1. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta}\, dx = \ldots\ldots\ldots + C$

(A) $2(\sin x + x \cos\theta)$ (B) $2(\sin x - x \cos\theta)$ (C) $2(\sin x + 2x \cos\theta)$ (D) $2(\sin x - 2x \cos\theta)$

જવાબ **(A) $2(\sin x + x \cos\theta)$**

➥ $I = \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}\, dx$

$= \int \frac{2\cos^2 x - 1 - 2\cos^2\theta - 1}{\cos x - \cos\theta}\, dx$

$= 2 \int \frac{\cos^2 x - \cos^2\theta}{\cos x - \cos\theta}\, dx$

$= 2 \int (\cos x + \cos\theta)\, dx$

$\therefore\ I = 2\ (\sin x + x\cos\theta) + C$

2. $\int \frac{dx}{\sin(x-a)\sin(x-b)} = \ldots\ldots\ldots + C$

(A) $\sin(b-a) \log \left|\frac{\sin(x-b)}{\sin(x-a)}\right|$ (B) $\text{cosec}(b-a) \log \left|\frac{\sin(x-a)}{\sin(x-b)}\right|$

(C) $\text{cosec}(b-a) \log \left|\frac{\sin(x-b)}{\sin(x-a)}\right|$ (D) $\sin(b-a) \log \left|\frac{\sin(x-a)}{\sin(x-b)}\right|$

જવાબ **(C) $\text{cosec}(b-a) \log \left|\frac{\sin(x-b)}{\sin(x-a)}\right|$**

➥ $I = \int \frac{dx}{\sin(x-a)\sin(x-b)}$

$= \frac{1}{\sin(b-a)} \int \frac{\sin(b-a)}{\sin(x-a)\sin(x-b)}\, dx$

$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a-x+b)}{\sin(x-a)\sin(x-b)}\, dx$

$= \frac{1}{\sin(b-a)} \int \frac{\sin\{(x-a)-(x-b)\}}{\sin(x-a)\sin(x-b)}\, dx$

$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)\cos(x-b) - \cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}\, dx$

$= \frac{1}{\sin(b-a)} \int [\cot(x-b) - \cot(x-a)]\, dx$

$= \frac{1}{\sin(b-a)} [\log|\sin(x-b)| - \log|\sin(x-a)|] + C$

$= \text{cosec}\,(b-a) \log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$

3. $\int \tan^{-1}\left(\sqrt{x}\right) dx = \ldots\ldots\ldots$

(A) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$ (B) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(C) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$ (D) $\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + C$

જવાબ **(A) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$**

➡ $I = \int \tan^{-1}\left(\sqrt{x}\right) dx$

અહીં $\sqrt{x} = t$ આદેશ લેતાં,

$\therefore x = t^2$

$dx = 2t\ dt$

$\therefore I = 2\int t\ \tan^{-1}t\ dt$

$u = \tan^{-1}t,\quad v = t$ લેતાં,

$\therefore \frac{du}{dt} = \frac{1}{1+t^2},\ \int v\ dt = \frac{t^2}{2}$

$I = 2\left\{u\int v\,dt - \int (u')\,v\,dt \int dt\right\}$

(ખંડશઃ સંકલનનો નિયમ)

$\therefore I = 2\left\{\frac{(\tan^{-1}t)\,t^2}{2} - \frac{1}{2}\int \frac{t^2}{1+t^2}\,dt\right\}$

$= t^2(\tan^{-1}t) - \int \frac{(1+t^2)-1}{1+t^2}\,dt$

$= t^2\tan^{-1}t - \int\left(1 - \frac{1}{1+t^2}\right)dt$

$= t^2\ \tan^{-1}t - t + \tan^{-1}t$

$I = (t^2+1)\ \tan^{-1}t - t + C$

$= (x+1)\ \tan^{-1}(\sqrt{x}) - \sqrt{x} + C$

4. $\int \frac{x^9}{(4x^2+1)^6}\,dx = \text{..........} + C$

(A) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^5$ (B) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}$ (C) $\frac{1}{10x}\left(1+4x^2\right)^{-5}$ (D) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}$

જવાબ **(D) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}$**

➡ $I = \int \frac{x^9}{(4x^2+1)^6}$

$= \int \frac{x^9}{\left[x^2\left(4+\frac{1}{x^2}\right)\right]^6}$

$= \int \frac{x^9}{x^{12}\left(4+\frac{1}{x^2}\right)^6}\,dx$

$= \int \frac{1}{x^3\left(4+\frac{1}{x^2}\right)^6}\,dx$

ધારો કે $4 + \frac{1}{x^2} = t$

$\therefore -\frac{2}{x^3}\, dx = dt$

$= -\frac{1}{2}\int \frac{-2}{x^3\left(4 + \frac{1}{x^2}\right)^6}\, dx$

$= -\frac{1}{2}\int \frac{dt}{t^6}$

$I = -\frac{1}{2}\left[\frac{t^{-6+1}}{-6+1}\right]$

$= -\frac{1}{2}\left(\frac{t^{-5}}{-5}\right) + C$

$\therefore I = \frac{1}{10}\left(4 + \frac{1}{x^2}\right)^{-5} + C$

5. $\int \frac{dx}{(x+2)(x^2+1)} = a \log|1 + x^2| + b \tan^{-1}x + \frac{1}{5}\log|x+2| + C$ **હોય તો .... વિકલ્પ સત્ય છે.**

(A) $a = -\frac{1}{10}, b = -\frac{2}{5}$ (B) $a = \frac{1}{10}, b = -\frac{2}{5}$ (C) $a = -\frac{1}{10}, b = \frac{2}{5}$ (D) $a = \frac{1}{10}, b = \frac{2}{5}$

જવાબ **(C)** $a = -\frac{1}{10}, b = \frac{2}{5}$

➡ $I = \int \frac{1}{(x+2)(x^2+1)}\, dx$

$\therefore \frac{1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx + C}{x^2+1}$

$\therefore 1 = A(x^2+1) + (Bx + C)(x+2)$

$\therefore 1 = (A + B)x^2 + (2B + C)x + (A + 2C)$

બંને બાજુ x^2, x અને અચળપદના સહગુણકો સરખાવતાં,

$A + B = 0,\ 2B + C = 0,\ A + 2C = 1$

આ પરિણામોને ઉકેલતાં $A = \frac{1}{5}, B = -\frac{1}{5}$ અને $C = \frac{2}{5}$ મળશે.

$\therefore I = \int \frac{\frac{1}{5}}{x+2}\, dx + \int \frac{-\frac{1}{5}x + \frac{2}{5}}{x^2+1}\, dx$

$= \frac{1}{5}\int \frac{1}{x+2}\, dx - \frac{1}{5}\int \frac{x}{x^2+1}\, dx + \frac{2}{5}\int \frac{1}{x^2+1}\, dx$

$I = \frac{1}{5}\log|x+2| - \frac{1}{10}\log|x^2+1| + \frac{2}{5}\tan^{-1}x + C$

હવે આપેલ જમણી બાજુ સાથે સરખાવો.

$\therefore a = -\frac{1}{10},\ b = \frac{2}{5}$

6. $\int \frac{x^3}{x+1}\, dx = ... + C$

(A) $x + \frac{x^2}{2} + \frac{x^3}{3} - \log|1 - x|$

(B) $x + \frac{x^2}{2} - \frac{x^3}{3} - \log|1 - x|$

(C) $x - \frac{x^2}{2} - \frac{x^3}{3} - \log|1 + x|$

(D) $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1 + x|$

જવાબ **(D)** $x - \frac{x^2}{2} + \frac{x^3}{3} - \log|1 + x|$

➡ $I = \int \frac{x^3}{x+1}\,dx$

$= \int \frac{x^3 + 1 - 1}{x+1}\,dx$

$= \int \left(\frac{x^3+1}{x+1} - \frac{1}{x+1}\right) dx$

$\therefore I = \int \left(x^2 - x + 1 - \frac{1}{x+1}\right) dx$

$= \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + C$

$\therefore I = x - \frac{x^2}{2} + \frac{x^3}{3} - \log|x+1| + C$

7. $\int \frac{x + \sin x}{1 + \cos x}\,dx = \text{..........} + C$

(A) $\log|1 + \cos x|$ (B) $\log|x + \sin x|$ (C) $x - \tan\left(\frac{x}{2}\right)$ (D) $x \cdot \tan\left(\frac{x}{2}\right)$

જવાબ **(D)** $x \cdot \tan\left(\frac{x}{2}\right)$

➡ $I = \int \frac{x + \sin x}{1 + \cos x}\,dx$

$= \int \left(\frac{x}{1+\cos x} + \frac{\sin x}{1 + \cos x}\right) dx$

$= \int \frac{x}{2\cos^2\left(\frac{x}{2}\right)}\,dx + \int \frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\,dx$

$= \int x\left(\frac{1}{2}\sec^2\left(\frac{x}{2}\right)\right) dx + \int \tan\left(\frac{x}{2}\right) dx$

હવે પ્રથમ સંકલિતમાં $u = x$, $v = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$ લઈને ખંડશઃ સંકલન કરતાં,

$I = u\int v\,dx - \int(u' \int v\,dx)\,dx + \int \tan\frac{x}{2}\,dx$

હવે $u = x \Rightarrow u' = 1$

અને $\int v\,dx = \int \frac{1}{2}\sec^2\left(\frac{x}{2}\right) dx = \tan\left(\frac{x}{2}\right)$

$\therefore I = x\tan\left(\frac{x}{2}\right) - \int \tan\left(\frac{x}{2}\right) dx + \int \tan\left(\frac{x}{2}\right) dx$

$\therefore I = x\tan\left(\frac{x}{2}\right) + C$

8. $\int \frac{x^3}{\sqrt{1+x^2}}\,dx = a(1+x^2)^{\frac{3}{2}} + b \cdot \sqrt{1+x^2} + C$ હોય તો .... મળે.

(A) $a = \frac{1}{3}$ અને $b = 1$ (B) $a = -\frac{1}{3}$ અને $b = 1$ (C) $a = -\frac{1}{3}$ અને $b = -1$ (D) $a = \frac{1}{3}$ અને $b = -1$

જવાબ **(D)** $a = \frac{1}{3}$ અને $b = -1$

➡ $I = \int \frac{x^3}{\sqrt{1+x^2}}\, dx$

$= \int \frac{x^2 \cdot x}{\sqrt{1+x^2}}\, dx$

હવે $1 + x^2 = t^2$ આદેશ મૂકો.

$\therefore\ 2x\, dx = 2t\, dt$

$\therefore\ x\, dx = t\, dt$

$\therefore \int \frac{(t^2 - 1)t\, dt}{t}$

$= \int (t^2 - 1)\, dt$

$= \frac{t^3}{3} - t + C$

$= \frac{1}{3}(1 + x^2)^{\frac{3}{2}} - 1\sqrt{1 + x^2} + C$

આપેલ મૂલ્ય સાથે સરખાવો,

$\therefore\ a = \frac{1}{3}$ અને $b = -1$ મળે.

9. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \cos(2x)}\, dx = ...$

(A) 1 (B) 2 (C) 3 (D) 4

જવાબ **(A) 1**

➡ $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \cos(2x)}\, dx$

$= 2\int_{0}^{\frac{\pi}{4}} \frac{1}{2\cos^2 x}\, dx$

$\left(\because \int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx.\right.$ જો $f(x)$ યુગ્મ વિધેય હોય તથા $1 + \cos 2\theta = 2\cos^2\theta$ થશે.)

$= \int_{0}^{\frac{\pi}{4}} \sec^2 x\, dx$

$= (\tan x)_{0}^{\frac{\pi}{4}}$

$= \tan\left(\frac{\pi}{4}\right) - \tan 0$

$= 1 - 0$

$\therefore\ I = 1$

10. $\int^{\frac{\pi}{2}} \sqrt{1 - \sin(2x)}\, dx = ...$

0

(A) $2\sqrt{2}$ (B) $2(\sqrt{2}+1)$ (C) 2 (D) $2(\sqrt{2}-1)$

જવાબ **(D) $2(\sqrt{2}-1)$**

➡ $I = \int_0^{\frac{\pi}{2}} \sqrt{1-\sin(2x)}\, dx$

$= \int_0^{\frac{\pi}{2}} \sqrt{(\cos x - \sin x)^2}\, dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{(\cos x - \sin x)^2}\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{(\sin x - \cos x)^2}\, dx$

$\left(\because x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)\right.$ માટે $\sin x > \cos x$ છે.$\left.\right)$

$= (\sin x + \cos x)_0^{\frac{\pi}{4}} + (-\cos x - \sin x)_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1\right) + \left(0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

$= 4\left(\frac{1}{\sqrt{2}}\right) - 2$

$= 2\sqrt{2} - 2 = 2(\sqrt{2}-1)$

11. $\int_0^{\frac{\pi}{2}} \cos x \cdot e^{\sin x}\, dx =$ **..........**

(A) $e+1$ (B) $e-1$ (C) e (D) $-e$

જવાબ **(B) $e-1$**

➡ $I = \int_0^{\frac{\pi}{2}} \cos x \cdot e^{\sin x}\, dx$

હવે $\sin x = t$ આદેશ લેતાં,

$\therefore \cos x\, dx = dt$ તથા $x = \frac{x}{2} \Rightarrow t = 1$

અને $x = 0 \Rightarrow t = 0$

$I = \int_0^1 e^t\, dt$

$= \left(e^t\right)_0^1$

$= e^1 - e^0$

$I = e - 1$

12. $\int \left(\frac{x+3}{(x+4)^2}\right) e^x\, dx =$ **.......... + C**

(A) $e^x\left(\frac{1}{x+4}\right)$ (B) $e^{-x}\left(\frac{1}{x+4}\right)$ (C) $e^{-x}\left(\frac{1}{x-4}\right)$ (D) $e^{2x}\left(\frac{1}{x-4}\right)$

જવાબ (A) $e^x\left(\frac{1}{x+4}\right)$

➡ $I = \int\left(\frac{x+3}{(x+4)^2}\right)e^x\,dx$

$I = \int \frac{(x+4)-1}{(x+4)^2}e^x\,dx$

$= \int e^x\left(\frac{1}{x+4} + \frac{(-1)}{(x+4)^2}\right)dx$

$= \int e^x\left(f(x) + f'(x)\right)dx$

જ્યાં $f(x) = \frac{1}{x+4} \Rightarrow f'(x) = \frac{-1}{(x+4)^2}$

$= e^x f(x) + C$

$\therefore I = e^x\left(\frac{1}{x+4}\right) + C$