Magnetically Coupled Circuit

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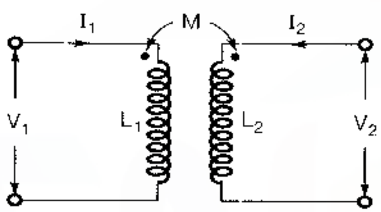
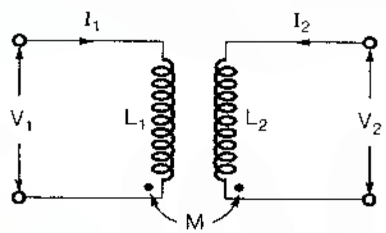
- When the two inductor are placed physically close to each other, then due to current flow in the first inductor there will be some magnetic flux, part of which will link with the second inductor. Due to rate of change of magnetic flux and by using Faraday's law of electromagnetic induction some induced voltage is produced between the two terminals of the second inductor.
- The polarity of the induced voltage depends upon the relative sense of windings of the two inductors and therefore, the induced voltage may be additive or substractive in nature.

Cases:

(A)



(Case-1)



(Case-2)

For above two cases, the effect of mutual inductance is positive. Also induced emf (e_{ind}) is additive in nature:

KVL in time domain:

$$V_1 = L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt}$$

$$V_2 = M\frac{dI_1}{dt} + L_2 \frac{dI_2}{dt}$$

KVL in s-Domain:

$$V_1(s) = L_1 s\, I_1(s) + Ms\, I_2(s)$$

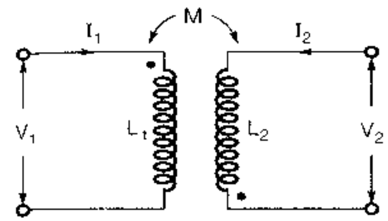
$$V_2(s) = Ms\, I_1(s) + L_2 s\, I_2(s)$$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} sL_1 & sM \\ sM & sL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

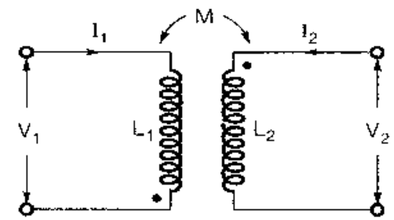
For sinusoidal excitation:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(B)



(Case-3)



(Case-4)

For above two cases, effect of the mutual coupling is negative. So e_{ind} is substractive in nature.

KVL in time domain:

$$V_1 = L_1 \frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

$$V_2 = -M\frac{dI_1}{dt} + L_2 \frac{dI_2}{dt}$$

KVL in s-domain:

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} sL_1 & -sM \\ -sM & sL_2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

For sinusoidal excitation:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & -j\omega M \\ -j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Representation of Mutual Inductance

Case-1: In terms of M.

Case-2: In terms of 'K' i.e. coefficient of coupling.

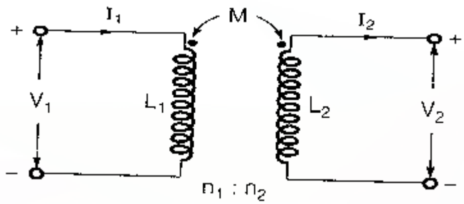
$$K = \frac{M}{\sqrt{L_1 L_2}}$$

(a) If $K < 1$; loose coupling (under coupling circuit).

(b) If $K = 1$ i.e. $M = \sqrt{L_1 L_2}$; (critical coupling)

(c) $K > 1$; tight coupling (over coupled circuit).

Case-3: In terms of turns ratio



$$\boxed{\frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{I_2}{I_1}}$$

and

$$\boxed{\frac{n_1}{n_2} \cong \frac{L_1}{M} \cong \frac{M}{L_2} = \sqrt{\frac{L_1}{L_2}}}$$