 0 the plank starts moving along the + x direction with an acceleration 1.5 m/s2. At the same instant a stone is projected from the origin with a velocity ur as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the X–Y plane. Find ur and the time after which the stone hits the object. Take g = 10 m/s (2000 - 10 Marks)



Ans. $\vec{u} = (3.75\hat{i} + 6.25\hat{j})\ \text{m/s},\ t = 1\ \text{sec.}$

Solution.

Let 't ' be the time after which the stone hits the object and θ be the angle which the velocity vector u makes with horizontal.



According to question, we have following three conditions. (i) Vertical displacement of stone is 1.25 m.

Therefore, $1.25 = (u\sin\theta)t - \frac{1}{2}gt^2$

where g = 10 m/s^2

or (u sinθ) t = 1.25 + 5t^2

(ii) Horizontal displacement of stone = 3 + displacement of object A.

Therefore, (u cos θ) t= 3 + 1/2 at^2

where a = 1.5 m/s^2

or(u cos q)t = 3 + 0.75 t^2 ... (ii)

Horizontal component of velocity of stone = vertical component (because velocity vector is inclined at 45° with horizontal.) Therefore (u cos θ) = gt – (u sinθ) ... (iii)

(The right hand side is written gt – u sinθ because the stone is in its downward motion. Therefore, gt > u sinθ .

In upward motion usinθ > gt). Multiplying equation (iii) with t we can write, (u cosθ) t + (u sin q) t = 10 t^2 ... (iv)

Now, (iv) – (ii) – (i) gives 4.25 t^2 – 4.25 = 0 or t = 1s Substituting t = 1s in (i) and (ii), we get, u sin q = 6.25 m/s or u_y = 6.25

m/s and u cos q = 3.75 m/s.

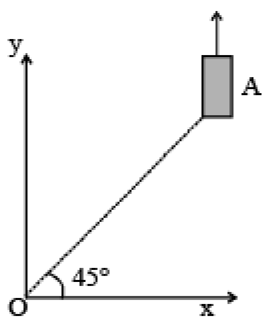
or $u_x = 3.75$ m/s therefore $\vec{u} = u_x\hat{i} + u_y\hat{j}$

or $\vec{u} = (3.75\hat{i} + 6.25\hat{j})$ m/s

Q.9. On a frictionless horizontal surface, assumed to be th e x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of (√ 3 -1) m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle f with the x-axis and it hits the trolley.

(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle q made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, ifφ = 4θ/4. (2002 - 5 Marks)



Ans. 45°, 2 m/sec.

Solution. (a) Let the ball strike the trolley at B. Let

$\vec{v}_{BG}$ = velocity of ball w.r.t. ground

$\vec{v}_{TG}$ = velocity of trolley w.r.t. ground

∴ Velocity of ball w.r.t. trolley

$\vec{v}_{BT} = \vec{v}_{BG} - \vec{v}_{TG}$ ...(i)

From triangle OAB

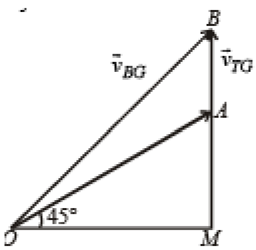
$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\therefore \quad \overrightarrow{OA} + \vec{v}_{TG} = \vec{v}_{BG}$

$\therefore \quad \overrightarrow{OA} = \vec{v}_{BG} - \vec{v}_{TG} \quad$ ...(ii)

From (i) and (ii) $\overrightarrow{OA} = \vec{v}_{BT}$

⇒ velocity of ball w.r.t. trolley makes an angle of 45° with the X-axis



(b) Here q = 45°

$\therefore \quad \phi = \frac{4\theta}{3} = \frac{4 \times 45}{3} = 60°$

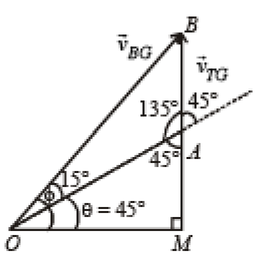
In ΔOMA,

θ = 45°⇨∠OAM = 45°

∴ ∠OAB = 135°

Also∠BOA = 60° – 45° = 15°

Using sine law in ΔOBA



$$\frac{v_{BG}}{\sin 135°} = \frac{v_{TG}}{\sin 15°} \Rightarrow v_{BG} = 2 \text{ m/s}$$

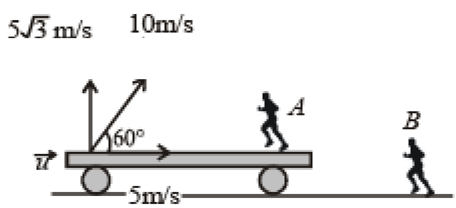
Integer Value

Q.1. A train is moving along a straight line with a constant acceleration ‘a’. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s2, is

Ans. 5

Solution. From t h e perspective of observer A, consider in g vertical motion of the ball from the point of throw till it reaches back at the initial height.

$U_y = +\,5\sqrt{3}$ m/s, $S_y = 0$, $a_y = -10\text{m/s}^2$, $t = ?$



Applying $S = ut + \frac{1}{2}at^2 \Rightarrow 0 = 5\sqrt{3}t - 5t^2$

$\therefore \quad t = \sqrt{3}$ sec

Considering horizontal motion from the perspective of observer B. Let u be the speed of train at the time of throw.

The horizontal distance travelled by the ball = (u + 5) √3 .

The horizonal distance travelled by the boy

$$= \left[u\sqrt{3} + \frac{1}{2}a(\sqrt{3})^2\right] + 1.15$$

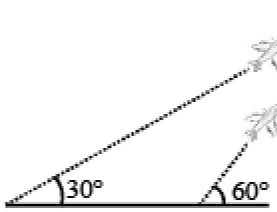
As the boy catches the ball therefore

$$(u+5)\sqrt{3} = u\sqrt{3} + \frac{3}{2}a + 1.15$$

$\therefore\ 5\sqrt{3} = 1.5a + 1.15 \quad \therefore\ 7.51 = 1.5a$

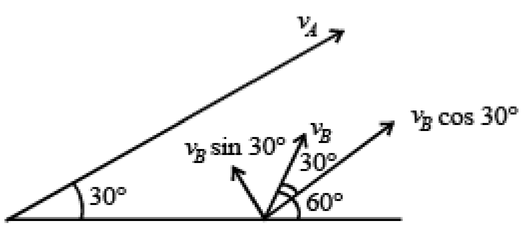
$\therefore\ a \approx 5\ m/s^2$

Q.2. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is 100 √3 m/s. At time t = 0 s, an observer in A finds B at a distance of 500 m. The observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is (JEE Adv. 2014)



Ans. 5

Solution.



Here

$v_A = v_B \cos 30°$

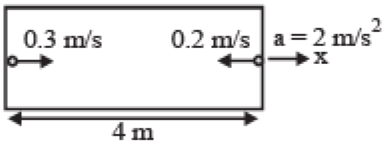
$\therefore\ 100\sqrt{3} = v_B \times \frac{\sqrt{3}}{2}$

$\therefore v_B = 200\ ms^{-1}$

$\text{Time} = \frac{\text{displacement}}{\text{velocity}}$

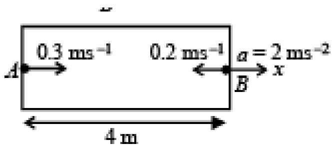
$\therefore\ t_0 = \frac{500}{v_B \sin 30°} = \frac{500}{200 \times \sin 30°} = 5\ \text{sec}$

Q.3. A rocket is moving in a gravity free space with a constant acceleration of 2 m/s^2 along +x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in +x direction with a speed of 0.3 m/s relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 m/s from its right end relative to the rocket. The time in seconds when the two balls hit each other is (JEE Adv. 2014)



Ans. 8

Solution.



For ball A

$u_1 = 0.3\ \text{ms}^{-1},\ a_1 = -2\text{ms}^{-2}, s_1 = x, t_1 = t$

$\therefore \quad s_1 = u_1t_1 + \frac{1}{2}a_1t_1^2$

$x = 0.3t - t^2$ ...(1)

For ball B

$u_2 = 0.2\ \text{ms}^{-1}, a_2 = 2\text{ms}^{-2}, s_2 = 4 - x, t_2 = t$

$\therefore \quad s_2 = u_2t_2 + \frac{1}{2}a_2t_2^2$

$4 - x = 0.2\ t + t^2$ ...(2)

From (1) and (2) t = 8 sec