7. Vectors

- The quantity which involves only one value, i.e. magnitude, is called a scalar quantity. For example: time, mass, distance, energy, etc.
- The quantity which has both magnitude and a direction is called a vector quantity. For example: force, momentum, acceleration, etc.
- A line with a direction is called a directed line. Let $\overrightarrow{AB}$ be a directed line along direction B.

  Here,

  - The length of the line segment AB represents the magnitude of the above directed line. It is denoted by $\left|\overrightarrow{AB}\right|$ or $\left|\vec{a}\right|$ or *a*.
  - $\overrightarrow{AB}$ represents the vector in the direction towards point B. Therefore, the vector represented in the above figure is $\overrightarrow{AB}$. It can also be denoted by $\vec{a}$.
  - The point A from where the vector $\overrightarrow{AB}$ starts is called its initial point and the point B where the vector $\overrightarrow{AB}$ ends is called its terminal point.
- The angles *a*, *b*, and *g* made by the vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ with the positive directions of the *x*-axis, *y*-axis, and *z*-axis respectively are called its direction angles. The cosines of the angle made by the vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ with the positive directions of *x*, *y*, and *z* axes are its direction cosines. These are usually denoted by *l* = cos*a*, *m* = cos*b*, and *n* = cos*g*. Also, $l^2 + m^2 + n^2 = 1$

Example: Write the direction ratio's of the vector $\vec{r} = 2\hat{i} - \hat{j} - 2\hat{k}$ and hence calculate its direction cosines.

Solution: The direction ratio's *a, b, c* of a vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ are the respective components *x, y* and *z* of the vector.
The direction ratio's of the given vector are *a* = 2, *b* = –1 and *c* = –2
If *l, m* and *n* are the direction cosines of the given vector, then

$$l = \frac{a}{|\vec{r}|}, m = \frac{b}{|\vec{r}|}, n = \frac{c}{|\vec{r}|}$$

$$|\vec{r}| = \sqrt{(2)^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3$$

$$\therefore l = \frac{2}{3}, m = \frac{-1}{3} \text{ and } n = \frac{-2}{3}$$

- The direction cosines (*l*, *m*, *n*) of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ are

  $l = \frac{a}{r}, m = \frac{b}{r}, n = \frac{c}{r}$, where *r* = magnitude of the vector $a\hat{i} + b\hat{j} + c\hat{k}$

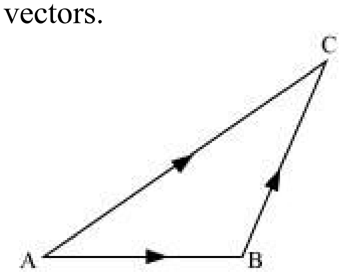
- The various types of vectors are given as follows:

- Zero vector: A vector whose initial and terminal points coincide is called a zero vector (or null vector). It is denoted as $\vec{0}$. The vectors $\overrightarrow{AA}$, $\overrightarrow{BB}$ represent zero vectors.
- Unit vector: A vector whose magnitude is unity, i.e. 1 unit, is called a unit vector. The unit vector in the direction of any given vector $\vec{a}$ is denoted by $\hat{a}$ and it is calculated by

Note: that if *l*, *m*, and *n* are direction cosines of a vector, then $l\hat{i} + m\hat{j} + n\hat{k}$ is the unit vector in the direction of that vector.

Example: To find the unit vector along the direction of a vector $\vec{r} = 16\hat{i} - 15\hat{j} + 12\hat{k}$, we may proceed as follows:

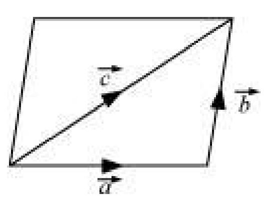
- The sum of two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is given by,
  $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$
- The difference of two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is given by
  $\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$
- **Triangle law of vector addition:** If two vectors are represented by two sides of a triangle in order, then the third closing side of the triangle in the opposite direction of the order represents the sum of the two vectors.



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

Note: The vector sum of the three sides of a triangle taken in order is $\vec{0}$.

- **Parallelogram law of vector addition:** If two vectors are represented by two adjacent sides of a parallelogram in order, then the diagonal of the parallelogram in the opposite direction of the order represents the sum of two vectors.



$\vec{c} = \vec{a} + \vec{b}$

- The properties of vector addition are given as follows:
  - Commutative property: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
  - Associative property: $\vec{a} + (b + c) = (a + b) + \vec{c}$

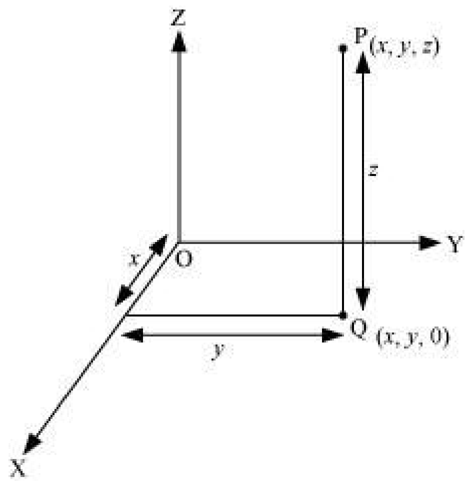
- Existence of additive identity: The vector $\vec{0}$ is additive identity of a vector $\vec{a}$, since
  $\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$
- Existence of additive inverse: The vector $-\vec{a}$ is called additive inverse of $\vec{a}$, since
  $\vec{a} + (-\vec{a}) = (-\vec{a}) + \vec{a} = 0$

- The multiplication of vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ by any scalar l is given by,

$\lambda\vec{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$

- The magnitude of the vector $\lambda\vec{a}$ is given by $|\lambda\vec{a}| = |\lambda||\vec{a}|$
- The vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are equal, if and only if $a_1 = b_1$, $a_2 = b_2$, and $a_3 = b_3$
- Let $\vec{a_1}$ and $\vec{a_2}$ be two vectors, and k_1 and k_2 be any scalars, then the following are the distributive laws of addition and multiplication of a vector by a scalar:
  - $k_1\vec{a_1} + k_2\vec{a_1} = (k_1 + k_2)\vec{a_1}$
  - $k_1(k_2\vec{a_1}) = (k_1k_2)\vec{a_1}$
  - $\vec{k_1}(\vec{a_1} + \vec{a_2}) = k_1a_1 + k_1a_2$
- Collinear vectors:
  - Two vectors $\vec{a}$ and $\vec{b}$ are collinear, if and only if there exists a non-zero scalar l such that
  - Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear, if and only if

- **Three-dimensions coordinate planes**
  - The coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called *x*, *y*, and *z*-axes.
  - The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-planes.
  - The three coordinate planes divide the space into eight parts known as octants.
  - In three-dimensional geometry, the coordinates of a point P are always written in the form of triplets i.e., (x, y, z). Here, x, y, and z are the distances from the YZ, ZX and XY-planes. Also, the coordinates of the origin are (0, 0, 0).



- The sign of the coordinates of a point determine the octant in which the point lies. The following table shows the signs of the coordinates in the eight octants.

Octants → / **Coordinates ↓**	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

Example: The point (–5, 6, –7) lies in the VI octant.

- In Coordinates of points lying on different axes:
  - Any point on the *x*-axis is of the form (*x*, 0, 0)
  - Any point on the *y*-axis is of the form (0, *y*, 0)
  - Any point on the *z*-axis is of the form (0, 0, *z*)
- Coordinates of points lying in different planes:
  - Coordinates of a point in the YZ-plane are of the form (0, *y*, *z*)
  - Coordinates of a point in the XY-plane are of the form (*x*, *y*, 0)
  - Coordinates of a point in the ZX-plane are of the form (*x*, 0, *z*)

Example: The points (–5, 6, 0), (0, –5, 6), (–5, 0, 6) lies in the XY-plane, YZ-plane and ZX-plane respectively.

- **distance formula**

Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example: Find the point(s), lying on the *z*-axis, whose distance from point (2, –1, 3) is 3 units.
Solution: Let the required point be (0, 0, *z*).
We know that the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1) + (z_2 - z_1)^2}$.

Therefore,

$$\sqrt{(2-0)^2+(-1-0)^2+(3-z)^2}=3$$

On squaring both the sides, we get

$$4+1+9+z^2-6z=9$$

$$\Rightarrow z^2-6z+5=0$$

$$\Rightarrow z^2-5z-z+5=0$$

$$\Rightarrow z(z-5)-1(z-5)=0$$

$$\Rightarrow z=1, 5$$

Thus, the required points on the *z*-axis are (0, 0, 1) and (0, 0, 5).

- The position vector of a point P(x, y, z) with respect to the origin (0, 0, 0) is given by $\overrightarrow{OP}=x\hat{i}+y\hat{j}+z\hat{k}$. This form of any vector is known as the component form.

  Here,

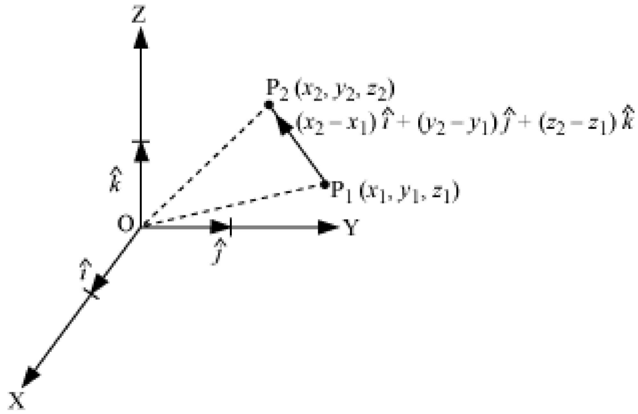
  - $\hat{i}$, $\hat{j}$, and $\hat{k}$ are called the unit vectors along the x-axis, y-axis, and z-axis respectively.
  - x, y, and z are the scalar components (or rectangular components) along x-axis, y-axis, and z-axis respectively.
  - $x\hat{i}+y\hat{j}+z\hat{k}$ are called vector components of $\overrightarrow{OP}$ along the respective axes.
  - The magnitude of $\overrightarrow{OP}$ is given by $\left|\overrightarrow{OP}\right|=\sqrt{x^2+y^2+z^2}$
- The scalar components of a vector are its direction ratios and represent its projections along the respective axes.

  The direction ratios of a vector $\vec{p}=a\hat{i}+b\hat{j}+c\hat{k}$ are a, b, and c.
  Here, a, b, and c respectively represent projections of $\vec{p}$ along x-axis, y-axis, and z-axis.

Vector Joining Two Points

The vector joining two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, represented as $\overrightarrow{P_1P_2}$, is calculated as

$$\overrightarrow{P_1P_2}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$$



The magnitude of $\overrightarrow{P_1P_2}$ is given by $\left|\overrightarrow{P_1P_2}\right| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Section Formula

If point *R* (position vector $\vec{r}$) lies on the vector $\overrightarrow{PQ}$ joining two points *P* (position vector $\vec{a}$) and *Q* (position vector $\vec{b}$) such that *R* divides $\overrightarrow{PQ}$ in the ratio *m: n* $\left[\text{i.e.} \frac{\overrightarrow{PR}}{\overrightarrow{RQ}} = \frac{m}{n}\right]$

Internally, then $\vec{r} = \dfrac{m\vec{b}+n\vec{a}}{m+n}$

Externally, then $\vec{r} = \dfrac{m\vec{b}-n\vec{a}}{m-n}$

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- The scalar product of two non-zero vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\cdot\vec{b}$ and it is given by the formula $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where *q* is the angle between $\vec{a}$ and $\vec{b}$ such that $0 \leq q \leq$ p

If either $\vec{a} = 0$ or $\vec{b} = 0$, then in this case, θ is not defined and $\vec{a}\cdot\vec{b} = 0$

- The following are the observations related to the scalar product of two vectors:
  - $\vec{a}\cdot\vec{b}$ is a real number.
  - The angle *q* between vectors $\vec{a}$ and $\vec{b}$ is given by,

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$$

- Let $\vec{a}$ and $\vec{b}$ be any two non-zero vectors , then $\vec{a} \cdot \vec{b} = 0$, if and only if $\vec{a} \perp \vec{b}$
- If $q = 0$, then $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$
- If $q = $ p, then $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$
- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1, \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

- If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
- The properties of scalar product are as follows:
  - Commutative property: $\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{a}$
  - Distributivity of scalar product over addition: $\hat{a} \cdot (\hat{b} + \hat{c}) = \hat{a} \cdot \hat{b} + \hat{a} \cdot \hat{c}$

Example: Find the angle between the vectors $8\hat{i} - 4\hat{j} - \hat{k}$ and $3\hat{i} - 6\hat{j} + 2\hat{k}$.

Solution:

Let $\vec{a} = 8\hat{i} - 4\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Angle between $\vec{a}$ and $\vec{b}$ is given by,

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)$$

However, $\vec{a} \cdot \vec{b} = 8 \times 3 + (-4) \times (-6) + (-1) \times 2 = 46$

$$\left|\vec{a}\right| = \sqrt{(8)^2 + (-4)^2 + (-1)^2} = 9$$

$$\left|\vec{b}\right| = \sqrt{(3)^2 + (-6)^2 + (2)^2} = 7$$

$$\therefore \theta = \cos^{-1}\left(\frac{46}{9 \times 7}\right) = \cos^{-1}\left(\frac{46}{63}\right)$$

- Projection of a vector:
  - If $\hat{p}$ is the unit vector along a line *l*, then the projection of a vector $\vec{a}$ on the line *l* is given by $\vec{a} \cdot \hat{p}$.
  - Projection of a vector $\vec{a}$ on other vector $\vec{b}$ is given by $\vec{a} \cdot \hat{b}$ or $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$.

Example: Find the projection of the vector $3\hat{i} - 8\hat{j} + 6\hat{k}$ on the vector $2\hat{i} - 3\hat{j} - 6\hat{k}$.

Solution:

Let $\vec{a} = 3\hat{i} - 8\hat{j} + 6\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} - 6\hat{k}$

Then, the projection of $\vec{a}$ on $\vec{b}$ is given by,

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{(3\hat{i}-8\hat{j}+6\hat{k})\cdot(2\hat{i}-3\hat{j}-6\hat{k})}{\sqrt{(2)^2+(-3)^2+(-6)^2}}$$

$$= \frac{6+24-36}{7}$$

$$= -\frac{6}{7}$$

- The vector product (or cross product) of two non-zero vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\times\vec{b}$ and is defined by $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$, where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$, and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.
- If $\vec{a} = a_1\hat{i} - a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} - b_2\hat{j} + b_3\hat{k}$ are two vectors, then their cross product $\vec{a}\times\vec{b}$, is defined by $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
- The following are the observations made by the vector product of two vectors:
  - $\vec{a}\times\vec{b} = \vec{0}$, if and only if $\vec{a} \parallel \vec{b}$
  - $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

    $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$
    $\hat{j}\times\hat{i} = -\hat{k}, \hat{k}\times\hat{j} = -\hat{i}, \hat{i}\times\hat{k} = -\hat{j}$
  - In terms of vector product, the angle θ between two vectors $\vec{a}$ and $\vec{b}$ is given by $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$ or
  - If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then its area is given as $\frac{1}{2}|\vec{a}\times\vec{b}|$.

Example:
Find the area of a triangle having the points A (1, 2, 3), B (1, –1, – 3) and C (–1, 1, 2) as its vertices

Solution:
$\overrightarrow{AB} = (1-1)\hat{i} + (-1-2)\hat{j} + (-3-3)\hat{k} = -3\hat{j} - 6\hat{k}$
$\overrightarrow{AC} = (-1-1)\hat{i} + (1-2)\hat{j} + (2-3)\hat{k} = -2\hat{i} - \hat{j} - \hat{k}$
The area of the given triangle is $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -6 \\ -2 & -1 & -1 \end{vmatrix}$$

$$= \hat{i}(3-6) - \hat{j}(0-12) + \hat{k}(0-6)$$

$$= -3\hat{i} + 12\hat{j} - 6\hat{k}$$

$$\therefore \left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-3)^2 + (12)^2 + (-6)^2} = \sqrt{9 + 144 + 36} = \sqrt{189}$$

Thus, the required area is $\frac{1}{2}\sqrt{189}$.

- If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram, then its area is given as $\left|\vec{a} \times \vec{b}\right|$.

- The properties of vector product are as follows:
  - Not commutative: $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

    However, $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
  - Distributivity of vector product over addition:
    - $\vec{a} \times \left(\vec{b} + \vec{c}\right) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
    - $\lambda\left(\vec{a} \times \vec{b}\right) = (\lambda \vec{a}) \times \vec{b} = \vec{a} \times \left(\lambda \vec{b}\right)$

Example: If the position vectors of vertices P, Q, R, and S of quadrilateral PQRS are $-\hat{i} + 2\hat{j} + \hat{k}, \hat{i} - 2\hat{j} + 5\hat{k}, 4\hat{i} - 7\hat{j} + 8\hat{k}, \text{ and } 2\hat{i} - 3\hat{j} + 4\hat{k}$ respectively, then find the area of quadrilateral PQRS.

Solution:

$$\overrightarrow{PQ} = (1+1)\hat{i} + (-2-2)\hat{j} + (5-1)\hat{k} \quad = 2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\overrightarrow{QR} = (4-1)\hat{i} + (-7+2)\hat{j} + (8-5)\hat{k} \quad = 3\hat{i} - 5\hat{j} + 3\hat{k}$$

$$\overrightarrow{RS} = (2-4)\hat{i} + (-3+7)\hat{j} + (4-8)\hat{k} \quad = -2\hat{i} + 4\hat{j} + 4\hat{k}$$

$$= -\left(2\hat{i} - 4\hat{j} + 4\hat{k}\right)$$

$$= -\overrightarrow{RS}$$

$$\overrightarrow{SP} = (-1-2)\hat{i} + (2+3)\hat{j} + (1-4)\hat{k} \quad = -3\hat{i} + 5\hat{j} - 3\hat{k}$$

$$= -\left(3\hat{i} - 5\hat{j} + 3\hat{k}\right)$$

$$= -\overrightarrow{QR}$$

Clearly, $\overrightarrow{PQ} \parallel \overrightarrow{RS}$ and $\overrightarrow{QR} \parallel \overrightarrow{SP}$. Hence, PQRS is a parallelogram.

Therefore, area (PQRS) = $\left|\overrightarrow{PQ} \times \overrightarrow{QR}\right|$

Now,

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 4 \\ 3 & -5 & 3 \end{vmatrix}$$

$$= (-12+20)\hat{i} - (6-12)\hat{j} + (-10+12)\hat{k}$$

$$= 8\hat{i} + 6\hat{j} + 2\hat{k}$$

$$\therefore \left|\overrightarrow{PQ} \times \overrightarrow{QR}\right| = \sqrt{(8)^2 + (6)^2 + (2)^2} = 2\sqrt{26}$$

Hence, area of the quadrilateral PQRS is $2\sqrt{26}$ square units.

Simple Applications of Product of Vectors

(a) Resultant of forces acting at a point :

If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$, ..., $\overrightarrow{F_n}$ are *n* forces acting at a point, then their resultant force $\overrightarrow{R}$ is defined as

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \ldots + \overrightarrow{F_n}$

Note:

- Forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$, ..., $\overrightarrow{F_n}$ are said to be in equilibrium if $\overrightarrow{R} = \overrightarrow{0}$.

- The parallelogram law of vectors and the expressions for the magnitude and the direction of the resultant vector are applicable to the forces also.

(b) Resolved part of a force :

Resolved part of $\vec{F}$ along a unit vector $\hat{a}$ is $\vec{F}\cos\theta\,\hat{a}$.

Note:

- The resolved part of $\vec{F}$ along x, y and z axes are $\vec{F}\cdot\hat{i}\,\hat{i}$, $\vec{F}\cdot\hat{j}\,\hat{j}$ and $\vec{F}\cdot\hat{k}\,\hat{k}$ respectively.
- If $\theta=\frac{\pi}{2}$, then $\vec{F}\cdot\hat{a}=0$. Hence, the resolved part of a force along a direction perpendicular to itself is zero.
- The sum of the resolved parts of a number of forces acting at a point along any direction is equal to the resolved part of their resultant along the same direction.

(c) Work done by a force :

The work done by the force $\vec{F}$ during displacement $\vec{r}$ is defined as

$$w = \vec{F}\cdot\vec{r}$$

Note:

- The work done by a force is a scalar quantity.
- The work done by a force in displacing the particle perpendicular to its own direction is zero.
- Total work done during some displacement by a number of forces acting on a particle is equal to the work done by the resultant force during the same displacement.

(d) Moment of a force about a point :

The tendency of a force that causes a body to rotate about a specific point is called moment of the force.

Let a force $\vec{F}$ act at a point A on the body. If it causes rotation of the body about B, the vector $\vec{BA}\times\vec{F}$ is called the moment of the force about the point B.

i.e. $\vec{M} = \vec{BA}\times\vec{F} = \vec{r}\times\vec{F}$

Note:

- Moment of force $\vec{M}$ is a vector quantity.
- $\vec{M} = \vec{BA}\times\vec{F} = \vec{BA}\,\vec{F}\sin\theta$
- $\vec{M}$ is perpendicular to the plane of $\vec{BA}$ and $\vec{F}$.
- If B lies on the line of action of $\vec{F}$, i.e. if $\vec{BA}\parallel\vec{F}$, then $\vec{M}=\vec{0}$.
- The moment of $\vec{F}$ about B is independent of the choice of A, i.e. A can be any point in the line of action of $\vec{F}$.

Varignon's Theorem : The sum of moments of a number of concurrent forces about any point in their plane is equal to the moment of their resultant about the same point.