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Network Laws and Theorems

Ohm's Law

The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing them is constant, provided the temperature of the conductor does not change.

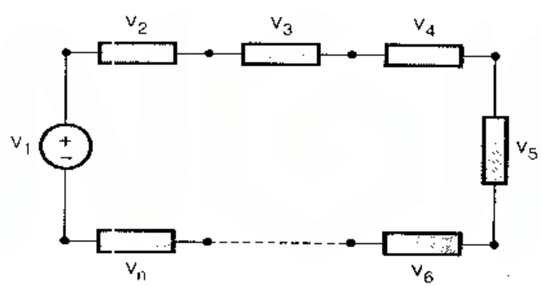
$$\frac{V}{I} = \text{constant} \quad \text{or} \quad \frac{V}{I} = R$$

Where, R is the resistance of the conductor between the two points considered.

Kirchoff's Laws

1. Kirchoff's Voltage Law (KVL)

For any closed path in a network, the algebraic sum of the voltages is zero.

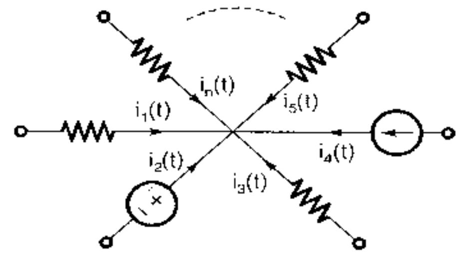


$$\sum_{k=1}^{n} v_k(t) = 0$$ ....in a closed loop

where, v_k is the voltage drop or voltage gain across k^{th} element

2. Kirchoff's Current Law (KCL)

The algebraic sum of the currents at a node is zero. Alternatively the sum of the currents entering a node is equal to the sum of the currents leaving that node.



$$\sum_{k=1}^{n} i_k(t) = 0$$ ..... at any node

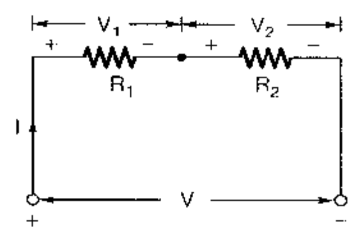
where $i_k(t)$ is the current through k^{th} branch

Note:

- A network is an interconnection of elements or devices, whereas a circuit is a network providing one or more closed paths.
- Number of KVL equations = b – (n – 1)
- Number of KCL equations = (n – 1)

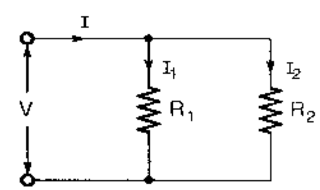
  where, b is number of branches and n is number of nodes.
- At node, current changes and in branch, current remains same.

Voltage Division Equations



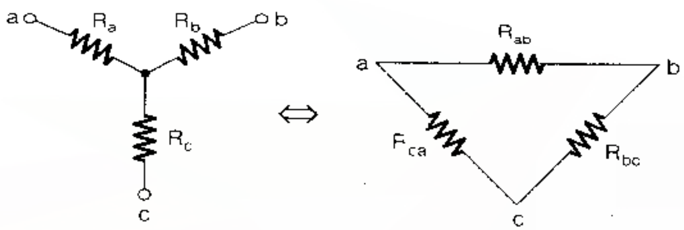
$$V_1 = \frac{R_1}{R_1 + R_2} V \quad ; \quad V_2 = \frac{R_2}{R_1 + R_2} V$$

Current Division Equations



$$I_1 = \frac{R_2 I}{R_1 + R_2} \quad ; \quad I_2 = \frac{R_1 I}{R_1 + R_2}$$

Star to Delta Transformation



$$R_{ab} = \frac{\Delta}{R_c}, R_{bc} = \frac{\Delta}{R_a}, R_{ca} = \frac{\Delta}{R_b}$$

where, $\Delta = (R_aR_b + R_bR_c + R_cR_a)$

Delta to Star Transformation

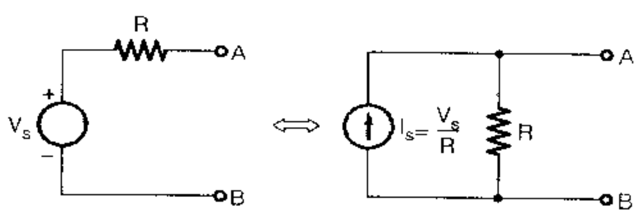
$$R_a = \frac{R_{ca}R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_b = \frac{R_{ab}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_c = \frac{R_{bc}R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

Source Transformation

Transformation of a resistive voltage source to a resistive current source or vice-versa.



Network Theorems

1. Super Position Theorem

The response in any element of a linear, bilateral RLC network containing more than one independent voltage or current source is the algebric sum of responses produce by the independent source when each of them acting alone with

(a) All other independent voltage sources are short circuited (S.C.).

(b) All other independent current sources are open circuited (O.C.).

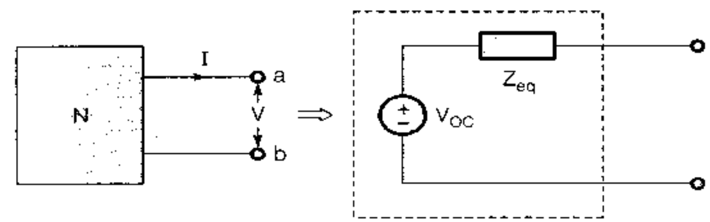
(c) All dependent voltage and current sources remain as they are and therefore, they are neither S.C. nor O.C.

Note:

- The theorem is not applicable to the network containing
  - (a) Non linear elements.
  - (b) Unilateral elements such as diode or BJT.
- The theorem is not applicable to power since it is a non linear parameter.
- The theorem is also applicable for circuit having initial condition.

2. Thevenin's Theorem

A linear active RLC network which contains one or more independent or dependent voltage or current sources can be replaced by a single voltage source V_{OC} in series with equivalent impedance Z_{eq}.



where, V_{OC} = Open circuit voltage between a and b (when I = 0).

Z_{eq} = Equivalent impedance between a and b, when

(a) All independent sources are replaced by their internal impedances.

(b) All dependent voltage and current sources are remain as they are.

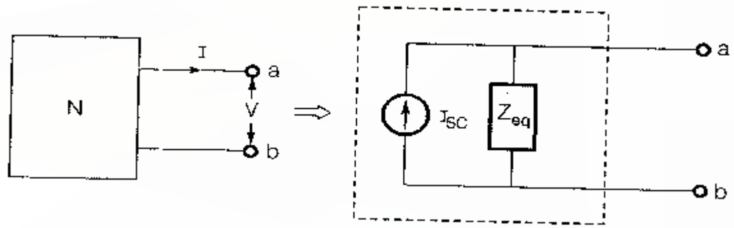
Note:

Theorem is not applicable to the network containing:

- Non linear element.
- Unilateral element.

3. Norton's Theorem

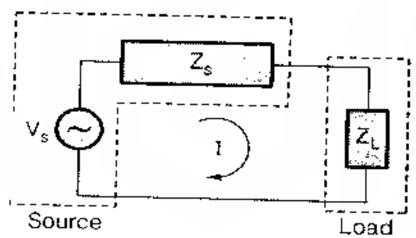
A linear, active RLC network which contains one or more independent or dependent voltage or current sources can be replaced by a single current source I_{SC} in shunt with equivalent impedance Z_{eq}.



where, I_{SC} = Short circuit current between a and b (when V = 0)

Z_{eq} = Same as that of Thevenin's theorem

4. Maximum Power Transfer Theorem



$\boxed{Z_L = Z_s^*}$ ......for maximum power transfer

Case 1: If $Z_s = R_s + jX_s$ and $Z_L = R_L + jX_L$

then $\boxed{R_L = R_s}$ and $\boxed{X_L = -X_s}$

Case 2: If $Z_s = R_s + jX_s$ and $Z_L = R_L$

then $\boxed{R_L = \sqrt{R_s^2 + X_s^2}}$

Case 3: If $Z_L = R_L$ and $Z_s = R_s$

then $\boxed{R_L = R_s}$

5. Tellegan's Theorem

- In any network, the sum of instantaneous power consumed by various elements of the branches is always equal to zero.
- Total power given out by different voltage sources is equal to total power consumed by various passive elements in various branches of the network.

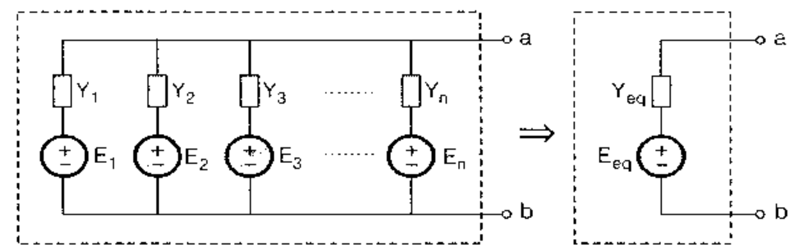
$$\boxed{\sum_{k=1}^{b} v_k \cdot i_k = 0}$$

where, b = Number of branches

Note:

The theorem is valid for any type of network so long as KVL and KCL equations are valid.

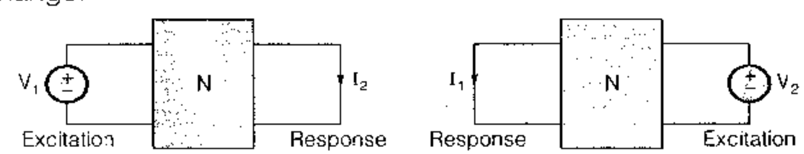
6. Millman's Theorem



$$\boxed{E_{eq} = \frac{\sum_{i=1}^{n} E_i Y_i}{\sum_{i=1}^{n} Y_i}} \qquad \boxed{Y_{eq} = \sum_{i=1}^{n} Y_i}$$

7. Reciprocity Theorem

In a linear bilateral single source network, the ratio of excitation to the response is constant when the position of excitation and response are interchange.



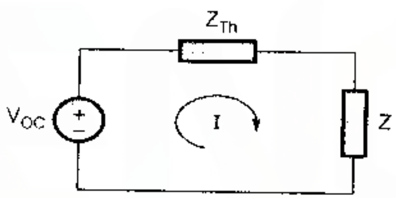
$$\boxed{\frac{V_1}{I_2} = \frac{V_2}{I_1}} \; ; \; \boxed{Z_{12} = Z_{21}}$$

Note:

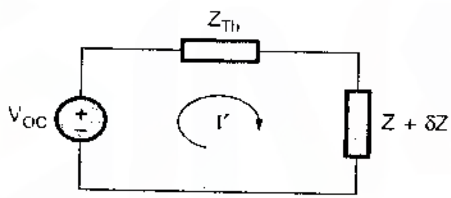
- $\left.\begin{array}{l} z_{12} = z_{21} \\ z_{13} = z_{31} \\ z_{23} = z_{32} \end{array}\right\}$ ··· for reciprocal network.
- The basis of the theorem is the symmetry of impedance or admittance of matrix.
- The theorem is valid for network in which linear and bilateral elements are present.
- The theorem is valid only when single independent voltage or current source is present.
- The initial conditions are assumed to be zero in reciprocity theorem.

8. Compensation Theorem

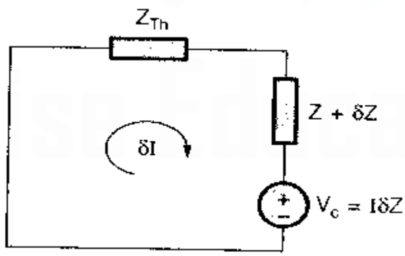
If impedance 'z' of any branch of a network is changed by 'δz', then the incremental current 'δI' in such branch is that which will be produced by a compensating voltage source $V_c = I\,\delta z$ introduced in the same branch with polarity opposing the original direction of current I.



(a) Compensation network



(b) Z changes to Z + δZ



Ideal voltage source V_c connected in series