JEE (Main)-2025 (Online) Session-2

Memory Based Question with & Solutions

(Physics, Chemistry and Mathematics)

4th April 2025 (Shift-1)

Time: 3 hrs. **M.M.: 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – APRIL, 2025

(Held On Friday 4th April, 2025) TIME : 9 : 00 AM to 12 : 00 PM

PHYSICS

SECTION-A

1. Find the dimension of $\frac{\phi_E}{\phi_B} = c$ where, ϕ_E represents electric flux and ϕ_B represents magnetic flux. Then dimension of c is given by $M^a L^b C^c$:-

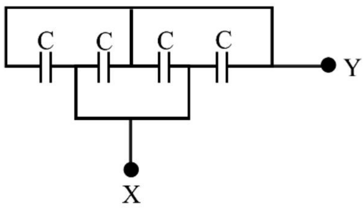
(1) a = 1, b = 1, c = –1 (2) a = 0, b = 1, c = –1

(3) a = 1, b = 2, c = –1 (4) a = 1, b = 2, c = 2

Ans. **(2)**

Sol. $\left(\frac{\phi_E}{\phi_B}\right) = \left[\frac{EA}{BA}\right] = LT^{-1}$

2. Find the equivalent capacitance between X and Y, where C = 16 μF

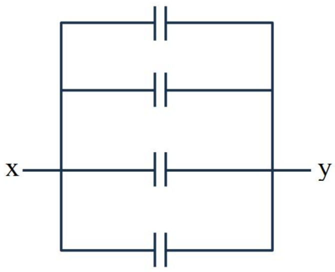


(1) 8 μF (2) 16 μF

(3) 64 μF (4) 32 μF

Ans. **(3)**

Sol.



$C_{eq} = 4C$

$= 64\ \mu F$

3. Mean free path for an ideal gas is to be observed 20 μm while average speed of molecules of gas is observed to be 600 m/s, then frequency (Hz) of collision is near by

(1) 2×10^{-7} (2) 3×10^{7}

(3) 4.2×10^{-7} (4) 6×10^{7}

Ans. **(2)**

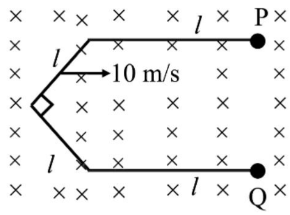
Sol. $\lambda = 20\mu m$

$v = 600\ m/s$

$\tau = \frac{\lambda}{v}$

$f = \frac{1}{\tau} = 3 \times 10^{7}\ Hz$

4. 4 rods of equal length are joined as shown in the figure. Combined system is moving with speed 10 m/s in a perpendicular magnetic field of $\frac{1}{\sqrt{2}}$ tesla. Find emf induced between point P and Q (l = 10 cm)



(1) $\sqrt{2}$ Volt (2) 1 Volt

(3) 0.1 Volt (4) 2 Volt

Ans. **(2)**

Sol. $E = vBL = 10 \times \frac{1}{\sqrt{2}} \times \frac{10\sqrt{2}}{100} = 1\ volts$

5. The current in a AC circuit is given as $i = 100\sqrt{2}\sin(100\pi t)A$. Find rms current and frequency.

(1) 100 A, 100 Hz (2) 200 A, 50 Hz

(3) 100 A, 50 Hz (4) $50\sqrt{2}$ A, 200 Hz

Ans. **(3)**

Sol. $i_{rms} = 100A$

$f = \frac{\omega}{2\pi} = 50Hz$

6. A real object placed in front of a spherical mirror forms an image whose magnification is $-\frac{1}{3}$. If the distance between the image and object is 30 cm. The focal length of the mirror is _____ cm.

(1) –22.5 cm (2) –11.25 cm

(3) –45 cm (4) –50 cm

Ans. **(2)**

Sol. $m = \frac{-1}{3}$

$v = \frac{u}{3}$

$v - u = 30$

$u = -45$ cm

$v = -15$ cm

$f = \frac{-45}{4}$ cm

7. Dipole of length 20 cm and charge 20 μC is placed in an electric field of infinite sheet of charge density 200 C/m², making an angle 30° with electric field, find torque experienced by dipole.

(1) $\frac{3}{\varepsilon_0} \times 10^{-4}$ N-m (2) $\frac{4}{\varepsilon_0} \times 10^{-4}$ N-m

(3) $\frac{2}{\varepsilon_0} \times 10^{-4}$ N-m (4) $\frac{12}{\varepsilon_0} \times 10^{-4}$ N-m

Ans. **(3)**

Sol. $\tau = PE\sin\theta$

$\tau = \left(20\times10^{-6}\times0.2\right)\times\left(\frac{200}{2\varepsilon_0}\right)\sin 30$

$\tau = \frac{2\times10^{-4}}{\varepsilon_0}$

8. Statement-I : The minimum kinetic energy required to take a body of mass m from surface of earth to infinity is mgR.

Statement-II: Potential energy at surface of earth is zero.

(1) Statement-I is correct, statement-II is correct and statement-II is correct explanation of statement-I.

(2) Statement-I is correct, statement-II is correct and statement-II is not the correct explanation of statement-I.

(3) Statement-I is correct and statement-II is incorrect.

(4) Statement-I is incorrect and statement-II is correct.

Ans. **(3)**

Sol. Theoretical

9. If slit width is doubled then % change in fringe width

(1) Remain same (2) 150%

(3) 75% (4) 50%

Ans. **(4)**

Sol. $\beta = \frac{\lambda D}{d}$

10. Longitudinal sound waves travel in three different gases namely helium, methane and carbon dioxide. Mean temperature of three gases are equal then ratio of speeds of waves in 3 gases respectively is

(1) $\frac{1}{\sqrt{3}}:\frac{1}{\sqrt{5}}:\frac{1}{2}$ (2) $\sqrt{5}: 1: \sqrt{\frac{21}{55}}$

(3) $\sqrt{3}: \sqrt{5}: \frac{1}{\sqrt{11}}$ (4) $\sqrt{5}: \sqrt{7}: \frac{1}{\sqrt{11}}$

Ans. **(2)**

Sol. $V_s = \sqrt{\frac{\gamma RT}{M}}$

11. **Assertion :** In photoelectric effect, if intensity of monochromatic light is increased then stopping potential increases.

Reason : Increased intensity results in increment of photocurrent.

(1) A is correct, R is correct and R is explanation of A

(2) A is correct, R is correct and R is not explanation of A

(3) A is incorrect and R is correct

(4) A is correct and R is incorrect

Ans. **(3)**

Sol. Theoretical

12. If $\frac{1}{5}th$ of volume of closed organ pipe is filled in water. Then % change in frequency

(1) 50% (2) 100%

(3) 25% (4) 400%

Ans. **(3)**

Sol. $f = \frac{v}{4L}$

$\frac{\Delta f}{f}\times100 = \frac{\left(\frac{5}{4}-1\right)}{1}\times100$

= 25 %

13. Given I = 0.02 t + 0.01 A. Find charge flown between t = 1 sec to t = 2 sec.

(1) 0.04 C (2) 0.05 C

(3) 0.02 C (4) 0.03 C

Ans. **(1)**

Sol. $q = \int i\,dt$

$= \int_1^2 (0.02t + 0.01)dt$

= 0.04 C

14. Which of the following is incorrect expression for torque

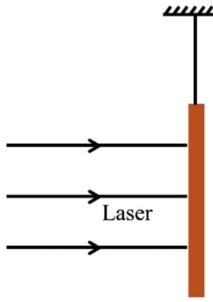
(1) $\vec{\tau} = \vec{r} \times \vec{L}$ (2) $\frac{d}{dt}(\vec{r} \times \vec{p})$

(3) $\vec{r} \times \frac{d}{dt}(\vec{p})$ (4) $\vec{\tau} = \vec{r} \times \vec{F}$

Ans. **(1)**

Sol. Theoretical

15. Laser ray having power P falls on a mirror having mass m. Find angle of deviation of mirror :-



(1) $\tan^{-1}\left(\frac{4P}{Cmg}\right)$ (2) $\tan^{-1}\left(\frac{P}{2Cmg}\right)$

(3) $\tan^{-1}\left(\frac{P}{Cmg}\right)$ (4) $\tan^{-1}\left(\frac{2P}{Cmg}\right)$

Ans. **(4)**

Sol. $\tan\theta = \frac{\frac{2P}{c}}{mg}$

16. Two simple pendulums with amplitudes θ_1 and θ_2 having length of strings ℓ_1 and ℓ_2 respectively. Choose the correct options if the maximum angular accelerations are same.

(1) $\theta_1\ell_2^2 = \theta_2\ell_1^2$ (2) $\theta_1\ell_1^2 = \theta_2\ell_2^2$

(3) $\theta_1\ell_2 = \theta_2\ell_1$ (4) $\theta_1\ell_1 = \theta_2\ell_2$

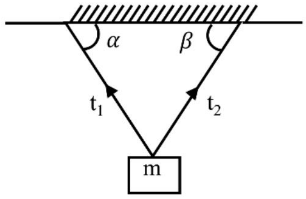
Ans. **(3)**

Sol. $\alpha = -\omega^2\theta$

$\omega_1^2\theta_1 = \omega_2^2\theta_2$

$\omega = \sqrt{\frac{g}{\ell}}$

17. A block of mass $m\,kg$ is connected to two strings as shown. If $T_1 = \sqrt{3}T_2$, then find ratio of angle α and angle β



(1) 2 (2) $\frac{1}{2}$

(3) 1 (4) 4

Ans. **(1)**

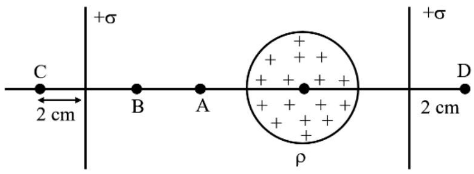
Sol. $T_1\cos\alpha = T_2\cos\beta$

$\sqrt{3} = \frac{\cos\beta}{\cos\alpha}$

$T_1\sin\alpha + T_2\sin\beta = mg$

$\beta = 30°$ $\alpha = 60°$

18. A non-conducting sphere with volume charge density ρ is placed between two non-conducting plane sheets with charge density σ as shown. Choose the correct relation between the magnitude of electric fields at A, B, C and D. Point A is at the middle of two sheets.



(1) $E_A = E_B,\ E_C \neq E_D$ (2) $E_A \neq E_B,\ E_C = E_D$

(3) $E_A > E_B,\ E_C = E_D$ (4) $E_A > E_B,\ E_C \neq E_D$

Ans. **(4)**

Sol. $E_B < E_A$

$E_C \neq E_D$

19. The Boolean expression $Y = A\bar{B}C + \bar{A}\bar{C}$ can be realised with which of the following gate configurations

(1) One-3 input *AND* gate, 3 NOT gate and one-2 input *OR* gate, one-2 input *AND* gate

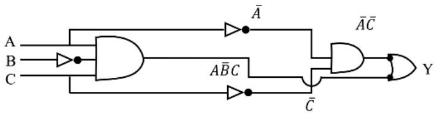
(2) 3-input *AND* gate, 3 NOT gates and one 2-input *OR* gate

(3) 3-input *OR* gate, 3 NOT gates and one 2-input *AND* gate

(4) One-3 input *AND* gate, 1 NOT gate, one-2 input *NOR* gate and one-2 input *OR* gates

Ans. **(1)**

Sol.



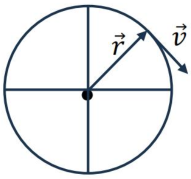
20. $\vec{L}$ and $\vec{p}$ are angular momentum about origin and linear momentum of a particle. If position vector of particle is given as $\vec{r} = a(\sin\omega t\hat{i} + \cos\omega t\hat{j})$ then direction of force is

(1) Opposite to $\vec{L} \times \vec{r}$ (2) Opposite to $\vec{p} \times \vec{r}$

(3) Opposite to $\vec{L}.\vec{r}$ (4) Opposite to $\vec{p} \times \vec{L}$

Ans. **(4)**

Sol.



SECTION – B

1. A ring and a solid sphere released from rest from same height on sufficient rough inclined surface. Ratio of their speed when they reach bottom is $\sqrt{\frac{7}{x}}$ m/s, then x is _____.

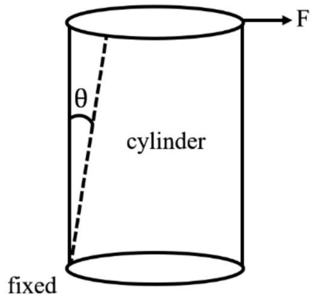
Ans. **(10)**

Sol. $a = \dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}}$

$v = \sqrt{2as}$

$\dfrac{V_{ring}}{V_{sphere}} = \dfrac{\sqrt{\frac{7}{5}}}{\sqrt{2}} = \sqrt{\dfrac{7}{10}}$

2. Two different cylinders experience shear forces. If $d_1 = 2d_2$, $\theta_1 = 2\theta_2$, $F_1 = F_2$, $\eta_1 = 4 \times 10^9, \eta_2 = x \times 10^9$. Find x :-



Ans. **(32)**

Sol. $\eta = \dfrac{F}{\phi A}$

CHEMISTRY

SECTION-A

1. Which of the following is the ratio of 5th Bohr orbit (r_5) of He^+ & Li^{2+} ?

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$

(3) $\frac{9}{4}$ (4) $\frac{4}{9}$

Ans. **(2)**

Sol. $r = 0.529 \times \frac{n^2}{z}$

$$\frac{(r_5)_{He^+}}{(r_5)_{Li^{+2}}} = \frac{0.529 \times \frac{25}{2}}{0.529 \times \frac{25}{3}} = \frac{3}{2}$$

2. Which of the following pair of ions have equal number of unpaired electrons

(1) V^{2+} and Ni^{2+} (2) Cr^{2+} and Mn^{2+}

(3) Fe^{2+} and Sc^{2+} (4) Mn^{3+} and Fe^{2+}

Ans. **(4)**

Sol. V^{2+} = [Ar] $3d^3 4s^0$

3d

↑	↑	↑		

No. of unpaired electrons = 3

Ni^{2+} = [Ar] $3d^8 4s^0$

↑↓	↑↓	↑↓	↑	↑

No. of unpaired electrons = 2

Cr^{2+} = [Ar] $3d^4 4s^0$

↑	↑	↑	↑	

No. of unpaired electrons = 4

Mn^{2+} = [Ar] $3d^5 4s^0$

↑	↑	↑	↑	↑

No. of unpaired electrons = 5

Fe^{2+} = [Ar] $3d^6 4s^0$

↑↓	↑	↑	↑	↑

No. of unpaired electrons = 4

Mn^{3+} = [Ar] $3d^4 4s^0$

↑	↑	↑	↑	

No. of unpaired electrons = 4

Sc^{2+} = [Ar] $3d^1 4s^0$

↑				

No. of unpaired electrons = 1

3. Incorrect order of atomic radius is

(1) B < Al (2) In < Tl

(3) Al < Ga (4) Ga < In

Ans. **(3)**

Sol. Size order $\Rightarrow$ B < Al > Ga < In < Tl

The radius of Ga is smaller than Al. Due to poor shielding effect of d-electrons in Ga

4. One mole of an ideal gas expands from 10 dm^3 to 20 dm^3 through isothermal reversible process. Find ΔU, q & w

(1) $\Delta U = 0, q = 0, w = 0$

(2) $\Delta U = 0, q \neq 0, w \neq 0$

(3) $\Delta U \neq 0, q = 0, w \neq 0$

(4) $\Delta U \neq 0, q \neq 0, w = 0$

Ans. **(2)**

Sol. Isothermal reversible expansion of an ideal gas

$\therefore \quad \Delta U = 0$

$q = -w$

$$w = -nRT \ln \frac{V_2}{V_1}$$

$\therefore \quad w \neq 0, \quad q \neq 0$

5. The rate of a chemical reaction is $K[A]^n [B]^m$. If concentration of A is doubled and concentration of B is halved, then change of rate will be:

(1) 2^{n-m} (2) 2^{m-n}

(3) 2^{2n-2m} (4) 2^{2m-n}

Ans. **(1)**

Sol. $r_1 = k[A]^n[B]^m$ ...(1)

$$r_2 = k[2A]^n \left[\frac{B}{2}\right]^m \quad \text{...(2)}$$

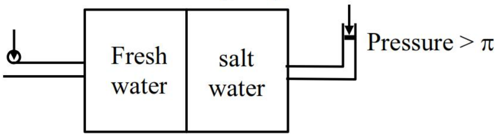
(1) Divided by (2)

$$\frac{r_2}{r_1} = \frac{k[2A]^n \left[\frac{B}{2}\right]^m}{k[A]^n [B]^m}$$

$$\frac{r_2}{r_1} = [2]^n \left[\frac{1}{2}\right]^m$$

$$r_2 = r_1 (2)^{(n-m)}$$

6. Observe the following diagram.



For reverse osmosis, which of the following can be used for porous membrane?

(1) Cellulose acetate

(2) Porous silicate

(3) Silicone

(4) Glass memebrane

Ans. (1)

Sol. Cellulose acetate is used as porous membrane for reverse osmosis.

[NCERT Based]

7. Which of the following is correct option regarding 1s orbital

(1) It is symmetrical

(2) It is non-symmetrical

(3) It is directional

(4) It has two radial nodes

Ans. (1)

Sol. 1s orbital ⇒ Symmetrical
Non-directional
No radial node

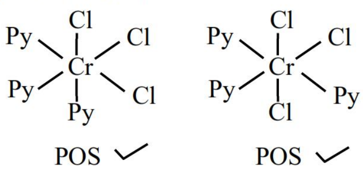
8. Total number of stereoisomers possible for complexes $[Cr(Cl)_3(Py)_3]$ and $[CrCl_2(C_2O_4)_2]$ respectively are

(1) 2,3 (2) 3,2

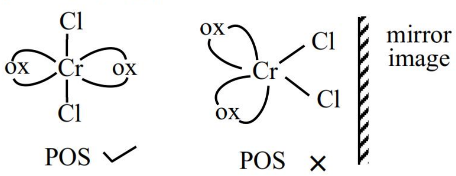
(3) 3,3 (4) 2,2

Ans. (1)

Sol. $[Cr(Cl)_3(Py)_3]$



$[CrCl_2(C_2O_4)_2]$



9. Among the following complexes

$[Fe(CN)_6]^{4-}$, $[FeF_6]^{4-}$,

$[Co(NH_3)_6]^{3+}$, $[Mn(SCN)_6]^{4-}$

The complexes having CFSE equals to 0 and having magnetic moment of 5.92 BM.

(1) $[Fe(CN)_6]^{4-}$ (2) $[FeF_6]^{4-}$

(3) $[Co(NH_3)_6]^{3+}$ (4) $[Mn(SCN)_6]^{4-}$

Ans. (4)

(1) $[Fe(CN)_6]^{4-} \Rightarrow Fe^{+2} \Rightarrow d^6$

$CN^- \rightarrow SFL$ $d^6 \Rightarrow$ [⇅ | ⇅ | ⇅ | |]

n = 0

(2) $[FeF_6]^{4-} \Rightarrow Fe^{+2} \Rightarrow d^6$

$F^- \rightarrow WFL$ [⇅ | 1 | 1 | 1 | 1]

n = 4

(3) $[Co(NH_3)_6]^{3+} \Rightarrow Co^{+3} \Rightarrow d^6$

$NH_3 \rightarrow SFL$ [⇅ | ⇅ | ⇅ | |]

n = 0

(4) $[Mn(SCN)_6]^{4-} \Rightarrow Mn^{+2} \Rightarrow d^5$

$SCN^{\ominus} \rightarrow WFL$ [1 | 1 | 1 | 1 | 1]

n = 5

$\mu = \sqrt{35} = 5.92$

10. $KMnO_4$ oxidises others in acidic medium, difference between two oxidation states of Mn is x. Neutral $FeCl_3$ reacts with oxalate to form a complex compound having y-d-electrons. Find x + y.

(1) 5 (2) 10

(3) 6 (4) 8

Ans. (2)

Sol. In acidic medium –

$$\overset{(+7)}{KMnO_4} \xrightarrow{H^+} Mn^{2+}$$

Change in oxidation state of Mn = 5

x = 5

$$\overset{+3}{Fe}Cl_3 + 3C_2O_4^{2-} \longrightarrow \overset{+3}{[Fe(C_2O_4)_3]}^{3-}$$

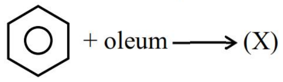
$Fe^{3+} = 3d^5\,4s^0$ [1 | 1 | 1 | 1 | 1]

No. of d-electons = 5

y = 5

⇒ x + y = 10

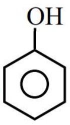
11. In the reaction sequence

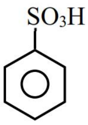


$(X) \xrightarrow{\text{(I) NaOH, (II) } H^+, \Delta} (Y)$

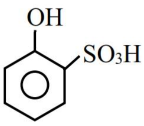
$(Y) \xrightarrow{\text{Zn,dust}} (Z)$

The compound (Z) is

(1) 

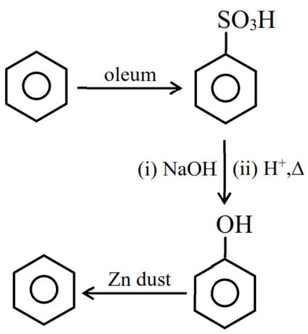
(2) 

(3) 

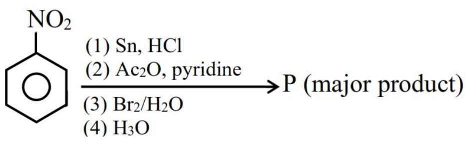
(4) 

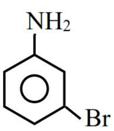
Ans. **(3)**

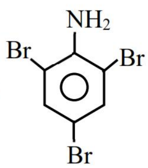
Sol.

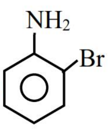


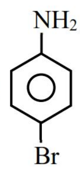
12. In the reaction sequence



(1) 

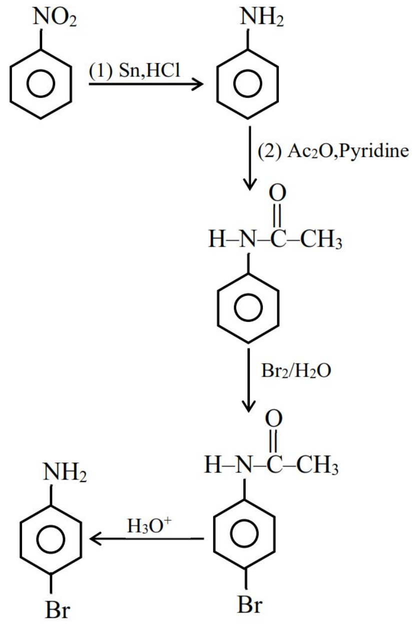
(2) 

(3) 

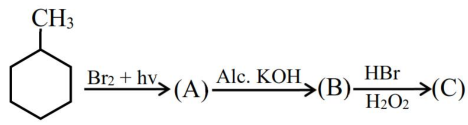
(4) 

Ans. **(4)**

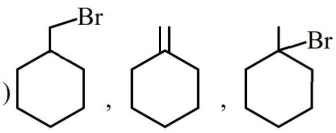
Sol.

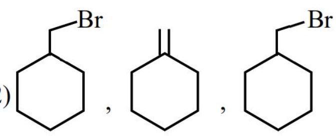


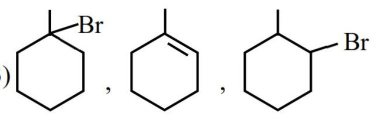
13.

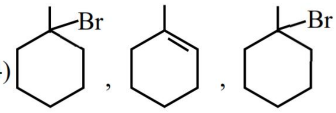


Identify (A), (B) and (C).

(1) 

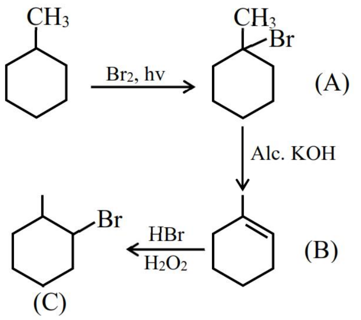
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



14. Consider the two products

$$CH_3-CH_2-CH_2-\overset{\overset{\large O}{||}}{C}_{a}-H \text{ and } CH_3-CH=CH-\overset{\overset{\large O}{||}}{C}_{b}-H$$

(A) (B)

The correct order of dipole moment and bond length order will be :

(1) A > B ; a > b (2) A < B ; a < b

(3) A < B ; a > b (4) A > B ; a < b

Ans. **(3)**

Sol.

$$CH_3-CH_2-CH_2-\overset{\overset{\large O}{||}}{C}_{a}-H \quad CH_3-CH=CH-\overset{\overset{\large O}{||}}{C}_{b}-H$$

(A) (B)

⥮ ⥮

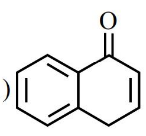
$$CH_3-CH_2-CH_2-\overset{\overset{\large O^{\ominus}}{|}}{C^{\oplus}}_{a}-H \quad CH_3-\overset{\oplus}{C}H-CH=\overset{\overset{\large O^{\ominus}}{|}}{C}_{b}-H$$

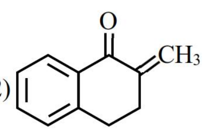
(A) < (B)

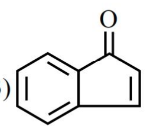
(more polar)

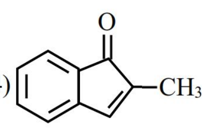
Bond length (b) < (a)

15. Which of the following compound is not a product of intramolecular aldol condensation reaction?

(1) 

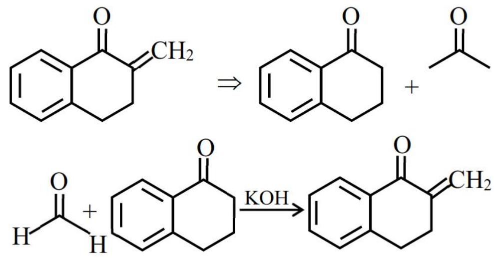
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



16. In following sequence of reaction. A is converted to D

$$\underset{A}{C_3H_6O} \xrightarrow{H_2/Pd} B \xrightarrow{HBr} C \xrightarrow{Mg/Ether} D$$

D is treated with A followed by hydrolysis to give 2, 3-dimethyl-butan-2-ol. Then identify A, B, C.

(1) A = CH_3COCH_3, B = $CH_3-CH(OH)CH_3$, C = $CH_3-CH(Br)CH_3$

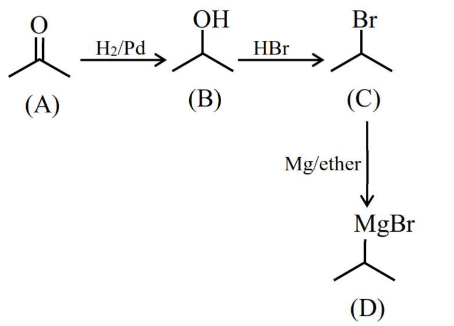
(2) A = CH_3CH_2CHO, B = $CH_3CH_2CH_2OH$, C = $CH_3CH_2CH_2$ Br

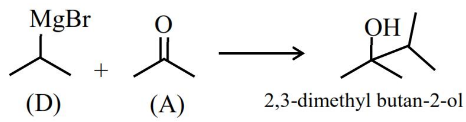
(3) A = $CH_2=CH-CH_2OH$, B = $CH_3CH_2CH_2OH$, C = $CH_3CH_2CH_2Br$

(4) A = Cyclopropanol, B = Cyclopropenone, C = Bromo propane

Ans. **(1)**

Sol.





17. The activation energy of forward reaction and backward reaction is 100 kJ/mole and 180 kJ/mole respectively. Find the correct statement if catalyst is added under same condition of temperature.

(1) Catalyst does not change ΔG of reaction

(2) Catalyst can make non-spontaneous reaction spontaneous

(3) Catalyst changes ΔH of reaction

(4) Enthalpy of reaction (ΔH) is 280 kJ/mole

Ans. **(1)**

Sol. $\Delta H = E_{af} - E_{ab}$

$= 100 - 180 = -80$ kJ/mol

SECTION-B

18. Among the following, the number of paramagnetic molecules are :

O_2, N_2, F_2, B_2, Cl_2

Ans. **(2)**

Sol. $O_2 : \sigma_{1s}^2 \sigma^{*2}_{1s}\, \sigma_{2s}^2 \sigma^{*2}_{2s}\, \sigma_{2p}^2$

$(\pi_{2p}^2 = \pi_{2p}^2)(\pi^{*1}_{2p} = \pi^{*1}_{2p})$; Paramagnetic

$N_2 :\ \sigma_{1s}^2 \sigma^{*2}_{1s}\, \sigma_{2s}^2 \sigma^{*2}_{2s}\, (\pi_{2p}^2 = \pi_{2p}^2)(\sigma_{2p}^2)$:

Diamagnetic

$F_2 :\ \sigma_{1s}^2 \sigma^{*2}_{1s}\, \sigma_{2s}^2 \sigma^{*2}_{2s}\, \sigma_{2p}^2 (\pi_{2p}^2 = \pi_{2p}^2)$

$(\sigma^{*2}_{2p} = \sigma^{*2}_{2p})$ Diamagnetic

$B_2 :\ \sigma_{1s}^2 \sigma^{*2}_{1s}\, \sigma_{2s}^2 \sigma^{*2}_{2s}\, (\pi_{2p}^1 = \pi_{2p}^1)$

Paramagnetic

Cl_2 : Diamagnetic

19. 0.01 M HX ($K_a = 4 \times 10^{-10}$) is diluted till the solution has pH = 6. If the new concentration is $x \times 10^{-4}$ M then find x.

Ans. **(25)**

Sol. pH = 6

$[H^+] = 10^{-6}$

$[H^+] = \sqrt{k_a c}$

$10^{-6} = \sqrt{4 \times 10^{-10} \times c}$

$4 \times 10^{-10} \times c = (10^{-6})^2$

$c = \dfrac{10^{-12}}{4 \times 10^{-10}}$

$c = 25 \times 10^{-4}$

MATHEMATICS

1. Foci of ellipse are (2,5) and (2, –3), eccentricity is $\frac{4}{5}$. Find the length of latus rectum.

Ans. $\frac{18}{5}$

Sol. 2ae = 5 + 3 = 8

$e = \frac{4}{5} \Rightarrow$ a = 5

$b^2 = 25\left(1 - \frac{16}{25}\right) = 9 \Rightarrow$ b = 3

$L = \frac{2b^2}{a} = \frac{18}{5}$

2. Solve $\int_{-1}^{1} \frac{1+2x}{e^{-x}+e^{x}} dx$

(1) $2\left(\tan^{-1} e - \frac{\pi}{4}\right)$ (2) $2\left(\tan^{-1} e - \frac{\pi}{3}\right)$

(3) $2\left(\tan^{-1} e - \frac{\pi}{2}\right)$ (4) $2\left(\frac{\pi}{2} - \tan^{-1} e\right)$

Ans. **(1)**

Sol. $I = \int_{-1}^{1} \frac{1+2x}{e^{-x}+e^{x}} dx$

Apply Kings rule,

$I = \int_{-1}^{1} \frac{1-2x}{e^{x}+e^{-x}} dx$

$2I = \int_{-1}^{1} \frac{(1+2x)+(1-2x)}{e^{x}+e^{-x}} dx$

$2I = 2\int_{-1}^{1} \frac{dx}{e^{x}+e^{-x}}$

$I = 2\int_{0}^{1} \frac{dx}{e^{x}+e^{-x}}$

$\Rightarrow 2\int_{0}^{1} \frac{ex}{(e^{x})^2+1} dx$

$e^x = t$

$= 2\int_{1}^{e} \frac{dt}{t^2+1}$

$= 2\left[\tan^{-1}\right]_1^e$

$\Rightarrow 2\left(\tan^{-1} e - \frac{\pi}{4}\right)$

3. The sum of the series $1 + 3 + 5^2 + 7 + 9^2 + \ldots$upto 40 terms is

(1) 41880 (2) 42880

(3) 41860 (4) 40860

Ans. **(1)**

Sol. $\left(1^2 + 5^2 + 9^2 + \ldots.20 \text{ terms}\right)$

$+ \left(3 + 7 + 11 + \ldots. + 20 \text{ terms}\right)$

$\Rightarrow \sum(4n-3)^2 + \sum(4n-1)$

$\sum(16n^2 - 24n + 9) + \sum 4n - 1$

$\Rightarrow 16\sum n^2 - 20\sum n + 8\sum 1$

$= 16\left\{\frac{20 \times 21 \times 41}{6}\right\} - 20\left\{\frac{20 \times 21}{2}\right\} + 8 \times 20$

$= 16\{2870\} - 4200 + 160$

= 41880

4. Let there be two A.P's with each having 2025 terms. Find the number of distinct terms in union of the two A.P's i.e., $A \cup B$ if first A.P. is 1,6,11,…and second A.P. is 9,16,23,…

(1) 3022 (2) 2025

(3) 4035 (4) 3761

Ans. **(4)**

Sol. Total number of terms : n(A) + n(B) = 2025 + 2025 = 4050

Now n(A ∩ B) = common terms of both A.Ps

Common difference = 35 (LCM of both A.P.)

Common A.P. : 16, 51, 86, ……

⇒ last term of smaller AP : 1 + (2024) (5)

= 10120

Now, 16 + (n – 1) (35) ≤ 10120

$(n-1) \le \frac{10104}{35}$

$(n-1) \le 288.68$

$n \le 289.68$

⇒ n = 289

Required ans.: 4050 - n(A ∩ B) = 4050 – 289

= 3761

5. If 10 $\sin^4\theta$ + 15 $\cos^4\theta$ = 6, then find the value of $\frac{27\text{cosec}^6\theta+8\sec^6\theta}{16\sec^8\theta}$

Ans. $\frac{2}{5}$

Sol. Let $\sin^2\theta = t$

$10t^2 + 15(1 + t^2 - 2t) = 6$

$25t^2 - 30t + 9 = 0$

$(5t - 3)^2 = 0$

$\Rightarrow \sin^2\theta = t = \frac{3}{5}$

$\cos^2\theta = \frac{2}{5}$

$$\frac{27\text{cosec}^2\theta+8\sec^6\theta}{16\sec^8\theta} = \frac{27\left(\frac{5}{3}\right)^3+8\left(\frac{5}{2}\right)^3}{16\left(\frac{5}{2}\right)^4} = \frac{125+125}{625} = \frac{2}{5}$$

6. Consider a committee of 12 memebers is formed randomly out of 4 Engineers, 2 Doctors and 10 Professors. Find the probability that the committee has exactly 3 Engineers and 1 Doctor.

(1) $\frac{15}{91}$ (2) $\frac{18}{71}$

(3) $\frac{18}{91}$ (4) $\frac{17}{91}$

Ans. **(3)**

Sol. Total cases = ${}^{16}C_{12}$

Favourable = ${}^{4}C_3 \times {}^{2}C_1 \times {}^{10}C_8$

$$P = \frac{4\times2\times\frac{10\times9}{2}\times4!}{16\times15\times14\times13} = \frac{18}{91}$$

7. The number of integral values of $n \in N$ for which the equation $x^2 + 4x - n = 0, n \in [20,100]$ have integral roots, is

(1) 7 (2) 5

(3) 4 (4) 6

Ans. **(4)**

Sol. $x^2 + 4x - n = 0$

$(x + 2)^2 - 4 - n = 0$

$(x + 2)^2 = 4 + n$

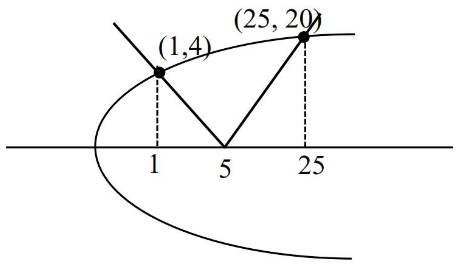
$x+2=\pm\sqrt{n+4}$

N = 21, 32, 45, 60, 77, 96

Six values.

8. Let $|x - 5| \le y \le 4\sqrt{x}$. If the area enclosed is A, then 3A equal to

Ans. **(368)**



Sol.

$y^2 = 16x = (x-5)^2$

$\Rightarrow x^2 - 26x + 25 = 0$

$\Rightarrow x = 1, 25$

$A = \int_1^{25} 4\sqrt{x}dx - \frac{1}{2}\cdot4\cdot4 - \frac{1}{2}\cdot20\cdot20$

$= \frac{8}{3}\left[x^{\frac{3}{2}}\right]_1^{25} - \frac{1}{2}(416)$

$= \frac{8}{3}(125-1) - 208$

$= \frac{8}{3}\times124 - 208$

$3A = 992 - 624 = 368$

9. In 10 balls, 3 are defective. If 2 are chosen at random, find variance (σ^2) of the defective balls.

Ans. $\frac{28}{75}$

Sol.

0	1	2
$\frac{{}^7C_2}{{}^{10}C_2}$	$\frac{{}^7C_1\cdot{}^3C_1}{{}^{10}C_2}$	$\frac{{}^3C_2}{{}^{10}C_2}$
$=\frac{7}{10}\cdot\frac{6}{9}$	$\frac{7.3.2}{10.9}$	$\frac{3}{10}.\frac{2}{9}=\frac{1}{15}$
$=\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

$\sigma^2 = \Sigma(x - \mu)^2P(x)$

$= \left(\frac{3}{5}\right)^2.\frac{7}{15}+\left(\frac{2}{5}\right)^2.\frac{7}{15}+\left(\frac{7}{5}\right)^2.\frac{1}{15}$

$= \frac{63+28+49}{375} = \frac{140}{375} = \frac{28}{75}$

$$\begin{Bmatrix} \sum x^2P(x) - \left(\sum xP(x)\right)^2 \\ \sum (x-\mu)^2 P(x) \\ \mu = 0 + \frac{7}{15} + \frac{2}{15} \\ = \frac{9}{15} = \frac{3}{5} \end{Bmatrix}$$

10. Let $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. Here $A^2 = A^T$. Then find trace $[(A+I)^3 + (A-I)^3 - 6A]$.

Ans. **(6)**

Sol. Here, A is orthogonal matrix

So, $A^T = A^{-1}$

$\Rightarrow A^2 = A^T \Rightarrow A^2 = A^{-1} \Rightarrow A^3 = I$

$B = (A+I)^3 + (A-I)^3 - 6A$

$= 2(A^3 + 3A) - 6A$

$= 2A^3$

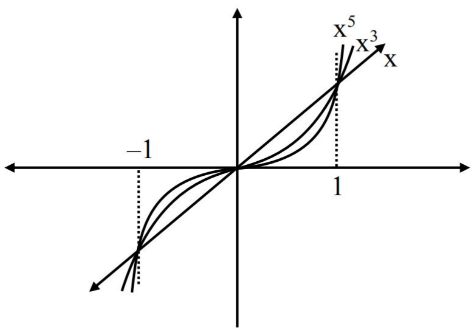
$= 2I$

$Tr(B) = 2 + 2 + 2 = 6$

11. $f(x) = \max\{x, x^3, x^5, \ldots, x^{21}\}$, if number of points where $f(x)$ is discontinuous $= p$ and number of points where $f(x)$ is not differentiable $= q$, then find the value of $p + q$

Ans. **(3)**

Sol.



$$f(x) = \begin{cases} x & : \quad x < -1 \\ x^{21} & : \quad -1 \le x < 0 \\ x & : \quad 0 \le x < 1 \\ x^{21} & : \quad 1 \le x \end{cases}$$

f(x) is always continuous $\therefore p = 0$

f(x) is not different at 3 points q = 3

$\therefore p + q = 3$

12. $\lim_{x\to 1^+} \frac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3} = -1$, where $\lambda, \mu \in R$. Then $\lambda + \mu$ is equal to

(1) 17 (2) 18

(3) 19 (4) 20

Ans. **(2)**

Sol. $\lim_{x\to 1^+} \frac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3} = -1$

$\Rightarrow \lim_{t\to 0^+} \frac{6t + \lambda t\cos t - \mu \sin t}{t^3} = -1$

$\Rightarrow \lim_{t\to 0^+} \frac{6t + \lambda t\left(1 - \frac{t^2}{2} + \frac{t^4}{24}\right) - \mu\left(t - \frac{t^3}{6} + \frac{t^5}{120}\right)}{t^3} = -1$

so, $\lambda + 6 - \mu = 0$ …(i)

and $\frac{\mu}{6} - \frac{\lambda}{2} = -1$ …(ii)

solving (i) and (ii)

we get, $\mu = 12,\ \lambda = 6$

so, $\lambda + \mu = 18$

13. Let $f, g : (1, \infty) \to R$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function $fog : [2,4] \to R$ is $[\alpha, \beta]$, then $\frac{1}{\beta - \alpha}$ is equal to

Ans. **(56)**

Sol. $g(2) = 4$

$g(4) = \frac{10}{3}$

$f(g(4)) = \frac{\frac{20}{3} + 3}{\frac{50}{3} + 2} = \frac{29}{56}$

$f(4) = \frac{11}{22} = \frac{1}{2}$

$f\left(\frac{10}{3}\right) = \frac{29}{56} = \beta$

So, $\frac{1}{\beta - \alpha} = \frac{1}{\left|\frac{29}{56} - \frac{1}{2}\right|} = \left|\frac{1}{\frac{29}{56} - \frac{1}{2}}\right| = 56$

14. Evaluate : $\int_{-1}^{1} \frac{[1+\sqrt{|x|-x}]e^x+(\sqrt{|x|-x})e^{-x}}{e^x+e^{-x}}dx$

Ans. $1+\frac{2\sqrt{2}}{3}$

Sol. Apply king property

$$I=\int_{-1}^{1}\frac{\left(1+\sqrt{|x|+x}\right)e^{-x}+\left(\sqrt{|x|+x}\right)e^{x}}{e^{x}+e^{-x}}dx$$

$$\therefore 2I=\int_{-1}^{1}\frac{\left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right)\left(e^{x}+e^{-x}\right)}{\left(e^{x}+e^{-x}\right)}dx$$

$$\Rightarrow 2I=\int_{-1}^{1}\left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right)dx$$

Apply odd even property

$$2I=2\int_{0}^{1}\left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right)dx$$

$$I=\int_{0}^{1}\left(1+\sqrt{2x}\right)dx$$

$$I=\left(x+\sqrt{2}\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_{0}^{1}$$

$$I=1+\frac{2\sqrt{2}}{3}$$

15. In the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, $n\in N$. If the ratio of 15th term from the beginning to the 15th term from the end is $\frac{1}{6}$, then find the value of ${}^{n}C_3$

Ans. **(2300)**

Sol. In the expansion of $(a+b)^n$

15th term from beginning : $T_{15}={}^{n}C_{14}a^{n-14}b^{14}$

15th term from the end : $T'_{15}={}^{n}C_{14}b^{n-14}a^{14}$

$$\therefore \frac{T_{15}}{T'_{15}}=\frac{1}{6}$$

$$\Rightarrow \frac{a^{n-14}b^{14}}{b^{n-14}a^{14}}=\frac{1}{6}$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-28}=\frac{1}{6}$$

$$\left(6^{\frac{1}{3}}\right)^{n-28}=6^{-1}$$

$$\Rightarrow \frac{n-28}{3}=-1$$

$$\Rightarrow n-28=-3$$

n = 25

$$\therefore {}^{25}C_3=2300$$

16. If $f(x)=1-2x+\int_0^x e^{x-t}f(t)\,dt,$ then the area bounded by the curve $y=f(x)$ and coordinate axes is (in square units)

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$

(3) 2 (4) 1

Ans. **(1)**

Sol. $y.e^{-x}=(1-2x)e^{-x}+\int_0^x e^{-t}f(t)dt$

$$y.e^{-x}(-1)+e^{-x}.y'=-e^{-x}+2xe^{-x}-2e^{-x}+e^{-x}.y$$

$$e^{-x}.y'-2ye^{-x}=(2x-3)e^{-x}$$

$$y'-2y=(2x-3)$$

$$e^{-2x}.y=\int(2x-3)e^{-2x}dx$$

$$=(2x-3)\left(-\frac{e^{-2x}}{2}\right)-2.\left(\frac{e^{-2x}}{4}\right)+c$$

$$=\frac{e^{-2x}}{2}(3-2x-1)$$

$$\Rightarrow e^{-2x}.y=e^{-2x}(1-x)+c$$

When x = 0, y = 1

So, 1 = (1 – 0) + c

$\Rightarrow$ c = 0

So, y = 1– x

So area bounded $=\frac{1}{2}\times1\times1=\frac{1}{2}$

17. The value of

$\sin^{-1}\left(\frac{\sqrt{3}x}{2}+\frac{1}{2}\sqrt{1-x^2}\right),-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$ is equivalent to

(1) $\frac{2\pi}{3}-\cos^{-1}x,-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

(2) $\pi-\cos^{-1}x,-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

(3) $\frac{\pi}{3}-\cos^{-1}x,-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

(4) $\frac{\pi}{2}-\cos^{-1}x,-\frac{1}{2}<x<\frac{1}{\sqrt{2}}$

Ans. **(1)**

Sol. Put x = sinθ

$$\sin^{-1}\left(\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}\right)$$

$$=\sin^{-1}\left[\sin\left(\theta+\frac{\pi}{6}\right)\right]$$

$$=\theta+\frac{\pi}{6}$$

$$=\sin^{-1}x+\frac{\pi}{6}$$

$$=\frac{\pi}{2}-\cos^{-1}x+\frac{\pi}{6}=\frac{2\pi}{3}-\cos^{-1}x$$

18. Let A and B two distinct points on the line $L:\frac{x-6}{3}=\frac{y-7}{3}=\frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{22}$ from the foot of the perpendicular drawn from the point (1, 2, 3)on the line L. If O is origin then $\overrightarrow{OA}\cdot\overrightarrow{OB}$ is equal to

Ans. **(18)**

Sol. $\overrightarrow{PM}=3\lambda+5,3\lambda+5,-2\lambda+4$

$\vec{L}=(3,3,-2)$

$\overrightarrow{PM}.\vec{L}=0=9\lambda+15+9\lambda+15+4\lambda-8=0$

$22\lambda+22=0 \Rightarrow \lambda=-1$

$M(3,4,9)$

Now,

Let $A(3\mu+6,3\mu+7,-2\mu+7)$

MA = $2\sqrt{22}$

$\Rightarrow(3\mu+3)^2+(3\mu+3)^2+(-2\mu-2)^2=4\times22$

$\Rightarrow(\mu+1)^2(9+9+4)=4\times22$

$\Rightarrow\mu+1=\pm2$

$\Rightarrow\mu=1,-3$

$A(9,10,5)$ $\quad B(-3,-2,13)$

$\overrightarrow{OA}\cdot\overrightarrow{OB}=-27-20+65=18$

19. Given two lines $L_1:\frac{x-3}{3}=\frac{y-\alpha}{1}=\frac{z+2}{-2}$ and $L_2:\frac{x+1}{2}=\frac{y+2}{1}=\frac{z-\beta}{-1}$. If shortest distance between L_1 and L_2 is $30\sqrt{3}$, then find the value of $|\alpha+\beta|$.

Ans. **(90)**

Sol. Point A(3, α, –2) B(–1, –2, β)

A, b, c, = 3, 1, –2 & a_2, b_2, c_2 = 2, 1, –1

A.T.Q.

$$\frac{1}{\sqrt{(-1+2)^2+(-3+4)^2+(3-2)^2}}\begin{vmatrix}3+1 & \alpha+2 & -2-\beta\\ 3 & 1 & -2\\ 2 & 1 & -1\end{vmatrix}=30\sqrt{3}$$

$$\Rightarrow\frac{1}{\sqrt{1+1+1}}\begin{vmatrix}4 & \alpha+2 & -2-\beta\\ 3 & 1 & -2\\ 2 & 1 & -1\end{vmatrix}=30\sqrt{3}$$

⇒ 4(–1 + 2) – (α + 2) (–3 + 4) + (–2 – β) (3 – 2) = 30 × 3

⇒ 4 – α – 2 – 2 – β = 90

⇒ |α + β| = 90

20. If $\vec{v}=2\hat{\imath}+\hat{\jmath}-\lambda\hat{k},(\lambda>0),\vec{u}=3\hat{\imath}-\hat{\jmath}$ and $\vec{v}_1$ is parallel to $\vec{u}$, $\overrightarrow{v_2}$ is perpendicular to $\vec{u}$ and $\vec{v}=\overrightarrow{v_1}+\overrightarrow{v_2}$. If angle between $\vec{v}$ and $\overrightarrow{v_1}$ is $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$, then $|\overrightarrow{v_1}|^2+|\overrightarrow{v_2}|^2$equals to

Ans. **(14)**

Sol. $|\vec{v}_1|=\lambda\vec{\mu}$

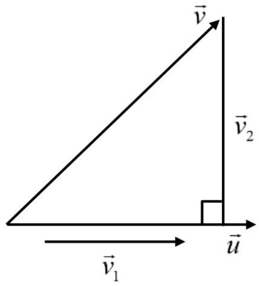
$\vec{v}_2\cdot\vec{\mu}=0$

$\vec{v}=\vec{v}_1+\vec{v}_2$

$\vec{v}_1=(\vec{v}.\hat{u})\hat{u}$

$\vec{v}_2=\vec{v}-(\vec{v}.\hat{u})\hat{u}$

$\frac{\vec{v}.\vec{u}}{|\vec{v}||\vec{u}|}=\frac{\sqrt{5}}{2\sqrt{7}}$



$\Rightarrow\frac{5}{\sqrt{4+1+\lambda^2}\sqrt{10}}=\frac{\sqrt{5}}{2\sqrt{7}}$

$\Rightarrow\frac{5}{\sqrt{5+\lambda^2}}=\frac{5}{\sqrt{14}}$

$\Rightarrow\lambda^2=9\Rightarrow\lambda=3\ ;\ \lambda>0$

$|\vec{v}_1|^2+|\vec{v}_2|^2=|\vec{v}|^2$

$=2^2+1^2+\lambda^2$

= 5 + 9

= 14