

# CHAPTER : 3

## LAWS OF MOTION

In the previous lesson you learnt to describe the motion of an object in terms of its displacement, velocity and acceleration. But an important question is : what makes an object to move? Or what causes a ball rolling along the ground to come to a stop? From our everyday experience we know that we need to push or pull an object if we wish to change its position in a room. Similarly, a football has to be kicked in order to send it over a large distance. A cricket ball has to be hit hard by a batter to send it across the boundary for a six. You will agree that muscular activity is involved in all these actions and its effect is quite visible.

There are, however, many situations where the cause behind an action is not visible. For example, what makes rain drops to fall to the ground? What makes the earth to go around the sun? In this lesson you will learn the basic laws of motion and discover that force causes motion. The concept of force developed in this lesson will be useful in different branches of physics. Newton showed that force and motion are intimately connected. The laws of motion are fundamental and enable us to understand everyday phenomena.

### OBJECTIVES

After studying this lesson, you should be able to :

- *explain the significance of inertia;*
- *state Newton's laws of motion and illustrate them with examples;*
- *explain the law of conservation of momentum and illustrate it with examples;*
- *understand the concept of equilibrium of concurrent forces;*
- *define coefficient of friction and distinguish between static friction, kinetic friction and rolling friction;*

- *suggest different methods of reducing friction and highlight the role of friction in every-day life; and*
- *analyse a given situation and apply Newton's laws of motion using free body diagrams.*

### 3.1 CONCEPTS OF FORCE AND INERTIA

We all know that stationary objects remain wherever they are placed. These objects cannot move on their own from one place to another place unless forced to change their state of rest. Similarly, an object moving with constant velocity has to be forced to change its state of motion. **The property of an object by which it resists a change in its state of rest or of uniform motion in a straight line is called inertia. Mass of a body is a measure of its inertia.**

In a way, inertia is a fantastic property. If it were not present, your books or classnotes could mingle with those of your younger brother or sister. Your wardrobe could move to your friend's house creating chaos in life. You must however recall that the state of rest or of uniform motion of an object are not absolute. In the previous lesson you have learnt that an object at rest with respect to one observer may appear to be in motion with respect to some other observer. Observations show that **the change in velocity of an object can only be brought, if a net force acts on it.**

You are very familiar with the term force. We use it in so many situations in our everyday life. We are exerting force when we are pulling, pushing, kicking, hitting etc. Though a force is not visible, its effect can be seen or experienced. Forces are known to have different kinds of effects :

- They may change the shape and the size of an object.** A balloon changes shape depending on the magnitude of force acting on it.
- Forces also influence the motion of an object.** A force can set an object into motion or it can bring a moving object to rest. A force can also change the direction or speed of motion.
- Forces can rotate a body about an axis.** You will learn about it in lesson seven.

#### 3.1.1 Force and Motion

Force is a vector quantity. For this reason, when several forces act on a body simultaneously, a net equivalent force can be calculated by vector addition, as discussed in lesson 1.

Motion of a body is characterised by its displacement, velocity etc. We come across many situations where the velocity of an object is either continuously

increasing or decreasing. For example, in the case of a body falling freely, the velocity of the body increases continuously, till it hits the ground. Similarly, in the case of a ball rolling on a horizontal surface, the velocity of the ball decreases continuously and ultimately becomes zero.

From experience we know that a net non-zero force is required to change the state of a body. ***For a body in motion, the velocity will change depending on the direction of the force acting on it.*** If a net force acts on a body in motion, its velocity will increase in magnitude, if the direction of the force and velocity are same. If the direction of net force acting on the body is opposite to the direction of motion, the magnitude of velocity will decrease. However, if a net force acts on a body in a direction perpendicular to its velocity, the magnitude of velocity of the body remains constant (see Sec 4.3). Such a force changes only the direction of velocity of the body. We may therefore conclude that ***velocity of a body changes as long as a net force is acting on it.***

### 3.1.2 First Law of Motion

When we roll a marble on a smooth floor, it stops after some time. It is obvious that its velocity decreases and ultimately it becomes zero. However, if we want it to move continuously with the same velocity, a force will have to be constantly applied on it.

We also see that in order to move a trolley at constant velocity, it has to be continuously pushed or pulled. Is there any net force acting on the marble or trolley in the situations mentioned here?

#### Motion and Inertia

Galileo carried out experiments to prove that in the absence of any external force, a body would continue to be in its state of rest or of uniform motion in a straight line. He observed that a body is accelerated while moving down an inclined plane (Fig. 3.1 a) and is retarded while moving up an inclined plane (Fig. 3.1 b). He argued that if the plane is neither inclined upwards nor downwards (i.e. if it is a horizontal plane surface), the motion of the body will neither be accelerated nor retarded. That is, on a horizontal plane surface, a body will move with a uniform speed/velocity (if there is no external force).

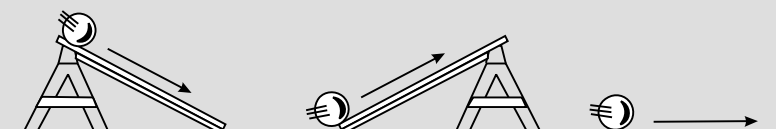


Fig. 3.1 : Motion of a body on inclined and horizontal planes

In another thought experiment, he considered two inclined planes facing each other, as shown in Fig. 3.2. The inclination of the plane PQ is same in all the three cases, whereas the inclination of the plane RS in Fig. 3.2 (a) is more than that in (b) and (c). The plane PQRS is very smooth and the ball is of marble. When the ball is allowed to roll down the plane PQ, it rises to nearly the same height on the face RS. As the inclination of the plane RS decreases, the balls moves a longer distance to rise to the same height on the inclined plane (Fig. 3.2b). When the plane RS becomes horizontal, the ball keeps moving to attain the same height as on the plane PQ, i.e. on a horizontal plane, the ball will keep moving if there is no friction between the plane and the ball.

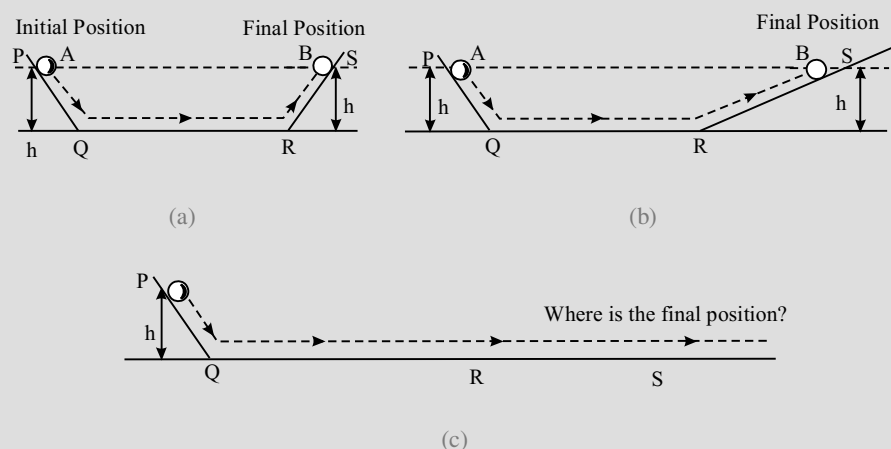


Fig. 3.2 : Motion of a ball along planes inclined to each other

### Sir Issac Newton (1642–1727)

Newton was born at Wollsthorpe in England in 1642. He studied at Trinity College, Cambridge and became the most profound scientist. The observation of an apple falling towards the ground helped him to formulate the basic law of gravitation. He enunciated the laws of motion and the law of gravitation. Newton was a genius and contributed significantly in all fields of science, including mathematics. His contributions are of a classical nature and form the basis of the modern science. He wrote his book “**Principia**” in Latin and his book on optics was written in English.



You may logically ask : Why is it necessary to apply a force continuously to the trolley to keep it moving uniformly? We know that a forward force on the cart is needed for balancing out the force of friction on the cart. That is, the force of friction on the trolley can be overcome by continuously pushing or pulling it.

Isaac Newton generalised Galileo's conclusions in the form of a law known as ***Newton's first law of motion***, which states that ***a body continues to be in a state of rest or of uniform motion in a straight line unless it is acted upon by a net external force.***

As you know, the state of rest or motion of a body depends on its relative position with respect to an observer. A person in a running car is at rest with respect to another person in the same car. But the same person is in motion with respect to a person standing on the road. For this reason, it is necessary to record measurements of changes in position, velocity, acceleration and force with respect to a chosen frame of reference.

A reference frame relative to which a body in translatory motion has constant velocity, if no net external force acts on it, is known as an ***inertial frame of reference***. This nomenclature follows from the property of inertia of bodies due to which they tend to preserve their state (of rest or of uniform linear motion). A reference frame fixed to the earth (for all practical purposes) is considered an inertial frame of reference.

Now you may like to take a break and answer the following questions.

### INTEXT QUESTIONS 3.1

1. Is it correct to state that a body always moves in the direction of the net external force acting on it?
2. What physical quantity is a measure of the inertia of a body?
3. Can a force change only the direction of velocity of an object keeping its magnitude constant?
4. State the different types of changes which a force can bring in a body when applied on it.

### 3.2 CONCEPT OF MOMENTUM

You must have seen that a fielder finds it difficult to stop a cricket ball moving with a large velocity although its mass is small. Similarly, it is difficult to stop a truck moving with a small velocity because its mass is large. These examples suggest that both, mass and velocity of a body, are important, when we study the effect of force on the motion of the body.

***The product of mass  $m$  of a body and its velocity  $\mathbf{v}$  is called its linear momentum  $\mathbf{p}$ . Mathmatically, we write***

$$\mathbf{p} = m\mathbf{v}$$

In SI units, momentum is measured in  $\text{kg ms}^{-1}$ . Momentum is a vector quantity. The direction of momentum vector is the same as the direction of velocity vector. Momentum of an object, therefore, can change on account of change in its magnitude or direction or both. The following examples illustrate this point.

**Example 3.1:** Aman weights 60 kg and travels with velocity  $1.0 \text{ m s}^{-1}$  towards Manoj who weights 40 kg, and is moving with  $1.5 \text{ m s}^{-1}$  towards Aman. Calculate their momenta.

**Solution :** For Aman

$$\begin{aligned}\text{momentum} &= \text{mass} \times \text{velocity} \\ &= (60 \text{ kg}) \times (1.0 \text{ m s}^{-1}) \\ &= 60 \text{ kg ms}^{-1}\end{aligned}$$

For Manoj

$$\begin{aligned}\text{momentum} &= 40 \text{ kg} \times (-1.5 \text{ ms}^{-1}) \\ &= -60 \text{ kg ms}^{-1}\end{aligned}$$

Note that the momenta of Aman and Manoj have the same magnitude but they are in opposite directions.

**Example 3.2:** A 2 kg object is allowed to fall freely at  $t = 0 \text{ s}$ . Calculate its momentum at (a)  $t = 0$ , (b)  $t = 1 \text{ s}$  and (c)  $t = 2 \text{ s}$  during its free-fall.

**Solution :** (a) As velocity of the object at  $t = 0 \text{ s}$  is zero, the initial momentum of the object will also be zero.

(b) At  $t = 1 \text{ s}$ , the velocity of the object will be  $9.8 \text{ ms}^{-1}$  [use  $v = v_0 + at$ ] pointing downward. So the momentum of the object will be

$$p_1 = (2 \text{ kg}) \times (9.8 \text{ ms}^{-1}) = 19.6 \text{ kg ms}^{-1} \text{ pointing downward.}$$

(c) At  $t = 2 \text{ s}$ , the velocity of the object will be  $19.6 \text{ m s}^{-1}$  pointing downward. So the momentum of the object will now be

$$p_2 = (2 \text{ kg}) \times (19.6 \text{ ms}^{-1}) = 39.2 \text{ kg ms}^{-1} \text{ pointing downward.}$$

Thus, we see that the momentum of a freely-falling body increases continuously in magnitude and points in the same direction. Now think what causes the momentum of a freely-falling body to change in magnitude?

**Example 3.3:** A rubber ball of mass 0.2 kg strikes a rigid wall with a speed of  $10 \text{ ms}^{-1}$  and rebounds along the original path with the same speed. Calculate the change in momentum of the ball.

**Solution :** Here the momentum of the ball has the same magnitude before and after the impact but there is a reversal in its direction. In each case the magnitude of momentum is  $(0.2 \text{ kg}) \times (10 \text{ ms}^{-1})$  i.e.  $2 \text{ kg ms}^{-1}$ .

If we choose initial momentum vector to be along  $+x$  axis, the final momentum vector will be along  $-x$  axis. So  $p_i = 2 \text{ kg ms}^{-1}$ ,  $p_f = -2 \text{ kg ms}^{-1}$ . Therefore, the change in momentum of the ball,  $p_f - p_i = (-2 \text{ kgms}^{-1}) - (2 \text{ kgms}^{-1}) = -4 \text{ kgms}^{-1}$ .

Here negative sign shows that the momentum of the ball changes by  $4 \text{ kg ms}^{-1}$  in the direction of  $-x$  axis. What causes this change in momentum of the ball?

In actual practice, a rubber ball rebounds from a rigid wall with a speed less than its speed before the impact. In such a case also, the magnitude of the momentum will change.

### 3.3 SECOND LAW OF MOTION

You now know that a body moving at constant velocity will have constant momentum. Newton's first law of motion suggests that no net external force acts on such a body.

In Example 3.2 we have seen that the momentum of a ball falling freely under gravity increases with time. Since such a body falls under the action of gravitational force acting on it, there appears to be a connection between change in momentum of an object, net force acting on it and the time for which it is acting. **Newton's second law of motion** gives a quantitative relation between these three physical quantities. It states that ***the rate of change of momentum of a body is directly proportional to the net force acting on the body. Change in momentum of the body takes place in the direction of net external force acting on the body.***

This means that if  $\Delta \mathbf{p}$  is the change in momentum of a body in time  $\Delta t$  due to a net external force  $\mathbf{F}$ , we can write

$$\mathbf{F} \propto \frac{\Delta \mathbf{p}}{\Delta t}$$

or 
$$\mathbf{F} = k \frac{\Delta \mathbf{p}}{\Delta t}$$

where  $k$  is constant of proportionality.

By expressing momentum as a product of mass and velocity, we can rewrite this result as

$$\mathbf{F} = k m \left( \frac{\Delta \mathbf{v}}{\Delta t} \right)$$

$$\mathbf{F} = k m \mathbf{a} \quad \left( \text{as } \frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a} \right) \quad (3.1)$$

The value of the constant  $k$  depends upon the units of  $m$  and  $\mathbf{a}$ . If these units are chosen such that when the magnitude of  $m = 1$  unit and  $a = 1$  unit, the magnitude of  $\mathbf{F}$  is also be 1 unit. Then, we can write

$$1 = k \cdot 1 \cdot 1$$

i.e.,  $k = 1$

Using this result in Eqn. (3.1), we get

$$\mathbf{F} = m \mathbf{a} \quad (3.2)$$

In SI units,  $m = 1 \text{ kg}$ ,  $a = 1 \text{ m s}^{-2}$ . Then magnitude of external force

$$\begin{aligned} F &= 1 \text{ kg} \times 1 \text{ ms}^{-2} = 1 \text{ kg ms}^{-2} \\ &= 1 \text{ unit of force} \end{aligned} \quad (3.3)$$

This unit of force (i.e.,  $1 \text{ kg m s}^{-2}$ ) is called one *newton*.

Note that the second law of motion gives us a unit for measuring force. The SI unit of force i.e., a newton may thus, be defined as the force which will produce an acceleration of  $1 \text{ ms}^{-2}$  in a mass of  $1 \text{ kg}$ .

**Example 3.3:** A ball of mass  $0.4 \text{ kg}$  starts rolling on the ground at  $20 \text{ ms}^{-1}$  and comes to a stop after  $10\text{s}$ . Calculate the force which stops the ball, assuming it to be constant in magnitude throughout.

**Solution :** Given  $m = 0.4 \text{ kg}$ , initial velocity  $u = 20 \text{ ms}^{-1}$ , final velocity  $v = 0 \text{ m s}^{-1}$  and  $t = 10\text{s}$ . So

$$\begin{aligned} |\mathbf{F}| = m|\mathbf{a}| &= \frac{m(v-u)}{t} = \frac{0.4 \text{ kg} (-20 \text{ ms}^{-1})}{10 \text{ s}} \\ &= -0.8 \text{ kg m s}^{-2} = -0.8 \text{ N} \end{aligned}$$

Here negative sign shows that force on the ball is in a direction opposite to that of its motion.

**Example 3.4:** A constant force of magnitude  $50 \text{ N}$  is applied to a body of  $10 \text{ kg}$  moving initially with a speed of  $10 \text{ m s}^{-1}$ . How long will it take the body to stop if the force acts in a direction opposite to its motion.

**Solution :** Given  $m = 10 \text{ kg}$ ,  $F = -50 \text{ N}$ ,  $v_0 = 10 \text{ ms}^{-1}$  and  $v = 0$ . We have to calculate  $t$ . Since

$$F = ma$$

we can write

$$F = m \left( \frac{v - v_0}{t} \right)$$

$$\therefore -50 \text{ N} = 10 \text{ kg} \left( \frac{0 - 10 \text{ m s}^{-1}}{t} \right)$$



or 
$$t = \frac{-100 \text{ kgms}^{-1}}{-50 \text{ N}} = \frac{100 \text{ kg ms}^{-1}}{50 \text{ kg m s}^{-2}} = 2 \text{ s.}$$

It is important to note here that Newton's second law of motion, as stated here is applicable to bodies having constant mass. Will this law hold for bodies whose mass changes with time, as in a rocket?

## INTEXT QUESTIONS 3.2

- Two objects of different masses have the same momentum. Which of them is moving faster?
- A boy throws up a ball with a velocity  $v_0$ . If the ball returns to the thrower with the same velocity, will there be any change in
  - momentum of the ball?
  - magnitude of the momentum of the ball?
- When a ball falls from a height, its momentum increases. What causes increase in its momentum?
- In which case will there be larger change in momentum of the object?
  - A 150 N force acts for 0.1 s on a 2 kg object initially at rest.
  - A 150 N force acts for 0.2 s on a 2 kg. object initially at rest.
- An object is moving at a constant speed in a circular path. Does the object have constant momentum? Give reason for your answer.

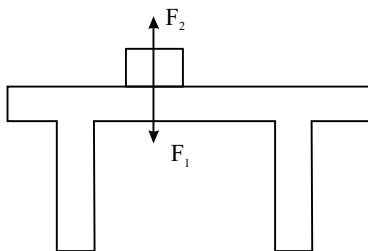
## 3.4 FORCES IN PAIRS

It is the gravitational pull of the earth, which allows an object to accelerate towards the earth. Does the object also pull the earth? Similarly when we push an almirah, does the almirah also push us? If so, why don't we move in the direction of that force? These situations compel us to ask whether a single force such as a push or a pull exists? It has been observed that actions of two bodies on each other are always mutual. Here, by action and reaction we mean 'forces of interaction'. So, whenever two bodies interact, they exert force on each other. One of them is called 'action' and the other is called 'reaction'. Thus, we can say that forces always exist in pairs.

### 3.4.1 Third Law of Motion

On the basis of his study of interactions between bodies, Newton formulated **third law of motion: To every action, there is an equal and opposite reaction.**

Here by ‘action’ and ‘reaction’ we mean force. Thus, when a book placed on a table exerts some force on the table, the latter, also exerts a force of equal magnitude on the book in the upward direction, as shown in Fig. 3.3. Do the forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  shown here cancel out? It is important to note that  $\mathbf{F}_1$  and  $\mathbf{F}_2$  are acting on different bodies and therefore, they do not cancel out.



**Fig 3.3 :** A book placed on a table exerts a force  $\mathbf{F}_1$  (equal to its weight  $mg$ ) on the table, while the table exerts a force  $\mathbf{F}_2$  on the book.

The action and reaction in a given situation appear as a pair of forces. Any one of them cannot exist without the other.

If one goes by the literal meaning of words, reaction always follows an action, whereas action and reaction introduced in Newton’s third law exist simultaneously. For this reason, it is better to state Newton’s third law as ***when two objects interact, the force exerted by one object on the other is equal in magnitude and opposite in direction to the force exerted by the latter object on the former.***

Vectorially, if  $\mathbf{F}_{12}$  is the force which object 1 experiences due to object 2 and  $\mathbf{F}_{21}$  is the force which object 2 experiences due to object 1, then according to Newton’s third law of motion, we can write

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \quad (3.4)$$

### 3.4.2 Impulse

The effect of force applied for a short duration is called impulse. Impulse is defined as the product of force ( $\mathbf{F}$ ) and the time duration ( $\Delta t$ ) for which the force is applied.

i.e.,  $\text{Impulse} = \mathbf{F} \cdot \Delta t$

If the initial and final velocities of body acted upon by a force  $\mathbf{F}$  are  $\mathbf{u}$  and  $\mathbf{v}$  respectively then we can write

$$\begin{aligned} \text{Impulse} &= \frac{m\mathbf{v} - m\mathbf{u}}{\Delta t} \cdot \Delta t \\ &= m\mathbf{v} - m\mathbf{u} \\ &= \mathbf{p}_f - \mathbf{p}_i \\ &= \Delta \mathbf{p} \end{aligned}$$

That is, impulse is equal to change in linear momentum.

Impulse is a vector quantity and its SI unit is  $\text{kg ms}^{-1}$  (or  $\text{N s}$ ).

### INTEXT QUESTIONS 3.3

1. When a high jumper leaves the ground, where does the force which throws the jumper upwards come from?
2. Identify the action - reaction forces in each of the following situations:
  - (a) A man kicks a football
  - (b) Earth pulls the moon
  - (c) A ball hits a wall
3. “A person exerts a large force on an almirah to push it forward but he is not pushed backward because the almirah exerts a small force on him”. Is the argument given here correct? Explain.

### 3.5 CONSERVATION OF MOMENTUM

It has been experimentally shown that if two bodies interact, the vector sum of their momenta remains unchanged, provided the force of mutual interaction is the only force acting on them. The same has been found to be true for more than two bodies interacting with each other. Generally, a number of bodies interacting with each other are said to be forming a system. If the bodies in a system do not interact with bodies outside the system, the system is said to be a closed system or an isolated system. ***In an isolated system, the vector sum of the momenta of bodies remains constant.*** This is called the ***law of conservation of momentum.***

Here, it follows that it is the total momentum of the bodies in an isolated system remains unchanged but the momentum of individual bodies may change, in magnitude alone or direction alone or both. You may now logically ask : What causes the momentum of individual bodies in an isolated system to change? It is due to mutual interactions and their strengths.

Conservation of linear momentum is applicable in a wide range of phenomena such as collisions, explosions, nuclear reactions, radioactive decay etc.

#### 3.5.1 Conservation of Momentum as a Consequence of Newton's Laws

According to Newton's second law of motion, Eqn. (3.1), the change in momentum  $\Delta \mathbf{p}$  of a body, when a force  $\mathbf{F}$  acts on it for time  $\Delta t$ , is

$$\Delta \mathbf{p} = \mathbf{F} \Delta t$$

This result implies that if no force acts on the body, the change in momentum of the body will be zero. That is, the momentum of the body will remain unchanged. This argument can be extended to a system of bodies as well.

**Newton's third law** can also be used to arrive at the same result. Consider an isolated system of two bodies A and B which interact with each other for time  $\Delta t$ . If  $\mathbf{F}_{AB}$  and  $\mathbf{F}_{BA}$  are the forces which they exert on each other, then in accordance with Newton's third law

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

or 
$$\frac{\Delta \mathbf{p}_A}{\Delta t} = -\frac{\Delta \mathbf{p}_B}{\Delta t}$$

or 
$$\Delta \mathbf{p}_A + \Delta \mathbf{p}_B = 0$$
 or

or 
$$\Delta \mathbf{p}_{\text{total}} = 0$$

or 
$$\mathbf{p}_{\text{total}} = \text{constant}$$

That is, there is no change in the momentum of the system. In other words, the momentum of the system is conserved.

### 3.5.2 A Few Illustrations of Conservation of Momentum

**a) Recoil of a gun :** When a bullet is fired from a gun, the gun recoils. The velocity  $v_2$  of the recoil of the gun can be found by using the law of conservation of momentum. Let  $m$  be the mass of the bullet being fired from a gun of mass  $M$ . If  $v_1$  is the velocity of the bullet, then momentum will be said to be conserved if the velocity  $v_2$  of the gun is given by

$$m\mathbf{v}_1 + M\mathbf{v}_2 = 0$$

or 
$$m\mathbf{v}_1 = -M\mathbf{v}_2$$

or 
$$\mathbf{v}_2 = -\frac{m}{M}\mathbf{v}_1 \quad (3.5)$$

Here, negative sign shows that  $\mathbf{v}_2$  is in a direction opposite to  $\mathbf{v}_1$ . Since  $m \ll M$ , the recoil velocity of the gun will be considerably smaller than the velocity of the bullet.

**b) Collision :** In a collision, we may regard the colliding bodies as forming a system. In the absence of any external force on the colliding bodies, such as the force of friction, the system can be considered to be an isolated system. The forces of interaction between the colliding bodies will not change the total momentum of the colliding bodies.

Collision of the striker with a coin of carrom or collision between the billiard balls may be quite instructive for the study of collision between elastic bodies.

**Example 3.5 :** Two trolleys, each of mass  $m$ , coupled together are moving with initial velocity  $v$ . They collide with three identical stationary trolleys coupled together and continue moving in the same direction. What will be the velocity of the trolleys after the impact?

**Solution :** Let  $v'$  be the velocity of the trolleys after the impact.

Momentum before collision =  $2mv$

Momentum after collision =  $5mv'$

In accordance with the law of conservation of momentum, we can write

$$2mv = 5mv'$$

or 
$$v' = \frac{2}{5}v$$

**c) Explosion of a bomb :** A bomb explodes into fragments with the release of huge energy. Consider a bomb at rest initially which explodes into two fragments A and B. As the momentum of the bomb was zero before the explosion, the total momentum of the two fragments formed will also be zero after the explosion. For this reason, the two fragments will fly off in opposite directions. If the masses of the two fragments are equal, the velocities of the two fragments will also be equal in magnitude.

**d) Rocket propulsion :** Flight of a rocket is an important practical application of conservation of momentum. A rocket consists of a shell with a fuel tank, which can be considered as one body. The shell is provided with a nozzle through which high pressure gases are made to escape. On firing the rocket, the combustion of the fuel produces gases at very high pressure and temperature. Due to their high pressure, these gases escape from the nozzle at a high velocity and provide thrust to the rocket to go upward due to the conservation of momentum of the system. If  $M$  is the mass of the rocket and  $m$  is the mass of gas escaping per second with a velocity  $\mathbf{v}$ , the change in momentum of the gas in  $t$  second =  $m\mathbf{v}t$ .

If the increase in velocity of the rocket in  $t$  second is  $\mathbf{V}$ , the increase in its momentum =  $M\mathbf{V}$ . According to the principle of conservation of momentum,

$$m\mathbf{v}t + M\mathbf{V} = 0$$

or 
$$\frac{\mathbf{V}}{t} = \mathbf{a} = -\frac{m\mathbf{v}}{M}$$

i.e., the rocket moves with an acceleration

$$\mathbf{a} = -\frac{m\mathbf{v}}{M}$$

### 3.5.3 Equilibrium of Concurrent Forces

A number of forces acting simultaneously at a point are called **Concurrent Forces**. Such forces are said to be in equilibrium, if their resultant is zero.

Let  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  be three concurrent forces acting at a point  $P$ , as shown in Fig. 3.4.

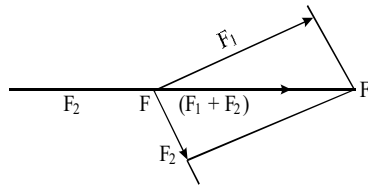


Fig. 3.4

The resultant of  $\mathbf{F}_1$  and  $\mathbf{F}_2$ , obtained by the parallelogram law, is shown by  $\mathbf{PA}$  (i.e.  $\mathbf{PA} = \mathbf{F}_1 + \mathbf{F}_2$ )

For equilibrium, the sum  $(\mathbf{F}_1 + \mathbf{F}_2)$  must be equal and opposite to  $\mathbf{F}_3$  i.e.

$$\mathbf{F}_3 = -(\mathbf{F}_1 + \mathbf{F}_2) \text{ or } \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$$

Or, the sum or resultant of two forces must be equal and opposite to the third force or for equilibrium, their vector sum must be zero.

### 3.6 FRICTION

You may have noticed that when a batsman hits a ball to make it roll along the ground, the ball does not continue to move forever. It comes to rest after travelling some distance. Thus, the momentum of the ball, which was imparted to it during initial push, tends to be zero. We know that some force acting on the ball is responsible for this change in its momentum. Such a force, called the **frictional force**, exists whenever bodies in contact tend to move with respect to each other. It is the force of friction which has to be overcome when we push or pull a body horizontally along the floor to change its position.

**Force of friction is a contact force and always acts along the surfaces in a direction opposite to that of the motion of the body.** It is commonly known that friction is caused by roughness of the surfaces in contact. For this reason deliberate attempts are made to make the surfaces rough or smooth depending upon the requirement.

Friction opposes the motion of objects, causes wear and tear and is responsible for loss of mechanical energy. But then, it is only due to friction that we are able to walk, drive vehicles and stop moving vehicles. Friction thus plays a dual role in our lives. It is therefore said that friction is a necessary evil.

#### 3.6.1 Static and Kinetic Friction

We all know that certain minimum force is required to move an object over a surface. To illustrate this point, let us consider a block resting on some horizontal

surface, as shown in Fig.3.5. Let some external force  $\mathbf{F}_{ext}$  be applied on the block. Initially the block does not move. This is possible only if some other force is acting on the block. The force is called the force of **static friction** and is represented by symbol  $f_s$ . As  $\mathbf{F}_{ext}$  is increased,  $f_s$  also increases and remains equal to  $\mathbf{F}_{ext}$  in magnitude until it reaches a critical value  $f_s^{(max)}$ . When  $\mathbf{F}_{ext}$  is increased further, the block starts to slide and is then subject to **kinetic friction**. It is common experience that the force needed to set an object in motion is larger than the force needed to keep it moving at constant velocity. **For this reason, the maximum value of static friction  $f_s$  between a pair of surfaces in contact will be larger than the force of kinetic friction  $f_k$  between them.** Fig. 3.6 shows the variation of the force of friction with the external force.

For a given pair of surfaces in contact, you may like to know the **factors** on which  $f_s^{(max)}$  and  $f_k$  depend? It is an experimental fact that  $f_s^{(max)}$  is directly proportional to the normal force  $\mathbf{F}_N$ . i.e.

$$f_s^{(max)} \propto \mathbf{F}_N \quad \text{or} \quad f_s^{(max)} = \mu_s \mathbf{F}_N \quad (3.6)$$

where  $\mu_s$  is called the **coefficient of static friction**. The normal force  $\mathbf{F}_N$  of the surface on the block can be found by knowing the force with which the block presses the surface. Refer to Fig. 3.5. The normal force  $\mathbf{F}_N$  on the block will be  $mg$ , where  $m$  is mass of the block.

Since  $f_s = \mathbf{F}_{ext}$  for  $f_s \leq f_s^{(max)}$ , we can write

$$f_s \leq \mu_s \mathbf{F}_N.$$

It has also been experimentally found that **maximum force of static friction between a pair of surfaces is independent of the area of contact.**

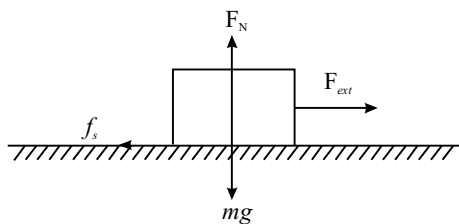


Fig. 3.5 : Forces acting on the block

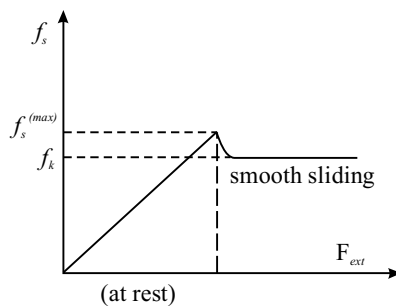


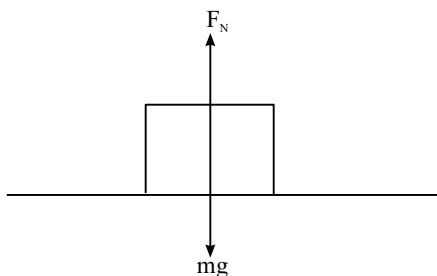
Fig. 3.6 : Variation of force of friction with external force

Similarly, we can write

$$f_k = \mu_k \mathbf{F}_N$$

where  $\mu_k$  is the **coefficient of kinetic friction**. In general,  $\mu_s > \mu_k$ . Moreover, coefficients  $\mu_s$  and  $\mu_k$  are not really constants for any pair of surfaces such as

wood on wood or rubber on concrete, etc. *Values of  $\mu_s$  and  $\mu_k$  for a given pair of materials depend on the roughness of surfaces, their cleanliness, temperature, humidity etc.*



**Fig. 3.7 :** Normal force on the block

**Example 3.6:** A 2 kg block is resting on a horizontal surface. The coefficient of static friction between the surfaces in contact is 0.25. Calculate the maximum magnitude of force of static friction between the surfaces in contact.

**Solution :**

Here  $m = 2$  kg and  $\mu_s = 0.25$ . From Eqn. (3.6), we recall that

$$\begin{aligned} f_s^{(\max)} &= \mu_s F_N \\ &= \mu_s mg \\ &= (0.25) (2 \text{ kg}) (9.8 \text{ ms}^{-2}) \\ &= 4.9 \text{ N}. \end{aligned}$$

**Example 3.7:** A 5 kg block is resting on a horizontal surface for which  $\mu_k = 0.1$ . What will be the acceleration of the block if it is pulled by a 10 N force acting on it in the horizontal direction?

**Solution :**

As  $f_k = \mu_k F_N$  and  $F_N = mg$ , we can write

$$\begin{aligned} f_k &= \mu_k mg \\ &= (0.1) (5 \text{ kg}) (9.8 \text{ ms}^{-2}) \\ &= 4.9 \text{ kg ms}^{-2} = 4.9 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Net force on the block} &= F_{ext} - f_k \\ &= 10 \text{ N} - 4.9 \text{ N} \\ &= 5.1 \text{ N} \end{aligned}$$

Hence,

$$\text{acceleration} = a = \frac{F_{net}}{m} = \frac{5.1 \text{ N}}{5 \text{ kg}} = 1.02 \text{ ms}^{-2}$$

So the block will have an acceleration of  $1.02 \text{ ms}^{-2}$  in the direction of externally applied force.



### 3.6.2 Rolling Friction

It is a common experience that pushing or pulling objects such as carts on wheels is much easier. The motion of a wheel is different from sliding motion. It is a rolling motion. The friction in the case of rolling motion is known as **rolling friction**. For the same normal force, rolling friction is much smaller than sliding friction. For example, when steel wheels roll over steel rails, rolling friction is about  $1/100^{\text{th}}$  of the sliding friction between steel and steel. Typical values for coefficient of rolling friction  $\mu_r$  are 0.006 for steel on steel and 0.02 – 0.04 for rubber on concrete.

We would now like you to do a simple activity :

#### ACTIVITY 3.1

Place a heavy book or a pile of books on a table and try to push them with your fingers. Next put three or more pencils below the books and now push them again. In which case do you need less force? What do you conclude from your experience?

### 3.6.3 Methods of Reducing Friction

Wheel is considered to be greatest invention of mankind for the simple reason that rolling is much easier than sliding. Because of this, ball bearings are used in machines to reduce friction. In a ball-bearing, steel balls are placed between two co-axial cylinders, as shown in Fig.3.8. Generally one of the two cylinders is allowed to turn with respect to the other. Here the rotation of the balls is almost frictionless. Ball-bearings find application in almost all types of vehicles and in electric motors such as electric fans etc.

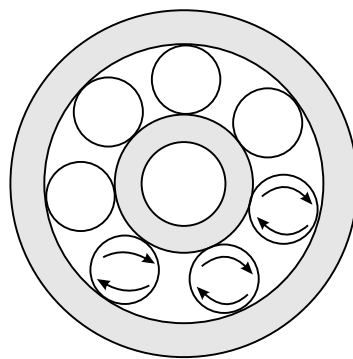


Fig. 3.8 : Balls in the ball-bearing

Use of **lubricants** such as grease or oil between the surfaces in contact reduces friction considerably. In heavy machines, oil is made to flow over moving parts. It reduces frictional force between moving parts and also prevents them from getting overheated. In fact, the presence of lubricants changes the nature of friction from dry friction to fluid friction, which is considerably smaller than the former.

**Flow of compressed and purified air** between the surfaces in contact also reduces friction. It also prevents dust and dirt from getting collected on the moving parts.

### Fluid Friction

Bodies moving on or through a liquid or gas also face friction. Shooting stars (meteors) shine because of the heat generated by air-friction. Contrary to solid friction, fluid friction depends upon the shape of the bodies. This is why fishes have a special shape and fast moving aeroplanes and vehicles are also given a fish-like shape, called a stream-line shape. Fluid friction increases rapidly with increase in speed. If a car is run at a high speed, more fuel will have to be burnt to overcome the increased fluid (air) friction. Car manufactures advise us to drive at a speed of 40-45 km h<sup>-1</sup> for maximum efficiency.

### 3.7 THE FREE BODY DIAGRAM TECHNIQUE

Application of Newton's laws to solve problems in mechanics becomes easier by use of the **free body diagram technique**. A diagram which shows all the forces acting on a body in a given situation is called a free body diagram (FBD). The procedure to draw a free body diagram, is described below :

1. Draw a simple, neat diagram of the system as per the given description.
2. Isolate the **object of interest**. This object will be called the **Free Body** now.
3. Consider all **external forces** acting on the free body and mark them by arrows touching the free body with their line of action clearly represented.
4. Now apply Newton's second law  $\Sigma \mathbf{F} = m \mathbf{a}$

(or  $\Sigma F_x = m a_x$  and  $\Sigma F_y = m a_y$ )

**Remember :** (i) A net force must be acting on the object along the direction of motion. (ii) For obtaining a complete solution, you must have as many independent equations as the number of unknowns.

**Example 3.8 :** Two blocks of masses  $m_1$  and  $m_2$  are connected by a string and placed on a smooth horizontal surface. The block of mass  $m_2$  is pulled by a force  $\mathbf{F}$  acting parallel to the horizontal surface. What will be the acceleration of the blocks and the tension in the string connecting the two blocks (assuming it to be horizontal)?

**Solution :** Refer to Fig. 3.9. Let  $\mathbf{a}$  be the acceleration of the blocks in the direction of  $\mathbf{F}$  and let the tension in the string be  $\mathbf{T}$ . On applying  $\Sigma \mathbf{F} = m\mathbf{a}$  in the component form to the free body diagram of system of two bodies of masses  $m_1$  and  $m_2$ , we get

$$N - (m_1 + m_2)g = 0$$

$$\text{and} \quad F = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2}$$

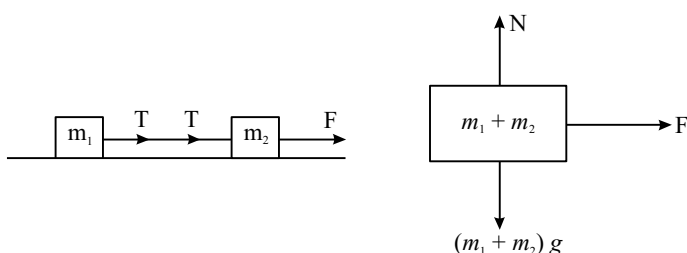


Fig 3.9: Free body diagram for two blocks connected by a string

On applying  $\Sigma F = ma$  in the component form to the free body diagram of  $m_1$  we get

$$N_1 - m_1g = 0 \quad \text{and} \quad T = m_1a$$

$$\Rightarrow T = m_1 \left( \frac{F}{m_1 + m_2} \right)$$

$$\text{or} \quad T = \left( \frac{m_1}{m_1 + m_2} \right) \cdot F$$

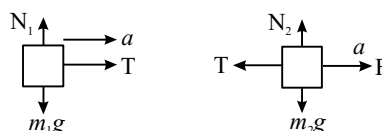


Fig 3.10

Apply  $\Sigma F = ma$  once again to the free body diagram of  $m_2$  and see whether you get the same expressions for  $a$  and  $T$ .

**Example 3.9 :** Two masses  $m_1$  and  $m_2$  ( $m_1 > m_2$ ) are connected at the two ends of a light inextensible string that passes over a light frictionless fixed pulley. Find the acceleration of the masses and the tension in the string connecting them when the masses are released.

**Solution :** Let  $a$  be acceleration of mass  $m_1$  downward. The acceleration of mass  $m_2$  will also be  $a$  only but upward. (Why?). Let  $T$  be the tension in the string connecting the two masses.

On applying  $\Sigma F = ma$  to  $m_1$  and  $m_2$  we get

$$m_1g - T = m_1a$$

$$T - m_2g = m_2a$$

On solving equations (1) and (2) for  $a$  and  $T$  we get

$$a = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \cdot g \quad T = \left( \frac{2m_1m_2}{m_1 + m_2} \right) a$$

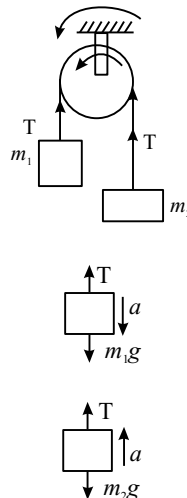


Fig 3.11

At this stage you can check the prediction of the results thus obtained for the extreme values of the variables (i.e.  $m_1$  and  $m_2$ ). Either take  $m_1 = m_2$  or  $m_1 \gg m_2$  and see whether  $a$  and  $T$  take values as expected.

**Example 3.10 :** A trolley of mass  $M = 10 \text{ kg}$  is connected to a block of mass  $m = 2 \text{ kg}$  with the help of massless inextensible string passing over a light frictionless pulley as shown in Fig. 3.12 (a). The coefficient of kinetic friction between the trolley and the surface ( $\mu_k$ ) = 0.02. Find,

- acceleration of the trolley, and
- tension in the string.

**Solution :** Fig (b) and (c) shows the free body diagrams of the trolley and the block respectively. Let  $a$  be the acceleration of the block and the trolley.

For the trolley,  $F_N = Mg$  and

$$T - f_k = Ma \text{ where } f_k = \mu_k F_N \\ = \mu_k Mg$$

$$\text{So } T - \mu_k Mg = Ma \quad \dots(1)$$

$$\text{For the block } mg - T = ma \quad \dots(2)$$

On adding equations (1) and (2) we get  $mg - \mu_k Mg = (M + m) a$

$$\text{or } a = \frac{mg - \mu_k Mg}{M + m} = \frac{(2 \text{ kg})(9.8 \text{ ms}^{-2}) - (0.02)(10 \text{ kg})(9.8 \text{ ms}^{-2})}{(10 \text{ kg} + 2 \text{ kg})} \\ = \frac{19.6 \text{ kg ms}^{-2} - 1.96 \text{ kg ms}^{-2}}{12 \text{ kg}} = 1.47 \text{ ms}^{-2}$$

$$\text{So } a = 1.47 \text{ ms}^{-2}$$

From equation (2)  $T = mg - ma = m(g - a)$

$$= 2 \text{ kg}(9.8 \text{ ms}^{-2} - 1.47 \text{ ms}^{-2}) \\ = 2 \text{ kg}(8.33 \text{ ms}^{-2})$$

$$\text{So } T = 16.66 \text{ N}$$

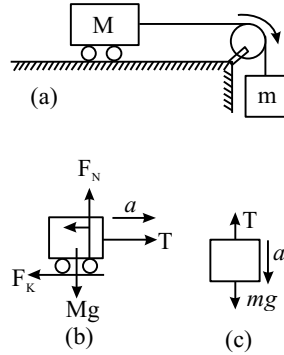


Fig. 3.12

### INTEXT QUESTIONS 3.4

- A block of mass  $m$  is held on a rough inclined surface of inclination  $\theta$ . Show in a diagram, various forces acting on the block.
- A force of 100 N acts on two blocks A and B of masses 2 kg and 3 kg respectively, placed in contact on a smooth horizontal surface as shown. What is the magnitude of force which block A exerts on block B?

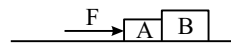


Fig. 3.13

3. What will be the tension in the string when a 5kg object suspended from it is pulled up with
- (a) a velocity of  $2\text{ms}^{-1}$ ?
  - (b) an acceleration of  $2\text{ms}^{-2}$ ?

### 3.8 ELEMENTARY IDEAS OF INERTIAL AND NON INERTIAL FRAMES

To study motion in one dimension (i.e. in a straight line) a reference point (origin) is enough. But, when it comes to motions in two and three dimensions, we have to use a set of reference lines to specify the position of a point in space. This set of lines is called **frame of reference**.

Every motion is described by an observer. The description of motion will change with the change in the state of motion of the observer. For example, let us consider a box lying on a railway platform. A person standing on the platform will say that the box is at rest. A person in a train moving with a uniform velocity  $v$  will say that the box is moving with velocity  $-v$ . But, what will be the description of the box by a person in a train having acceleration ( $a$ ). He/she will find that the box is moving with an acceleration ( $-a$ ). Obviously, the first law of motion is failing for this observer.

Thus a frame of reference is fixed with the observer to describe motion. If the frame is stationary or moving with a constant velocity with respect to the object under study (another frame of reference), then in this frame law of inertia holds good. Therefore, such frames are called inertial frames. On the other hand, if the observer's frame is accelerating, then we call it non-inertial frame.

For the motion of a body of mass  $m$  in a non-inertial frame, having acceleration ( $a$ ), we may apply second law of motion by involving a psuedo force  $m a$ . In a rotating body, this force is called centrifugal force.

#### INTEXT QUESTIONS 3.5

1. A glass half filled with water is kept on a horizontal table in a train. Will the free surface of water remain horizontal as the train starts?
2. When a car is driven too fast around a curve it skids outwards. How would a passenger sitting inside explain the car's motion? How would an observer standing on a road explain the event?
3. A tiny particle of mass  $6 \times 10^{-10} \text{ kg}$  is in a water suspension in a centrifuge which is being rotated at an angular speed of  $2\pi \times 10^3 \text{ rad s}^{-1}$ . The particle is

at a distance of 4 cm from the axis of rotation. Calculate the net centrifugal force acting on the particle.

4. What must the angular speed of the rotation of earth so that the centrifugal force makes objects fly off its surface? Take  $g = 10 \text{ m s}^{-2}$ .
5. In the reference frame attached to a freely falling body of mass 2 kg, what is the magnitude and direction of inertial force on the body?



## WHAT YOU HAVE LEARNT

- The **inertia** of a body is its tendency to resist any change in its state of rest or uniform motion.
- **Newton's first law** states that a body remains in a state of rest or of uniform motion in a straight line as long as net external force acting on it is zero.
- For a single particle of mass  $m$  moving with velocity  $\mathbf{v}$  we define a vector quantity  $\mathbf{p}$  called the linear momentum as  $\mathbf{p} = m \mathbf{v}$ .
- **Newton's second law** states that the time rate of change of momentum of a body is proportional to the resultant force acting on the body.
- According to Newton's second law, acceleration produced in a body of constant mass is directly proportional to net external force acting on the body :  $\mathbf{F} = m \mathbf{a}$ .
- **Newton's third law** states that if two bodies A and B interact with each other, then the force which body A exerts on body B will be equal and opposite to the force which body B exerts on body A.
- According to the law of conservation of momentum, if no net external force acts on a system of particles, the total momentum of the system will remain constant regardless of the nature of forces between them.
- A number of forces acting simultaneously at a point are called concurrent force. Such forces are said to be in equilibrium if their resultant is zero.
- Frictional force is the force which acts on a body when it attempts to slide, or roll along a surface. The force of friction is always parallel to the surfaces in contact and opposite to the direction of motion of the object.
- The maximum force of static friction  $f_s^{(\max)}$  between a body and a surface is proportional to the normal force  $\mathbf{F}_N$  acting on the body. This maximum force occurs when the body is on the verge of sliding.
- For a body sliding on some surface, the magnitude of the force of kinetic friction  $f_k$  is given by  $f_k = \mu_k \mathbf{F}_N$  where  $\mu_k$  is the coefficient of kinetic friction for the surfaces in contact.

- Use of rollers and ball-bearings reduces friction and associated energy losses considerably as rolling friction is much smaller than kinetic friction.
- Newton's laws of motion are applicable only in an inertial frame of reference. An inertial frame is one in which an isolated object has zero acceleration.
- For an object to be in static equilibrium, the vector sum of all the forces acting on it must be zero. This is a necessary and sufficient conditions for point objects only.

## ANSWERS TO INTEXT QUESTIONS

### 3.1

1. No. The statement is true only for a body which was at rest before the application of force.
2. Inertial mass
3. Yes, as in uniform circular motion.
4. A force can change motion. It can also deform bodies.

### 3.2

1. Object of smaller mass
2. (a) Yes (b) No.
3. Momentum of the falling ball increases because gravitational force acts on it in the direction of its motion and hence velocity increases.
4. In case (b) the change in momentum will be larger. It is the  $F \Delta t$  product that gives the change in momentum.  $\left( \text{as } F \propto \frac{\Delta p}{\Delta t} \right)$
5. No. Though the speed is constant, the velocity of the object changes due to change in direction. Hence its momentum will not be constant.

### 3.3

1. The jumper is thrown upwards by the force which the ground exerts on the jumper. This force is the reaction to the force which the jumper exerts on the ground.
2. (a) The force with which a man kicks a football is action and the force which the football exerts on the man will be its reaction.  
(b) The force with which earth pulls the moon is action and the force which the moon exerts on the earth will be its reaction.  
(c) If the force which the ball exerts on the wall is the action then the force which the wall exerts on the ball will be its reaction.

3. No. The argument is not correct. The almirah moves when the push by the person exceeds the frictional force between the almirah and the floor. He does not get pushed backward due to a large force of friction that he experiences due to the floor. On a slippery surface, he will not be able to push the almirah forward.

### 3.4

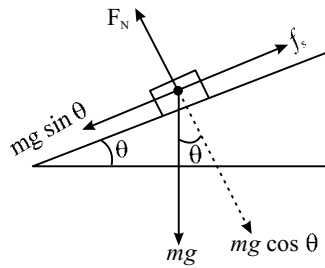


Fig. 3.17

2. 40 N
3. (a)  $(5 \times 9.8)$  N
- (b)  $F = (5 \times 2)$  N +  $(5 \times 9.8)$  N = 59 N

### 3.5

- (1) When the train starts it has an acceleration, say  $a$ . Thus the total force acting on water in the frame of reference attached to the train is

$$\mathbf{F} = m \mathbf{g} - m \mathbf{a}$$

where  $m$  is the mass of the water and the glass. (Fig. 3.16). The surface of the water takes up a position normal to  $\mathbf{F}$  as shown.

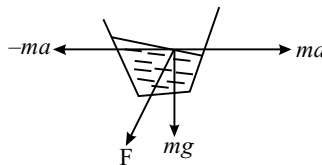


Fig. 3.18

- (2) To the passenger sitting inside, a centrifugal force  $(-mv^2/r)$  acts on the car. The greater  $v$  is the larger  $r$  would be. To an observer standing on the road, the car moving in a curve has a centripetal acceleration given by  $v^2/r$ . Once again, the greater is  $v$ , the larger will be  $r$ .
- (3) The net centrifugal force on the particle is  $\mathbf{F} = m\omega^2 r = (6 \times 10^{-10} \text{ kg}) \times (2\pi \times 10^3 \text{ rad s}^{-1})^2 \times (0.04 \text{ m}) = 9.6 \times 10^{-4} \text{ N}$ .
- (4) For an object to fly off centrifugal force (= centripetal force) should be just

more than the weight of a body. If  $r$  is the radius of the earth then  $\frac{mv^2}{r} = mg$

as  $v = r\omega$

$$\frac{r^2 \omega^2}{r} = g$$

or, angular speed  $\omega = \sqrt{g/r}$

$\therefore$  Any angular speed more than  $\sqrt{g/r}$  will make objects fly off.

5. Zero (as it is a case of free fall of a body).