Mathematics (Code–041)

FOR CLASSES IX-X

Project Based Learning

Developing children's abilities for Mathematisation is the main goal of mathematics education. The narrow aim of school mathematics is to develop 'useful' capabilities, particularly those relating to numeracy–numbers, number operations, measurements, decimals and percentages. Aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and the ability and the attitude to formulate and solve problems. *–NCF 2005*

Project Based Learning engages students in applications of Mathematics, which may help them to transfer their Mathematical knowledge and skills to other disciplines and to solve the real-world problems. Teaching scholastic subjects through projects has been one of the methods to generate interest in the subject and make school students actively involved and enthusiastic about the subject. It allows students to decide and apply mathematics problem-solving strategies and skills to real life situations. This also enables them to master mathematical concepts and to retain the knowledge gained to a longer period, avoiding the short-term gains of rote memorisation. Students gain valuable skills in ***problem-solving, reasoning and communicating ideas in Mathematics*** while learning, how to conduct research, manage resources, and collaborate with others, which are some of the prerequisite skills of the work place/life today.

The important segment of mathematics - the ability to reason and think clearly - is extremely useful in every aspect of life. It deals with data, measurement, observations from science, mathematical models of natural phenomenon including human behaviour and social system, Mathematics offers a distinctive mode of thought which is versatile and powerful, including mathematical modeling, optimisation, logical analysis, inference from data and use of symbols.

Mathematics is a powerful way of investigating and understanding the world.

Some problems in school Mathematics education

1. A majority of children have a sense of fear and failure regarding Mathematics. Hence, they give up early on, and drop out of serious mathematical learning.
2. The curriculum is disappointing not only to this non-participating majority, but also to the talented minority by offering them no challenges.
3. Problems, exercises and methods of evaluation are mechanical and repetitive with too much emphasis on computation. Areas of Mathematics such as spatial thinking are not developed enough in the curriculum.
4. Teachers lack confidence, preparation and support. –NCF 2005

In order to overcome such problems, schools can adopt project-based learning as one of the methodologies. This innovative approach would develop interest in the subject and create happiness in children. Activities under Project Based Learning, offer the greatest experiences for learning and hands-on, minds-on tasks i.e., students are actively engaged physically and mentally. It provides an opportunity for students to learn to be a part of a team and communicate effectively with others.

Aims and Objectives

The main objectives of project based learning in Mathematics are to:

- create awareness about association of Mathematics with various changes and challenges that are faced in life.
- popularise the interdisciplinary approach of teaching – learning of Mathematics in the context of resolving real life issues and problems.
- sensitise teacher/students regarding the use of Mathematics in identifying and predicting various aspects of environment.
- encourage the problem-solving approach in various aspects of real life issues and problems.
- develop creative thinking, habit of exploration and manipulative skills in the students through their self-devised projects.

Project-Based Learning in Mathematics will help students

- inculcate a spirit of enquiry and research.
- develop confidence, self-direction and time management through independent work.
- encourage collaboration and cooperative learning.
- obtain a deeper knowledge of the subject.
- engage themselves in the applications of Mathematics.
- develop lasting interest in Mathematics discipline.
- grow into logical, rational and analytical individuals.

Guidelines to Teachers and Schools

- Define the project and the objectives carefully and clearly. Have the objectives aligned with both process and content standards.
- Give students a timeline so that they know exactly what is expected and when the project is due to be submitted. Monitor the work status, reports, etc.
- Design an assessment plan in advance. Share the rubrics with the students before they begin the project. If possible, show students samples of what we expect, including project documentation and the end product.
- Consider teaming-up with teachers in other subject areas.
- If students need a particular skill for the project (such as graphing data) teach mini-lessons along the way.
- Provide appropriate resources to the student like: web sites, books, people to guide, computer software, including various programmes for helping students to present their projects.
- Give students enough time in the class to complete the necessary steps such as brainstorming, writing an outline, drafting a report, editing and revising the report of the peers.
- Provide specific feedback regarding their ideas and plans, to execute the project.

Guidelines to students

Project-based learning engages a student with an in-depth investigation of a Mathematical Concept, which leads to critical thinking and reasoning skills.

Project-based learning in Mathematics requires students to

- develop a plan to solve Mathematical problems posed by a project.
- collect information from a variety of sources.
- develop a plan for solving the problem.
- analyse all data and information collected.
- carry out the plan.
- communicate findings as to how the problem was solved to complete the project.

Learning outcomes

Students are able to–

- retain content and change their attitudes towards learning.
- enhance their creative skills and critical thinking.
- improve their problem-solving and collaborative skills.
- inculcate the use of higher - order thinking skills.
- develop meaningful learning and connects it to their past performances.
- express his/her ideas and opinions freely.
- gain a deeper understanding of the concepts and basic facts.
- address community issues, explore careers and interact with adult mentors.
- use technology, and present their work beyond the classroom.

Assessment approach

Allocation of marks (10)

The assessment can be done under following heads:

Parameter	Observation points	Marks
Topic/content	Selection of topic, Content concepts	1
Originality	Collection of data and information	2
Presentation	Creativity, Method of presentation, Methodology	2
Analysis and suggestions	Logical reasoning, Analytical skills Ability to draw Inferences	2
Conclusion	Inference and Solutions	1
Viva-Voce	Oral skills /presentation of project to class	2
	Total	**10**

Mathematics (Code-041)
Exemplary Project
Class-IX

Project 1: "Pick and ride your dream car using Linear Equations in 2 variables".

Key Concepts

- A linear equation is made up of two expressions set equal to each other.
- It has one or two variables.
- No variable in a linear equation is raised to a power greater than 1 or used as the denominator of a fraction.
- When you find pairs of values that make the linear equation true and plot those pairs on a coordinate grid, all the points for any one equation lie on the same line.
- Linear equations graph as straight lines.
- A linear equation in two variables describes a relationship in which the value of one of the variables depends on the value of the other variable.
- In a linear equation in x and y, x is called the independent variable and y depends on it.
- When you assign a value to the independent variable x, you can compute the value of the dependent variable y. You can then plot the points named by each (x, y) pair on a coordinate grid.
- An equation of the type $y = m\,x$, represents a line passing through the origin.

Gist of the project

Students will compare the prices of various automobiles and their mileage, to decide which automobile would be best to purchase when they get their driver's license. They will calculate cost using mileage, budgeting, graphing, writing and solving equations, x and y intercepts.

Linear equations are one of the fundamentals of upper-level mathematics. They have many mathematical applications, as the formulas that many are familiar with, are just linear equations with specific variables.

Introduction

There are many aspects that need to be considered when purchasing a car, such as mileage, lifestyle, depreciation, affordability, etc. Purchasing a car requires that you save and budget your money so that you can afford it, when the time comes. You will need a down payment, insurance coverage, sales tax, licenses before you even purchase the vehicle! You may not be ready to purchase a vehicle right now, but you may need to start planning and preparing for the purchase now.

Historical background

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable.

Linear equations can have one or more variables. Linear equations occur with great regularity in applied mathematics. They arise quite naturally when modeling many phenomena, they are particularly useful. Since many non-linear equations may be reduced to linear equations by assuming that quantities of interest vary, to only a small extent, from some "background" state.

Aims and objectives

Students will be able to:

- ☆ understand, frame and solve linear equations in two variables.
- ☆ solve real life problems involving the concept of linear equations.
- ☆ make graphical representation of linear equations in two variables and their interpretation.
- ☆ develop oral and written communication skills and confidence.

Essential questions

1. How will a person choose to buy a car so that it fits his/her style and budget?
2. What is budget and affordability?
3. What is a down payment?
4. What aspects need to be considered while buying a car?
5. What equations need to be framed and how do we solve them?
6. How do you do a cost comparison on the vehicles?

Procedure/methodology/project design

1. Each student will document the miles he/she has travelled in an automobile for one week.
2. Tabulate the information as under

No. of weeks	1	2	3	4	8	12	16	20	24	28	32	36	40	44	48	56
Miles																

   a) Do you see a pattern in the table? If so, what is it?

   b) Write an equation using the table if applicable.

3. Your job is to find a vehicle that fits to your budget and lifestyle.
4. To do this, each group will survey car showrooms of different companies.
5. You will compare the cost and attributes of various vehicles within your group, based on the criteria that are important to you.

6. Some of the comparisons you make will require mathematical calculations, writing /solving equations and creating/interpreting graphical representations.
7. Now let each student document the number of litres of fuel consumed, if he/she travelled in an automobile for one year and tabulate this information as under.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Litres												

   a) How did you use the miles you drive, to find the litres of fuel used?
   b) Do you see a pattern in the table? If so, what is it?
   c) Write an equation using the table if applicable.
   d) What do you need to know to calculate the fuel costs?

8. Calculate the fuel cost and tabulate it as under

Month	1	2	3	4	5	6	7	8	9	10	11	12
Cost of fuel												

   a) Write a linear equation to find the total cost of fuel for any given time.
   b) What do you need to find the total cost of the vehicle?

9. Calculate the cost of the vehicle and tabulate it as under

Years	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Cost of vehicle											

   a) How did the initial purchase of the vehicle affect the total cost?
   b) Write a linear equation to find the total cost of your vehicle.
   c) Graph the total cost of your vehicle over the year. Be sure to label the axes, include a title and be consistent with your numbering. Each group member must use a different colour.

10. Use your research and calculations to create a proposal to convince your parents that you should get this particular brand vehicle.

Tools used

Survey method, questionnaire, opinion poll etc.

Technology

- A computer-generated document

Analysis and Data Interpretation

Individual Graph and Equations

The individual graph includes

- correct equations for graphs.
- initial cost.
- vehicle cost (including gas) plotted correctly over the year.
  - each group member used a different colour when graphing
  - labels for title, scale and for each axis.

Group Graph and Equations

The group graph includes the following:

- Each group member's individual graph is correctly combined on one graph.
- Graph colours are consistent with individual graph colours.
- Each graph is labelled with its corresponding linear equation.

Suggestions

- Data collected by the students must be authenticated through showroom visits.
- Change in prices of fuel must be adjusted while finding the cost over the year.
- The pattern must be clearly understood, framing and solving of equation must be correct to get the desired result.

Learning Outcomes

Students will–

- understand, frame and solve linear equations in two variables.
- solve real life problems involving the concept of linear equations.
- prepare a graphical representation of linear equations in two variables and their interpretation.
- plan to select a car which fits in their budget and suits their requirement.

Web references

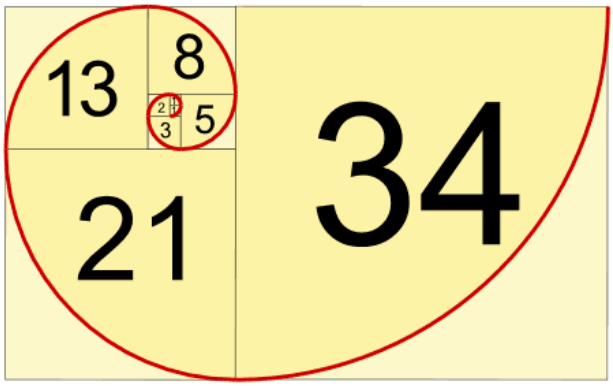
- http://www.bie.org
- http://www.google.co.in/webhp?source=search_app#hl=enandsclient=psy-abandq=linear+equations+in+two+variablesandoq=Linear+Equationsandgs_l

Mathematics (Code-041)
Class-IX
Suggested Projects

CHAPTER–1: NUMBER SYSTEMS

Key Concepts

- The Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 . . .) occurs throughout the worlds of nature, art, music and mathematics.
- The sequence takes its name from a famous thirteenth-century European mathematician, Leonard of Pisa (1170-1250) also called Fibonacci.
- Each term in the series is produced by adding together the two previous terms, so that 1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5 and so on.
- The division of any two adjacent numbers gives the amazing Golden Number e.g. 34 / 55 = 0.618 or inversely 55 /34 = 1.618.



- When you make squares with the widths of fibonaci series, you get a nice spiral. You see how the squares fit neatly together. For example 5 and 8 make 13, 8 and 13 make 21 and so on.

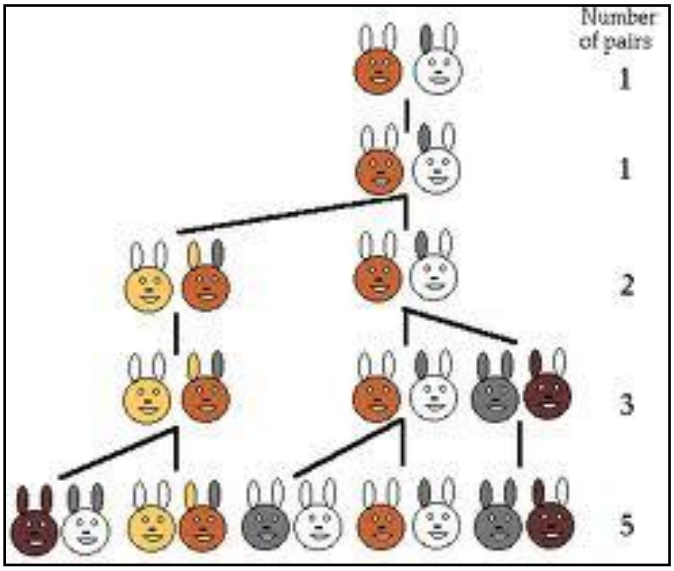
Aims and Objectives

Students will–

- explore Fibonacci numbers.
- locate Fibonacci numbers in nature.
- know about the mathematician who gave the concept.
- find connection between Fibonacci numbers and the aspects of engineering, architecture, painting and music.
- understand how Fibonacci spirals are helpful in the formation of galaxies.

Essential Questions

1. Define Fibonacci numbers.
2. Who was Fibonacci?
3. How were Fibonacci numbers discovered?
4. List first 26 Fibonacci numbers.
5. Explore in nature the things that correspond to Fibonacci numbers by making pictures.



Procedure/Methodology

1. For each number in the Fibonacci sequence, add all the previous Fibonacci numbers. Is there a pattern to the result?
2. Add alternate Fibonacci numbers starting with 0 and 1, with 1 and 2.

   Compare the results.
3. Create a table to record the following results
   a) Multiply each Fibonacci number with the next one in the sequence.
   b) Multiply each Fibonacci number with itself. Add the squared Fibonacci numbers. Create a table and compare the results.
4. Create a table and list the first thirty numbers in the Fibonacci numbers. Use different colours to shade the multiples of the Fibonacci numbers 2,3,5,8.

   Does a pattern emerge?

   Choose one of the research topics below and discover how it relates to Fibonacci sequence.

   a) The Golden Ratio
   b) The Lucas Sequence

c) Perfect Rectangles
d) Fibonacci Spirals
e) The Mona Lisa
f) Fibonacci Rabbit Problem

Learning Outcomes

The students will–

- ☆ understand Fibonacci Numbers and will explore the same in nature.
- ☆ get to know about the mathematician who gave the concept.
- ☆ relate Fibonacci Sequence to several topics.

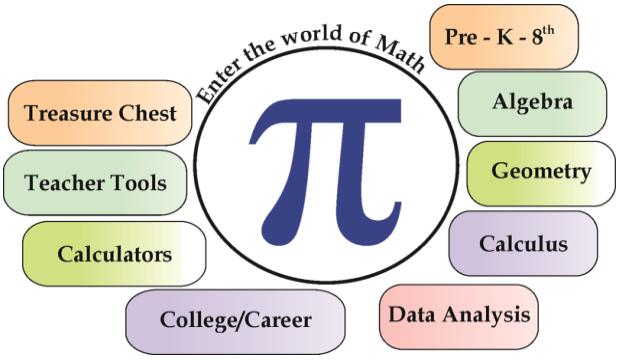
CHAPTER–2: NUMBER SYSTEMS

Key Concepts

- ☆ A number can be placed into several categories based on its properties.
  - Is it prime or composite?
  - Is it imaginary or real?
  - Is it transcendental or algebraic?
- ☆ These questions help define a number's behaviour in different situations. In order to understand where π fits in to the world of mathematics, one must understand several of its properties: π is irrational and π is transcendental.
- ☆ π is defined as the 16th letter of the Greek alphabet.
- ☆ The symbol 'pi' denotes the ratio of the circumference of a circle to its diameter: the ratio itself.
- ☆ Transcendental number, value to eight decimal places is 3.14159265.

Aims and Objectives

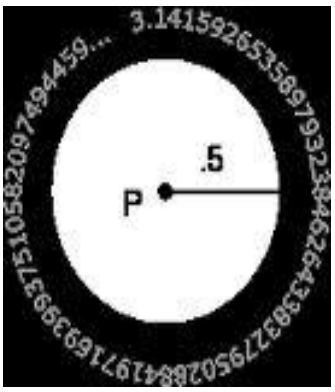
- ☆ To know about $\boldsymbol{\pi}$
- ☆ Investigate the various historical aspects of the number $\boldsymbol{\pi}$



Essential Questions

1. What is the sixteenth letter of the Greek alphabets?
2. What is the value of π?
3. Who gave the concept of π?

Procedure/Methodology



1. Find out about π in various ancient civilizations.
2. Give different approximations for π.
3. Relate circle with π.
4. State the famous mathematical problems featuring π.

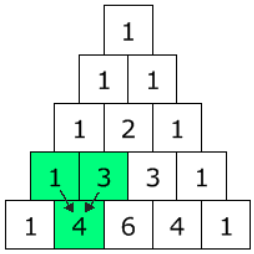
Learning Outcomes

The students will be able to–

- collect study material on the history of mathematics.
- organise their results and findings.
- describe all the resources collected to compile the data.

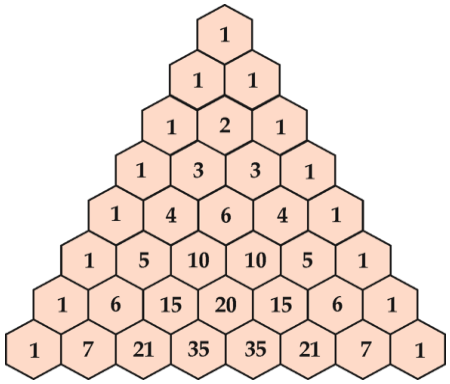
CHAPTER–3: POLYNOMIALS

Key Concepts



- One of the most interesting number patterns is Pascal's Triangle (named after *Blaise Pascal*, a famous French Mathematician and Philosopher).
- To build the triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern.
- Each number is just the two numbers above it added together (except for the edges, which are all "1").

Aims and Objectives



Students will be able to:

- know about Pascal.
- explore Pascal's Triangle (take up to 10 rows) and develop generalised patterns.
- find their existence in nature.
- relation of Pascal's triangle to identities.

Essential Questions

1. What is Pascal's Triangle? What is the history behind it?

2. How to construct different rows of Pascal's triangle?
3. How is Pascal's triangle useful in deriving different identities like
   a) $(a \pm b)^2$;
   b) $(a \pm b)^3$

Procedure/Methodology

1. Explore the history of Pascal's Triangle and present the same.
2. Sum up the properties of Pascal's Triangle as given below:
   (a) The sum of the numbers in any row is 2n, where n is the number of the row.
   (b) Property related to prime number.
   (c) Hockey stick pattern.
   (d) Fibonacci sequence located through Pascal's triangle.
3. Make a model of Pascal's Triangle.

Learning Outcomes

Students will be able to–

- ☆ identify growing numeric/non-numeric patterns.
- ☆ describe growing numeric/non-numeric patterns.
- ☆ extend growing numeric/non-numeric patterns.

CHAPTER–4: COORDINATE GEOMETRY

Key Concepts

- ☆ In coordinate geometry, points are placed on the 'coordinate plane'.
- ☆ It has two scales - one running across the plane called the 'x axis' and another at a right angles to it, called the 'y axis'.
- ☆ The point where the axes cross is called the **origin** and is where both x and y are zero.
- ☆ On the x-axis, values to the right are positive and those to the left, are negative.
- ☆ On the y-axis, values above the origin are positive and those below are negative.
- ☆ A point's location on the plane is given by two numbers. The first tells where it is on the x-axis and the second which tells where it is on the y-axis. Together, they define a single, unique position on the plane.

Aims and Objectives

Students will–

- ☆ know about mathematicians who worked on and developed co-ordinate system.

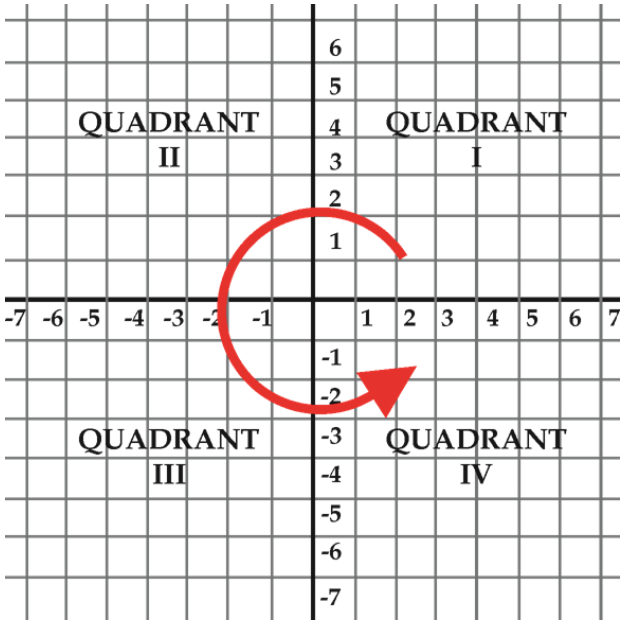
- ☆ know about coordinate axes, origin, quadrants, abscissa and ordinate.
- ☆ understand the co-ordinate system and its use in everyday real life.
- ☆ have fun with co-ordinate system.

Essential Questions

1. Identify thr mathematician who introduced the co-ordinate system.
2. What do you mean by origin?
3. Can you locate a point with reference to origin?
4. Describe the location of various landmarks of your locality in your own words.

Procedure/Methodology

1. Take a graph paper and draw a coordinate axis. Place each counter at (0, 0). No overshooting is allowed i.e. neither the abscissa nor the ordinate can be > 15.
2. Except (0, 0), coinciding is not allowed.
3. If a move results in overshooting, the player misses that turn.
4. If only one of the coordinates arrives at 15, the player counter goes to (0,0).
5. Each player throws 2 dice simultaneously. Record the numbers on each dice.
6. Prepare the table for both players and move the counters on graph accordingly.
7. The table must include the following things for each throw and for each player:
   a) number on white dice(x),
   b) number on green dice(y),
   c) current position of counter on graph (x, y)



8. Repeat the steps until a player reaches (15,15).
9. An interesting game with the help of coordinate geometry.
10. A game for 2 persons to arrive at (15, 15) using 2 dice and graph paper with given conditions.

Learning Outcomes

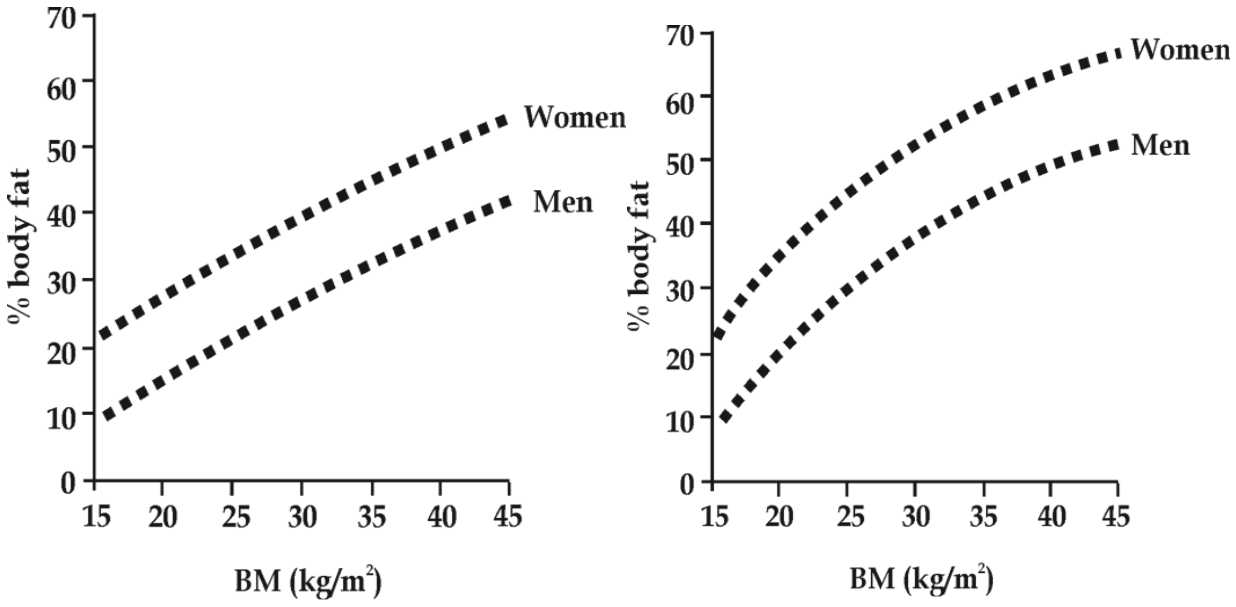
The students will be able to–

- ☆ understand the Cartesian system.
- ☆ locate the coordinates of the plotted point.
- ☆ plot the points on the graph.
- ☆ explore how coordinate system was developed.
- ☆ locate the position of the seats in the classroom with reference to rows and columns.

CHAPTER–5: LINEAR EQUATIONS IN TWO VARIABLES

Key Concepts

- ☆ A linear equation is made up of two expressions set equal to each other.
- ☆ Linear equations graph as straight lines.
- ☆ A linear equation in two variables describes a relationship in which the value of one of the variables depends on the value of the other variable.
- ☆ An equation of the type $y = m\,x$, represents a line passing through the origin.
- ☆ Know about the relationship between weight and height for males and females.
- ☆ BMI (Body Mass Index) and its relation with health status of an individual.



Aims and Objectives

Students will be able to-

- ☆ know about the association between a range of health and nutrition indicators.
- ☆ identify Body Mass Index (BMI) trajectories and to describe their association with subsequent academic and cognitive outcomes.
- ☆ investigate the health of students of a class based on the Body Mass Index (BMI).

Essential Questions

1. What is BMI?
2. What are the factors affecting BMI?
3. Is BMI different for males and females?
4. Can the BMI for males and females be represented in linear equations in two variables?
5. What do the variables in your equation represent?
6. What does the equation represent?
7. Was your data positively correlated, negatively correlated or neither?

Procedure/Methodology

1. Survey your neighbourhood (twenty families), find out their weight (in kg) and height (in m). Establish the relationship between height and weight for males and females separately. Draw your own conclusion.
2. Calculate their BMI and intimate their health status.
3. Repeat the same for the students of your class.
4. Establish the relationship between height and weight for boys and girls separately
5. Is it linear or not? Explain?
6. Give possible explanations for the relationships or absence of relationships that you see in the data.

Learning Outcomes

Students will be able to–

- ☆ establish the relationship between height and weight for males and females separately.
- ☆ determine whether or not they may have weight problem (Unhealthy over weight puts us at increased risk for heart disease, stroke and diabetes in future.)

CHAPTER–6: INTRODUCTION TO EUCLID'S GEOMETRY

Key Concepts

☆ In Euclidean geometry, a **Platonic solid** is a regular, convex polyhedron. The faces are congruent, regular polygons, with the same number of faces meeting at each vertex. There are exactly five solids which meet these criteria; each is named according to its number of faces.

Tetrahedron (four faces)	Cube or hexahedron (six faces)	Octahedron (eight faces)	Dodecahedron (twelve faces)	Icosahedron (twenty faces)
				

☆ They are named after the ancient Greek philosopher Plato .

☆ Euler's Formula.

For any polyhedron *that doesn't intersect itself*

F + V - E = 2 where F, V, E stand for Number of Faces, Vertices (corner points) and Edges respectively.

Aims and Objectives

Students will be able to–

☆ know about Plato.

☆ explore platonic solids in real world.

☆ verify Euler's Formula.

☆ construct the five Platonic Solids of Euclid's *Elements*.

Essential Questions

1. What are Platonic solids?
2. How are the Platonic solids used as art motifs?
3. What is Euler's formula?
4. Can you verify Euler's formula for all platonic solids?

Procedure/Methodology

1. Mention five platonic solids and its properties.
2. Explore/learn the history of platonic solids.

3. Write the procedure of making Platonic solids.
4. Make the same using transparent sheet.
5. Tabulate the number of faces, vertices and edges of each solids.
6. Verify Euler's formula for each of the solid.

Learning Outcomes

Students will be able to–

- ☆ describe the making of platonic solids.
- ☆ make platonic solids.
- ☆ verify Euler's Formula for each of the solid.

CHAPTER–7: LINES AND ANGLES

Key Concepts

- ☆ In order to continue the study of geometry, it is necessary to learn and recapitulate the different properties of lines, angles and their measures. Different classifications of angles (acute, obtuse, right, or straight) as well as some special properties of pairs of angles.
- ☆ If a ray stands on a line, then the sum of two adjacent angles so formed is 180^0.
- ☆ Conversely if the sum of two adjacent angles is 180^0, then a ray stands on a line (i.e., the non-common arms form a line).
- ☆ If the sum of two adjacent angles is 180^0, then the non-common arms of the angles form a line. It is called Linear Pair Axiom.
- ☆ If two lines intersect each other, then the vertically opposite angles are equal.

Aims and Objectives

Students will be able to–

- ☆ explore Lines, Angles, Triangles and their properties.
- ☆ understand and use Angles, Lines and Triangles in a Geometrical map.

Essential Questions

1. How are lines and angles related to each other?
2. How are lines and angles used in practical life?

Procedure/Methodology

1. Design a map that includes different kinds of lines, angles and triangles.
2. Your proposed map can be of a town, your neighborhood or a made up place. It must however include the following:

a) Two sets of streets that are parallel

b) Two sets of streets that are perpendicular

c) One street that intersects another street to form an obtuse angle

d) One street intersects another to form an acute angle

e) One street that is a line segment

f) One street that is a line

g) One street that is a ray

h) An ice cream parlour in the shape of an equilateral triangle

i) A pool that is in the shape of a scalene triangle

j) A pizza place in the shape of a isosceles triangle

k) A circular fountain area

Once your map is complete, write five directions from one place to another. Each direction must have one of these terms: parallel, intersecting or perpendicular.

3. Using different colours, show the land marks on the map representing:

   a) a pair of corresponding angles

   b) a pair of alternate interior / exterior angles

   c) a pair of co-interior angles.

   d) linear pair of angles

   e) vertically opposite angles

4. Find out how angles are used in the professional field e.g. construction workers, surgeons, athletes and military personnel.

5. Learn about the contribution of Russian Painter Kazimir Malevich to Geometric arts.

Learning Outcomes

Students will be able to–

- ☆ understand the concept of lines and angles.
- ☆ consolidate the concept by identifying/marking different pairs of angles on a locality map.
- ☆ apply the concept learnt in practical life.

CHAPTER–8: TRIANGLES

Key Concepts

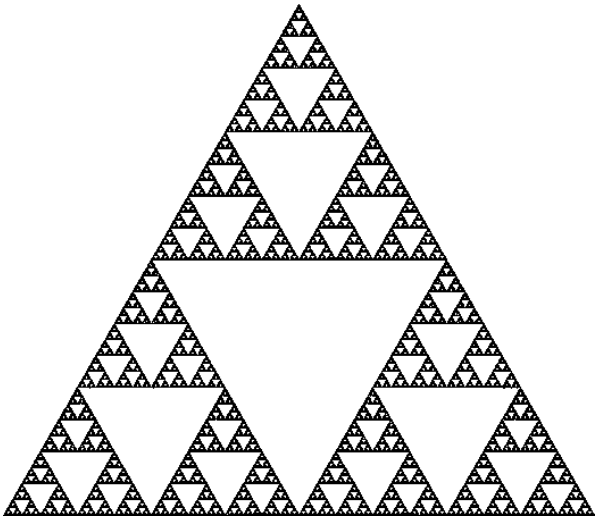
- ☆ Sierpinski triangle is a fractal named after Waclaw Sierpinski who described it in 1915.
- ☆ The Sierpinski triangle or the Sierpinski carpet is equivalent repetitive tiling arrangements, it is evident that similar structures can be built into any reptile arrangements.

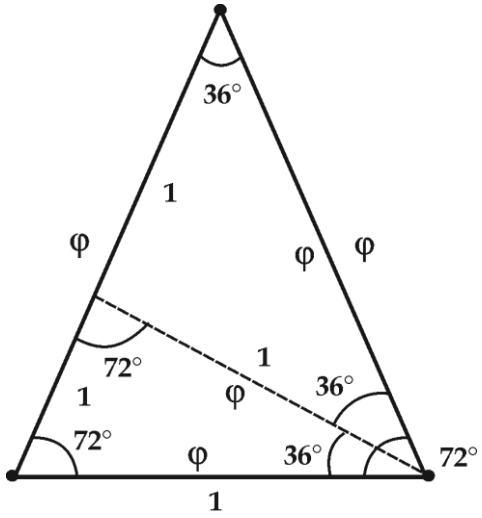
- Golden triangles are found in the nets of several stellations of dodecahedrons and icosahedrons
- A golden triangle, also known as the sublime triangle, is an isosceles triangle in which the smaller side is in golden ratio with its adjacent side.

Aims and Objectives

To understand and use–

- Scavenger triangle hunt
- Sierpinski Triangle
- Golden Triangle





Essential Questions

1. Name famous individuals associated with the theories of triangles.
2. What is the importance of triangular shaped structures in construction?
3. What is Sierpinski Triangle"?
4. What is "Golden Triangle"?

Procedure/Methodology

1. The students work in groups for this project.
2. Study the biographies of the famous individuals associated with the theories of triangles.
3. Find the basic components of Napoleon's theorem or Morley's theorem.
4. Explore the history of how people have used triangles throughout time.
5. Write a research paper using reputable references that can either be submitted as a report, or PowerPoint Presentation.
6. Find out how triangles help to make structures stronger and how they are used in construction.

7. Find a building which has used triangles in its composition, either in its structure or in its facade.
8. Make a paper model of the building, highlighting its use of triangles and include a scale with the finished product.
9. Find out a list of triangular items, their angles or side lengths common to your classroom or school grounds.
10. Explore and draw "The Sierpinski Triangle" and "Golden Triangle"

This project works well for individuals and teams alike.

Learning Outcomes

Students will be able to–

- ☆ carry out practical and investigational works and undertake extended pieces of work.
- ☆ use mathematical and other instruments to measure and construct figures to an acceptable degree of accuracy.
- ☆ use mathematics in daily life by recognising and applying appropriate mathematical problem-solving strategies.
- ☆ recognize and use patterns, relationships and sequences and make generalisations.

CHAPTER–9: CIRCLES

Key Concepts

- ☆ A circle is a shape with all points the same distance from its centre. A circle is named by its centre. Some real world examples of a circle are a wheel, a dinner plate and (the surface of) a coin.
- ☆ Half a circle is, a closed shape consisting of half a circle and a diameter of that circle
- ☆ The area of a semicircle is half the area of the circle from which it is made. Recall that the area of a circle is πR^2, where R is the radius.

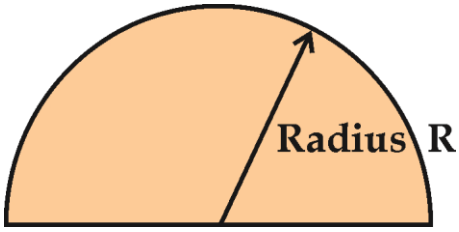
  So, the formula for the area of a semicircle is:

$$\text{area} = \frac{\pi R^2}{2}$$

  where:

  R is the radius of the semicircle

  π is Pi, approximately 3.142



Aims and Objectives

Students will be able to–

- ☆ understand circles and perimeters of semi circles.

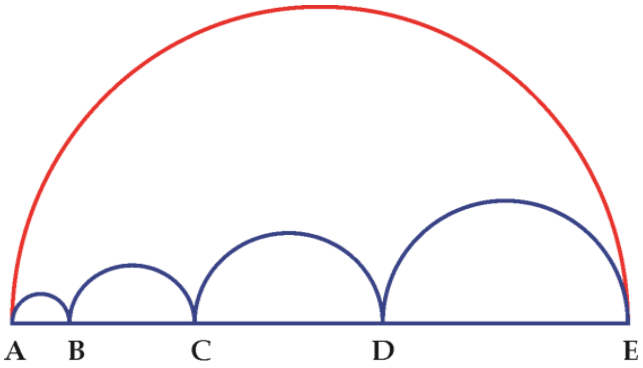
- prove that the sum of the perimeters of the inscribed semicircles is equal to the perimeter of the outside semicircle.

Essential Questions

1. What is the perimeter of a semicircle?
2. What is the area of a semicircle?

Procedure/Methodology

1. Draw a figure with semicircle (AE) with a series of smaller semicircles (AB, BC, CD, DE,) constructed inside it.
2. Ensure that the sum of the diameters of the four smaller semicircles is equal to the diameter of the large semicircle.
3. Establish the relationship between the area and perimeter of the larger semicircle and the sum of the four smaller semicircles.



4. In addition to your background research and your proof, make a model of the figure with coloured paper. Use a compass and ruler to construct the semicircles. Cut pieces of string or yarn equal to the arc-lengths of the semicircles. You can use these to demonstrate that the perimeter lengths are indeed equal. Repeat for 3 different measurements of semi circles.
5. Tabulate your findings mathematically, prove the result.
6. Present your work in handwritten pages.

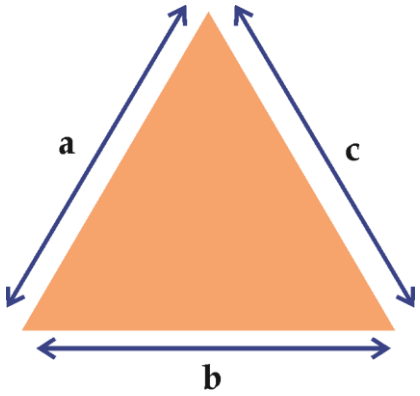
Learning Outcomes

The students will be able to–

- define circle, circumference.
- recognise that 'Pi' is constant for any circle.
- restate the formula and procedure for finding the circumference/ area of a circle.
- recognise that a circle has many different radii and diameters passing through its centre.
- compute the circumference and area of a circle, given its diameter/ radius.
- give examples of circumference/ area of circle from the real world.
- apply circumference and area concepts and formulas to solve real-world problems.
- recognise the difference between circumference and area of a circle.

CHAPTER–10: HERON'S FORMULA

Key Concepts

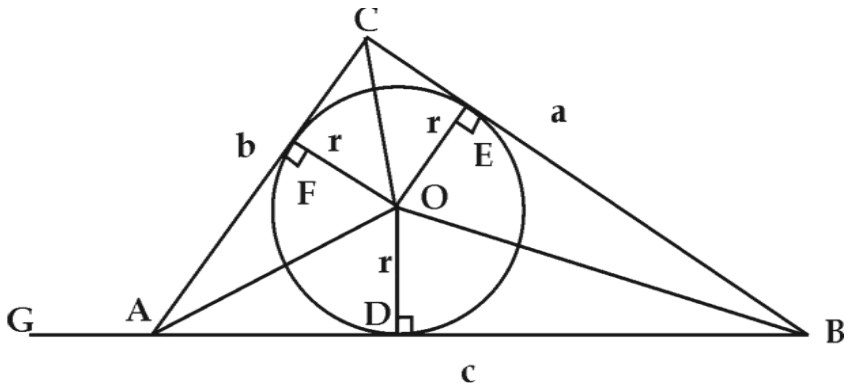


- Heron's Formula
- Area of a triangle from its sides
- You can calculate the area of a triangle if you know the lengths of all three sides, using a formula that has been known for nearly 2,000 years
- It is called "Heron's Formula" after Hero of Alexandria.

Aims and Objectives

Applications of Heron's Formula in different triangles to–

- understand the incentre of a triangle.
- make conjectures and describe relationships between the area of triangles using Heron's theorem and the traditional formula.



Essential Questions

1. Does Heron's Formula work for all acute, right, obtuse triangles?
2. Why was this formula discovered to find the area of triangle?

Procedure/Methodology

1. Find the in centre to develop part of Heron's Formula.
2. Join the in centre to the three vertices to get three smaller triangles.
3. Find the altitude to a side and use the formula.

   A = ½ b x h for calculating the area of three smaller triangles.
4. Find the area of original triangle.
5. Find the relation between area of original triangle and the smaller triangles.

6. Establish the same relation for different types of triangles.
7. Considering one side of a triangle as a diagonal, draw a rectangle and establish the relation that the area of triangle is half the area of rectangle.

Learning Outcomes

Students will be able to–

- ☆ find out the in centre of a triangle.
- ☆ apply Heron's Formula in different types of triangles.
- ☆ find out the area of a quadrilateral using Heron's Formula.

CHAPTER–11: SURFACE AREAS AND VOLUMES

Key Concepts

Formula for calculation of area

Square	$side^2$
Rectangle	length × width
Parallelogram	base × height
Triangle	base × height / 2
Circle	pi × $radius^2$
Cube (surface)	6 × $side^2$
Sphere (surface)	4 × pi × $radius^2$
Cylinder (surface of side)	perimeter of circle × height/length
	2 × pi × radius × height/length
Cylinder (whole surface)	Areas of top and bottom circles × Area of the side
	2(pi × $radius^2$) + 2 × pi × radius × height pi × radius × side
Cube	$side^3$
Sphere	(4/3) × pi × $radius^3$
Cylinder	pi × radius2 × height/length
Cone	(1/3) × pi × $radius^2$ × height/length

Aims and Objectives

Students will be able to–

- ☆ familiarise themselves with formulas for area and volume of different shapes .
- ☆ apply these formulas in daily life.
- ☆ design and draw an eco-friendly house/floor plan with a given area using appropriate scale.
- ☆ relate with quality and cost of construction.

Essential Questions

1. What would your dream home look like?
2. What are the materials required to make an eco-friendly and beautiful house?
3. What are the different 2-D and 3-D shapes you see in a house?
4. What are the details regarding locations of different rooms, space ideas and interiors?
5. How do you construct and interpret scale drawings?
6. How do you apply the principles of ratio, proportion, and symmetry in a scale drawing?

Procedure/Methodology

1. Explore and list quality and quantity of the material required for construction of an eco-friendly house.
2. Bring architectural blueprints or floor plan drawings from home design magazines.
3. Discuss the size of your dream house with the class, review square footage, the key unit of measure used by architects.
4. Discuss the available money and building codes (restrictions or requirements about what can be included in a new building) which plays a big role in the way architects design a house.
5. Keep in mind that while you have many choices in designing the space, you are required to explore and follow the building norms listed by the government.
6. Discuss the required items and options that can be included in the house.
7. Reread the project and think about how you want to create the space according to the guidelines.
8. Practice drawing any geometric shapes of your choice for your house, on the grid paper. Also, tabulate their surface areas and volumes.

9. Draw room designs and spaces using the grid paper and the scale 2 feet = ¼ inch or any other appropriate scale.
10. Have a set of symbols to use for the floor plan drawings.
11. Sketch out a rough draft floor plan for your dream home on a sheet of paper. Label all walls and rooms with dimension sizes.
12. Compare the construction cost of an eco friendly house to a normal house.

Learning Outcomes

The students will be able to–

- ☆ understand the concept of surface area and volume and their application in real life.
- ☆ apply the concept in designing an eco-friendly house/floor plan with given area using appropriate scale.

CHAPTER–12: STATISTICS

Key Concepts

- ☆ Graphs are pictures that help us understand amounts. These amounts are called data. There are many kinds of graphs, each having special parts.
- ☆ Histograms represent a popular means for feature representation.
- ☆ Histogram is a graphical presentation of substantial information grouped into data sets or classes, for which frequencies or occurrences are counted.

Aims and Objectives

Students will be able to–

- ☆ about primary and secondary data.
- ☆ familiarise themselves with the collection and tabulation of data.
- ☆ apply the statistics in daily life.
- ☆ understand the concepts of statistics and use them to find the rate of growth of population.

Essential Questions

- ☆ How fast is our population growing?
- ☆ Has it always grown at the same rate?
- ☆ Is the population of different countries/ states growing differently?
- ☆ How can we use statistical data and graphs to chart and understand this growth?
- ☆ How can we predict the population in the future?

Procedure/Methodology

1. Collect data on the population of your state and country for at least last 5 years from the internet or any other reliable source.
2. Tabulate the above collected information and draw inferences regarding the increase or decrease of the same.
3. Draw a line graph depicting the growth of population in your state as well as for your country.
4. Explore the factors promoting the growth.
5. Now, conduct a Group discussion/slogan writing activity on "Should population growth be brought under control?"
6. Dramatise the topic "The after effects of excessive population" in the classroom.
7. Compare annual increase in your country's population to population-estimates of your state.
8. Identify regions of the country that have had significant population growth in the last decade and speculate reasons for this.
9. Determine the best representation for displaying rate of change of population.
10. Determine current and future growth patterns of your country's population.
11. Make predictions of population growth using histograms.
12. Identify states with different growth rates based on the shapes of their population histograms.
13. Present your findings through PowerPoint Presentation so that the message is conveyed to the entire school.

Learning Outcomes

Students will be able to–

- ☆ imbibe the concept of statistics by collecting primary and secondary data.
- ☆ tabulate and present the data in graphical form.

CHAPTER–13: PROBABILITY

Key Concepts

- ☆ **Probability** -How something is likely to happen.
- ☆ Many events can't be predicted with total certainty. The best we can say is how likely they are to happen, using the idea of probability.
- ☆ In general: Probability of an event happening = $\frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$

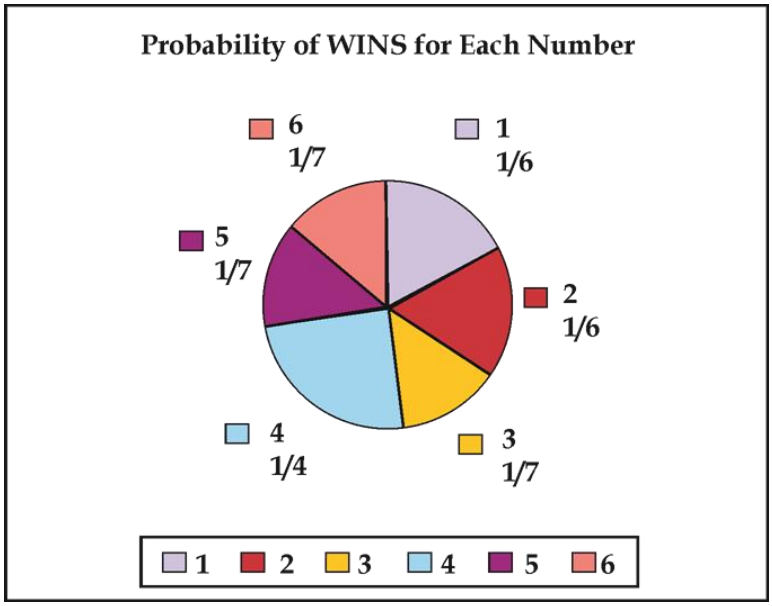
Aims and Objectives

- ☆ To find the probability of occurrence of some particular diseases like cholera, typhoid, jaundice, swine flu etc.

- To define the terms ***experiment***, ***outcome***, ***event***, ***probability*** and ***equally likely***.
- Restate the formula for finding the probability of an event.
- Recognise the difference between outcomes that are equally likely and not equally likely to occur.
- Define the terms ***certain event***, ***impossible event***.
- Examine the probabilities of experiments with certain outcomes.
- Examine the probabilities of experiments with impossible outcomes.
- Define the term ***sample space***.
- Recognise that the sum of the probabilities of the distinct outcomes within a sample space is one.
- Compare and operate with fractions, decimals and percents.
- Use graphs to organise, display and interpret data.

Essential Questions

- Is life fair to everyone?
- What is the likelihood that certain events will occur?
- What determines fairness?
- What is probability?
- How do you measure the likelihood of an event?
- How do you determine and represent probable outcomes?
- What is the difference between experimental and theoretical probability?



Procedure/Methodology

1. Divide students in different groups.
2. Ask them to survey different localities and make a survey sheet indicating the
   a) family details (age, gender, number of family members, number of children).
   b) the locality details (area, type of water, type of rainwater harvesting system).
   c) affected person's details (age, gender, affected by, previously affected by any disease).
   d) divide your report into 5 categories of families – Family of 5+, 5, 4, 3, or 2 members.
3. Tabulate the information specifying the diseases in different localities, number. of people affected and probability of others catching the disease.
4. Repeat the same for different categories of families.
5. Draw a pie chart showing which kind of family has maximum number of people affected.
6. Represent the same information by means of a bar graph.

Learning Outcomes

The students will be able to–

- ☆ talk about chance and classify events as ***certain, possible***, or ***impossible.***
- ☆ compare and order the likelihood of simple events.
- ☆ recognise that not all things occur with same likelihood.
- ☆ observe some things are fairer than others.
- ☆ make predictions based on data collected.
- ☆ identify all possible outcomes of an event.
- ☆ determine the experimental probability of simple events using frequency tables.
- ☆ compare theoretical and experimental probabilities.
- ☆ explain why probability is not an accurate predictor of events.
- ☆ interpret the graphs.

Mathematics (Code-041)
Exemplary Project: Class-X

Co-Ordinate Geometry

Key Concepts

- Review the concepts of co-ordinate geometry done earlier including graphs of linear equations.
- Awareness of geometrical representation of quadratic polynomials.
- Distance between two points and section formula.
- Area of a triangle = ½ [x1 (y2-y3) + x2 (y3 – y1) + x3 (y1-y2)] where (x1, y1), (x2, y2), (x3, y3) are the vertices of the triangle.

Gist of the Project

In this project, the students use various Mathematical Skills to do the following:

- Referral map of an area, taking a house as origin, locate the position (along x and y axis) of five different landmarks (Temple, Bus Stand, Petrol pump, Hospital, Police Station) within a radius of 3 km from your house by taking an appropriate scale. Draw conclusions from the same.
- Find mirror images of land marks plotted along x axis and y axis.
- Find the distance between any landmark and its mirror image along x axis and y axis and also distance between any two landmarks.
- Find the distance of a house from both the landmarks already taken and calculate the area enclosed between the triangles so formed.

Introduction

Everything in this world is made up of points and we can locate a specific place with points. That is how the coordinate system was developed. Coordinate system is used across various spheres of life from air trafficking to ocean navigation. Whenever you look at a bridge or a building, you may see the results of coordinate geometry. If you need to travel the shortest distance between two points, you use coordinate geometry. This Mathematics discipline describes the position of points on a plane using an ordered pair of numbers.

Historical Background

The Greek Mathematician Menaechmus solved problems and proved theorems by using a method that had a strong resemblance to the use of coordinates and it has sometimes been maintained that he introduced coordinate geometry. Another pioneer in this field was Pierre Fermat. Although not published in his lifetime, a manuscriptform of Ad Locos Planos et Solidos Isagoge (Introduction to Plane and Solid Loci) was circulating in Paris in 1637, just prior to the publication of Descartes' Discourse. Clearly written and well received, the introduction also laid the groundwork for coordinate geometry. Fermat always started with an algebraic equation and then described the

geometric curve which satisfied it, while Descartes starts with geometric curves and produces their equations as one of the several properties of the curves. As a consequence of this approach, Descartes had to deal with more complicated equations and he had to develop the methods to work with polynomial equations of higher degree.

Aims and Objectives

Students will–

- become aware of the use of coordinate geometry in real life.
- learn how to collect useful information by surveying the neighbourhood or by surfing the internet.
- find the shortest distance to be travelled between any two places.

Essential Questions

1. How do you locate your seat in the cinema hall in examination hall with the help of your ticket/hall ticket and find out how the seat number is written on it?
2. Name the Mathematician who introduced the **"Co-ordinate System".**
3. What do you mean by origin and how do you locate a point with reference to origin?
4. Describe the location of various landmarks of your locality, in your own words.
5. In locating mirror image with respect to x and y axis, which coordinate remains the same?

Procedure/Methodology/Project Design

1. The teacher should initiate the project by asking questions related to axis, quadrants, Cartesian plane, origin, distance between two points and area of triangle to recapitulate the concept of coordinate geometry already studied.
   a) Find out the places where and how coordinate geometry is used.
   b) If you are told to describe the location of your house, how will you initiate the same?
   c) Find the distance between various landmarks of your locality with respect to your house and draw inferences from the same.
2. The child should survey the locality around his house to find out the different landmarks his house in his/her locality (Temple, Bus stand, Petrol pump, Hospital, Police station).
3. By Referring/drawing map of an area, mark different landmarks and locate their positions, taking a house as origin.
4. Plot the represented landmarks on the Cartesian coordinate system taking appropriate scale on a graph paper.
5. Find and plot the mirror images of the above represented landmarks, with respect to 'x' axis and 'y' axis.

6. Visualize and plot the mirror images with respect to 'x' axis and 'y' axis of some of the English alphabets.
7. Find distance between all the landmarks and also their distance from the house.
8. Does finding distance help you to find out the shortest distance to be travelled between any two places?

Tools Used: Proper survey conducted, appropriate scale was taken,

Conclusion/End Product

The students will understand coordinate geometry by applying its concepts in real life situations.

Learning Outcomes

Students will be able to–

- find the shortest distance to be travelled between two places.
- become aware of the use of coordinate geometry in real life.

Mathematics (Code-041)
Class-X
Suggested Projects

CHAPTER–1: REAL NUMBERS

Key Concepts

- The number system is a mathematical notation which is used to represent numbers of a given set. The number system which we are most familiar with is the decimal (base-10) system. It is also known as system of numeration or numeral system with which helps us to express numbers using digits or other symbols.
- Basically, the number system represents a meaningful set of numbers including integers, rational numbers. Each and every number shows the algebraic and arithmetic numbers.
- Following points should be exploited
  - History
  - Mathematicians associated
  - Types of numbers
  - Properties of numbers
  - Area of interaction

Aims and Objectives

Students will–

- learn about Number Systems.
- understand the development and use of Number Systems.
- make presentation of collected / explored Number Systems.

Essential Questions

1. What is Number System?
2. How the evolution of Number Systems took place in different civilizations?

Procedure/Methodology:

Egyptian Hieroglyphic Numbers

= 1	= 1,000	= 1,000,000
= 10	= 10,000	
= 100	= 100,000	

1. Explore evolution of the following Number Systems:
   a) The Egyptian
   b) The Greek
   c) The Babylonian
   d) The Indian
2. Present your work in the form of ***Mathematics Journal,*** full of mysterious and amazing secrets and discoveries.
3. Use your imagination and art skills to draw pictures to support your write up for this journey.

Learning Outcomes

Students will be able to–

- ☆ know properties of Real Number Systems.
- ☆ support it with explanations and examples.

CHAPTER–2: PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Key Concepts

1. The most general form of a pair of linear equations is : $a_1x + b_1y + c_1 = 0$

   $a_2x + b_2y + c_2 = 0$ Where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers and $a_1^2 + b_1^2$ and

   $a_2^2 + b_2^2$ are not equal to 0
2. The graph of a pair of linear equations in two variables is represented by two lines.
   (i) If the lines intersect at a point, the pair of equations is consistent. The point of intersection gives the unique solution of the equation.
   (ii) If the lines coincide, then there are infinitely many solutions. The pair of equations is consistent. Each point on the line will be a solution.
   (iii) If the lines are parallel, the pair of the linear equation has no solution. The pair of linear equation is inconsistent.

Aims and Objectives

Students will–

- ☆ understand linear relationship, equation and linear equation in two variables.
- ☆ interpret that equations and lines (graphs) are different ways of expressing mathematical relationships.

Essential Questions

1. What is a Linear Equation in two variables?
2. State the standard form of linear equation.

3. How a linear equation is represented graphically?
4. What is the relation between coefficients and solutions of a linear equation?

Procedure/Methodology

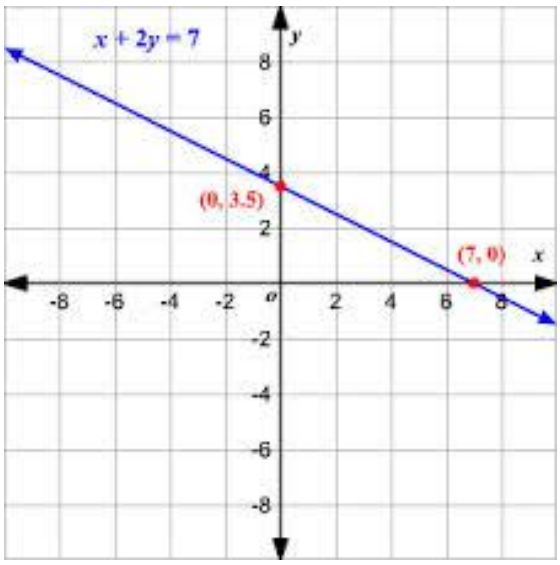
1. Think/analyse a situation in which the quantities have a linear relationship.

   For example: think about a nearly empty swimming pool that is being filled with water. The pool begins with a water depth of 10 cm; the water rises at the rate of 3 cm per minute until it is full.

2. Find out / visualise which quantities are related and how they are related? Which quantity depends on the other one in order to bring up dependent and independent variables?
3. Once the relationship has been expressed in words, use Mathematical symbols to express the relationship. For example:

   Height = 3 X no. of minutes or

   H = 3M or y = 3x



4. Represent it graphically.
5. Find out / collect at least five more life situations on which linear equation can be framed.

Learning Outcomes

Students will be able to–

- understand the linear relationship of a physical situation, a mathematical equation, a table of values, and a visual graph.

CHAPTER–3: ARITHMETIC PROGRESSIONS

Key Concepts

- **Sequence:** A set of numbers arranged in some definite order and formed according to some rules, is called a sequence.
- **Progression:** The sequence that follows a certain pattern is called **progression.**
- **Arithmetic Progression:** A sequence in which the difference obtained by subtracting any term from its proceeding term is constant throughout, is called an arithmetic sequence or **Arithmetic Progression** (A.P.).
- The general form of an A.P. is a, a + d, a + 2d, ... (a: first term d : common difference).

- General Term: If 'a' is the first term and 'd' is common difference in an A.P., then nth term (general term) is given by $a_n = a + (n - 1) d$.
- Sum up of n terms of an A.P. : If 'a' is the first term and 'd' is the common difference of an A.P., then sum of first n terms is given by $S_n = n/2 [2a + (n-1) d]$.
- Exploring A.P.: Derivation of standard results of finding the nth term.

or

- 'Sum of first n terms' and its application in solving daily life problems.
- Arithmetic Progressions in architectural designs of monuments

Aims and Objectives

Students will be able to–

- identify arithmetic progression in architectural monuments and in nature.
- explore the general term of an arithmetic progression using the given pattern.
- explore the sum of first n terms of an arithmetic progression.
- understand and apply arithmetic progressions in study of different floor designs.

Essential Questions

- Can you find Geometric shapes in buildings and structures?
- How were early arithmetic, algebra and geometry used in constructing temples, stupas, churches, mosques and gardens?
- How are Mathematics and architecture correlated?
- What is an A.P. (Arithmetic Progression)?
- How do we use the concept of A.P. in creating various geometrical designs?

Procedure/Methodology

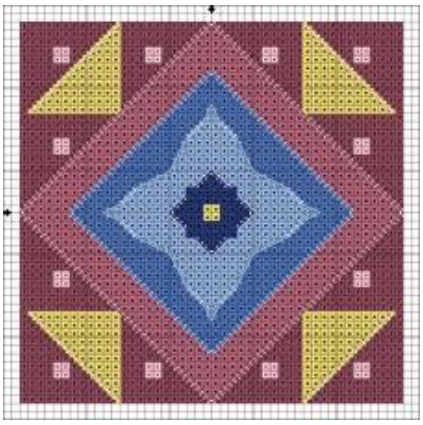
I) Arithmetic Progressions in monuments

1. Visit monuments of your state/city according to your choice and feasibility.
2. Locate **Arithmetic Progression** patterns on floor and walls and click photographs of the same.
3. Draw the observed patterns and try drawing some more of them.
4. Find a picture or a postcard of a famous building or skyscraper (like the Parthenon of ancient Greece) and discover the three-dimensional shapes within.

II) Arithmetic Progressions in Rangoli patterns

1. Make a 2X2 square on a coloured paper. Using different colours, make squares of 3X3, 4X4 and so on. Place them adjacently in increasing/decreasing order and infer whether the squares are in **Arithmetic Progressions**.

2. Repeat this activity with different shapes like triangle/ cube/pentagon/rhombus/circles/semicircles and see how beautifully different designs in **Arithmetic Progressions**, can be formed.
3. Make a Rangoli in your classroom using these patterns with eco-friendly colours.



III) Arithmetic Progressions in floor planning your classroom:

1. Students can work in teams.
2. Sketch the classroom's shape and the location of its walls, windows, doorways and closets.
3. Make a scale drawing of your classroom.
4. Now, make floor plan of your classroom using **Arithmetic Progressions** patterns.

Learning Outcomes

Students will be able to–

- ☆ recognise arithmetic sequences in a variety of contexts.
- ☆ recognise sequences that are not arithmetic.
- ☆ apply their knowledge of arithmetic sequences in a variety of contexts.
- ☆ apply the relevant formula in both theoretical and practical contexts.
- ☆ calculate the value of the first term **a,** the common difference **d** and the general term **Tn,** of an arithmetic sequence from the information given about the sequence.

CHAPTER–4: COORDINATE GEOMETRY

Key Concepts

- ☆ Review the concepts of co-ordinate geometry including graphs of linear equations.
- ☆ Awareness of geometrical representation of quadratic polynomials.
- ☆ Study distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ where (x_1, y_1) and (x_2, y_2) are the coordinates of Cartesian plane.
- ☆ Application of Section formula and Midpoint formula.

Aims and Objectives

Students will be able to–

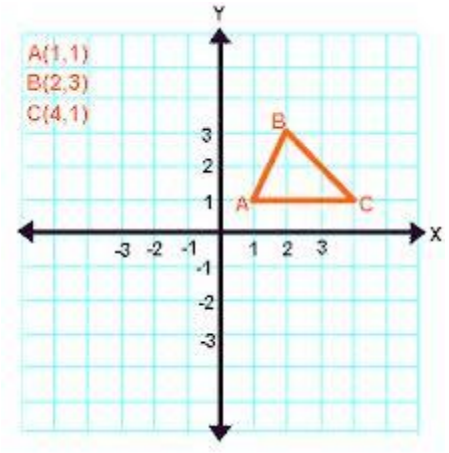
1. study the distance between different points of a geometrical figure when it is displaced and/or rotated.
2. enhance familiarity with co-ordinate geometry.

Essential Questions

1. Find the Mathematician who introduced the Co-ordinate System.
2. What do you mean by origin and what do you understand by sections?
3. How can you divide a line segment into sections?
4. How can you locate a point with reference to origin?
5. How can you find the distance between the two coordinates?
6. What is the section formula?
7. What is the midpoint formula?
8. How can you derive the Midpoint formula from section formula?

Procedure/Methodology

1. Place a cut out of a geometrical figure such as a triangle on a rectangular sheet of paper marked with x and y-axis.
2. Note the co-ordinates of the vertices of the triangle and its centroid.
3. Displace the triangular cut out along x-axis, along y-axis or along any other direction.
4. Note down the new co-ordinates of the vertices and centroid.
5. Repeat the procedure by rotating the triangle as well as displacing it. Note the new co-ordinate of vertices and centroid again.
6. Using the distance formula, find distance between the vertices of the triangle obtained for the triangle in original position and in various displaced and noted positions.
7. Using the new co-ordinates of the vertices and the centroids, obtain the ratio in which the centroid divides the medians for various displaced and rotated positions of the triangles.



Learning Outcomes

Students will be able to–

- verify that under any displacement and rotation of a triangle the displacement between verticals remain unchanged, also the centroid divides the medians in the ratio 2:1 in all cases.
- verify by the method of co-ordinate geometry, what is obvious geometrically, that the length of a triangle does not change when the triangle is displaced or rotated.
- develop familiarity with co-ordinates, distance formula and section formula of co-ordinate geometry.

CHAPTER–5: APPLICATIONS OF TRIGONOMETRY

Key Concepts

- Line of Sight : The line of sight is the line drawn from the eyes of an observer to a point in the object viewed by the observer.
- Angle of elevation: The angle of elevation is the angle formed by the line of sight with the horizontal, when it is above the horizontal level i.e. the case when we raise our head to look at the object.
- Angle of Depression : The angle of depression is the angle formed by the line of sight with the horizontal when it is below the horizontal i.e. case when we lower our head to look at the object.

Aims and Objectives

- To make a Clinometer.
- To find out the height of a school building.
- To apply trigonometry by using the principles to solve real life problems.

Essential Questions

1. What is a Clinometer?
2. How can you find the height of a school building?
3. How do you measure the height of the building using Clinometer?

Procedure/Methodology

1. Make a Clinometer using stiff card, drinking straw, thread and a weight.
2. Stand at a known distance from the building on the ground.
3. Look through the straw or a pipe at the top of the building. Make sure that the top of the building is perpendicular to the ground
4. Hold the Clinometer steady and let your partner record the angle the string makes, on the scale of the clinometer.
5. Using trigonometric ratio, find the height of the school building.
6. Change your distance from the building and note how the angle of elevation varies.
7. Conclude that although the distance and θ varies, the height comes out to be same.

Learning Outcomes

Students will be able to–

- ☆ find the height of a tall building e.g. School building.
- ☆ understand the concept and the applications of Trigonometry

CHAPTER–6: AREAS RELATED TO CIRCLES

Key Concepts

- ☆ Circumference of a circle= 2 πr where r is the radius of the circle.
- ☆ Area of a circle = πr^2

Aims and Objectives

- ☆ To find out the circumference and area of a circle.
- ☆ To find out what dimensions of a ball is suited to which particular sports and why?

Essential Questions

- ☆ What is the circumference of a circle?
- ☆ What is area of a circle?
- ☆ How are circumference and area helpful in determining the suitability of a ball for a particular sport?

Procedure/Methodology

1. Using basketballs, soccer balls, tennis balls and volleyballs, let students measure and find the circumference for each ball.
2. Let students record their results, including radius, diameter and circumference of each ball.
3. Inform students that they are to draw each ball, on a poster board showing their results.
4. Have your students find the new circumference of each ball if the diameter were changed by 10, 15 and 20 centimeters.
5. Let them explore relationship (if any) between circumference and area.

Learning Outcomes

Students will be able to–

- ☆ find circumference and area of different balls.
- ☆ find suitability of a ball for a particular sport.

CHAPTER–7: AREAS RELATED TO CIRCLES

Key Concepts

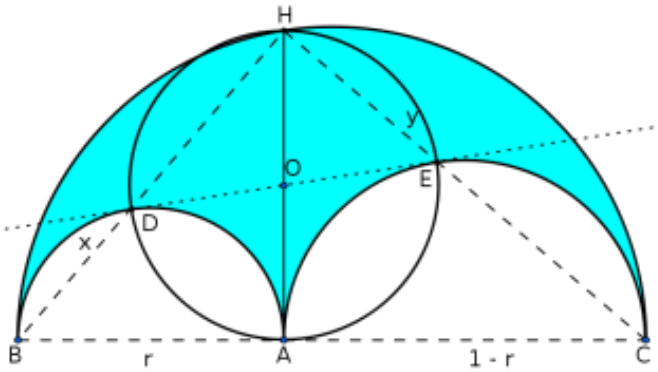
- Circles and various terms related to it
- In geometry, an arbelos is a plane region bounded by a semicircle of diameter 1, connected at the corners to semicircles of diameters r and (1 − r), all rising above a common baseline
- Area of arbelos = $\pi r/4 - \pi r^2/4$ = Area of circle

Aims and Objectives

- To explore the area of an arbelos.
- To find that the area of the arbelos (white shaded region) is equal to the area of circle with radius HA.

Essential Questions

1. What are the area and circumference of the circle? Are they related?
2. What are concentric circles?
3. What is an arbelos?
4. Which instrument of the same shape is popularly used?



Procedure/Methodology

1. Make a sketch of an arbelos.
2. Collect pictures for the same.
3. Find out about the mathematician who did research on the same.
4. Organise known facts. Spend some time in thinking about the problem and you should be able to come up with the proof.
5. Present findings on pages in a handwritten report.

Learning Outcomes

Students will be able to–

- know what is arbelos
- interpret their findings

CHAPTER–8: SURFACE AREAS AND VOLUMES

Key Concepts

- ☆ Total surface area of cube of side a units = $6a^2$ units.
- ☆ Volume of cube of side a units = a^3 cubic units.
- ☆ Total surface area of cuboid of dimensions l, b and h = 2(l × b + b × h + h × l) square units.
- ☆ Volume of cuboid of dimensions l, b and h = l × b × h cubic units.
- ☆ Curved surface area of cylinder of radius r and height h = $2\pi rh$ square units.
- ☆ Total surface area of cylinder of radius r and height h = $2\pi r$ (r + h) square units.
- ☆ Volume of cylinder of radius r and height h = $\pi r^2 h$ cubic units.
- ☆ Curved surface area of cone of radius r height h and slant height l = πrl square units
- ☆ Total surface area of cone = πr (l + r) sq. units.
- ☆ Volume of cone = $1/3\ \pi r^2 h$ cu.units.
- ☆ Total curved surface area of sphere of radius r units = $4\pi r^2$ sq. units.
- ☆ Curved surface area of hemisphere of radius r units= $2\pi r^2$ sq. units.
- ☆ Total surface area of hemisphere of radius r units = $3\pi r^2$ sq. units.
- ☆ Volume of sphere of radius r units = $4/3\ \pi r^3$ cubic units.
- ☆ Volume of hemisphere of radius r units =$2/3\pi r^3$ cubic units.

Aims and Objectives

- ☆ To understand the changes in surface areas and volumes of cuboids with respect to each other.

Essential Questions

- ☆ How will you find the volume of cuboids and cube?
- ☆ How will you find the surface area of cube and cuboids?
- ☆ Is there any relation between surface area and volume of cubes and cuboids?

Procedure/Methodology

1. Make the cuboids having equal volumes and following dimensions
   (i) L = 20cm, B = 10cm, H = 5cm.
   (ii) L = 10cm, B = 10cm, H = 10cm.
   (iii) L = 100cm, B = 5cm, H = 2cm.
2. Calculate the volume and surface area for the same.
3. Let the students conclude that the surface of cuboid, which is a cube, is minimum.

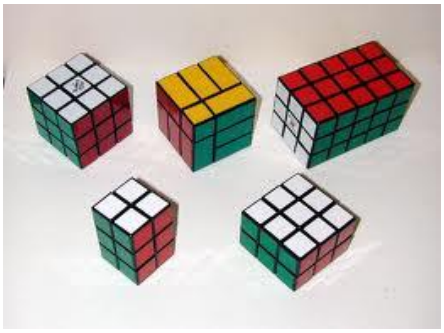
4. Make the cuboids having equal volumes and following dimensions

   (i) L = 14cm, B = 6cm, H = 5.4cm.

   (ii) L = 8cm, B = 8cm, H = 8cm.

   (iii) L= 16cm, B=4cm, H = 4cm.

5. Let the students conclude that the cuboid, which is a cube, has largest volume.



Learning Outcomes

Students will learn that–

- ☆ of all the cuboids with equal volumes, the cube has the minimum surface area.
- ☆ of all the cuboids with equal surface areas, the cube has the maximum volume.

CHAPTER–9: SURFACE AREAS AND VOLUMES

Key Concepts

- ☆ Curved surface area of cylinder of radius r and height h = $2\pi rh$ square units.
- ☆ Total surface area of cylinder of radius r and height h = $2\pi r\ (r + h)$ square units.
- ☆ Volume of cylinder of radius r and height h = $\pi r^2 h$ cubic units.
- ☆ Curved surface area of cone of radius r height h and slant height l = πrl square units
- ☆ Total surface area of cone = $\pi r\ (l + r)$ sq. units.
- ☆ Volume of cone = $1/3\ \pi r^2\ h$ cu.units.
- ☆ Total curved surface area of sphere of radius r units = $4\pi r^2$ sq. units.
- ☆ Curved surface area of hemisphere of radius r units= $2\pi r^2$ sq. units.
- ☆ Total surface area of hemisphere of radius r units = $3\pi r^2$ sq. units.
- ☆ Volume of sphere of radius r units = $4/3\ \pi r^3$ cubic units.
- ☆ Volume of hemisphere of radius r units =$2/3\pi r^3$ cubic units.

Aims and Objectives

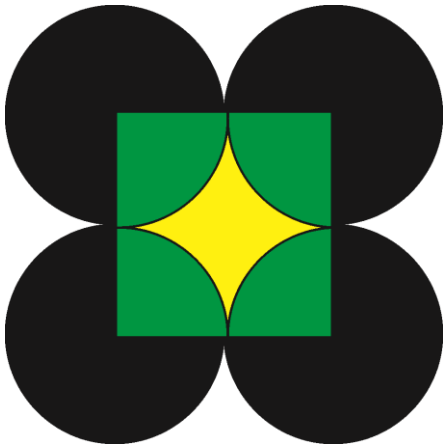
- ☆ To investigate the efficiency of packing of objects, with different shapes in a cuboid box.

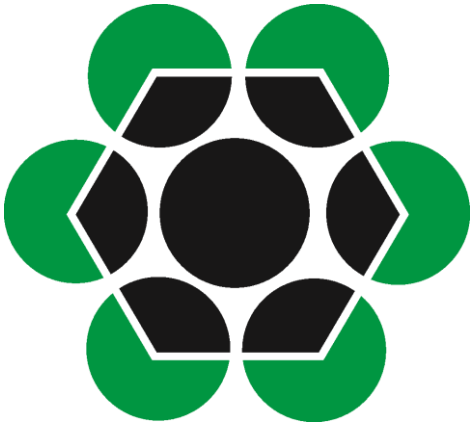
Essential Questions

1. What is the efficiency of packing of objects with different shapes in a cuboid box?

Procedure/Methodology

1. Take a certain number of cylindrical tins and pack them in a cuboid container. Illustration- Take 81 tins, 64 tins, 49 tins.
2. Place the cylindrical tins in two different ways.
   (a) Square packing.
   (b) Hexagonal packing.
3. Find out which of the two is more efficient.
4. Uunderstand the difference between the two packings refer the following figures.





Learning Outcomes

Students will be able to–

- ☆ find out the efficiency of packing of objects, with different shapes in a cuboid box.

CHAPTER–10: STATISTICS

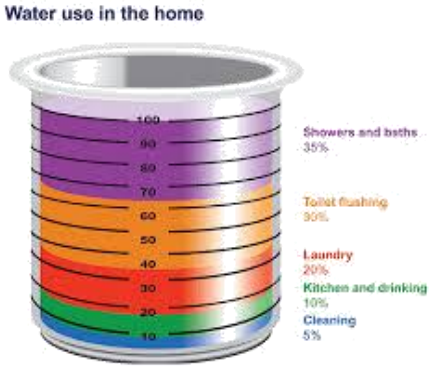
Key Concepts

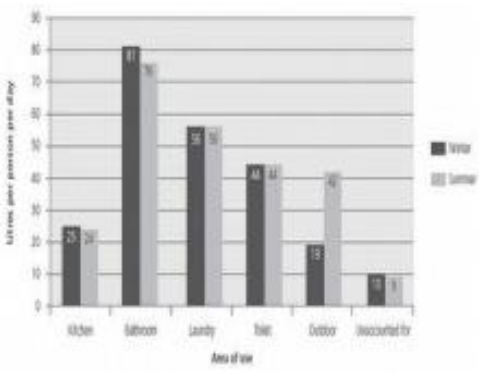
- ☆ Statistical formula for finding mean, median and mode
- ☆ Empirical Formula : Mode = 3 median - 2 mean
- ☆ Cumulative frequency curve or an Ogive
- ☆ Ogive is the graphical representation of the cumulative frequency distribution
  (i) Less than type Ogive: • Construct a cumulative frequency table
  • Mark the upper class limit on the x axis
  (ii) More than type Ogive: • Construct a frequency table
  • Mark the lower class limit on the x-axis
- ☆ To obtain the median of frequency distribution from the graph

- Locate point of intersection of less than type Ogive and more than type Ogive. Draw a perpendicular from this point on x- axis. The point at which it cuts the x-axis gives us the median.
- Histograms represent a popular means for feature representation.

Aims and Objectives

- To analyse daily water usage in a home or school.
- To apply math skills to estimate the amount used per week.
- To explore how much water could be conserved by becoming 'water smart.'
- To enlist the average consumption of water in a house under different heads like washing, cleaning, gardening, cooking, drinking etc.
- To collect the same information from their neighbourhood.
- To find out the house consuming minimum amount of water and to create awareness for conserving it.
- To find different ways of saving water.





Essential Questions

- How is water important in life? Mention its uses.
- Describe how water using habits affect the availability of water in the future.
- Use Mathematics skills to calculate total water usage and there by estimate savings.
- List ways people can personally conserve water.
- Explain ways in which families can conserve water.
- What are the different sources of obtaining portable water in urban and rural areas?
- What necessary steps can be taken to conserve water?
- Which measure of central tendency (mean, median or mode) is applied in completing the project?

Procedure/Methodology

1. List down the water consumption in your house under different heads such as cooking and washing etc.
2. Repeat the same activity by surveying twenty families in your neighbourhood and tabulate the information.
3. Find out the house with minimum and maximum overall water consumption and also under different heads.
4. Prepare posters on **"SAVE WATER, SAVE YOUR FUTURE".**
5. **Find out different ways of saving water.**
6. Locate rainwater harvesting units in your locality.
7. Find out whether the water being harvested is sufficient to meet the water requirement of your locality.
8. Find out the mean water consumption of the neighbourhood and also the mean consumption under different heads.
9. Find out the median and mode of the above data.
10. Draw conclusions from the above activities and represent the same graphically.

Learning Outcomes

Students will be able to–

- ☆ analyse daily water consumption (own/ household).
- ☆ find the ways of saving it for future.
- ☆ find mean, median, mode for the same.
- ☆ Represent the data graphically.

CHAPTER–11: PROBABILITY

Key Concepts

- ☆ The Theoretical probability of an event E written as (E) is

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

- ☆ The sum of the probability of all the elementary events of an experiment is 1.
- ☆ The probability of a sure event is 1 and probability of an impossible event is 0.
- ☆ If E is an event, in general, it is true that P(E) + P (not E) = 1.
- ☆ From the definition of the probability, the numerator is always less than or equal to the denominator therefore $0 \leq P(E) \leq 1$.
- ☆ There is a 50% chance that two people will have the same birthday.

Aims and Objectives

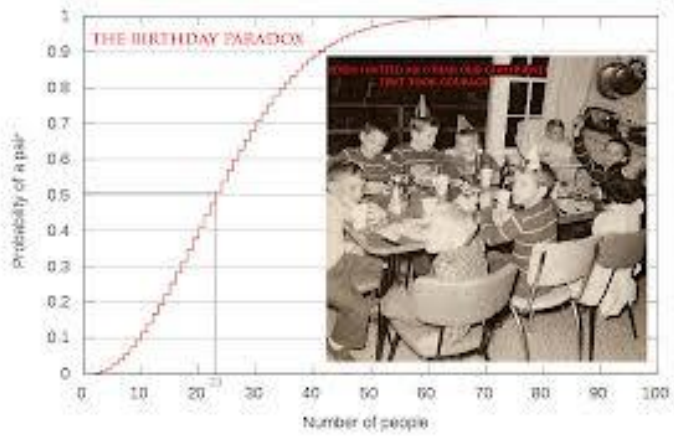
- To understand the Birthday Paradox.
- To prove whether or not the Birthday Paradox holds true by looking at random groups of twenty three or more people.

Essential Questions

1. What is the probability of a certain event?
2. What is the probability of an impossible event?
3. If you're sitting in a room with forty people in it, what are the chances that two of those people have the same birthday?
4. What is Birthday Paradox? Who invented the Birthday Paradox?
5. What is the formula for calculating Birthday Paradox?

Procedure/Methodology

1. Collect birth dates for 10-12 groups of twenty three or more people, so that you have enough groups to compare.
2. Take the class lists of about twelve sections. Pass these to each of these classes and collect the birth date data.
3. Use the birth dates of players on major teams using internet.
4. Next you will need to sort through all the birth dates you have collected and see if the Birthday Paradox holds true for the random groups of people you collected.
5. How many of your groups have two or more people with the same birthday?
6. Based on the Birthday Paradox, estimate how many groups would you expect to find, that have two people with the same birthday.
7. Tabulate/Organise your data and findings.



Learning Outcomes

The students will be able to–

- find the probability of an event.
- find the validity of Birthday Paradox.

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