UNIT – V

Trigonometry

CHAPTER

10

INTRODUCTION TO TRIGONOMETRY AND TRIGONOMETRIC IDENTITIES

Syllabus

- *Introduction to Trigonometry : Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined) motivate the ratios, which are defined at 0° and 90°. Values (with proofs) of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.*
- *Trigonometric Identities : Proof and applications of the identity, $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given. Trigonometric ratios of complementary angles.*

Chapter Analysis

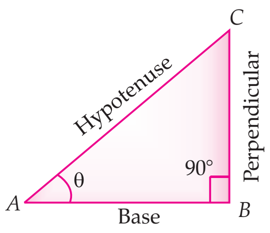
List of Topics	2016			2017			2018
	Delhi	Outside Delhi	Foreign	Delhi	Outside Delhi	Foreign	Delhi & Outside Delhi
Question based on Trigonometric Ratios	Summative Assessment-I						1 Q (1 M) 1 Q (3 M)
Question based on Trigonometric Identities							1 Q (4 M)

TOPIC-1

Trigonometric Ratios and Trigonometric Ratios of Complementary Angles

Revision Notes

- In fig., a right triangle ABC right angled at B is given and $\angle BAC = \theta$ is an acute angle. Here side AB which is adjacent to $\angle A$ is base, side BC opposite to $\angle A$ is perpendicular and the side AC is hypotenuse which is opposite to the right angle B.



Know the Formulae

The trigonometric ratios of $\angle A$ in right triangle ABC are defined as

$$\text{sine of } \angle A = \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\text{cosine of } \angle A = \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{tangent of } \angle A = \tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\text{cosecant of } \angle A = \operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC} = \frac{1}{\sin\theta}$$

$$\text{secant of } \angle A = \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{1}{\cos\theta}$$

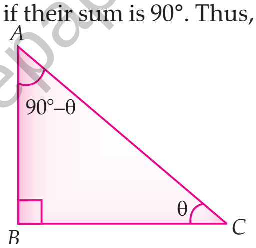
$$\text{cotangent of } \angle A = \cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{1}{\tan\theta}$$

It is clear from the above ratios that cosecant, secant and cotangent are the reciprocals of sine, cosine and tangent respectively.

Also, $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

and $$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

- The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.
- The value of trigonometric ratio of an angle does not depend on the size of the triangle but depends on the angle only.
- **Complementary Angles :**
  Two angles are said to be complementary if their sum is 90°. Thus, (in fig.) $\angle A$ and $\angle C$ are complementary angles.



- **Trigonometric Ratios of Complementary Angles :**
  We have, BC = Base, AB = Perpendicular, and AC = Hypotenuse, with respect to θ.

$\therefore$ $$\sin\theta = \frac{AB}{AC},\ \cos\theta = \frac{BC}{AC},\ \tan\theta = \frac{AB}{BC}$$

and $$\operatorname{cosec}\theta = \frac{AC}{AB},\ \sec\theta = \frac{AC}{BC},\ \cot\theta = \frac{BC}{AB}.$$

Again, with respect to the angle (90° – θ), BC= Perpendicular, AB = Base and AC = Hypotenuse

$\therefore$ $$\sin(90° - \theta) = \frac{BC}{AC} = \cos\theta$$

$$\cos(90° - \theta) = \frac{AB}{AC} = \sin\theta$$

$$\tan(90° - \theta) = \frac{BC}{AB} = \cot\theta$$

$$\operatorname{cosec}(90° - \theta) = \frac{AC}{BC} = \sec\theta$$

$$\sec(90° - \theta) = \frac{AC}{AB} = \operatorname{cosec}\theta$$

$$\cot(90° - \theta) = \frac{AB}{BC} = \tan\theta$$

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

How it is done on GREENBOARD?

Q. If $2\sin\theta - 1 = 0$, then prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

Sol. Step I : Given $2\sin\theta - 1 = 0$

or, $2\sin\theta = 1$

or, $\sin\theta = \frac{1}{2}$ ...(i)

Step II : $\sin 30° = \frac{1}{2}$ ...(ii)

From (i) and (ii), $\theta = 30°$

Step III : L.H.S. $= \sin 3\theta$

$= \sin(3 \times 30°)$

$= \sin 90$

$= 1$

R.H.S. $= 3\sin\theta - 4\sin^3\theta$

$= 3 \times \sin 30° - 4(\sin\theta)^3$

$= 3 \times \frac{1}{2} - 4 \times \frac{1}{8}$

$= \frac{3}{2} - \frac{1}{2}$

$= \frac{2}{2}$

$= 1 =$ L.H.S.

Objective Type Questions

(1 mark each)

[A] Multiple choice Questions :

Q. 1. If $\cos A = \frac{4}{5}$ then the value of $\tan A$ is :

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$

(c) $\frac{4}{3}$ (d) $\frac{1}{8}$

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Given, $\cos A = \frac{4}{5}$

$$\therefore \sin A = \sqrt{1-\cos^2 A} \quad \left[\begin{array}{l}\because \sin^2 A + \cos^2 A = 1 \\ \therefore \sin A = \sqrt{1-\cos^2 A}\end{array}\right]$$

$$\sin A = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\text{Now, } \tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

Q. 2. If $\sin A = \frac{1}{2}$ then the value of $\cot A$ is :

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

R [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Given, $\sin A = \frac{1}{2}$

$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{1}{2}\right)^2}$$

$$\cos A = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \quad \left[\begin{array}{l} \because \sin^2 A + \cos^2 A = 1 \\ \Rightarrow \cos A = \sqrt{1-\sin^2 A} \end{array}\right]$$

$$\text{Now, } \cot A = \frac{\cos A}{\sin A} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Q. 3. The value of the expression [cosec (75° + θ) – sec (15° – θ) – tan (55° + θ) + cot (35° – θ)] is :

(a) –1 **(b)** 0

(c) 1 **(d)** $\frac{3}{2}$

R **[NCERT Exemp.]**

Sol. Correct option : (b)

Explanation : cosec (75° + θ) – sec (15° – θ) – tan (55° + θ) + cot (35° – θ)

$$= \text{cosec}[90° - (15° - \theta)] - \sec(15° - \theta) - \tan(55° + \theta) + \cot[90° - (55° + \theta)]$$

$$= \sec(15° - \theta) - \sec(15° - \theta) - \tan(55° + \theta) + \tan(55° + \theta)$$

$$= 0$$

Q. 4. Given that sin $\theta = \frac{a}{b}$ then cos θ is equal to

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ **(b)** $\frac{b}{a}$

(c) $\frac{\sqrt{b^2 - a^2}}{b}$ **(d)** $\frac{a}{\sqrt{b^2 - a^2}}$

R **[NCERT Exemp.]**

Sol. Correct option : (c)

Explanation : Given, $\sin\theta = \frac{a}{b}$

$$\left[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}\right]$$

$$\cos\theta = \sqrt{1-\left(\frac{a}{b}\right)^2} = \sqrt{1-\frac{a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$$

Q. 5. If cos ($\alpha + \beta$) = 0, then sin ($\alpha - \beta$) can be reduced to

(a) cos β **(b)** cos 2β

(c) sin α **(d)** sin 2α

R **[NCERT Exemp.]**

Sol. Correct option : (b)

Explanation : cos ($\alpha + \beta$) = 0

$$\cos(\alpha + \beta) = \cos 90°$$
$$\alpha + \beta = 90°$$
$$\alpha = 90° - \beta$$
$$\sin(\alpha - \beta) = \sin(90° - \beta - \beta)$$
$$= \sin(90° - 2\beta)$$
$$= \cos 2\beta$$

Q. 6. The value of (tan 1° tan 2° tan 3°... tan 89°) is :

(a) 0 **(b)** 1

(c) 2 **(d)** $\frac{1}{2}$

U **[NCERT Exemp.]**

Sol. Correct option : (b)

Explanation : (tan 1° tan 2° tan 3° ... tan 89°)

$$= (\tan 1° \tan 89°)(\tan 2° \tan 88°)(\tan 3° \tan 87°) \ldots(\tan 45° \tan 45°)$$

$$= [\tan 1° \tan (90° - 1)][\tan 2° \tan (90° - 2)][\tan 3° \tan (90° - 3)] \ldots [\tan 45° \tan (90° - 45°)]$$

$$= \tan 1° \cot 1° \tan 2° \cot 2° \tan 3° \cot 3° \ldots \tan 45° \cot 45°$$

$$= \tan 1° \times \frac{1}{\tan 1°} \tan 2° . \frac{1}{\tan 2°} \tan 3° . \frac{1}{\tan 3°} \ldots \frac{\tan 45°}{\tan 45°}$$

$$= 1 \cdot 1 \cdot 1 \cdot 1 \ldots . 1 \cdot 1$$

$$= 1$$

Q. 7. If cos 9α = sin α and 9α < 90°, then the value of tan 5α is :

(a) $\frac{1}{\sqrt{3}}$ **(b)** $\sqrt{3}$

(c) 1 **(d)** 0

U **[NCERT Exemp.]**

Sol. Correct option : (c)

Explanation : cos 9α = sin α

cos 9α = cos (90° – α)

On comparing both sides, we have

9α = 90° – α

10α = 90°

α = 9°

$\therefore$ tan 5α = tan 5 × 9° = tan 45° = 1

Q. 8. If ΔABC is right angled at C, then the value of cos $(A+B)$ is :

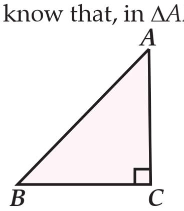
(a) 0 **(b)** 1

(c) $\frac{1}{2}$ **(d)** $\frac{\sqrt{3}}{2}$

U **[NCERT Exemp.]**

Sol. Correct option : (a)

Explanation : We know that, in ΔABC,



sum of three angles = 180°

i.e., $\angle A + \angle B + \angle C = 180°$

$\angle C = 90°$ [Given]

$\angle A + \angle B + 90° = 180°$

$\Rightarrow$ $A + B = 90°$

$\therefore$ $\cos(A+B) = \cos 90° = 0$

Q. 9. If sin A + sin^2 A = 1, then the value of the expression (cos^2 A + cos^4 A) is :

(a) 1 **(b)** $\frac{1}{2}$

(c) 2 **(d)** 3

R **[NCERT Exemp.]**

Sol. Correct option : (a)

Explanation : Given, $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A$ $[\because \sin^2\theta + \cos^2\theta = 1]$

On squaring both sides, we get

$$\sin^2 A = \cos^4 A$$

$$\Rightarrow \quad 1 - \cos^2 A = \cos^4 A$$

$$\Rightarrow \cos^2 A + \cos^4 A = 1$$

Q. 10. Given that $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is :

(a) $0°$ (b) $30°$
(c) $60°$ (d) $90°$

R [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : Given, $\sin\alpha = \frac{1}{2} = \sin 30°$

$$\left[\because \sin 30° = \frac{1}{2}\right]$$

$$\Rightarrow \alpha = 30°$$

And, $\cos\beta = \frac{1}{2} = \cos 60°$

$$\Rightarrow \quad \beta = 60°$$

$$\therefore \quad \alpha + \beta = 30° + 60° = 90°$$

$$\left[\because \cos 60° = \frac{1}{2}\right]$$

Q. 11. The value of the expression

$$\left[\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°\right]$$ **is :**

(a) 3 (b) 2
(c) 1 (d) 0

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Given expression,

$$\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$$

$$= \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2(90° - 68°) + \cos^2 68°} + \sin^2 63° + \cos 63° \sin(90° - 63°)$$

$$= \frac{\sin^2 22° + \cos^2 22°}{\cos^2 68° + \sin^2 68°} + \sin^2 63° + \cos 63° \cdot \cos 63°$$

$$[\because \sin(90° - \theta) = \cos\theta \text{ and } \cos(90° - \theta) = \sin\theta]$$

$$= \frac{1}{1} + (\sin^2 63° + \cos^2 63°) \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= 1 + 1 = 2$$

Q. 12. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

R [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : Given, $4\tan\theta = 3$

$$\Rightarrow \tan\theta = \frac{3}{4} \qquad \text{...(i)}$$

$$\therefore \frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} \quad \left[\begin{array}{l}\text{Divided by } \cos\theta \text{ in} \\ \text{both numerator and} \\ \text{denominator}\end{array}\right]$$

$$= \frac{4\tan\theta - 1}{4\tan\theta + 1} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2} \qquad \left[\begin{array}{l}\text{Put } \tan\theta = \frac{3}{4} \\ \text{from equation (i)}\end{array}\right]$$

Q. 13. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is :

(a) 1 (b) $\frac{3}{4}$
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

R [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : Given $\sin\theta - \cos\theta = 0$

$$\Rightarrow \sin\theta = \cos\theta = \frac{\sin\theta}{\cos\theta} = 1$$

$$\Rightarrow \tan\theta = 1 \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \tan 45° = 1\right]$$

$$\Rightarrow \tan\theta = \tan 45°$$

$$\therefore \quad \theta = 45°$$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45°$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 \qquad \left[\because \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Q. 14. $\sin(45° + \theta) - \cos(45° - \theta)$ is equal to :

(a) $2\cos\theta$ (b) 0
(c) $2\sin\theta$ (d) 1

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : $\sin(45° + \theta) - \cos(45° - \theta)$
$= \sin(45° + \theta) - \cos[90° - (45° + \theta)]$
$= \sin(45° + \theta) - \sin(45° + \theta)$
$= 0$

Q. 15. $\frac{2\tan 30°}{1 + \tan^2 30°} =$

(a) $\sin 60°$ (b) $\cos 60°$
(c) $\tan 60°$ (d) $\sin 30°$

R [NCERT Exemp.]

Sol. Correct option : (a)

Explanation :

$$\frac{2\tan 30°}{1 + \tan^2 30°} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$\frac{\sqrt{3}}{2} = \sin 60$

Q. 16. $\frac{1-\tan^2 45°}{1+\tan^2 45°} =$

(a) $\tan 90°$ (b) 1

(c) $\sin 45°$ (d) 0

R [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : $\frac{1-\tan^2 45°}{1+\tan^2 45°} = \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Q. 17. $\sin 2A = 2 \sin A$ **is true when** $A =$

(a) 0° (b) 30°

(c) 45° (d) 60°

R [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : As $\sin 2A = \sin 0° = 0$

$2\sin A = 2\sin 0° = 2(0) = 0$

Q. 18. $\frac{2\tan 30°}{1-\tan^2 30°} =$

(a) $\cos 60°$ (b) $\sin 60°$

(c) $\tan 60°$ (d) $\sin 30°$

R [NCERT Exemp.]

Sol. Correct option : (c)

Explanation :

$$\frac{2\tan 30°}{1-\tan^2 30°} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

$\tan 60° = \sqrt{3}$

[B] Very Short Answer Type Questions :

Q. 1. In a triangle ABC**, write** $\cos\left(\frac{B+C}{2}\right)$ **in terms of angle** A**.** R [Board Term-1, 2016, Set-O4YP6G7]

Sol. $A + B + C = 180°$

or, $B + C = 180° - A$

$\therefore \cos\left(\frac{B+C}{2}\right) = \cos\left[\frac{180° - A}{2}\right]$

$= \cos\left(90 - \frac{A}{2}\right)$

$= \sin\frac{A}{2}$ 1

[CBSE Marking Scheme, 2016]

Q. 2. If sec θ.sin θ = 0, then find the value of θ.

R [Board Term-1, 2016, Set-O4YP6G7]

Sol. Given, $\sec\theta.\sin\theta = 0$

or, $\frac{\sin\theta}{\cos\theta} = 0$

or, $\tan\theta = 0 = \tan 0°$

$\therefore \theta = 0°$ 1

[CBSE Marking Scheme, 2016]

Q. 3. If $A + B = 90°$ **and sec A =** $\frac{2}{3}$**, then find the value of cosec** B**.**

R [Board Term-1, 2016, Set-ORDAWEZ]

Sol. Given, $A + B = 90°$ and

$\sec A = \frac{2}{3}$

or, $\sec(90° - B) = \frac{2}{3}$.

$\therefore \text{cosec } B = \frac{2}{3}$ 1

[CBSE Marking Scheme, 2016]

Q. 4. If $\tan 2A = \cot(A + 60°)$**, find the value of** A **where** $2A$ **is an acute angle.**

U [Board Term-1, 2016, Set-LGRKRO]

Sol. Given $\tan 2A = \cot(A + 60°)$

or, $\cot(90° - 2A) = \cot(A + 60°)$

or, $90° - 2A = A + 60°$

or, $3A = 30°$

$\therefore A = 10°$ 1

[CBSE Marking Scheme, 2016]

Q. 5. Find the value of $\frac{\sin 25°}{\cos 65°} + \frac{\tan 23°}{\cot 67°}$

U [Board Term-1, 2015, Set-FHN8MGD]

Sol. $\frac{\sin 25°}{\cos 65°} + \frac{\tan 23°}{\cot 67°} = \frac{\sin 25°}{\sin(90° - 25°)} + \frac{\tan 23°}{\tan(90° - 23°)}$

$= 1 + 1 = 2$ 1

[CBSE Marking Scheme, 2015]

Q. 6. If $\cos 2A = \sin(A - 15°)$**, find** A**.**

U [Board Term-1, 2015, Set-FHN8MGD]

Sol. $\sin(90° - 2A) = \sin(A - 15°)$

or, $90° - 2A = A - 15°$

or, $3A = 105°$

$\therefore A = 35°$ 1

[CBSE Marking Scheme, 2015]

Q. 7. If $\tan(3x + 30°) = 1$**, then find the value of** x**.**

U [Board Term-1, 2015, Set-WJQZQBN]

Sol. $\tan(3x + 30°) = 1 = \tan 45°$

or, $3x + 30° = 45°$ or, $x = 5°$ 1

[CBSE Marking Scheme, 2015]

Q. 8. What happens to value of cos θ when θ increases from 0° to 90° ?

A [Board Term-1, 2015, Set-WJQZQBN]

Sol. cos θ decreases from 1 to 0. 1

[CBSE Marking Scheme, 2015]

Q. 9. If A **and** B **are acute angles and** $\sin A = \cos B$**, then find the value of** $A + B$**.**

U [Board Term-1, 2016, Set-MV98HN3]

Sol. Given, $\sin A = \cos B$

or, $\sin A = \sin(90° - B)$

or, $A = 90° - B$

$\therefore$ $A + B = 90°$ 1

Q. 10. What is the value of $(\cos^2 67° - \sin^2 23°)$? U

Sol. $\because$ $\cos^2 67° = \cos^2 (90° - 23°) = \sin^2 23$

$\therefore$ $\sin^2 23 - \sin^2 23 = 0$

[CBSE Marking Scheme, 2018] 1

Detailed Answer :

$$\cos^2 67° - \sin^2 23° = \cos^2 67° - \{\sin 23°\}^2$$

$$= \cos^2 67° - \{\sin(90 - 67)°\}^2$$

$$[\because \sin(90° - \theta) = \cos\theta]$$

$$= \cos^2 67° - \cos^2 67°$$

$$= 0$$

$\therefore$ $\cos^2 67° - \sin^2 23° = 0$ 1

Q. 11. If $\sin\theta = \cos\theta$, then find the value of $2\tan\theta + \cos^2\theta$. U [CBSE SQP-2018]

Sol. Given, $\sin\theta = \cos\theta$ $\theta = 45°$

$$2\tan\theta + \cos^2\theta = 2 + \frac{1}{2} = \frac{5}{2}$$

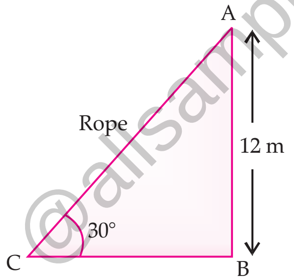
[CBSE Marking Scheme, 2018]

Q. 12. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground . The height of the pole is 12 m and the angle made by the rope with ground level is 30°.

(i) Calculate the distance covered by the artist in climbing the top of the pole.

(ii) Which mathematical concept is used in this problem ? AE

Sol.



(i) Clearly, distance covered by the artist is equal to the length of the rope AC. Let AB be the vertical pole of height 12 m.

It is given that $\angle ACB = 30°$

Thus, in right-angled triangle ABC,

$$\sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{12}{AC}$$

$\because$ $AC = 24$ m. 1

Hence, the distance covered by the circus artist is 24 m.

(ii) Height and Distance. 1

[C] True / False :

Q. 1. State whether the following are true or false. Justify your answer.

(a) The value of $\tan A$ is always less than 1.

(b) $\sec A = \frac{12}{5}$ for some value of angle A.

(c) $\cos A$ is the abbreviation used for the cosecant of angle A.

(d) $\cot A$ is the product of cot and A.

(e) $\sin\theta = \frac{4}{3}$ for some angle θ.

[NCERT Exemp.]

Sol. (a) False, the value of $\tan 90°$ is greater than 1.

(b) True, $\sec A = \frac{12}{5} = \cos A = \frac{5}{12}$ as 12 is the hypotenuse which is the largest side of triangle.

(c) False, $\cos A$ is the abbreviation used for cosine of $\angle A$.

(d) False, $\cot A$ is not the product of cot and A. It is the cotangent of $\angle A$.

(e) False, in a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of $\sin\theta$ is not possible.

Q. 2. State whether the following are true or false. Justify your answer.

(a) $\sin(A + B) = \sin A + \sin B$

(b) The value of $\sin\theta$ increases as θ increases.

(c) The value of $\cos\theta$ increases as θ increases.

(d) $\sin\theta = \cos\theta$ for all values of θ.

(e) $\cot A$ is not defined for $A = 0°$.

[NCERT Exemp.]

Sol. (a) False, $\sin(A+B) = \sin A + \sin B$

Let, $A = 30°$ and $B = 60°$

LHS $\sin(A+B) = \sin(30° + 60°)$

$$= \sin 90° = 1$$

RHS $\sin A + \sin B = \sin 30° + \sin 60°$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

Clearly, $\sin(A+B) \neq \sin A + \sin B$

(b) True, the value of $\sin\theta$ increases as θ increases in the interval of $0° < \theta < 90°$.

(c) False, the value of $\cos\theta$ decreases as θ increases in the interval of $0° < \theta < 90°$.

(d) False, it is true when $\theta = 45°$ not for all other values of θ.

(e) True, $\cot A$ is not defined for $A = 0°$

Q. 3. State whether the following are true or false. Justify your answer.

(a) $\frac{\tan 47°}{\tan 43°} = 1$

(b) The value of the expression $(\sin 80° - \cos 80°)$ is negative.

[NCERT Exemp.]

Sol. (a) True,

$$= \frac{\tan 47°}{\cot 43°}$$

$$= \frac{\tan 47°}{\cot (90° - 47°)}$$

$$= \frac{\tan 47°}{\tan 47°} = 1$$

(b) False,

80° is near to 90°, sin 90° = 1 and cos 90° = 0

So, the given expression sin 80° – cos 80° > 0

So, the value of the given expression is positive.

Short Answer Type Questions-I　(2 marks each)

Q. 1. Evaluate :

$$\frac{3\tan^2 30° + \tan^2 60° + \text{cosec}\, 30° - \tan 45°}{\cot^2 45°}$$

U [Board Term-1, 2016, Set-MV98HN3]

Sol. $$\frac{3\tan^2 30° + \tan^2 60° + \text{cosec}\, 30° - \tan 45°}{\cot^2 45°}$$

$$= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2} \quad \mathbf{1}$$

$$= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1}$$

$$= 1 + 3 + 2 - 1 = 5 \quad \mathbf{1}$$

Commonly Made Error

- Sometimes students get confused with the values of trigonometric angles. They substitute wrong values which leads to the wrong result.

Answering Tip

- Memorize the values of trigonometric angles properly and practice more such problems to not to get confused.

Q. 2. If sin $(A + B) = 1$ and sin $(A - B) = \frac{1}{2}, 0 \le A + B = 90°$ and $A > B$, then find A and B.

U [Board Term-1, 2016, Set-O4YP6G7]

Sol. Here, $\sin (A + B) = 1 = \sin 90°$

or, $A + B = 90°$ ...(i)

$\sin (A - B) = \frac{1}{2} = \sin 30°$ **1**

or, $A - B = 30°$ ...(ii)

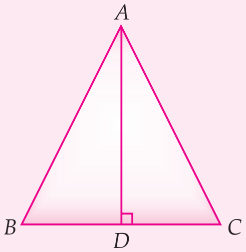
Solving eq. (i) and (ii),

$A = 60°$ and $B = 30°$ **1**

Q. 3. Find cosec 30° and cos 60° geometrically.

U [Board Term-1, 2015, Set-FHN8MGD]

Sol.



Let a triangle ABC with each side equal to $2a$. ½

$\angle A = \angle B = \angle C = 60°$

Draw AD perpendicular to BC

$\Delta BDA \cong \Delta CDA$ by RHS ½

$BD = CD$

$\angle BAD = \angle CAD = 30°$ by $CPCT$

In ΔBDA, $\text{cosec}\, 30° = \frac{AB}{BD} = \frac{2a}{a} = 2$ ½

and $\cos 60° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$ ½

[CBSE Marking Scheme, 2015]

Q. 4. Evaluate : $\frac{\sin 90°}{\cos 45°} + \frac{1}{\text{cosec} 30°}$

R [Board Term-1, 2013, Set-FFC]

Sol. $$\frac{\sin 90°}{\cos 45°} + \frac{1}{\text{cosec}\, 30°} = \frac{1}{\frac{1}{\sqrt{2}}} + \frac{1}{2} \quad \mathbf{1}$$

$$= \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2} \quad \mathbf{1}$$

Q. 5. If sin (36 + θ)° = cos (16 + θ)°, then find θ, where (36 + θ)° and (16 + θ)° are both acute angles.

U [Board Term-1, 2012, Set–68]

Sol. $\sin (36 + \theta)° = \cos (16 + \theta)°$

or, $\cos [90° - (36 + \theta)°] = \cos (16 + \theta)°$ **1**

or, $90° - 36° - \theta = 16° + \theta$

or, $2\theta = 90° - 36° - 16° = 38°$

∴ $\theta = \frac{38°}{2} = 19°.$ **1**

Q. 6. If $\sqrt{2} \sin \theta = 1$, find the value of $\sec^2 \theta - \text{cosec}^2 \theta$.

R [Board Term-1, 2012, Set–67]

Sol. Given, $\sqrt{2} \sin \theta = 1$

or, $\sin \theta = \frac{1}{\sqrt{2}} = \sin 45°$

∴ $\theta = 45°$ **1**

Now, $\sec^2 \theta - \text{cosec}^2 \theta = \sec^2 45° - \text{cosec}^2 45°$

$= \left(\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2$

$= 0.$ **1**

Q. 7. If 4 cos θ = 11 sin θ, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$. U [Board Term-1, 2012, Set–50]

Sol. Given : $4\cos\theta = 11\sin\theta$

or, $\cos\theta = \frac{11}{4}\sin\theta$

Now, $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11\times\frac{11}{4}\sin\theta - 7\sin\theta}{11\times\frac{11}{4}\sin\theta + 7\sin\theta}$ 1

$= \frac{\sin\theta\left(\frac{121}{4} - 7\right)}{\sin\theta\left(\frac{121}{4} + 7\right)}$

$= \frac{121-28}{121+28} = \frac{93}{149}$. 1

Q. 8. If tan (*A* + *B*) = $\sqrt{3}$**, tan (*A* – *B*) =** $\frac{1}{\sqrt{3}}$**, 0° < *A* + *B* ≤ 90°, *A* > *B*, then find *A* and *B*.** A [Board Term-1, 2012, Set–69]

Sol. ∵ $\tan(A+B) = \sqrt{3} = \tan 60°$

Hence, $A + B = 60°$ ...(i) ½

Again, $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30°$

or, $A - B = 30°$ ...(ii) ½

Adding equations (i) and (ii),

$2A = 90°$

∴ $A = \frac{90°}{2} = 45°$

Putting this value of *A* in equation (i),

$B = 60° - A = 60° - 45° = 15°$

Hence, $A = 45°$ and $B = 15°$. 1

Q. 9. If cos (*A* – *B*) = $\frac{\sqrt{3}}{2}$ **and sin (*A* + *B*) =** $\frac{\sqrt{3}}{2}$**, find *A* and *B*, where (*A* + *B*) and (*A* – *B*) are acute angles.** A [Board Term-1, 2012, Set–70]

Sol. $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30°$

or, $A - B = 30°$ ...(i) ½

$\sin(A+B) = \frac{\sqrt{3}}{2} = \sin 60°$

or, $A + B = 60°$ ...(ii) ½

Adding equations (i) and (ii),

$2A = 90°$

∴ $A = 45°$ ½

From eqn. (ii), $B = 60° - A = 60° - 45° = 15°$ ½

[CBSE Marking Scheme, 2012]

Q. 10. Express cos 68° + tan 76° in terms of the angles between 0° and 45°. U [Board Term-1, 2012, Set–64]

Sol. $\cos 68° + \tan 76° = \cos(90° - 22°) + \tan(90° - 14°)$ 1

$= \sin 22° + \cot 14°$, 1

[∵ $\cos(90° - \theta) = \sin\theta$ and $\tan(90° - \theta) = \cot\theta$]

Q. 11. Find the value of cos 2θ, if 2 sin 2θ = $\sqrt{3}$**.** U [Board Term-1, 2012, Set–25]

Sol. Given, $2\sin 2\theta = \sqrt{3}$

or, $\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60°$ 1

or, $2\theta = 60°$

Hence, $\cos 2\theta = \cos 60° = \frac{1}{2}$. 1

[CBSE Marking Scheme, 2012]

Q. 12. Find the value of :
sin 30°. cos 60° + cos 30°. sin 60°
Is it equal to sin 90° or cos 90° ? U [Board Term-1, 2016, Set-ORDAWEZ]

Sol. $\sin 30° \cos 60° + \cos 30° \sin 60°$

$= \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$ 1

$= \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4} = 1$ 1

It is equal to sin 90° = 1 but not equal to cos 90° as cos 90° = 0.

Q. 13. Evaluate : $\frac{6\sin 23° + \sec 79° + 3\tan 48°}{\operatorname{cosec} 11° + 3\cot 42° + 6\cos 67°}$ U [Board Term-1, 2012, Set–55]

Sol. $\frac{6\sin 23° + \sec 79° + 3\tan 48°}{\operatorname{cosec} 11° + 3\cot 42° + 6\cos 67°}$

$= \frac{6\sin(90° - 67°) + \sec(90° - 11°) + 3\tan(90° - 42°)}{\operatorname{cosec} 11° + 3\cot 42° + 6\cos 67°}$ 1

$= \frac{6\cos 67° + \operatorname{cosec} 11° + 3\cot 42°}{\operatorname{cosec} 11° + 3\cot 42° + 6\cos 67°}$

$= 1.$ 1

Q. 14. If $\sqrt{3}$ **sin θ – cos θ = 0 and 0°< θ < 90°, find the value of θ.** U [Board Term-1, 2012, Set–35]

Sol. Here $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0° < \theta < 90°$

or, $\sqrt{3}\sin\theta = \cos\theta$

or, $\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$ 1

or, $\tan\theta = \frac{1}{\sqrt{3}}$

$= \tan 30°$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

∴ $\theta = 30°$. 1

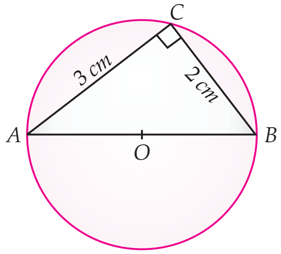
Q. 15. Evaluate : $\frac{\cos 45°}{\sec 30°} + \frac{1}{\sec 60°}$ U [Board Term-1, 2012, Set–63]

Sol. $$\frac{\cos 45°}{\sec 30°}+\frac{1}{\sec 60°} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}}+\frac{1}{2}$$

$$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{2} = \frac{\sqrt{6}}{4}+\frac{1}{2} \quad 1$$

$$= \frac{\sqrt{6}+2}{4} \quad 1$$

Q. 16. **In the given figure, *AOB* is a diameter of a circle with centre *O*. Find tan *A* tan *B*.**



U **[Board Term-1, 2012, Set–52]**

Sol. In ΔABC, $\angle C = 90°$ (Angle in a semi-circle)

$$\tan A = \frac{BC}{AC} = \frac{2}{3} \quad \tfrac{1}{2}$$

and $$\tan B = \frac{AC}{BC} = \frac{3}{2} \quad \tfrac{1}{2}$$

$\therefore$ $$\tan A.\tan B = \frac{2}{3}\times\frac{3}{2} = 1. \quad 1$$

Q. 17. ***A, B, C* are interior angles of *ABC*. Prove that cosec $\left(\frac{A+B}{2}\right) = \sec\frac{C}{2}$**

U **[CBSE Comptt. Set I, II, III-2018]**

Sol. $$A + B + C = 180°$$

$$\frac{A+B}{2} = 90° - \frac{C}{2} \quad 1$$

$$\text{cosec}\left(\frac{A+B}{2}\right) = \text{cosec}\left(90° - \frac{C}{2}\right) = \sec\frac{C}{2} \quad 1$$

[CBSE Marking Scheme, 2018]

Short Answer Type Questions-II

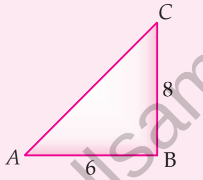
(3 marks each)

Q. 1. **If in a triangle *ABC* right angled at *B*, *AB* = 6 units and *BC* = 8 units, then find the value of sin *A*.cos *C* + cos *A*.sin *C*.**

U **[Board Term-1, 2016, Set-O4YP6G7]**

Sol. Here, $AC^2 = (8)^2 + (6)^2 = 100$

or, $AC = 10$



$\therefore$ $$\sin A = \frac{8}{10}, \cos A = \frac{6}{10} \quad 1$$

and $$\sin C = \frac{6}{10}, \cos C = \frac{8}{10} \quad 1$$

$\therefore$ sin A cos C + cos A sin C

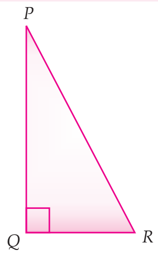
$$= \frac{8}{10}\times\frac{8}{10}+\frac{6}{10}\times\frac{6}{10}$$

$$= \frac{64}{100}+\frac{36}{100}$$

$$= \frac{100}{100} = 1. \quad 1$$

[CBSE Marking Scheme, 2016]

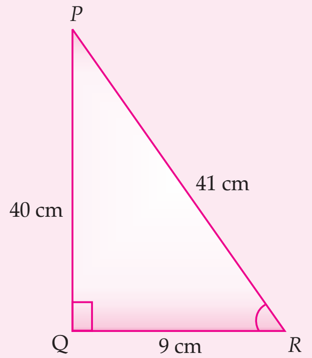
Q. 2.



In the given ΔPQR, right–angled at *Q*, *QR* = 9 cm and *PR* – *PQ* = 1 cm. Determine the value of sin *R* + cos *R*.

U **[Board Term-1, 2015, Set–FHN8MGD]**

Sol.



$$PQ^2 + QR^2 = PR^2$$

(By Pythagoras theorem)

or, $PQ^2 + 9^2 = PR^2$

or, $PQ^2 + 81 = (PQ + 1)^2$

or, $PQ^2 + 81 = PQ^2 + 1 + 2PQ$

or, $PQ = 40$

$PR - PQ = 1$ (Given)

or, $PR = 1 + 40$

or, $PR = 41$

$\therefore$ $$\sin R + \cos R = \frac{40}{41}+\frac{9}{41} = \frac{49}{41} \quad 3$$

[CBSE Marking Scheme, 2015]

Q. 3. **Evaluate :** $$\frac{5\cos^2 60°+4\cos^2 30°-\tan^2 45°}{\sin^2 30°+\cos^2 60°}$$

R **[Board Term-1, 2013, Set–LK–59]**

Sol. $\dfrac{5\cos^2 60° + 4\cos^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 60°}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \quad 1$$

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}} \quad 1$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2} \quad 1$$

Q. 4. If sin 3θ = cos (θ – 6°), where 3θ and θ – 6° are both acute angles, find the value of θ.

U [Board Term-1, 2011, Set–21]

Sol. According to the question,

$\sin 3\theta = \cos(\theta - 6°)$ 1

or, $\cos(90° - 3\theta) = \cos(\theta - 6°)$

or, $90° - 3\theta = \theta - 6°$ 1

or, $4\theta = 90° + 6° = 96°$

∴ $\theta = \dfrac{96°}{4} = 24°$ 1

[CBSE Marking Scheme, 2011]

Q. 5. Simplify :

$$\frac{\sin\theta\sec(90°-\theta)\tan\theta}{\operatorname{cosec}(90°-\theta)\cos\theta\cot(90°-\theta)} - \frac{\tan(90°-\theta)}{\cot\theta}.$$

U [Board Term-1, 2011, Set–66]

Sol. ∵ $\sec(90° - \theta) = \operatorname{cosec}\theta$,

$\tan(90° - \theta) = \cot\theta$,

$\cot(90° - \theta) = \tan\theta$,

$\operatorname{cosec}(90° - \theta) = \sec\theta$ 1

Hence,

$$\frac{\sin\theta\sec(90°-\theta)\tan\theta}{\operatorname{cosec}(90°-\theta)\cos\theta\cot(90°-\theta)} - \frac{\tan(90°-\theta)}{\cot\theta}$$

$$= \frac{\sin\theta\operatorname{cosec}\theta\tan\theta}{\sec\theta\cos\theta\tan\theta} - \frac{\cot\theta}{\cot\theta} \quad 1$$

$$= \frac{\sin\theta\times\frac{1}{\sin\theta}\times\tan\theta}{\frac{1}{\cos\theta}\times\cos\theta\tan\theta} - 1 = 1 - 1 = 0 \quad 1$$

Q. 6. If tan 2*A* = cot(*A* – 18°), where 2*A* is an acute angle, find the value of *A*.

C + U [CBSE Delhi/OD Set-2018]

Sol. $\tan 2A = \cot(A - 18°)$ 1

⇒ $90° - 2A = A - 18°$ 1

⇒ $3A = 108°$ 1

⇒ $A = 36°$

[CBSE Marking Scheme, 2018]

Commonly Made Error

- Generally conversion from tan to cot is not done and the angles are equated and simplified incorrectly.

Answering Tip

- The candidates should remember to convert the tan to cot before equating the angles.

Q. 7. If 4 tanθ = 3, evaluate $\left(\dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$

U [CBSE Delhi/OD Set-2018]

Sol. Given, $4\tan\theta = 3$

⇒ $\tan\theta = \dfrac{3}{4}$

⇒ $\sin\theta = \dfrac{3}{5}$ and $\cos\theta = \dfrac{4}{5}$ ½ + ½

∴ $\left(\dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right) = \dfrac{4\times\frac{3}{5} - \frac{4}{5} + 1}{4\times\frac{3}{5} + \frac{4}{5} - 1}$ 1

$= \dfrac{13}{11}$ 1

[CBSE Marking Scheme, 2018]

Commonly Made Error

- Mostly candidates do not find the values of sine and cosine. Some candidates do the wrong calculation.

Answering Tip

- Candidates should find the value of sinθ and cosθ by using Pythagoras theorem.

Q. 8. If sin (*A* + 2*B*) = $\dfrac{\sqrt{3}}{2}$ **and cos(*A* + 4*B*) = 0, *A* > *B*, and *A* + 4*B* ≤ 90°, then find *A* and *B*.**

C + U [CBSE Compt. Set I, II, III, 2018]

Sol. Given, $\sin(A + 2B) = \dfrac{\sqrt{3}}{2} \Rightarrow A + 2B = 60°$ 1

⇒ $\cos(A + 4B) = 0, \Rightarrow A + 4B = 90°$ 1

Solving, we get $A = 30$ and $B = 15°$ ½ + ½

[CBSE Marking Scheme, 2018]

Q. 9. If cos θ + sin θ = $\sqrt{2}\cos\theta$**, show that cos θ – sin θ =** $\sqrt{2}\sin\theta$**.**

U [Board Term-1, 2011, Set–74]

Sol. Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

or, $\sin\theta = \cos\theta(\sqrt{2} - 1)$

or, $\sin\theta = \dfrac{\cos\theta(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}$ 1

or, $\sin\theta = \dfrac{\cos\theta(2-1)}{\sqrt{2}+1}$

or, $(\sqrt{2} + 1)\sin\theta = \cos\theta$ 1 or, $\sqrt{2}\sin\theta + \sin\theta = \cos\theta$

or, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$. **Hence proved. 1**

Q. 10. Prove that : $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin A}{1-\cot A} = \sin A + \cos A.$

U [Board Term-1, 2013, FFC ; 2011, Set–74]

Sol.

$$\text{LHS} = \frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}$$

$$= \frac{\cos A}{1-\left(\dfrac{\sin A}{\cos A}\right)}+\frac{\sin A}{1-\left(\dfrac{\cos A}{\sin A}\right)} \quad 1$$

$$= \frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A}$$

$$= \frac{\cos^2 A}{\cos A-\sin A}-\frac{\sin^2 A}{\cos A-\sin A} \quad 1$$

$$= \frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$$

$$= \frac{(\cos A-\sin A)(\cos A+\sin A)}{(\cos A-\sin A)}$$

$= \cos A + \sin A$

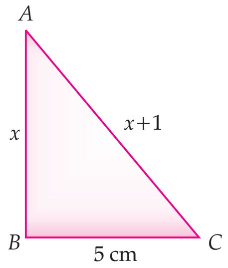
$= \sin A + \cos A$

= RHS **Hence proved. 1**

Q. 11. In ΔABC, $\angle B = 90°$, $BC = 5$ cm and $AC - AB = 1$, Evaluate : $\dfrac{1+\sin C}{1+\cos C}$.

U [Board Term-1, 2011, Set-52]

Sol.



Let $AB = x$

$\because$ $AC - AB = 1$

or, $AC = x + 1$

$\because$ $AC^2 = AB^2 + BC^2$

or, $(x + 1)^2 = x^2 + (5)^2$

or, $x^2 + 2x + 1 = x^2 + 25$

or, $2x = 24$

or, $x = \dfrac{24}{2} = 12$ cm **1**

Hence, $AB = 12$ cm and $AC = 13$ cm

$$\sin C = \frac{AB}{AC} = \frac{12}{13}$$

$$\cos C = \frac{BC}{AC} = \frac{5}{13} \quad 1$$

Now $$\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}. \quad 1$$

Long Answer Type Questions

(4 marks each)

Q. 1. Evaluate :

$$\frac{\tan^2 30° \sin 30° + \cos 60° \sin^2 90° \tan^2 60° - 2\tan 45° \cos^2 0°}{\sin 90°}$$

R [Board Term-1, 2015, Set-WJQZQBN]

Sol. $\tan^2 30° \sin 30° + \cos 60° \sin^2 90° \tan^2 60° - 2\tan 45° \cos^2 0° \sin 90°$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2 \times 1 \times 1^2 \times 1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 1 \times 3 - 2 \times 1 \times 1 \times 1.$$

$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1+9-12}{6}$$

$$= -\frac{2}{6} = -\frac{1}{3}$$ [CBSE Marking Scheme, 2015] **4**

Q. 2. Evaluate : $\sin^2 30° \cos^2 45° + 4\tan^2 30° + \dfrac{1}{2}\sin^2 90° - 2\cos^2 90° + \dfrac{1}{24}$

R [Board Term-1, 2013, LK–59]

Sol. $\sin^2 30° \cos^2 45° + 4\tan^2 30° + \dfrac{1}{2}\sin^2 90° - 2\cos^2 90° + \dfrac{1}{24}$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0)^2 + \frac{1}{24} \quad 1$$

$$= \frac{1}{4}\left(\frac{1}{2}\right) + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} \quad 1$$

$$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{3+32+12+1}{24} \quad 1$$

$$= \frac{48}{24} = 2. \quad 1$$

Q. 3. Evaluate : $4\left(\sin^4 30° + \cos^4 60°\right) - 3\left(\cos^2 45° - \sin^2 90°\right)$

R [Board Term-1, 2013, Set–FFC]

Sol. $4\left(\sin^4 30° + \cos^4 60°\right) - 3\left(\cos^2 45° - \sin^2 90°\right)$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right] \quad 1$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right] \quad 1$$

$$= 4 \times \frac{2}{16} - 3 \times -\frac{1}{2}$$

$$= \frac{1}{2} + \frac{3}{2} \quad 1$$

$$= \frac{4}{2} = 2. \quad 1$$

Q. 4. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$. U **[Board Term-1, 2012, Set–48]**

Sol. Let $\cot\theta = x$,

then $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$ becomes

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0 \quad 1$$

or, $\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$

or, $(x - \sqrt{3})(\sqrt{3}x - 1) = 0$

$\therefore$ $x = \sqrt{3}$ or $\frac{1}{\sqrt{3}}$ 1

or, $\cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$

$\therefore$ $\theta = 30°$ or $\theta = 60°$

If $\theta = 30°$, then

$$\cot^2 30° + \tan^2 30° = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{10}{3} \quad 1$$

If $\theta = 60°$, then $\cot^2 60° + \tan^2 60° = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$

$$= \frac{1}{3} + 3 = \frac{10}{3}. \quad 1$$

Q. 5. Evaluate the following :

$$\frac{2\cos^2 60° + 3\sec^2 30° - 2\tan^2 45°}{\sin^2 30° + \cos^2 45°}.$$

R **[Board Term-1, 2012, Set–43]**

Sol. $\dfrac{2\cos^2 60° + 3\sec^2 30° - 2\tan^2 45°}{\sin^2 30° + \cos^2 45°}$

$$= \frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} \quad 2$$

$$= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}} \quad 1$$

$$= \frac{10}{3}. \quad 1$$

Q. 6. Evaluate : $\dfrac{\cos 65°}{\sin 25°} - \dfrac{\tan 20°}{\cot 70°} - \sin 90° + \tan 5°$ $\tan 35° \tan 60° \tan 55° \tan 85°$.

U **[Board Term-1, 2012, Set–50]**

Sol. $\dfrac{\cos 65°}{\sin 25°} = \dfrac{\cos 65°}{\sin(90° - 65°)}$

$$= \frac{\cos 65°}{\cos 65°} = 1, \quad 1$$

$$\frac{\tan 20°}{\cot 70°} = \frac{\tan(90° - 70°)}{\cot 70°}$$

$$= \frac{\cot 70°}{\cot 70°} = 1 \quad 1$$

and $\sin 90° = 1$

$\tan 5° \tan 35° \tan 60° \tan 55° \tan 85°$

$= \tan(90° - 85°)\tan(90° - 55°)$

$\tan 55° \tan 60° \tan 85°$. 1

$= \cot 85° \tan 85° \cot 55° \tan 55° . \sqrt{3}$

$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$

$\therefore$ Given expression $= 1 - 1 - 1 + \sqrt{3}$

$= \sqrt{3} - 1$. 1

[CBSE Marking Scheme, 2012]

Q. 7. Prove that :

$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta.$$

U **[Board Term-1, 2012, Set-48]**

Sol. LHS $= \dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta}$

$$= \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta} \quad \left(\because \tan\theta = \frac{1}{\cot\theta}\right)$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1 - \tan\theta)\tan\theta} \quad 1\tfrac{1}{2}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta} \quad [\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)] \quad 1$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)} = \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta} \quad 1$$

$= \tan\theta + 1 + \cot\theta$

$=$ RHS. **Hence proved.** ½

Commonly Made Error

- Sometimes students don't apply this formula : $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$. They directly simplify equation which leads to incorrect result.

Q. 8. In an acute angled triangle ABC, if $\sin(A + B - C) = \frac{1}{2}$ and $\cos(B + C - A) = \frac{1}{\sqrt{2}}$, find $\angle A$, $\angle B$ and $\angle C$. A [Board Term-1, 2012, Set–39]

Sol. We have

$$\sin(A + B - C) = \frac{1}{2} = \sin 30°$$

or, $$A + B - C = 30° \quad ...(i)\ \mathbf{1}$$

and $$\cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45°$$

or, $$B + C - A = 45° \quad ...(ii)\ \mathbf{1}$$

Adding eqns. (i) and (ii), we get

$$2B = 75°$$

or, $$B = 37.5°$$

Now subtracting eqn. (ii) from eqn. (i),

$$2(A - C) = + 15°$$

or, $$A - C = 7.5° \quad ...(iii)$$

$\because$ $$A + B + C = 180° \quad \mathbf{1}$$

or, $$A + C = 142.5° \quad ...(iv)$$

Adding eqns. (iii) and (iv),

$$2A = 150°$$

or, $$A = 75°$$

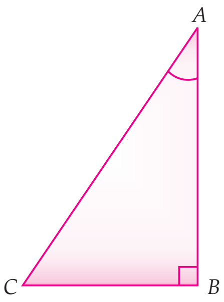
and $$C = 67.5°$$

Hence, $$\angle A = 75°, \angle B = 37.5° \text{ and } \angle C = 67.5° \quad \mathbf{1}$$

TOPIC-2
Trigonometric Identities

Revision Notes

- An equation is called an identity if it is true for all values of the variable(s) involved.
- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.



In ΔABC, right-angled at B, By Pythagoras Theorem,

$$AB^2 + BC^2 = AC^2 \quad ...(i)$$

Dividing each term of (i) by AC^2,

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

or $$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

or $$(\cos A)^2 + (\sin A)^2 = 1$$

or $$\cos^2 A + \sin^2 A = 1 \quad ...(ii)$$

This is true for all values of A such that $0° \le A \le 90°$. So, this is a trigonometric identity. Now divide eqn.(1) by AB^2.

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or $$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

or $$1 + \tan^2 A = \sec^2 A \quad ...(iii)$$

Is this equation true for $A = 0°$? Yes, it is. What about $A = 90°$? Well, $\tan A$ and $\sec A$ are not defined for $A = 90°$. So, eqn. (iii) is true for all values of A such that $0° \le A < 90°$.

dividing eqn. (i) by BC^2.

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

or $$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

or $$\cot^2 A + 1 = \mathrm{cosec}^2 A \quad ...(iv)$$

Note that cosec A and cot A are not defined for all $A = 0°$. Therefore eqn. (iv) is true for all value of A such that $0° < A \leq 90°$.

Using these identities, we can express each trigonometric ratio in terms of other trigonometric ratios, *i.e.*, if any one of the ratios is known, we can also determine the values of other trigonometric ratios.

Know the Formulae

- **(i)** $\sin^2 A + \cos^2 A = 1$ **(ii)** $1 + \tan^2 A = \sec^2 A$

  (iii) $1 + \cot^2 A = \text{cosec}^2 A$

How it is done on GREENBOARD?

Q. Prove that. $\dfrac{1}{\sec\theta - \tan\theta} = \dfrac{1 + \sin\theta}{\cos\theta}$.

Sol. : Step I : L.H.S. $= \dfrac{1}{\sec\theta - \tan\theta}$

Multiplying with $\sec\theta + \tan\theta$

L.H.S. $= \dfrac{1}{\sec\theta - \tan\theta} \times \dfrac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$

$= \dfrac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= \sec\theta + \tan\theta$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1 + \sin\theta}{\cos\theta}$ = R.H.S.

Objective Type Questions (1 mark each)

[A] Multiple Choice Questions :

Q. 1. $9\sec^2 A - 9\tan^2 A =$

(a) 1 **(b)** 9

(c) 8 **(d)** 0

A **[NCERT Exemp.]**

Sol. Correct option : (b)

Explanation :

$9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$

$= 9\,(1)$ $[\because \sec^2 A - \tan^2 A = 1]$

$= 9$

Q. 2. $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta) =$

(a) 0 **(b)** 1

(c) 2 **(d)** –1

A **[NCERT Exemp.]**

Sol. Correct option : (c)

Explanation : $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\theta)$

$= \left(1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)\left(1 + \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)$

$= \left(\dfrac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\dfrac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$

$= \dfrac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$

$= \dfrac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$

$= \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$

Q. 3. $(\sec A + \tan A)(1 - \sin A) =$

(a) $\sec A$ **(b)** $\sin A$

(c) $\text{cosec}\, A$ **(d)** $\cos A$

A **[NCERT Exemp.]**

Sol. Correct option : (d)

Explanation :

$(\sec A + \tan A)(1 - \sin A) = \left(\dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}\right)(1 - \sin A)$

$= \left(\dfrac{1 + \sin A}{\cos A}\right)(1 - \sin A)$

$= \left(\dfrac{1 - \sin^2 A}{\cos A}\right) = \dfrac{\cos^2 A}{\cos A}$

$= \cos A$

[B] Very Short Answer Type Questions :

Q. 1. Evaluate : $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

Sol. $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{1 + \tan^2 A}{1 + \dfrac{1}{\tan^2 A}}$

$$= \frac{\tan^2 A(1+\tan^2 A)}{(\tan^2 A+1)}$$

$$= \tan^2 A$$

Q. 2. If $k + 1 = \sec^2\theta\ (1 + \sin\theta)\ (1 - \sin\theta)$, then find the value of k. C + U [Board Term-1, 2015, Set-JJOQ]

Sol. $k + 1 = \sec^2\theta\ (1 + \sin\theta)\ (1 - \sin\theta)$

or, $k + 1 = \sec^2\theta\ (1 - \sin^2\theta)$

or, $k + 1 = \sec^2\theta.\cos^2\theta$ [$\because \sin^2\theta + \cos^2\theta = 1$]

or, $k + 1 = \sec^2\theta \times \frac{1}{\sec^2\theta}$

or, $k + 1 = 1$ ½

or, $k = 1 - 1$ ½

$\therefore$ $k = 0$. [CBSE Marking Scheme, 2015]

Q. 3. Write the value of $\cot^2\theta - \frac{1}{\sin^2\theta}$ U [SQP-2018]

Sol. $\cot^2\theta - \frac{1}{\sin^2\theta} = \cot^2\theta - \text{cosec}^2\theta$

$= -1$

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-I (2 marks each)

Q. 1. Express the trigonometric ratio of sec A and tan A in terms of sin A.

U [Board Term-1, 2015, Set-FHN8MGD]

Sol. $\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}}$ 1

and $\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$ 1

[CBSE Marking Scheme, 2015]

Q. 2. Prove that :

$$\frac{(\sin^4\theta+\cos^4\theta)}{1-2\sin^2\theta\cos^2\theta} = 1$$

A [Board Term-1, 2015, Set-WJQZQBN

Sol. LHS $= \frac{(\sin^4\theta+\cos^4\theta)}{1-2\sin^2\theta\cos^2\theta}$

$= \frac{(\sin^2\theta)^2+(\cos^2\theta)^2}{1-2\sin^2\theta\cos^2\theta}$

$= \frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{1-2\sin^2\theta\cos^2\theta}$

$= \frac{1-2\sin^2\theta\cos^2\theta}{1-2\sin^2\theta\cos^2\theta} = 1 =$ RHS 2

[CBSE Marking Scheme, 2015]

Commonly Made Error

- Common errors are found in simplification.

Answering Tip

- Follow step by step simplification to avoid errors.

Q. 3. Prove that : $-1 + \frac{\sin A \sin\ (90^\circ - A)}{\cot\ (90^\circ - A)} = -\sin^2 A$

U [Board Term-1, 2012, Set–62]

Sol. LHS $= -1 + \frac{\sin A \sin\ (90^\circ - A)}{\cot\ (90^\circ - A)}$

[$\because \sin(90^\circ - \theta) = \cos\theta$]

[$\because \cot(90^\circ - \theta) = \tan\theta$]

$= -1 + \frac{\sin A\cos A}{\tan A}$ ½

$= -1 + \sin A \cos A \times \cot A$ ½

[$\because \cot\theta = \frac{\cos\theta}{\sin\theta}$]

$= -1 + \sin A \cos A \times \frac{\cos A}{\sin A}$ ½

[$\because \sin^2\theta + \cos^2\theta = 1$]

$= -1 + \cos^2 A = -(1 - \cos^2 A)$ ½

$= -\sin^2 A =$ RHS **Hence proved.**

[CBSE Marking Scheme, 2012]

Q. 4. Prove that : $\sqrt{\frac{1-\cos A}{1+\cos A}} = \text{cosec}\ A - \cot A$

U [Board Term-1, 2012, Set–74]

Sol. LHS $= \sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1-\cos A}{1-\cos A}}$ 1

$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$

$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \text{cosec}\ A - \cot A =$ RHS **Hence proved. 1**

Q. 5. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta + \cos\theta$. U [Board Term-1, 2013, Set-FFC]

Sol. $\because$ $\sin\theta - \cos\theta = \frac{1}{2}$

On squaring both sides,

$$(\sin\theta - \cos\theta)^2 = \left(\frac{1}{2}\right)^2$$

or, $\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \frac{1}{4}$

or, $1 - 2\sin\theta\cos\theta = \frac{1}{4}$

or, $2\sin\theta\cos\theta = 1 - \frac{1}{4} = \frac{3}{4}$ **1**

Again, $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$

$= 1 + \frac{3}{4} = \frac{7}{4}$

$\therefore \quad \sin\theta + \cos\theta = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$ **1**

Q. 6. If θ be an acute angle and 5cosec θ = 7, then evaluate $\sin\theta + \cos^2\theta - 1$.

U [Board Term-1, 2012, Set–43]

Sol. Given, $5\,\text{cosec}\,\theta = 7$

or, $\text{cosec}\,\theta = \frac{7}{5}$

or, $\sin\theta = \frac{5}{7}$ $\left[\because \text{cosec}\,\theta = \frac{1}{\sin\theta}\right]$ **1**

$\sin\theta + \cos^2\theta - 1 = \sin\theta - (1 - \cos^2\theta)$

$= \sin\theta - \sin^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{5}{7} - \left(\frac{5}{7}\right)^2 = \frac{35-25}{49} = \frac{10}{49}$ **1**

[CBSE Marking Scheme, 2012]

Q. 7. If $\sin A = \frac{\sqrt{3}}{2}$, then find the value of $2\cot^2 A - 1$.

U [Board Term-1, 2012, Set–21]

Sol. $2\cot^2 A - 1 = 2(\text{cosec}^2 A - 1) - 1$ **1**

$(\because \cot^2\theta = -1 + \text{cosec}^2\theta)$

$= \frac{2}{\sin^2 A} - 3$

$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3$

$\therefore \quad 2\cot^2 A - 1 = \frac{8}{3} - 3 = \frac{-1}{3}$ **1**

Short Answer Type Questions-II

(3 marks each)

Q. 1. Prove that :

$$\frac{\cos A}{1 + \tan A} - \frac{\sin A}{1 + \cot A} = \cos A - \sin A$$

U [Board Term-1, 2016, Set-MV98HN3]

Sol. Try yourself, Similar to Q. No. 10 in SATQ-II in Topic-1

Commonly Made Error

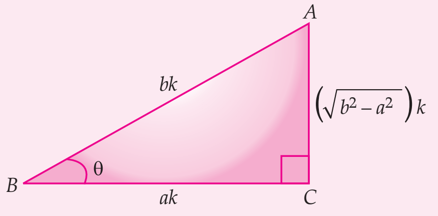
- Sometimes students work with both sides together.

Answering Tip

- Do simplify only one side at a time.

Q. 2. If $b\cos\theta = a$, then prove that $\text{cosec}\,\theta + \cot\theta = \sqrt{\frac{b+a}{b-a}}$. U [Board Term-1, 2015, Set-WJQZQBN]

Sol.



Given, $\cos\theta = \frac{a}{b}$

$AC^2 = AB^2 - BC^2$

$AC = \sqrt{b^2 - a^2}\,k$

$\text{cosec}\,\theta = \frac{b}{\sqrt{b^2 - a^2}}, \cot\theta = \frac{a}{\sqrt{b^2 - a^2}}$

$\text{cosec}\,\theta + \cot\theta = \frac{b+a}{\sqrt{b^2 - a^2}} = \sqrt{\frac{b+a}{b-a}}$ **3**

[CBSE Marking Scheme, 2015]

Q. 3. Prove that : $(\cot\theta - \text{cosec}\,\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

U [Board Term-1, 2015 Set JTOQ, 2015]

Sol. LHS $= (\cot\theta - \text{cosec}\,\theta)^2$

$= \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$

$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$

= RHS **Hence proved. 3**

Answering Tip

- Adequate practice of identities is necessary to avoid errors in simplification.

Q. 4. Show that :

$$\text{cosec}^2\theta - \tan^2(90° - \theta) = \sin^2\theta + \sin^2(90° - \theta)$$

U [Board Term-1, 2013, LK–59]

Sol.

$$\text{LHS} = \text{cosec}^2\theta - \tan^2(90° - \theta)$$
$$= \frac{1}{\sin^2\theta} - \frac{\sin^2(90°-\theta)}{\cos^2(90°-\theta)}$$
$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} \quad 1$$
$$= \frac{1-\cos^2\theta}{\sin^2\theta}$$
$$= \frac{\sin^2\theta}{\sin^2\theta} \quad 1$$
$$= 1$$
$$= \sin^2\theta + \cos^2\theta$$
$$= \sin^2\theta + \sin^2(90° - \theta)$$
$$= \text{RHS} \quad 1$$

Q. 5. Prove that :

$$\frac{\text{cosec}^2\theta}{\text{cosec}\,\theta - 1} - \frac{\text{cosec}^2\theta}{\text{cosec}\,\theta + 1} = 2\sec^2\theta$$

U [Board Term-1, 2013, FFC]

Sol.

$$\frac{\text{cosec}^2\theta}{\text{cosec}\,\theta - 1} - \frac{\text{cosec}^2\theta}{\text{cosec}\,\theta + 1}$$
$$= \frac{\text{cosec}^2\theta\,[\text{cosec}\theta + 1 - \text{cosec}\theta + 1]}{\text{cosec}^2\theta - 1}$$
$$= \frac{2\text{cosec}^2\theta}{\cot^2\theta}$$
$$= \frac{2}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$
$$= 2\sec^2\theta = \text{RHS} \quad 1$$

Q. 6. Prove that :

$$\frac{1}{\text{cosec}\,A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A + \cot A}.$$

U [Board Term-1, 2011, Set–66]

Sol.

$$\frac{1}{\text{cosec}\,A - \cot A} - \frac{1}{\sin A}$$
$$= \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A + \cot A}$$

Re-arranging above equation,

$$\Rightarrow \quad \frac{1}{\text{cosec}\,A - \cot A} + \frac{1}{\text{cosec}\,A + \cot A}$$
$$= \frac{2}{\sin A} \quad 1$$

Now, L.H.S.

$$= \frac{1}{\text{cosec}\,A - \cot A} + \frac{1}{\text{cosec}\,A + \cot A}$$
$$= \frac{\text{cosec}A + \cot A + \text{cosec}A - \cot A}{(\text{cosec}A - \cot A)(\text{cosec}A + \cot A)} \quad 1$$
$$= \frac{2\text{cosec}A}{\text{cosec}^2A - \cot^2A}$$
$$= \frac{\frac{2}{\sin A}}{1} = \frac{2}{\sin A} = \text{R.H.S.}$$

Hence Proved. 1

Q. 7. If $\sec\theta = x + \frac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$.

A [Board Term-1, 2011, Set–55]

Sol.

$$\sec\theta = x + \frac{1}{4x}$$
$$\sec^2\theta = x^2 + \frac{1}{16x^2} + 2\cdot x\cdot\frac{1}{4x}$$
$$1 + \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$
$$\tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$$
$$\tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$
$$\tan^2\theta = x^2 + \frac{1}{16x^2} - 2\cdot x\cdot\frac{1}{4x}$$
$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2 \quad 1$$

Taking square root of both sides

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

If $\tan\theta = x - \frac{1}{4x}$

Given, $\sec\theta = x + \frac{1}{4x}$

Now, $\tan\theta + \sec\theta = 2x$

If $\tan\theta = -\left(x - \frac{1}{4x}\right) = -x + \frac{1}{4x}$ 1

Given, $\sec\theta = x + \frac{1}{4x}$

Now, $\sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x}$ 1

Hence Proved.

Q. 8. Prove that : $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}.$

U [Board Term-1, 2011, Set–39]

Sol.

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)} \quad \mathbf{1}$$

$$= \frac{1+1}{\sin^2\theta - 1 + \sin^2\theta} \quad \mathbf{1}$$

$$= \frac{2}{2\sin^2\theta - 1} = \text{RHS} \quad \mathbf{1}$$

Hence proved.

Q. 9. If $x \sin^3\theta + y \cos^3\theta = \sin\theta\cos\theta$ and $x \sin\theta = y \cos\theta$, prove that $x^2 + y^2 = 1$.

A **[Board Term-1, 2011, Set-44]**

Sol. Given : $x \sin\theta = y \cos\theta$

or, $x = \dfrac{y\cos\theta}{\sin\theta}$ ...(i) **1**

and $x \sin^3\theta + y \cos^3\theta = \sin\theta\cos\theta$ ...(ii)

Substituting x from eqn. (i) in eqn. (ii),

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

or, $y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$

or, $y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$

or, $y\cos\theta \times 1 = \sin\theta\cos\theta$

or, $y = \sin\theta$ ...(iii) **1**

Substituting this value of y in eqn. (i),

$x = \cos\theta$ ...(iv)

$\therefore$ Squaring and adding eqn. (iii) and eqn. (iv), we get

$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$ **1**

Hence proved.

Q. 10. Prove that $\dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2.$

U **[Board Term-1, 2011, Set-40]**

Sol. LHS $= \dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$

$$+ \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)} \quad \mathbf{1}$$

$= (1 - \sin\theta\cos\theta) + (1 + \sin\theta\cos\theta)$ **1**

$= 2 - \sin\theta\cos\theta + \sin\theta\cos\theta$ **1**

$= 2 =$ RHS **Hence proved.**

Q. 11. Evaluate the following :

$$\frac{\sec^2(90° - \theta) - \cot^2\theta}{2(\sin^2 25° + \sin^2 65°)} - \frac{2\cos^2 60° \tan^2 28° \tan^2 62°}{3(\sec^2 43° - \cot^2 47°)}$$

U **[Board Term-1, 2011, Set–60]**

Sol. $\dfrac{\sec^2(90° - \theta) - \cot^2\theta}{2(\sin^2 25° + \sin^2 65°)} - \dfrac{2\cos^2 60° \tan^2 28° \tan^2 62°}{3(\sec^2 43° - \cot^2 47°)}$

$$= \frac{(\text{cosec}^2\theta - \cot^2\theta)}{2(\sin^2 25° + \cos^2 25°)}$$

$$- \frac{2 \times \frac{1}{2} \times \frac{1}{2} \tan^2 28° \times \cot^2 28°}{3[\sec^2 43° - \tan^2 43°]} \quad \mathbf{1}$$

$$= \frac{1}{2\times(1)} - \frac{\frac{1}{2} \times \tan^2 28° \times \frac{1}{\tan^2 28°}}{3} \quad \mathbf{1}$$

$$= \frac{1}{2} - \frac{1}{6} = \frac{1}{3} \quad \mathbf{1}$$

[CBSE Marking Scheme, 2011]

Q. 12. Evaluate : $\dfrac{\sec 41° \sin 49° + \cos 29° \text{cosec } 61° - \frac{2}{\sqrt{3}}(\tan 20° \tan 60° \tan 70°)}{3(\sin^2 31° + \sin^2 59°)}$.

U **[Board Term-1, 2011, Set–25]**

Sol. $\dfrac{\sec 41° \sin 49° + \cos 29° \text{cosec } 61° - \frac{2}{\sqrt{3}}(\tan 20° \tan 60° \tan 70°)}{3(\sin^2 31° + \sin^2 59°)}$

$$= \frac{\sec(90° - 49°)\sin 49° + \cos 29° \text{cosec }(90° - 29°) - \frac{2}{\sqrt{3}}[\tan 20° \sqrt{3} \tan(90° - 20°)]}{3[\sin^2 31° + \sin^2(90° - 31°)]} \quad \mathbf{1}$$

$$= \frac{\text{cosec } 49° \sin 49° + \cos 29° \sec 29° - \frac{2}{\sqrt{3}}[\tan 20° \sqrt{3} \cot 20°]}{3(\sin^2 31° + \cos^2 31°)} \quad \mathbf{1}$$

$$= \frac{1+1-2}{3} = \frac{2-2}{3} = 0 \quad \mathbf{1}$$

Q. 13. Evaluate : $\dfrac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta)\tan(30° - \theta)}$ + cosec $(75° + \theta)$ – sec $(15° - \theta)$

U **[Board Term-1, 2011, Set–40]**

Sol. $\dfrac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta)\tan(30° - \theta)}$ + cosec $(75° + \theta)$ – sec $(15° - \theta)$

$$= \frac{\cos^2(45°+\theta)+\sin^2(90°-45°+\theta)}{\tan(60°+\theta)\cot(90°-30°+\theta)} + \text{cosec}(75°+\theta) - \text{cosec}(90°-15°+\theta) \quad \mathbf{1}$$

$$= \frac{\cos^2(45°+\theta)+\sin^2(45°+\theta)}{\tan(60°+\theta)\cot(60°+\theta)} + \text{cosec}(75°+\theta) - \text{cosec}(75°+\theta) \quad \mathbf{1}$$

$$= \frac{1}{1} + 0 = 1 \quad \mathbf{1}$$

Q. 14. Express : sin *A*, tan *A* and cosec *A* in terms of sec *A*. A **[Board Term-1, 2011, Set–25]**

Sol. $\sin^2 A + \cos^2 A = 1$

(i) $\sin A = \sqrt{1-\cos^2 A}$

$$= \sqrt{1-\frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \quad \mathbf{1}$$

(ii) $\tan^2 A = \sec^2 A - 1$

or, $\tan A = \sqrt{\sec^2 A - 1}$ **1**

(iii) $\text{cosec}\, A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$ **1**

Q. 15. Find the value of the following without using trigonometric tables :

$$\frac{\cos 50°}{2\sin 40°} + \frac{4(\text{cosec}^2 59° - \tan^2 31°)}{3\tan^2 45°} - \frac{2}{3}\tan 12° \tan 78°. \sin 90°.$$

U **[Board Term-1, 2011, Set–21]**

Sol. Try yourself, Similar to Q. No. 12 in SATQ-II.

Q. 16. Evaluate :

$$\frac{\text{cosec}^2 63° + \tan^2 24°}{\cot^2 66° + \sec^2 27°} + \frac{\sin^2 63° + \cos 63° \cdot \sin 27° + \sin 27° \sec 63°}{2(\text{cosec}^2 65° - \tan^2 25°)}$$

A **[Sample Question Paper 2017-18]**

Sol. Try yourself, Similar to Q. No. 12 in SATQ-II.

Q. 17. If sin θ + cos θ = $\sqrt{2}$, the evaluate tan θ + cot θ.

A **[Sample Question Paper 2017-18]**

Sol. Given, $\sin\theta + \cos\theta = \sqrt{2}$

On squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = 2 - 1 = 1$$

$$\Rightarrow \quad \frac{1}{\sin\theta\cos\theta} = 2 \quad \text{...(i)} \ \mathbf{1}$$

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta} \quad \text{...(ii)} \ \mathbf{1}$$

From (i) and (ii) we get

$\tan\theta + \cot\theta = 2$ **1**

Q. 18. Prove that cot θ – tan θ = $\frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$ U **[SQP-2018]**

Sol. LHS $= \cot\theta - \tan\theta$ **1**

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \quad \tfrac{1}{2}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} \quad \mathbf{1}$$

$$= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta} \quad \tfrac{1}{2}$$

$$= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta} = \text{RHS}$$

[CBSE Marking Scheme, 2018]

Q. 19. Prove that sinθ (1 + tanθ) + cosθ (1 + cotθ) = secθ cosecθ U **[SQP-2018]**

Sol. LHS $= \sin\theta(1+\tan\theta) + \cos\theta(1+\cot\theta)$ **1**

$$= \sin\theta\left(1+\frac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(1+\frac{\cos\theta}{\sin\theta}\right)$$

$$= \sin\theta\left(\frac{\cos\theta+\sin\theta}{\cos\theta}\right) + \cos\theta\left(\frac{\sin\theta+\cos\theta}{\sin\theta}\right) \quad \mathbf{1}$$

$$= (\cos\theta + \sin\theta)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\cos\theta+\sin\theta}{\cos\theta\sin\theta} = \text{cosec}\theta + \sec\theta = \text{RHS} \quad \mathbf{1}$$

[CBSE Marking Scheme, 2018]

Long Answer Type Questions (4 marks each)

Q. 1. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if :
(i) $x = a\sec\theta$, $y = b\tan\theta$, or
(ii) $x = a\,\text{cosec}\,\theta$, $y = b\cot\theta$.

U **[Board Term-1, 2015, Set-WJQZQBN]**

Sol. (i) $\frac{x^2}{a^2} = \sec^2\theta$, $\frac{y^2}{b^2} = \tan^2\theta$

or, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1.$

$\therefore \quad b^2x^2 - a^2y^2 = a^2b^2$ **2**

(ii) $\frac{x^2}{a^2} = \text{cosec}^2\theta,\ \frac{y^2}{b^2} = \cot^2\theta$

or, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \text{cosec}^2\theta - \cot^2\theta = 1$

$\therefore\quad b^2x^2 - a^2y^2 = a^2b^2$ **2**

Q. 2. If cosec θ – cot θ = $\sqrt{2}$ cot θ, then prove that cosec θ + cot θ = $\sqrt{2}$ cosec θ.

U [Board Term-1, 2015, Set-WJQZQBN]

Sol. Given, $\text{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$

Squaring both the sides,

$\text{cosec}^2\theta + \cot^2\theta - 2\,\text{cosec}\,\theta\cot\theta = 2\cot^2\theta$

or, $\text{cosec}^2\theta - \cot^2\theta = 2\,\text{cosec}\,\theta\cot\theta$ **1**

$[\because a^2 - b^2 = (a+b)(a-b)]$

or, $(\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 2\,\text{cosec}\,\theta\cot\theta$ **1**

Given : $(\text{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta)$

or, $\text{cosec}\,\theta + \cot\theta = \frac{2\,\text{cosec}\,\theta\cot\theta}{\sqrt{2}\cot\theta}$ **1**

$\text{cosec}\,\theta + \cot\theta = \sqrt{2}\,\text{cosec}\,\theta$ **1**

Hence Proved.

[CBSE Marking Scheme, 2015]

Q. 3. Prove that :

$$\frac{\cot^3\theta.\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta.\cos^3\theta}{(\cos\theta+\sin\theta)^2} = \frac{\sec\theta\,\text{cosec}\,\theta - 1}{\text{cosec}\,\theta + \sec\theta}$$

U [Set-FHN8MGD, 2015]

Sol. LHS $= \frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2}$

$$= \frac{\frac{\cos^3\theta}{\sin^3\theta}\times\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta}\times\cos^3\theta}{(\cos\theta+\sin\theta)^2} \quad \mathbf{1}$$

$$= \frac{\cos^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta+\sin\theta)^2} \quad \mathbf{1}$$

$$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\sin\theta\cos\theta)}{(\cos\theta+\sin\theta)^2} \quad \mathbf{1}$$

$$= \frac{1-\sin\theta\cos\theta}{\cos\theta+\sin\theta}$$

$$= \frac{\text{cosec}\,\theta\ \sec\theta - 1}{\text{cosec}\,\theta+\sec\theta} \quad \mathbf{1}$$

(Divide numerator and denominator by sin θ cos θ)

= RHS. **Hence proved.**

[CBSE Marking Scheme, 2015]

Q. 4. Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\,\text{cosec}\,\theta.$

U [Board Term-1, 2012, Set–39]

Sol. LHS $= \sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

$$= \frac{(\sec\theta-1)+(\sec\theta+1)}{\sqrt{(\sec\theta+1)(\sec\theta-1)}} \quad \mathbf{1}$$

$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta-1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta} \quad \mathbf{1}$$

$(\because \tan^2\theta = \sec^2\theta - 1)$

$$= 2\times\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}$$

$$= 2\times\frac{1}{\sin\theta} \quad \mathbf{1}$$

$= 2\,\text{cosec}\,\theta$

= RHS. **1**

[CBSE Marking Scheme, 2012]

Q. 5. Prove that : $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\sec\theta+1}{\sec\theta-1}.$

U [Board Term-1, 2012, Set–21]

Sol. LHS $= \frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta} \quad \mathbf{1\tfrac{1}{2}}$$

$$= \frac{\sin\theta\left(\frac{1}{\cos\theta}+1\right)}{\sin\theta\left(\frac{1}{\cos\theta}-1\right)} \quad \mathbf{1\tfrac{1}{2}}$$

$= \frac{\sec\theta+1}{\sec\theta-1}$ = RHS. **Hence proved. 1**

[CBSE Marking Scheme, 2012]

Q. 6. Prove that : $\frac{\text{cosec}\,A}{\text{cosec}\,A-1} + \frac{\text{cosec}\,A}{\text{cosec}\,A+1} = 2\sec^2 A.$

U [Board Term-1, 2012, Set-62]

Sol. Try yourself, Similar to Q. No. 5 in SATQ-II.

Q. 7. If cosec θ + cot θ = *p*, then prove that

$\cos\theta = \frac{p^2-1}{p^2+1}.$ U [Board Term-1, 2012, Set–39]

[Board Term-1, 2016, Set–MV98HN3]

Sol. RHS $= \frac{p^2-1}{p^2+1}$

$$= \frac{(\text{cosec}\,\theta+\cot\theta)^2-1}{(\text{cosec}\,\theta+\cot\theta)^2+1} \quad \mathbf{1}$$

$$= \frac{\text{cosec}^2\theta+\cot^2\theta+2\,\text{cosec}\,\theta\cot\theta-1}{\text{cosec}^2\theta+\cot^2\theta+2\,\text{cosec}\,\theta\cot\theta+1} \quad \mathbf{1}$$

$$= \frac{1+\cot^2\theta+\cot^2\theta+2\,\text{cosec}\,\theta\cot\theta-1}{\text{cosec}^2\theta+\text{cosec}^2\theta-1+2\,\text{cosec}\,\theta\cot\theta+1} \quad \mathbf{1}$$

$$= \frac{2\cot\theta\,(\cot\theta+\text{cosec}\,\theta)}{2\,\text{cosec}\,\theta\,(\text{cosec}\,\theta+\cot\theta)}$$

$$= \frac{\cos\theta}{\sin\theta}\times\sin\theta = \cos\theta = \text{LHS}. \quad \mathbf{1}$$

Hence proved.

[CBSE Marking Scheme, 2012]

Q. 8. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $m^2 + n^2 = a^2 + b^2$

U **[Board Term-1, 2012, Set–58]**

Sol. Given,

$m^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta$ ...(i) **1**

and

$n^2 = a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta$ ...(ii) **1**

Adding equations (i) and (ii), **1**

$m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$= a^2(1) + b^2(1)$

$= a^2 + b^2 = \text{RHS}.$ **Hence proved. 1**

Q. 9. Prove that :

$$\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta} = 1 + \sin\theta\cos\theta.$$

U **[Board Term-1, 2012, Set–50]**

Sol. L.H.S $= \dfrac{\cos^2\theta}{1-\tan\theta}+\dfrac{\sin^3\theta}{\sin\theta-\cos\theta}$

$$= \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin^3\theta}{\sin\theta-\cos\theta} \quad \mathbf{1}$$

$$= \frac{\cos^3\theta}{\cos\theta-\sin\theta}-\frac{\sin^3\theta}{\cos\theta-\sin\theta} \quad \mathbf{1}$$

$$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta} \quad \mathbf{1}$$

$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{(\cos\theta-\sin\theta)}$$

$= 1 + \sin\theta\cos\theta = \text{R.H.S.}$ **Hence proved. 1**

[CBSE Marking Scheme, 2012]

Q. 10. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, prove that $q(p^2 - 1) = 2p$. U **[Board Term-1, 2012, Set–38]**

Sol. Given : $\cos\theta + \sin\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$

L.H.S $= q(p^2 - 1)$

$= (\sec\theta + \text{cosec}\,\theta)\,[(\cos\theta + \sin\theta)^2 - 1]$ **1**

$= (\sec\theta + \text{cosec}\,\theta)(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1)$

$= (\sec\theta + \text{cosec}\,\theta)\,[1 + 2\sin\theta\cos\theta - 1]$ **½**

$$= \left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta) \quad \mathbf{1}$$

$$= \frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\times 2\sin\theta\cos\theta$$

$= 2(\sin\theta + \cos\theta)$ **1**

$= 2p$

$= \text{R.H.S.}$ **Hence proved. ½**

Q. 11. If $x = r\sin A\cos C$, $y = r\sin A\sin C$ and $z = r\cos A$, then prove that $x^2 + y^2 + z^2 = r^2$.

A **[Board Term-1, 2012, Set–50]**

Sol. Since, $x^2 = r^2\sin^2 A\cos^2 C$

$y^2 = r^2\sin^2 A\sin^2 C$

and $z^2 = r^2\cos^2 A$ **1**

L.H.S. $= x^2 + y^2 + z^2$

$= r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$ **1**

$= r^2\sin^2 A(\cos^2 C + \sin^2 C) + r^2\cos^2 A$

$= r^2\sin^2 A + r^2\cos^2 A$ **1**

$= r^2(\sin^2 A + \cos^2 A)$

$= r^2.$ **1**

$= \text{R.H.S.}$ **Hence proved.**

Q. 12. Prove that : $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta.$

U **[Board Term-1, 2012, Set–40]**

Sol. L.H.S $= \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}\times\frac{(1+\sin\theta)}{(1+\sin\theta)}}+\sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}\times\frac{(1-\sin\theta)}{(1-\sin\theta)}} \quad \mathbf{1}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} \quad \mathbf{1}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}} \quad \mathbf{1}$$

$$= \frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$= \frac{2}{\cos\theta}$$

$= 2\sec\theta = \text{R.H.S.}$ **Hence proved. 1**

Q. 13. Prove that :

$(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta).$

U **[Board Term-1, 2012, Set–62]**

Sol. L.H.S $= (1 - \sin\theta + \cos\theta)^2$

$= 1 + \sin^2\theta + \cos^2\theta - 2\sin\theta - 2\sin\theta\cos\theta + 2\cos\theta$

$= 1 + 1 - 2\sin\theta - 2\sin\theta\cos\theta + 2\cos\theta$ **1**

$= 2 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta$ **1**

$= 2(1 + \cos\theta) - 2\sin\theta(1 + \cos\theta)$

$= (1 + \cos\theta)(2 - 2\sin\theta)$ **1**

$= 2(1 + \cos\theta)(1 - \sin\theta) = \text{R.H.S.}$ **1**

Q. 14. Prove that : $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta-1} = \sec\theta + \tan\theta.$

U **[Board Term-1, 2012, Set–43]**

Sol. L.H.S $= \dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$

$$= \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1},$$

$(\because 1 + \tan^2\theta = \sec^2\theta)$ **1½**

$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1} \quad \mathbf{1}$$

$$= \frac{(\tan\theta+\sec\theta)[1-\sec\theta+\tan\theta]}{\tan\theta-\sec\theta+1} \quad \mathbf{1}$$

$= \tan\theta + \sec\theta =$ R.H.S. **Proved. ½**

Q. 15. Prove that : $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

U [Board Term-1, 2012, Set–52]

Sol. L.H.S $= (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ **1**

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + 2\sin\theta \times \frac{1}{\sin\theta} + \sec^2\theta + 2\cos\theta \times \frac{1}{\cos\theta}$ **1**

$= 1 + (1 + \cot^2\theta) + 2 + (1 + \tan^2\theta) + 2$ **1**

$= 7 + \tan^2\theta + \cot^2\theta$

= R.H.S. **Hence proved. 1**

Q. 16. If $\sin\theta = \frac{c}{\sqrt{c^2+d^2}}$ and $d > 0$, find the value of $\cos\theta$ and $\tan\theta$.

U [Board Term-1, 2013, LK–59]

Sol. Given, $\sin\theta = \frac{c}{\sqrt{c^2+d^2}}$

Since, $\cos^2\theta = 1 - \sin^2\theta$

$= 1 - \left(\frac{c}{\sqrt{c^2+d^2}}\right)^2$ **1**

$= 1 - \frac{c^2}{c^2+d^2}$

$= \frac{c^2+d^2-c^2}{c^2+d^2} = \frac{d^2}{c^2+d^2}$

$\therefore \cos\theta = \frac{d}{\sqrt{c^2+d^2}}$ **1**

Again, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{c}{\sqrt{c^2+d^2}}}{\frac{d}{\sqrt{c^2+d^2}}}$

$\therefore \tan\theta = \frac{c}{d}$ **2**

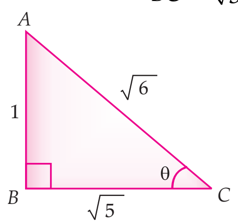
Q. 17. If $\tan\theta = \frac{1}{\sqrt{5}}$,

(i) Evaluate : $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$

(ii) Verify the identity : $\sin^2\theta + \cos^2\theta = 1$.

U [Board Term-1, 2012, Set–60]

Sol. $\tan\theta = \frac{AB}{BC} = \frac{1}{\sqrt{5}}$



In ΔABC, $AC^2 = AB^2 + BC^2 = 1 + 5 = 6$ **1**

or, $AC = \sqrt{6}$

(i) $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$, From figure

$= \frac{\left(\frac{\sqrt{6}}{1}\right)^2 - \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{6}}{1}\right)^2 + \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}$

$= \frac{6 - \frac{6}{5}}{6 + \frac{6}{5}}$

$= \frac{24}{36}$

$= \frac{2}{3}$

(ii) L.H.S $= \sin^2\theta + \cos^2\theta$

$= \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$ **1½**

$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1 =$ R.H.S

Hence proved.

Q. 18. Evaluate : $\frac{\cot(90° - \theta)\sin(90° - \theta)}{\sin\theta} + \frac{\cot 40°}{\tan 50°} - (\cos^2 20° + \cos^2 70°)$.

U [Board Term-1, 2012, Set–35]

Sol. Try yourself, Similar to Q. No. 11 in SATQ-II

Q. 19. Evaluate :

$$\frac{\text{cosec}^2(90° - \theta) - \tan^2\theta}{4(\cos^2 40° + \cos^2 50°)} - \frac{2\tan^2 30°\sec^2 52°\sin^2 38°}{3(\text{cosec}^2 70° - \tan^2 20°)}.$$

U [Board Term-1, 2012, Set–52]

Sol. Try yourself, Similar to Q. No. 11 in SATQ-II

Q. 20. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \frac{1}{p}$.

Hence, find the values of $\cos\theta$ and $\sin\theta$.

A [Board Term-1, 2015]

Sol. $\frac{1}{p} = \frac{1}{\sec\theta + \tan\theta} \times \frac{(\sec\theta - \tan\theta)}{\sec\theta - \tan\theta}$ ½

$\frac{1}{p} = \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \sec\theta - \tan\theta$ ½

Solving $\sec\theta + \tan\theta = p$ and $\sec\theta - \tan\theta = \frac{1}{p}$

We get $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2+1}{2p}$ **1**

and $\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right) = \frac{p^2-1}{2p}$ **1**

$\therefore \cos\theta = \frac{2p}{p^2+1}$ and $\sin\theta = \frac{p^2-1}{p^2+1}$ **1**

[CBSE Marking Scheme, 2015]

Q. 21. Prove that : $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

[Sample Question Paper 2017-18]

Sol. LHS $= \dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$

$= \dfrac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$ **1**

$= \dfrac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$

$= \dfrac{\sin\theta\cos\theta + \sin\theta - (1 - \cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$ **1**

$= \dfrac{\sin\theta(\cos\theta + 1) - [(1 - \cos\theta)(1 + \cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$

$= \dfrac{(1 + \cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$ **1**

$= \dfrac{(1 + \cos\theta)(\cos\theta + \sin\theta - 1)}{\sin\theta(\cos\theta + \sin\theta - 1)} = \dfrac{1 + \cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \operatorname{cosec}\theta + \cot\theta$ = RHS **Hence Proved 1**

Q. 22. Prove that : $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$.

U **[Board Term-I, 2012 Set 25]**

Sol. LHS $= (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$

$= \left(\sin A + \dfrac{1}{\cos A}\right)^2 + \left(\cos A + \dfrac{1}{\sin A}\right)^2$ **1**

$= \sin^2 A + \dfrac{1}{\cos^2 A} + 2\dfrac{\sin A}{\cos A} + \cos^2 A + \dfrac{1}{\sin^2 A} + 2\dfrac{\cos A}{\sin A}$

$= \sin^2 A + \cos^2 A + \dfrac{1}{\sin^2 A} + \dfrac{1}{\cos^2 A} + 2\left(\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}\right)$ **1**

$= 1 + \dfrac{\sin^2 A + \cos^2 A}{\sin^2 A\cos^2 A} + 2\left(\dfrac{\sin^2 A + \cos^2 A}{\sin A\cos A}\right)$

$= 1 + \dfrac{1}{\sin^2 A\cos^2 A} + \dfrac{2}{\sin A\cos A}$ **1**

$= \left(1 + \dfrac{1}{\sin A\cos A}\right)^2$

$= (1 + \sec A.\operatorname{cosec} A)^2$ **Hence Proved 1**

Q. 23. If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$, prove that each of the side is equal to ± 1.

U **[Board Term-I, 2012 Set 12]**

Sol. Given : $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$
$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ **1**

Multiply both the sides by
$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

or, $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \times (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$ **1**

or, $(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$
$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

or, $1 = [(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)]^2$

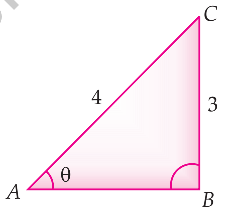
or, $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = \pm 1$ **1**

Similarly, multiply both sides by
$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$,
$\therefore (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1.$ **1**

Q. 24. If $4\sin\theta = 3$, find the value of x if $\sqrt{\dfrac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \dfrac{\sqrt{7}}{x} + \cos\theta$.

U **[Board Term-1, 2012, Set–40]**

Sol. $\sin\theta = \dfrac{3}{4}$ (Given)



In ABC, $\angle B = 90°$

Apply Pythagorus theorem,

$AB = \sqrt{7}$ **1**

$\cos\theta = \dfrac{\sqrt{7}}{4}$ **½**

and $\tan\theta = \dfrac{3}{\sqrt{7}}$ **½**

$\therefore \sqrt{\dfrac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \dfrac{\sqrt{7}}{x} + \cos\theta$

or, $\sqrt{\dfrac{1}{\tan^2\theta}} + 2 \times \dfrac{\sqrt{7}}{3} = \dfrac{\sqrt{7}}{x} + \dfrac{\sqrt{7}}{4}$ **½**

or, $\dfrac{1}{\tan\theta} + \dfrac{2\sqrt{7}}{3} = \dfrac{\sqrt{7}}{x} + \dfrac{\sqrt{7}}{4}$

or, $\dfrac{\sqrt{7}}{3} + \dfrac{2\sqrt{7}}{3} - \dfrac{\sqrt{7}}{4} = \dfrac{\sqrt{7}}{x}$ **½**

or, $\dfrac{4\sqrt{7} - \sqrt{7}}{4} = \dfrac{\sqrt{7}}{x}$

or, $\dfrac{3\sqrt{7}}{4} = \dfrac{\sqrt{7}}{x}$

$\therefore x = \dfrac{4}{3}$ **1**

Q. 25. Prove that : $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$

U **[CBSE Delhi/OD Set-2018]**

[Board Term-I, 2015, Set WJQ = QBN, FHN8MGD]

Sol. LHS $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \dfrac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$ **1**

$= \dfrac{\sin A(1-2(1-\cos^2 A))}{\cos A(2\cos^2 A - 1)}$ **1**

$= \tan A \dfrac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$ **1**

$= \tan A = \text{RHS}$ **1**

[CBSE Marking Scheme, 2018]

Commonly Made Error

- Many candidates do not able to identify the term sin*A* and cos*A* common to numerator and denominator respectively. Some do error in simplification.

Answering Tip

- Ensure adequate practice of sums based on identities.

Q. 26. If sec θ + tan θ = *p*, then find the value of cosec θ.

U **[CBSE SQP-2018]**

Sol. $\sec\theta + \tan\theta = p$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= 1 + \sin\theta = p\cos\theta$

$= p\sqrt{1-\sin^2\theta}$ **1**

$(1+\sin\theta)^2 = p^2(1-\sin^2\theta)$ **½**

$1 + \sin^2\theta + 2\sin\theta = p^2 - p^2\sin^2\theta$ **1**

$(1+p^2)\sin^2\theta + 2\sin\theta + (1-p^2) = 0$

$D = 4 - 4(1+p^2)(1-p^2)$

$= 4 - 4(1-p^4) = 4p^4$ **1**

$\sin\theta = \dfrac{-2 \pm \sqrt{4p^4}}{2(1+p^2)} = \dfrac{-1 \pm p^2}{(1+p^2)}$ **½**

$= \dfrac{p^2-1}{p^2+1}, -1$

$\therefore \quad \text{cosec}\,\theta = \dfrac{p^2+1}{p^2-1}, -1$ **1**

[CBSE Marking Scheme, 2018]

Q. 27. If 15 tan² θ + 4 sec² θ = 23, then find the value of (sec θ + cosec θ)² – sin² θ.

U **[Board Term-1, 2012, Set–38]**

Sol. $15\tan^2\theta + 4\sec^2\theta = 23$

$15\tan^2\theta + 4(\tan^2\theta + 1) = 23$

$(\because \sec^2\theta = 1 + \tan^2\theta)$ **1**

or, $15\tan^2\theta + 4\tan^2\theta + 4 = 23$

or, $19\tan^2\theta = 19$

or, $\tan\theta = 1 = \tan 45°$ **1**

$\therefore \quad \theta = 45°$

Now, $(\sec\theta + \text{cosec}\,\theta)^2 - \sin^2\theta$

$= (\sec 45° + \text{cosec}\,45°)^2 - \sin^2 45°$

$= \left(\sqrt{2}+\sqrt{2}\right)^2 - \left(\dfrac{1}{\sqrt{2}}\right)^2$ **1**

$= \left(2\sqrt{2}\right)^2 - \dfrac{1}{2}$

$= 8 - \dfrac{1}{2} = \dfrac{15}{2}$ **1**

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