Chapter 8: Linear Inequations

EXERCISE 8.1 [PAGE 116]

Exercise 8.1 | Q 1.1 | Page 116

Write the inequations that represent the interval and state whether the interval is bounded or unbounded $\left(-4\frac{7}{3}\right)$.

SOLUTION

The inequation is $-4 \leq x \leq \frac{7}{3}$

Here, x ∈ R can take all values between – 4 and $\frac{7}{3}$ (both inclusive).

∴ The interval is bounded (closed).

Exercise 8.1 | Q 1.2 | Page 116

Write the inequations that represent the interval and state whether the interval is bounded or unbounded (0, 0.9)

SOLUTION

The inequation is – 4 < x ≤ 0.9
Here, x ∈ R can take all values between 0
and 0.9 (excluding 0, but including 9)
∴ The interval is bounded (semi-right closed).

Exercise 8.1 | Q 1.3 | Page 116

Write the inequations that represent the interval and state whether the interval is bounded or unbounded $(-\infty, \infty)$

SOLUTION

The inequation is $-\infty < x < \infty$

Here, x ∈ R can take all values between $-\infty$

And ∞ (both exclusive)

∴ The interval is unbounded.

Exercise 8.1 | Q 1.4 | Page 116
Write the inequations that represent the interval and state whether the interval is bounded or unbounded $[5, \infty]$.

SOLUTION

The inequation is $5 \leq x < \infty$

Here, x ∈ R can take all values between 5 and ∞ (including 5, but excluding ∞)

∴ The interval is unbounded (semi left closed).

Exercise 8.1 | Q 1.5 | Page 116
Write the inequations that represent the interval and state whether the interval is bounded or unbounded (– 11, – 2).

SOLUTION

The inequation is – 11 < x < – 2
Here, x ∈ R can take all values between – 11 and – 2 (both exclusive).
∴ The interval is bounded (open).

Exercise 8.1 | Q 1.6 | Page 116
Write the inequations that represent the interval and state whether the interval is bounded or unbounded $(-\infty, 3)$.

SOLUTION

The inequation is $-\infty < x < 3$

Here, x ∈ R can take all values between $-\infty$ and 3 (both exclusive).

∴ The interval is unbounded.

Exercise 8.1 | Q 2.1 | Page 116
Solve the following inequation: 3x – 36 > 0

SOLUTION

3x – 36 > 0

⇒ 3x > 36

⇒ x > 12

$(12, \infty)$.

Exercise 8.1 | Q 2.2 | Page 116

Solve the following inequation: $7x - 25 \le -4$

SOLUTION

$7x - 25 \le -4$

$\Rightarrow 7x \le 25 - 4$

$\Rightarrow 7x \le 21$

$\Rightarrow x \le 3$

$(-\infty, 3)$.

Exercise 8.1 | Q 2.3 | Page 116

Solve the following inequation: $0 < \frac{x-5}{4} < 3$

SOLUTION

$$0 < \frac{x-5}{4} < 3$$

Multiply by 4, we get

$0 < x - 5 < 12$

i.e. $0 < x - 5$ and $x - 5 < 12$

i.e. $5 < x$ and $x < 17$

i.e. $x > 5$ and $x < 17$

i.e. $5 < x < 17$

$(5, 17)$.

Exercise 8.1 | Q 2.4 | Page 116

Solve the following inequation: $|7x - 4| < 10$

SOLUTION

If |x| < k, then – k < x < k

∴ – 10 < 7x – 4 < 10

Add 4 to each part of inequation.

– 10 + 4 < 7x – 4 + 4 < 10 + 4

∴ – 6 < 7x < 14

Divide by 7

$$-\frac{6}{7} < x < 2$$

$$\left(-\frac{6}{7}, 2\right).$$

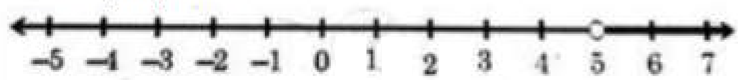
Exercise 8.1 | Q 3.1 | Page 116

Sketch the graph which represents the solution set for the following inequation x > 5.

SOLUTION

The solution set of the given inequation is $(5, \infty)$

∴ The graph is:



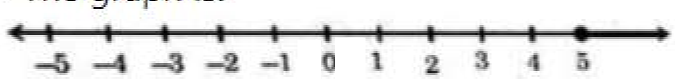
Exercise 8.1 | Q 3.2 | Page 116

Sketch the graph which represents the solution set for the following inequation x ≥ 5.

SOLUTION

The solution set of the given inequation is $(5, \infty)$

∴ The graph is:



Exercise 8.1 | Q 3.3 | Page 116

Sketch the graph which represents the solution set for the following inequation x < 3.

SOLUTION

The solution set of the given inequation is $(-\infty, 3)$

∴ The graph is:

Exercise 8.1 | Q 3.4 | Page 116

Sketch the graph which represents the solution set for the following inequation $x \le 3$.

SOLUTION

The solution set of the given inequation is $(-\infty, 3)$

∴ The graph is:

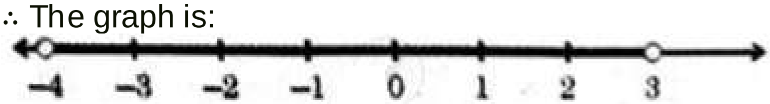
Exercise 8.1 | Q 3.5 | Page 116

Sketch the graph which represents the solution set for the following inequation $-4 < x < 3$.

SOLUTION

The solution set of the given inequation is $(-4, 3)$

∴ The graph is:



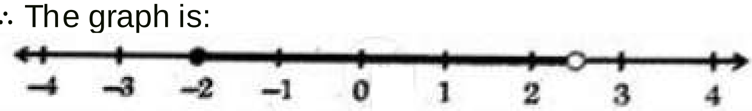
Exercise 8.1 | Q 3.6 | Page 116

Sketch the graph which represents the solution set for the following inequation $-2 \le x < 2.5$

SOLUTION

The solution set of the given inequation is $(-2, 2.5)$

∴ The graph is:

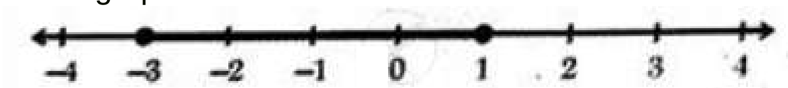


Exercise 8.1 | Q 3.7 | Page 116

Sketch the graph which represents the solution set for the following inequation $-3 \le x \le 1$.

SOLUTION

The solution set of the given inequation is (– 3, 1)
∴ The graph is:



Exercise 8.1 | Q 3.8 | Page 116

Sketch the graph which represents the solution set for the following inequation |x| < 4.

SOLUTION

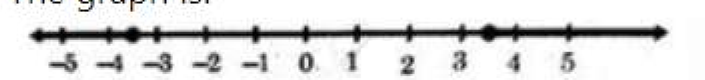
∵ |x| < 4
∴ – 4 < x < 4
The solution set of the given inequation is (– 4, 4)
∴ The graph is:

Exercise 8.1 | Q 3.9 | Page 116

Sketch the graph which represents the solution set for the following inequation |x| ≥ 3.5

SOLUTION

∵ |x| ≥ 3.5
∴ X ≥ 3.5 or X ≤ 3.5
The solution set of the given inequation is $(-\infty, 3.5] \cup [3.5, \infty)$
The graph is:



Exercise 8.1 | Q 4.01 | Page 116

Solve the inequation: 5x + 7 > 4 – 2x

SOLUTION

$\therefore$ 5x + 7 > 4 – 2x

$\therefore$ 5x + 2x > 4 – 7

$\therefore$ 7x > – 3

$\therefore x > \frac{-3}{7}.$

The solution set is unbounded open interval $\left(-\frac{3}{7}, \infty\right).$

Exercise 8.1 | Q 4.02 | Page 116

Solve the inequation: 3x + 1 ≥ 6x – 4

SOLUTION

3x + 1 ≥ 6x – 4

$\therefore$ 3x + 1 + 4 ≥ 6x

$\therefore$ 3x + 5 ≥ 6x

$\therefore$ 5 ≥ 3x

$\therefore \frac{5}{3} \geq x$

$\therefore x \leq \frac{5}{3}$

The solution set is unbounded open interval $\left(-\infty, \frac{5}{3}\right).$

Exercise 8.1 | Q 4.03 | Page 116

Solve the inequation: 4 – 2x < 3(3 – x)

SOLUTION

$4 - 2x < 3(3 - x)$

$\therefore 4 - 2 < 9 - 3x$

$\therefore 4 - 2x + 3x < 9$

$\therefore x < 9 - 4$

$\therefore x < 5$

The solution set is unbounded open interval $(-\infty, 5)$.

Exercise 8.1 | Q 4.04 | Page 116

Solve the inequation: $\frac{3}{4}x - 6 \leq x - 7$.

SOLUTION

$$\frac{3}{4}x - 6 \leq x - 7$$

$\Rightarrow 3x - 24 \leq 4 - 28$

$\Rightarrow -24 \leq 4x - 28 - 3x$

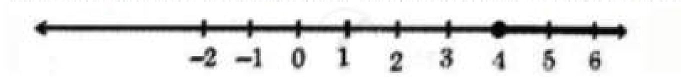
$\Rightarrow -24 + 28 \leq x$

$\Rightarrow 4 \leq x$

$\Rightarrow x \geq 4$

The solution Set of the inequation is the set of all real values of x which are greater than 4.

The solution set is unbounded set semi closed left interval $[4, \infty)$.



Exercise 8.1 | Q 4.05 | Page 116

Solve the inequation: $-8 \leq -(3x - 4) < 13$

SOLUTION

$-8 \le -(3x-4) < 13$

Multiply the inequation by -1, the sign of inequality changes.

$8 \ge 3x-5 > -13$

Add 5 : $8+5 \ge 3x-5 > -13+5$

$13 \ge 3x > -8$

$\frac{13}{3} \ge x > \frac{-8}{3} < x \le \frac{13}{3}$

The solution set of the inequation is the set of all values of x between $\frac{-8}{3}$ and $\frac{13}{3}$ excluding left boundary point and including right boundary point.

The solution set is semi right closed interval $\left(\frac{-8}{3}, \frac{13}{3}\right)$

Exercise 8.1 | Q 4.06 | Page 116

Solve the inequation: $-1 < 3 - \frac{x}{5} \le 1$

SOLUTION

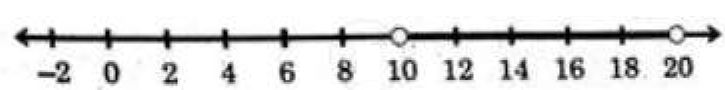
$$-1 < 3 - \frac{x}{5} \le 1$$

Multiplying the inequation by 5, the sign of inequality changes
5 > – 15 + x > – 5

Adding 15 on both the sides, we get
20 > x > 10
⇒ 10 < x < 20

The solution contains all the real values of x lying between 10 and 20, excluding the boundary value.

The solution set can be written in the form of open interval (10, 20).



Exercise 8.1 | Q 4.07 | Page 116

Solve the inequation: 2|4 – 5x| ≥ 9

SOLUTION

2|4 – 5x| ≥ 9

Dividing the inequality by 2, we get $|4-5x| \geq \frac{9}{2}$

We know that |x2| ≥ k implies x < – k or x > k

$\therefore 4-5x \geq -\frac{9}{2} \quad \text{or} \quad 4-5x \geq +\frac{9}{2}$

Multiplying the inequations by – 1, sign of inequation changes.

$-4+5x \geq \frac{9}{2} \quad \text{or} \quad -4+5x \leq -\frac{9}{2}$

$\therefore 5x \geq 4+\frac{9}{2} \quad \text{or} \quad 5x \leq 4-\frac{9}{2}$

$\therefore x \geq \frac{14}{10} \quad \text{or} \quad x \leq -\frac{1}{10}$

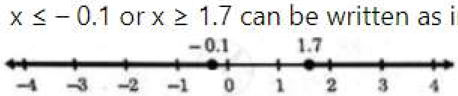
∴ x ≥ 1.7 or x ≤ – 0.1

∴ x ≤ – 0.1 or x ≥ 1.7

The solution set contains all real values of x which are either less than equal to – 0.1 or greater than or equal to 1.7

x ≤ – 0.1 means all the real values less than equal to – 0.1 i,e, interval $(-\infty, -0.1]$ and x ≥ 1.7 implies the interval $[1.7, \infty)$

∴ x ≤ – 0.1 or x ≥ 1.7 can be written as interval $(-\infty, -0.1] \cup [1.7, \infty)$



Exercise 8.1 | Q 4.08 | Page 116

Solve the inequation: |2x + 7| ≤ 25

SOLUTION

We know that |x| ≤ k implies – k ≤ x ≤ k
|2x + 7| ≤ 25 implies – 25 ≤ 2x + 7 ≤ 25
Thus, we have
– 25 ≤ 2x + 7 ≤ 25
Subtract 7 from each part,
25 – 7 ≤ 2x + 7 ≤ 25 – 7
– 32 ≤ 2x ≤ 18

Divide by 2,
The inequation |2x + 7| ≤ 25 has its solution as all real values of x lying between and including – 16 and 9
The interval of solution set is a closed interval [– 16, 9]

Exercise 8.1 | Q 4.09 | Page 116
Solve the inequation: 2|x + 3| > 1

SOLUTION

2|x + 3| > 1

Dividing the inequality by 2

∴ |x + 3| > 1

Now, |x| > k implies x < – k or x > k

$$\therefore (x+3) < \frac{-1}{2} \quad \text{or} \quad (x+3) > \frac{1}{2}$$

$$\therefore x < \frac{-1}{2} - 3 \quad \text{or} \quad x > \frac{1}{2} - 3$$

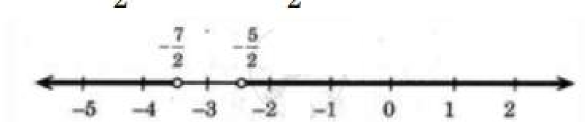
$$\therefore x < \frac{-1-6}{2} \quad \text{or} \ x > \frac{1-6}{2}$$

$$\therefore x < \frac{-7}{2} \quad \text{or} \ x > \frac{-5}{2}$$

The solution set contains all real values of x which are either less than $\frac{-7}{2}$ or greater than $\frac{-5}{2}$

$x < \frac{-7}{2}$ means all the real values less than $\frac{-7}{2}$, i.e. interval $\left(-\infty \frac{-7}{2}\right)$ and $x > \frac{-5}{2}$ implies the interval $\left(\frac{-5}{2}, \infty\right)$

$\therefore x < \frac{-7}{2}$ or $x > \frac{-5}{2}$ can be written as interval $\left(-\infty, \frac{-7}{2}\right) \cup \left(\frac{-5}{2}, \infty\right)$



Exercise 8.1 | Q 4.1 | Page 116

Solve the inequation: $\dfrac{x+5}{x-3} < 0$

SOLUTION

$$\frac{x+5}{x-3} < 0$$

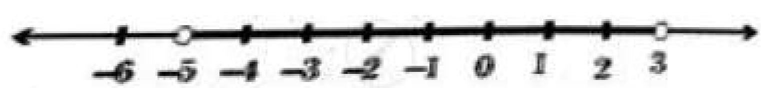
We know that if $\dfrac{a}{b} < 0$

Then either a > 0 or b < 0 or a < 0 or b > 0

∴ either x + 5 < 0 and x – 3 < 0
or x + 5 < 0 and x – 3 > 0

Case I:
x + 5 < 0 and x – 3 > 0
or x > – 5 and x < 3
Which is not possible as x can not be simultaneously less than – 5 and greater than 3

Case II:
x + 5 > 0 and x – 3 < 0
or x > – 5 and x < 3
or – 5 < x < 3
Which is an open interval (– 5, 3) where x can take any value between – 5 to 3.



Exercise 8.1 | Q 4.11 | Page 116

Solve the inequation: $\dfrac{x-2}{x+5} > 0$

SOLUTION

$$\frac{x-2}{x+5} > 0$$

If $\frac{a}{b} > 0$, then either $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$

Case I:

$\frac{x-2}{x+5} > 0 \Rightarrow x - 2 > 0$ and $x + 5 > 0$

$x > 2$ and $x > -5$

If $x > 2$ then already $x > -5$, therefore we can consider $x > 2$.

The solution set is $(2, \infty)$

Case II:

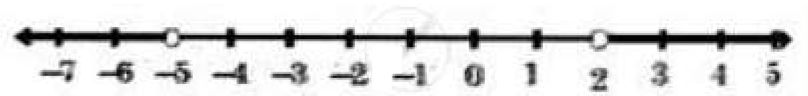
$\frac{x-2}{x+5} > 0 \Rightarrow x - 2 < 0$ and $x + 5 < 0$

$x < 2$ and $x < -5$

If , $x < -5$, then also $x < 2$

The solution set is $(-\infty, -5)$

Combining both the cases, we can take solution set as $(-\infty, -5) \cup (2, \infty)$ or $\{x;\ x < -5 \text{ or } x < 2\}$



Exercise 8.1 | Q 5 | Page 116

Rajiv obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

SOLUTION

Let Rajiv obtain 'x' marks in the third test.

Rajiv obtained 70 and 75 marks in the first two unit tests.

∴ Average of his marks in the three tests

$$= \frac{x + 70 + 75}{3}$$

$$= \frac{x + 145}{3}$$

∴ The average of marks should be at least 60

$$\therefore \frac{x + 145}{3} \geq 60$$

∴ $x + 145 \geq 180$

∴ $x \geq 180 - 145$

∴ $x \geq 35$

Rajiv should get minimum 35 marks in the third test.

Exercise 8.1 | Q 6 | Page 116

To receive Grade ‘A’ in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita’s marks in first four examinations are 87, 92,94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade ‘A’ in the course.

SOLUTION

Let Sunita obtain ‘x’ marks in the fifth examination. Sunita’s marks in the first four examination are 87,92 94, and 95.

Average of her marks in the five examinations

$= \frac{x + 87 + 92 + 94 + 95}{5}$

$= \frac{x + 368}{5}$

To receive grade A, Sunita's average should be 90 or more.

$\therefore \frac{x + 368}{5} \geq 90$

$\therefore x + 368 \geq 450$

$\therefore x \geq 450 - 368$

$\therefore x \geq 82$

∴ Sunita must get minimum 82 marks in the fifth examination.

Exercise 8.1 | Q 7 | Page 116

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

SOLUTION

Let the Consecutive odd positive integers be 'x' and 'x + 2'.
Sum of the numbers is more than 11
$\therefore x + (x + 2) > 11$
$\therefore 2x + 2 > 11$
$\therefore 2x > 11 - 2$
$\therefore 2x > 9$

$\therefore x > \frac{9}{2}$

$\therefore x > 4.5$

It is also stated that the numbers are less than 10. The immediate odd positive integer greater than 4.5 and less than 10 is 5.

When $x = 5$; $x + 2 = 5 + 2 = 7$
When $x = 7$; $x + 2 = 7 + 2 = 9$
When $x = 9$; $x + 2 = 9 + 2 = 11$

But, the numbers should be less than 10
$\therefore x + 2 = 11$ is discarded.
The pairs are (5, 7) and (7, 9).

Exercise 8.1 | Q 8 | Page 116

Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

SOLUTION

Let the consecutive even positive integers be 'x' and 'x + 2'
Sum of the members is less than 23
$\therefore x + x + 20 < 23$
$\therefore 2x + 2 < 23$
$\therefore 2x < 23 - 2$
$\therefore 2x < 21$

$\therefore x < \frac{21}{2}$

$\therefore x < 10.5$

It is also stated that the numbers are larger than 5.

The immediate even positive integer greater than 5 and less than 10.5 is 6.

When $x = 6$; $x + 2 = 6 + 2 = 8$

When $x = 8$; $x + 2 = 8 + 2 = 10$

When $x = 10$: $x + 2 = 10 + 2 = 12$

$\therefore$ The pairs are (6, 8),(8, 10) and (10, 12).

Exercise 8.1 | Q 9 | Page 116

The longest side of a triangle is twice the shortest side and the third side is 2cm longer than the shortest side. If the perimeter of the' triangle is more than 166 cm then find the minimum length of the shortest side.

SOLUTION

Let the sides of the triangle be a, b, c such that $a > b > c$.
The longest side is twice the shortest side
$\therefore a = 2c$

The third side is 2cm longer than the shortest side.
$\therefore b = c + 2$
$\therefore$ Perimeter of the triangle
$= a + b + c$
$= 2c + (C + 2) + c$
$= 4c + 2$

But, perimeter of the triangle is more than 166 cm.
$\therefore 4C + 2 > 166$

∴ 4c > 166 - 2
∴ 4c > 164

$$\therefore \ c > \frac{164}{4}$$

∴ > 41.

The minimum length of the shortest side is 41 cm.

EXERCISE 8.2 [PAGE 120]

Exercise 8.2 | Q 1.1 | Page 120

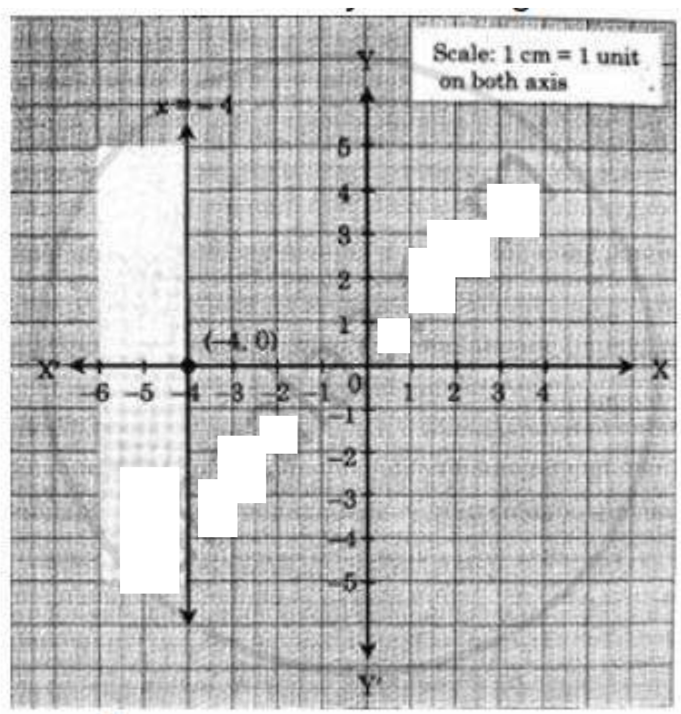
Solve the following inequations graphically in two-dimensional plane x ≤ – 4

SOLUTION

Consider the equation x = – 4

It is the equation of the line passing through (– 4, 0) and parallel to Y-axis. If we put x = 0, then 0 ≤ – 4 does not satisfy the inequation.

Solution set is away from origin.



Exercise 8.2 | Q 1.2 | Page 120

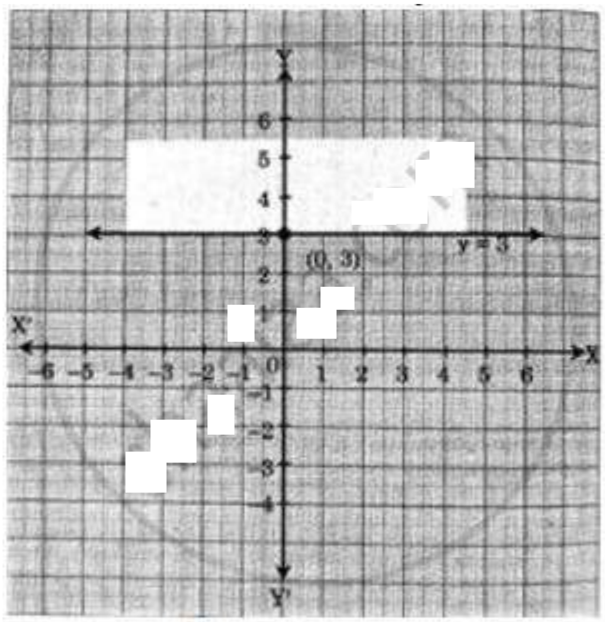
Solve the following inequations graphically in two-dimensional plane y ≥ 3

SOLUTION

Consider the equation y = 3.
It is the equation of a line parallel to Y-axis passing through the point. (0, 3) on the Y-axis.

Choose the point O(0, 0).
0 ≥ 3 does not satisfy the inequation.

The solution set is away from the origin.

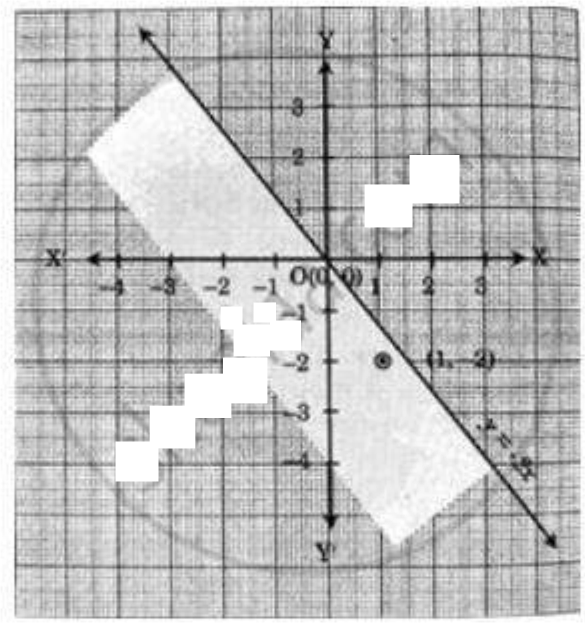


Exercise 8.2 | Q 1.3 | Page 120

Solve the following inequations graphically in two-dimensional plane y ≤ – 2x

Consider the equation y = 2x.

This is the equation of a line passing through origin in third and fourth quadrant inclined. more towards Y-axis.



Consider a point (2, 1).

Substitute in equation

$y \leq -2x$

$\Rightarrow 1 \leq 2(2)$

$\Rightarrow 1 \leq -4$

shows that point (2, 1) does not satisfy the inequation.

The solution is in the other plane away from that point.

Exercise 8.2 | Q 1.4 | Page 120

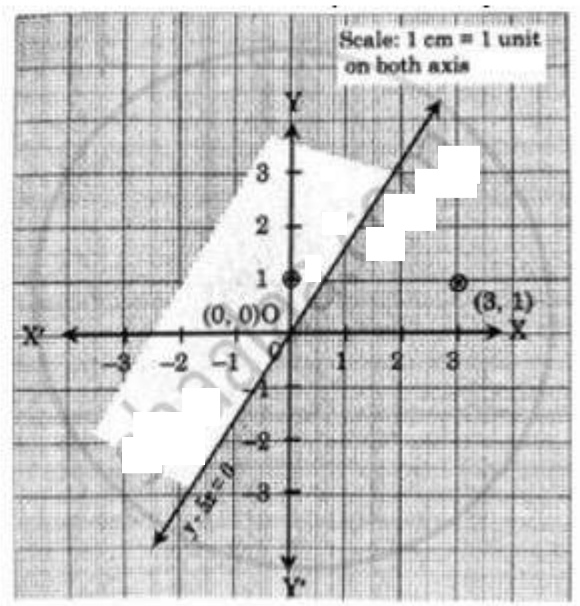
Solve the following inequations graphically in two-dimensional plane $y - 5x \geq 0$

SOLUTION

The equation $y - 5x \geq 0$ of the form $y = 5x$, which shows that the line passes through the origin in first and third quadrant more inclined towards Y-axis.

Since the line passes through the origin, it can not be taken as a landmark for the solution set.

Consider another point, say (3,1) on the graph.



The inequation is
$y - 5x \leq 0$

Put x = 3, y = 1 in the equation
$y - 5x \geq 0$
$1 - 5(3) = - 14 \geq 0$

Which shows that the solution set is not containing the point (3, 1).

The solution set does not contain the point (3, 1).

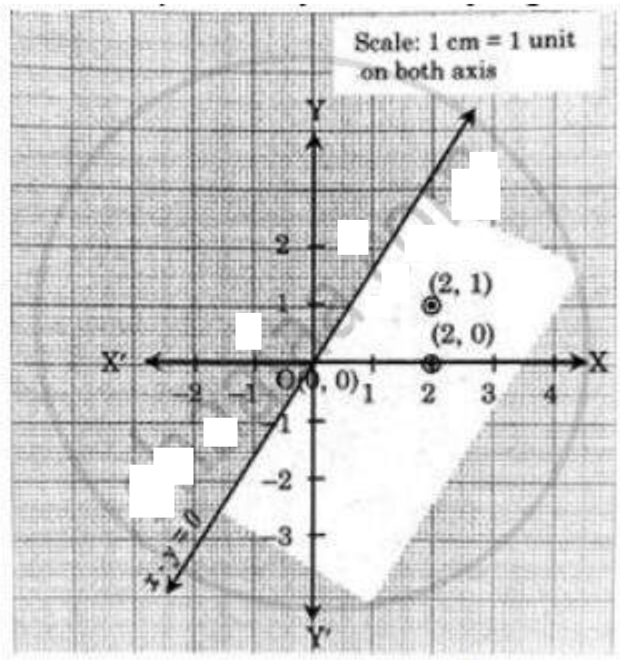
Exercise 8.2 | Q 1.5 | Page 120
Solve the following inequations graphically in two-dimensional plane $x - y \leq 0$

SOLUTION

Consider the equation $x - y = 0$

which shows that is the equation of the line passing through the origin and divides quadrants into equal parts.

So, the origin can not be considered as a landmark for the solution set. We have to consider another point say (2, 1) to judge the plane of the solution set.



The inequation is
x – y ≥ 0
2 – 1 ≥ 0
1 ≥ 0.

Exercise 8.2 | Q 1.6 | Page 120
Solve the following inequations graphically in two-dimensional plane x – y ≤ 2

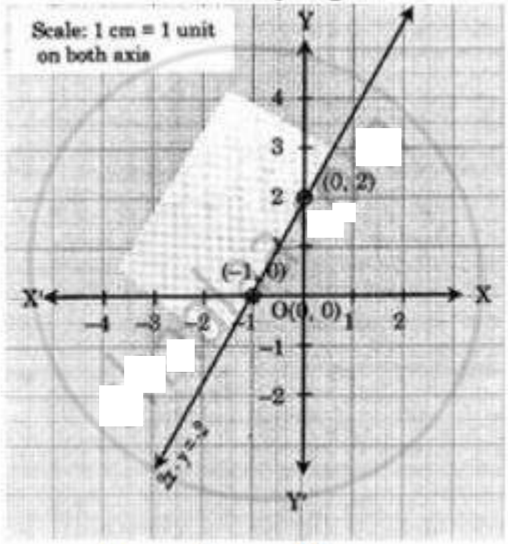
SOLUTION

Consider the equation 2x – y = – 2
Two points on the axes are given as

x	– 1	0
y	0	2

i.e. (– 1, 0) on the X-axis and (0, 2) on Y-axis.
If we put x = 0, y = 0 then.
2(0) – 0 ≤ – 2
0 ≤ – 2
Which shows that the origin is hot satisfying the inequation.

Solution set is lying on the line as well as in the plane away from the origin.



Exercise 8.2 | Q 1.7 | Page 120

Solve the following inequations graphically in two-dimensional plane 4x + 5y ≤ 40

SOLUTION

Consider the equation 4x + 5y ≤ 40

Two points required to draw the line are given by the table

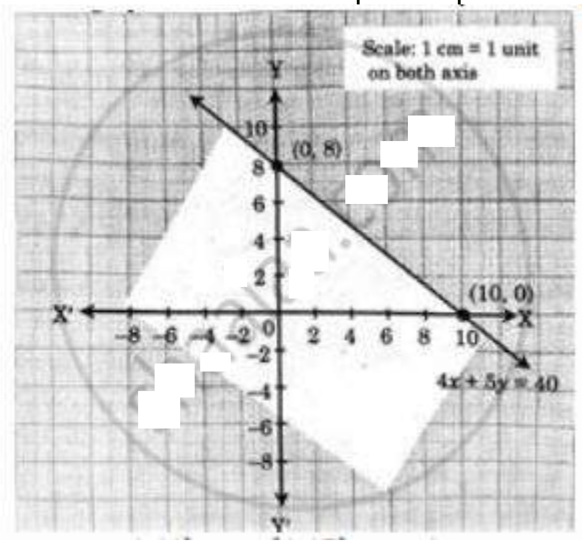
x	10	0
y	0	8

The two points on the coordinate axes are (10, 0) and (0, 8) respectively.

Since the inequality is 4x + 5y ≤ 40, the origin O (0, 0) will satisfy the inequation as 4x + 5y ≤ 40

i.e. 0 ≤ 40.

The solution set is in the part of plane towards the origin.



Exercise 8.2 | Q 1.8 | Page 120

Solve the following inequations graphically in two-dimensional plane

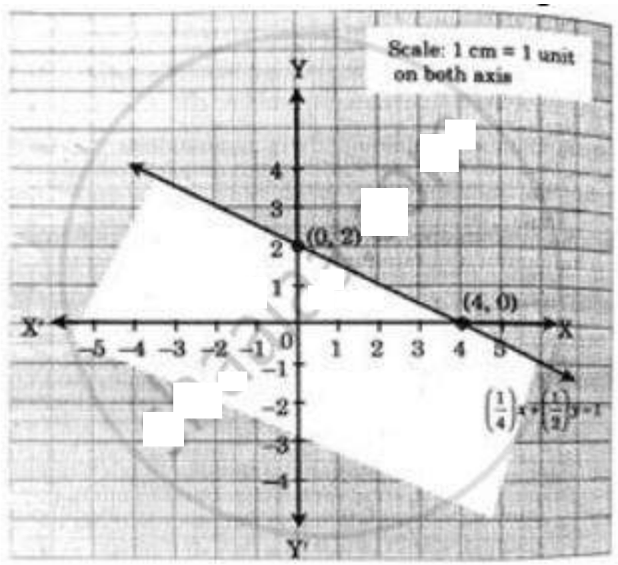
$$\frac{1}{4}x + \frac{1}{2}y \leq 1$$

SOLUTION

x + 2y ≤ 4
Consider the equation x + 2y – 4
The two points required to plotting the line on the graph are

x	4	0
y	0	2

(4, 0) and (0, 2) on the X and Y axe respectively.
Substitute x = 0, y = 0 in the inequation.
0 + 2(0) ≤ 4
i.e. 0 ≤ 4.
Origin satisfies the inequation on showing that the solution set contains origin.
The solution set is towards origin.



Exercise 8.2 | Q 2 | Page 120

Mr. Rajesh. Has Rs. 1,800/- to spend on fruits for a meeting. Grapes cost Rs. 150/- per kg and peaches cost Rs. 200/- per kg. Formulate and solve it graphically.

SOLUTION

The cost of grapes = Rs. 150/- per kg.
Let x kg of grapes be bought.

Then total cost of grapes = 150 x
The cost of peaches = Rs.200/- per kg.

Let y kg of peaches be bought.
Then total cost of peaches = 200 y

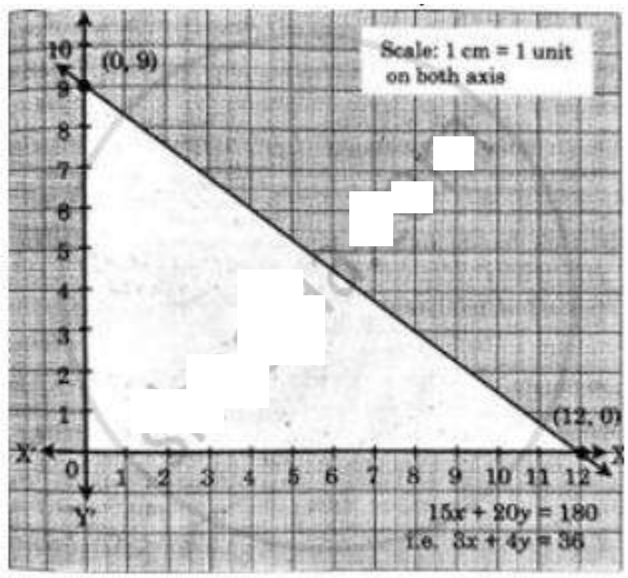
Since Mr. Rajesh has total amount Rs. 1800 to spend on fruits.
His total expenses 150 x + 200 y should be less than or equal to 1800.

Inequation is $150x + 200y \leq 1800$
$\Rightarrow 3x + 4y \leq 36$
$x, y \geq 0$ as the quantities of grapes and peaches can't be negative.

Points on axes are

x	12	0
y	0	9

(12, 0) on X-axis and (0, 9) on Y-axis.
Since origin satisfies the inequation, solution set is towards origin.
Since x and y are both positive, solution lies in first quadrant only.



Exercise 8.2 | Q 3 | Page 120

Diet of a sick person must contain at least 4000 units of vitamins. Each unit of food F1 contains 200 units of vitamins, where as each unit of food F2 contains 100 units of vitamins. Write an inequation to fulfil sick person's requirements. Represent the solution set graphically.

SOLUTION

Let x units of vitamins be consumed in food
F1 and y units of vitamins be consumed in food F2 by sick person.

One unit of food F1 contains
= 200 units of vitamins
One unit of food F2 contains
= 100 units of vitamins

Total vitamin consumption = 200x + 100y
As minimum requirement = 4000 units consumption will either greater than or equal to 4000.

The inequation is $200x + 100y \geq 4000$
or $2x + y \geq 20, x \geq 0, y \geq 0$

For drawing the graph, consider $2x + 4y = 20$.

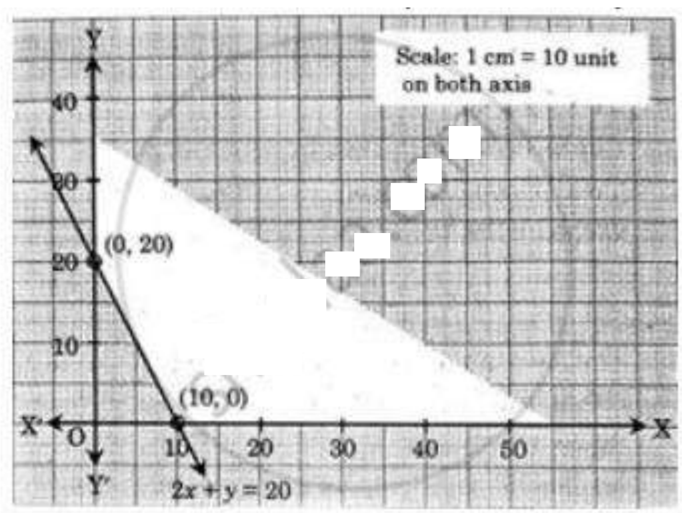
The two points required for the plotting the line are

x	10	0
y	0	20

(10, 0) on X-axis and (0, 20) on Y-axis.
Substitute the coordinate of origin x = 0,

y = 0 in the inequation
$2(0) + 0 \geq 20$
$\Rightarrow 0 \geq 20$

which shows that the origin is not satisfying the inequation.
∴ Plane containing solution 1s away from the origin.
Solution set is in first quadrant only.



EXERCISE 8.3 [PAGE 121]

Exercise 8.3 | Q 1 | Page 121

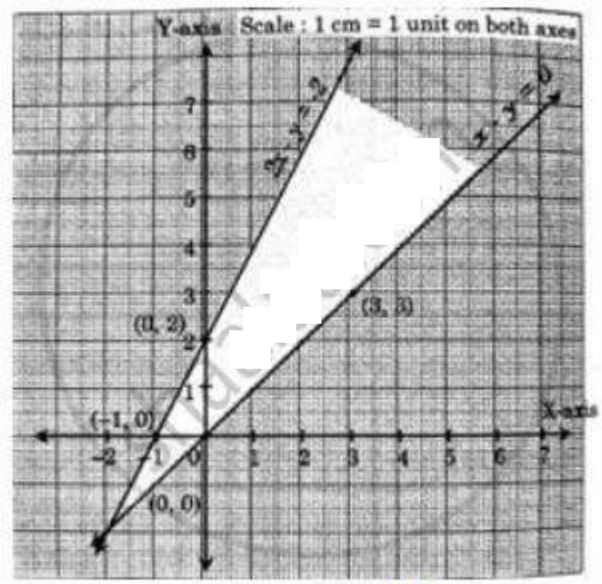
Find the graphical solution of the following system of linear inequations: $x - y \leq 0$, $2x - y \geq 2$

Writing the above inequalities as equations
$x - y = 0$

x	0	3
y	0	3
(x, y)	(0,0)	(3,3)

$2x - y = -2$

x	0	– 1
y	2	0
(x, y)	(0,2)	(– 1,0)



The inequality x – y ≤ 0 represents the region above the line, in dividing the points on the line x – y = 0.

The inequality 2x – y ≥ 2 represents the region below the line, including the points on the line 2x – y = – 2

∴ The shaded region between the lines represents the solution of the given inequations.

Exercise 8.3 | Q 2 | Page 121

Find the graphical solution of the following system of linear inequations: 2x + 3y ≥ 12; – x + y ≤ 3, x ≤ 4; y ≤ 3.

SOLUTION

The given equations in system are 2x + 3y ≥ 12; – x + y ≤ 3, x ≤ 4; y ≤ 3
Consider the equations:
2x + 3y = 12

x	6	0
y	0	4

The two points on the coordinate axes are (6, 0) and (0, 4).
Origin (0, 0) does not satisfy the inequation as

2(0) + 3(0) ≥ 12
i.e. 0 ≥ 12 which is not true.
∴ Solution set is always from the origin

– x + y = 3

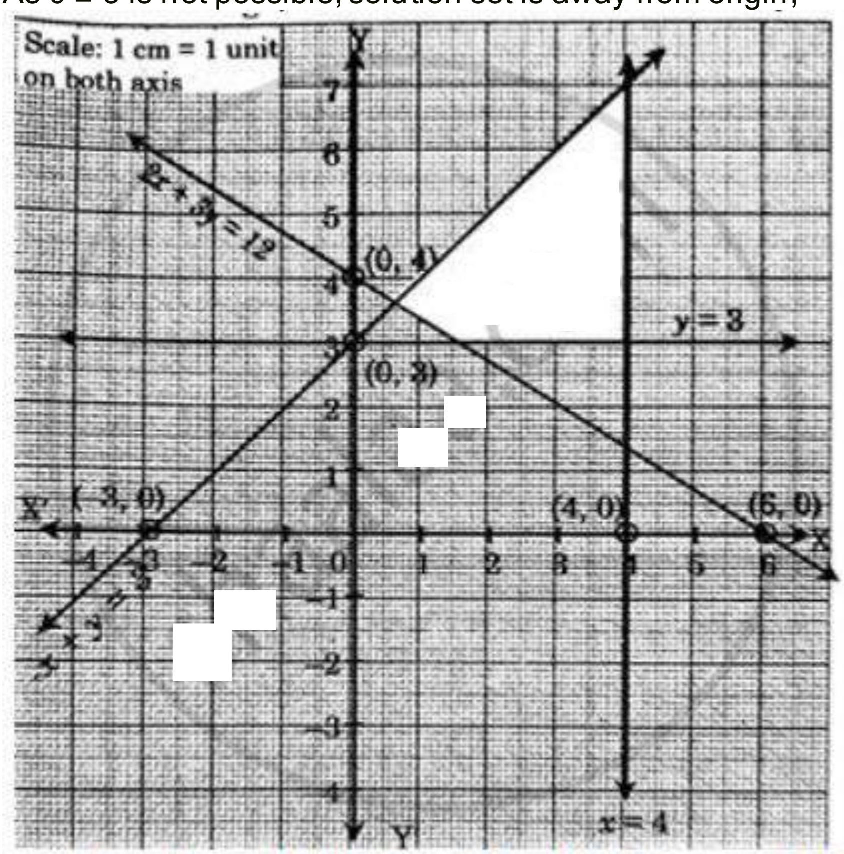
x	– 3	3
y	0	3

The points on the coordinate axes are
(3, 0) and (0, 3)

Origin (0, 0) satisfies the inequation as
– 0 + 0 ≤ 3 i.e. 0 ≤ 3 which is true.
Solution set is towards origin.

For x ≤ 4, consider x = 4. Equation of line which passes through the point (4, 0) and since 0 ≤ 4 implies solution is towards origin.

For y ≥ 3, consider y = 3 which is the equation of a line passing through (0, 3) and parallel to X-axis.
As 0 ≥ 3 is not possible, solution set is away from origin,



Exercise 8.3 | Q 3 | Page 121

Find the graphical solution of the following system of linear inequations: $3x + 2y \le 1800$; $2x + 7y \le 1400$, $0 \le x \le 350$; $0 \le y \le 150$.

SOLUTION

Inequations System of inequalities contains the
$3x + 2y \le 1800$; $2x + 7y \le 1400$, $0 \le x \le 350$; $0 \le y \le 150$.

Consider the equations:
$3x + 2y = 1800$

x	600	0
y	0	900

The two points required for plotting graph on the axes are (600, 0) and (0, 900) respectively.

$\because 3(0) + 2(0) \le 1800$.
i.e. $0 \le 1800$

Origin satisfies the inequation« showing that the solution set is towards origin.
$2x + 7y = 1400$

x	700	0
y	0	200

The two points on the axes are (700, 0) and (0, 200) respectively.
$\because 2(0) + 7(0) \le 1400$

$\Rightarrow 0 \le 1400$
Showing that the inequation satisfies the inequations solution set is towards origin.
The two double inequations are

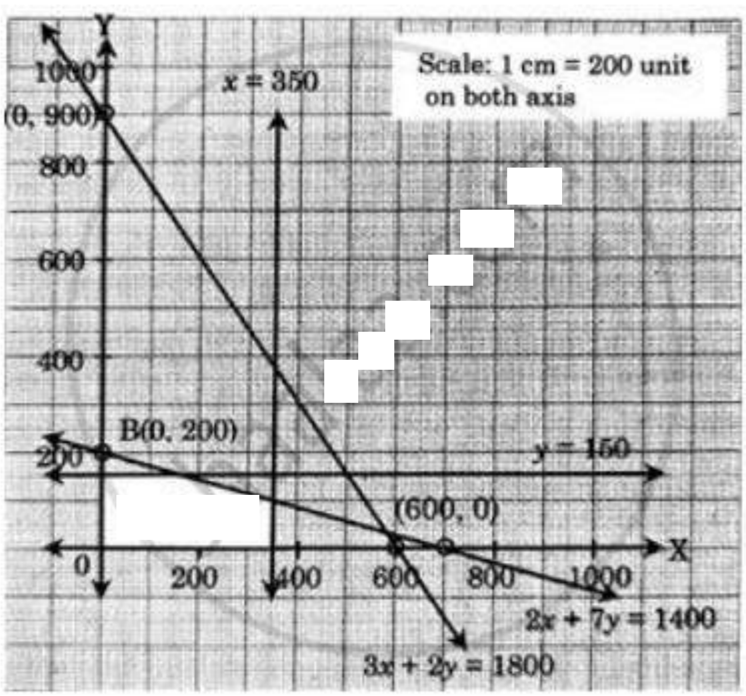
$0 \le x \le 350$ and $0 \le y \le 150$
Which can be written as:
$0 \le x$ and $x \le 350$

$0 \le y$ and $y \le 150$
For $x \le 350$, consider $x = 350$.

It is the equation of a line passing through (350, 0) and parallel to Y-axis.
For $y \le 150$, consider $y = 150$

The line passes through (0, 150) and parallel to X axis.
∵ Both the inequations $x \le 350$ and $y \le 150$ satisfy the origin.

∴ Solutions sets are towards the origin.
The conditions $x, y \le 0$ show that the common solution set of system lies in first quadrant.



The line $3x + 2y \le 1800$ has not contributed to solution set.
∴ It is known as redundant.

Exercise 8.3 | Q 4 | Page 121

Find the graphical solution of the following system of linear inequations:
$$\frac{x}{60} + \frac{y}{90} \le 1, \frac{x}{120} + \frac{y}{75} \le 1, y \ge 0, x \ge 0$$

SOLUTION

Consider the equations:

$$\frac{x}{60}+\frac{y}{90}\leq 1$$

x	60	0
y	0	20

The two points on the axes are (60, 0) and (0, 90) as in the equation 60 and 90 are x and y intercepts, (x and y are equivalent to X and Y axes.)

∵ The inequation $\frac{x}{60}+\frac{y}{90}\leq 1$ satisfies the origin.

Solution set is towards origin.

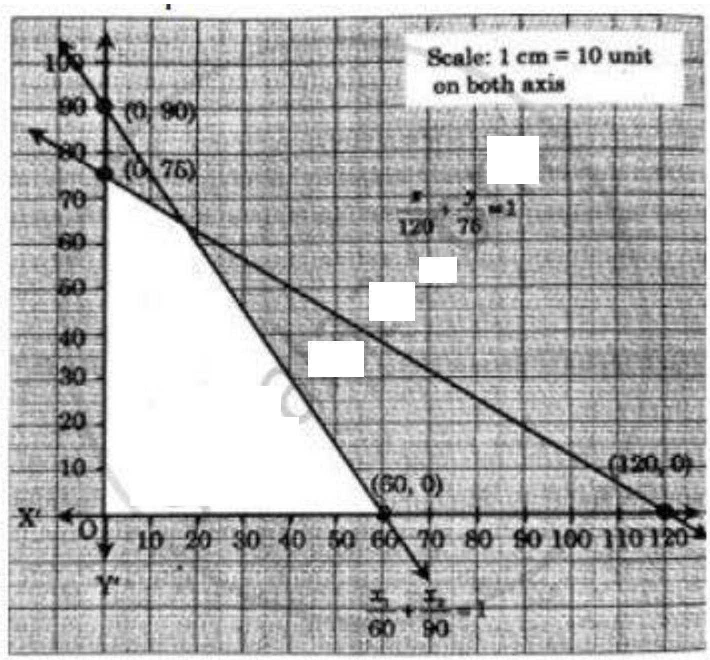
$$\frac{x}{120}+\frac{y}{75}\leq 1$$

x	120	0
y	0	75

The two points on the axes are (120, 0) and (0, 75) respectively.

The inequation $\frac{x}{120}+\frac{y}{75}\leq 1$ satisfies the origin.

Solution set of the inequation is towards origin x ≥ 0, y ≥0 are the inequations showing the conditions that the solutions set (common region) is in the first quadrant



Exercise 8.3 | Q 5 | Page 121

Find the graphical solution of the following system of linear inequations: 3x + 2y ≤ 24; 3x + y ≥ 15; x ≥ 4.

SOLUTION

Writing the above inequalities as equations
3x + 2y ≤ 24

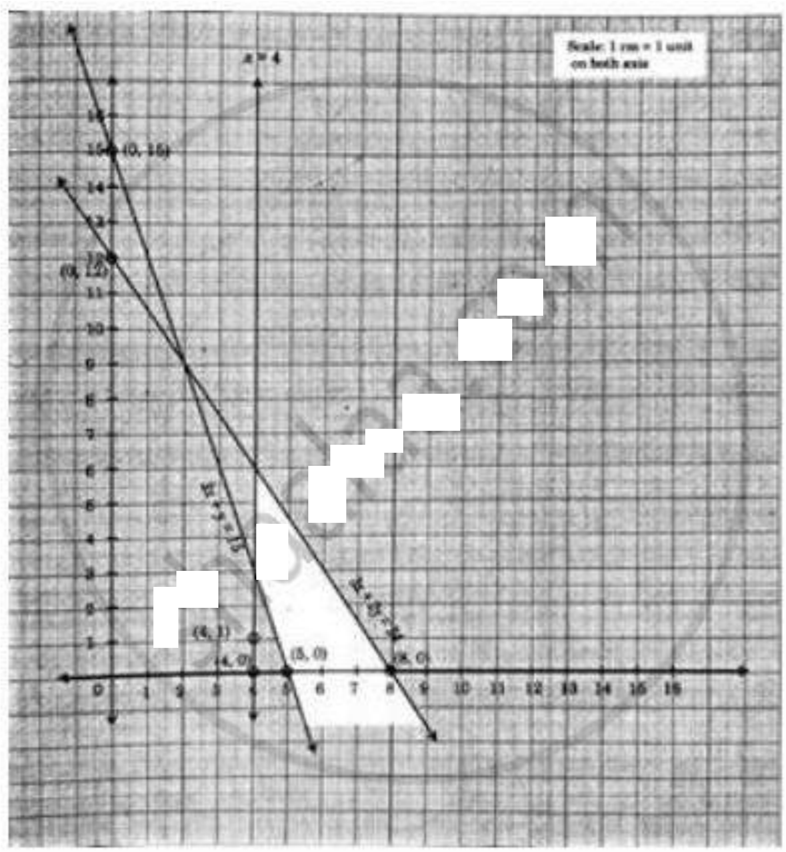
x	0	8
y	12	0
(x, y)	(0, 12)	(8, 0)

3x + y ≥ 15

x	0	5
y	15	0
(x, y)	(0, 15)	(5, 0)

x ≥ 4

x	4	4
y	0	1
(x, y)	(4, 0)	(4, 1)



The inequality 3x + 2y ≤ 24 represents the region below the line including the points on the line 3x + 2y = 24.

The inequality 3x + y ≥ 15 represents the region above the line, including the points on the line 3x + y = 15.

The inequality x ≥ 4 represents the region to the right of the line, including the points on the line x = 4.

∴ The shaded region between the lines represents the solution of the given inequality.

Exercise 8.3 | Q 6 | Page 121

Find the graphical solution of the following system of linear inequations: 2x + y ≥ 8; x + 2y ≥ 10; x ≥ 0; y ≥ 0

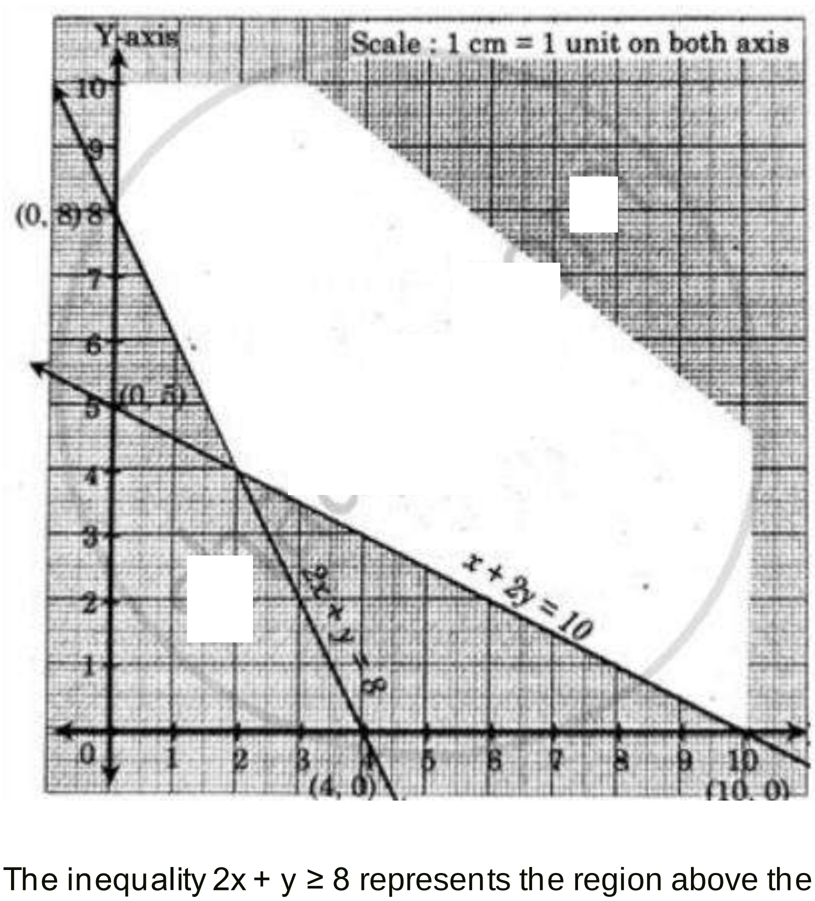
SOLUTION

Writing the above inequalities as equations
$2x + y = 8$

x	0	4
y	8	0
(x, y)	(0, 8)	(4, 0)

$x + 2y = 10$

x	0	10
y	5	0
(x, y)	(0, 5)	(10, 0)



The inequality $2x + y \geq 8$ represents the region above the line, including the points on the line $2x + y = 8$
The inequality $x + 2y \geq 10$ represents the region above the line, including the points on

the line x + 2y = 10

Since x ≥ 0; y ≥ 0, all points in the shaded region represents solution of the given system of inequalities.

MISCELLANEOUS EXERCISE 8 [PAGE 122]

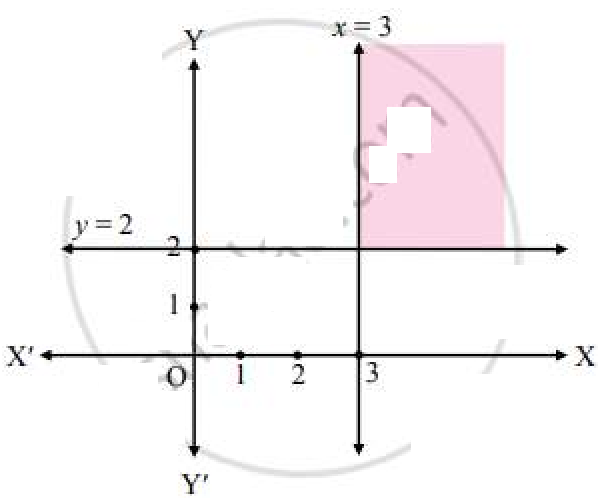
Miscellaneous Exercise 8 | Q 1 | Page 122

Solve the following system of inequalities graphically x ≥ 3, y ≥ 2

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Interpect form	Points (x, y)	Region
x ≥ 3	x = 3	–	–	0 ≱ 3 ∴ R.H.S. of line x=3
y ≥ 2	y = 2	–	–	0 ≱ 2 ∴ above the line y = 2



Shaded portion represents the graphical solution.

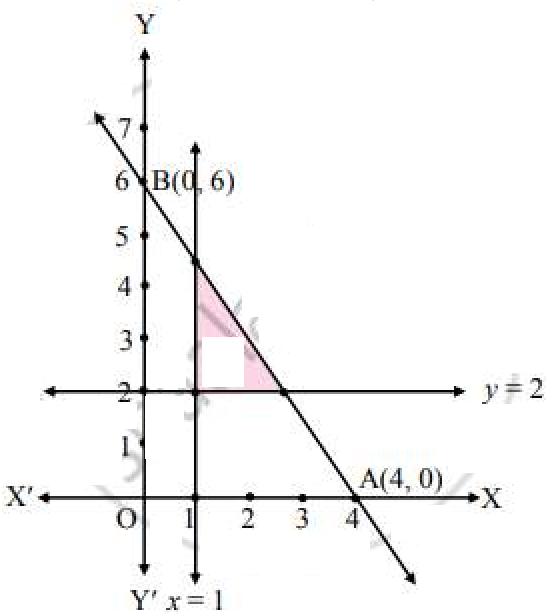
Miscellaneous Exercise 8 | Q 2 | Page 122

Solve the following system of inequalities graphically 3x + 2y ≤ 12, x ≥ 1, y ≥ 2

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
$3x + 2y \le 12$	$3x + 2y = 12$	$\frac{x}{4} + \frac{y}{6} = 1$	A(4, 0), B(0, 6)	$3(0) + 2(0) \le 12$ $\therefore 0 \le 12$ $\therefore$ origin side
$x \ge 1$	$x = 1$	–	–	$0 \not\ge 1$ $\therefore$ R.H.S. of line $x = 1$
$y \ge 2$	$y = 2$	–	–	$0 \not\ge 2$ $\therefore$ above line $y = 2$



Shaded portion represents the graphical solution.

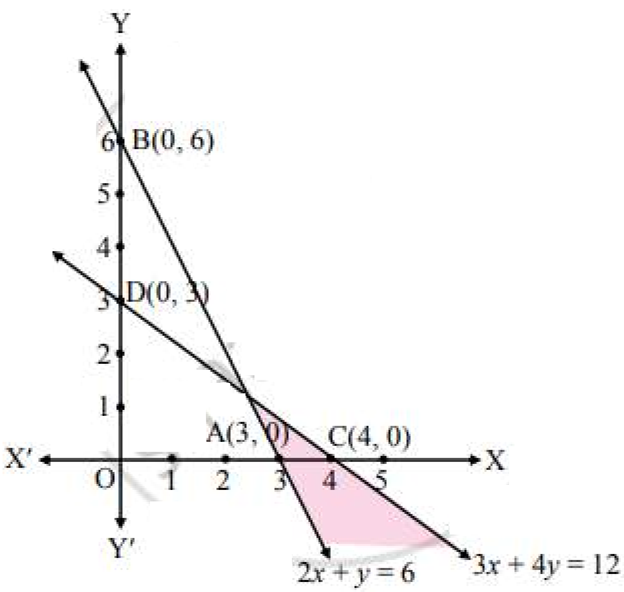
Miscellaneous Exercise 8 | Q 3 | Page 122

Solve the following system of inequalities graphically $2x + y \ge 6$, $3x + 4y \le 12$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
2x + y ≥ 6	2x + y = 6	$\frac{x}{3} + \frac{y}{6} = 1$	A (3, 0) B (0, 6)	2(0) + 0 ≱6 ∴ 0 ≱6 ∴ non-origin side
3x + 4y ≤ 12	3x + 4y = 12	$\frac{x}{4} + \frac{y}{3} = 1$	C (4, 0) D (0, 3)	3(0) + 4(0) ≤ 12 ∴ 0 ≤ 12 ∴ origin side



Shaded portion represents the graphical solution.

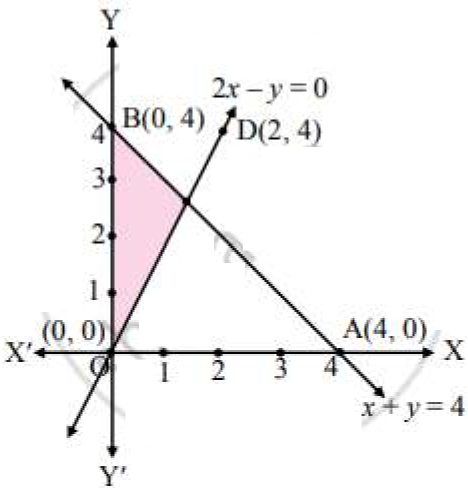
Miscellaneous Exercise 8 | Q 4 | Page 122

Solve the following system of inequalities graphically x + y ≥ 4, 2x – y ≤ 0

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
$x + y \geq 4$	$x + y = 4$	$\frac{x}{4} + \frac{y}{4} = 1$	A (4, 0), B (0, 4)	$0 + 0 \not\geq 4$ $\therefore 0 \not\geq 4$ ∴ non-origin side
$x - y \leq 0$	$2x - y = 0$	–	O (0, 0), D (2, 4)	$2(0) - 0 \leq 0$ $\therefore 0 \leq 0$ ∴ origin side



Shaded portion represents the graphical solution.

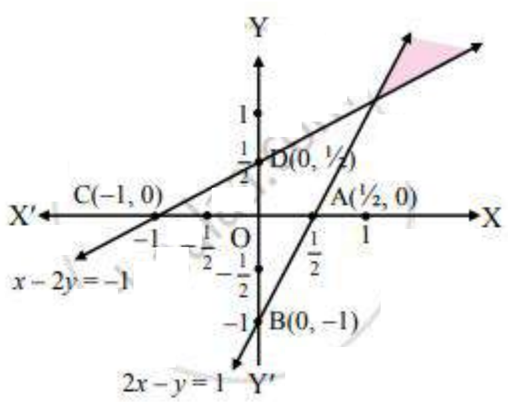
Miscellaneous Exercise 8 | Q 5 | Page 122

Solve the following system of inequalities graphically $2x - y \geq 1$, $x - 2y \leq -1$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
2x – y ≥ 1	2x – y = 1	$\frac{2x}{1} - \frac{y}{1} = 1$ i.e., $\frac{x}{\frac{1}{2}} + \frac{y}{-1} = 1$	A $\left(\frac{1}{2}, 0\right)$ B (0, – 1)	2(0) – 0 ≱ 1 ∴ 0 ≱ 1 ∴ non-origin side
x – 2y ≤ – 1	x – 2y = – 1	$\frac{x}{-1} - \frac{2y}{-1} = 1$ i.e., $\frac{x}{-1} + \frac{y}{\left(\frac{1}{2}\right)} = 1$	C (– 1, 0), D $\left(0, \frac{1}{2}\right)$	0 – 2(0) ≰ – 1 ∴ 0 ≰ – 1 ∴ non-origin side



Shaded portion represents the graphical solution.

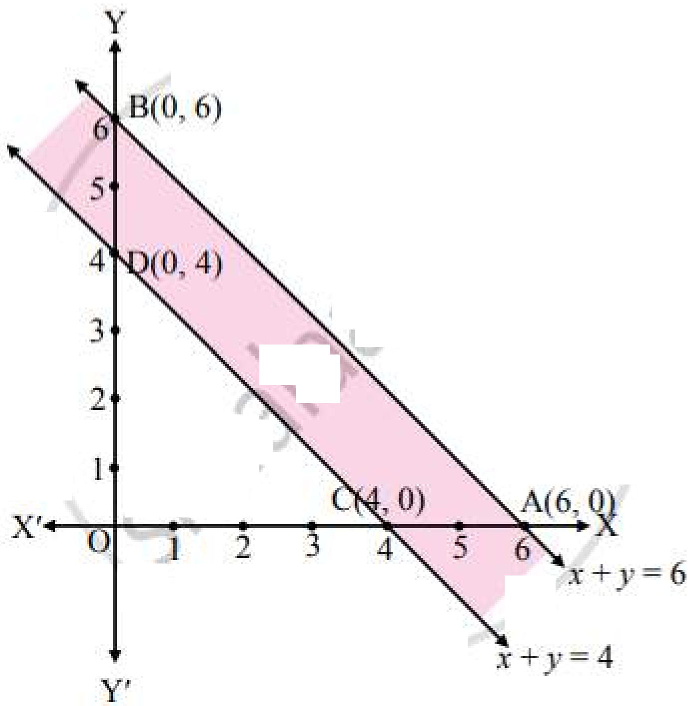
Miscellaneous Exercise 8 | Q 6 | Page 122

Solve the following system of inequalities graphically x + y ≤ 6, x + y ≥ 4

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
$x + y \le 6$	$x + y = 6$	$\frac{x}{6} + \frac{y}{6} = 1$	A (6, 0), B (0, 6)	$0 + 0 \le 6$ $\therefore 0 \le 6$ $\therefore$ origin side
$x + y \ge 4$	$x + y = 4$	$\frac{x}{4} + \frac{y}{4} = 1$	C (4, 0), D (0, 4)	$0 + 0 \not\ge 4$ $\therefore 0 \not\ge 4$ $\therefore$ non-origin side



Shaded portion represents the graphical solution.

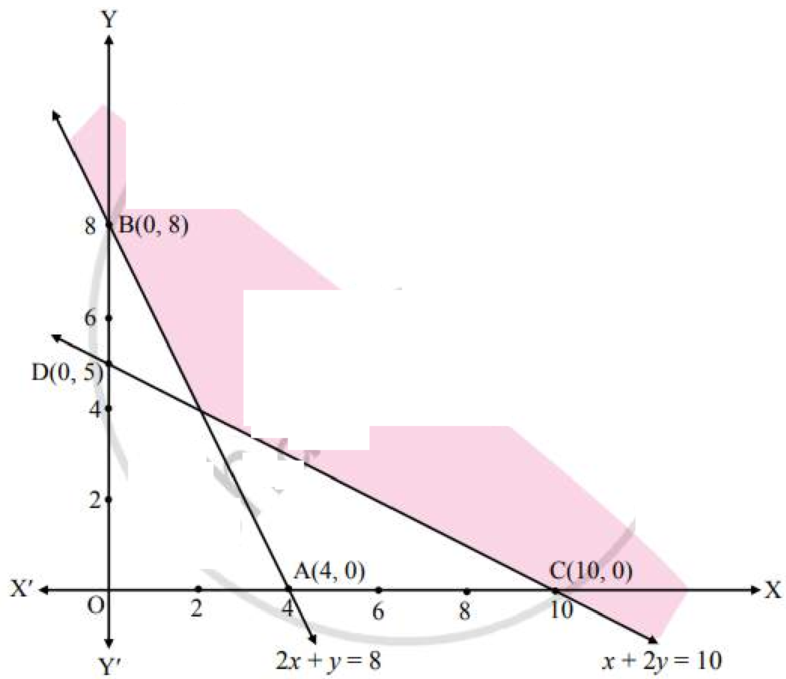
Miscellaneous Exercise 8 | Q 7 | Page 122

Solve the following system of inequalities graphically $2x + y \ge 8$, $x + 2y \ge 10$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
2x + y ≥ 8	2x + y = 8	$\frac{x}{4}+\frac{y}{8}=1$	A (4, 0), B (0, 8)	2(0) + 0 ≱ 8 ∴ 0 ≱ 8 ∴ non-origin side
x + 2y ≥ 10	x + 2y = 10	$\frac{x}{10}+\frac{y}{5}=1$	C (10, 0), D (0, 5)	0 + 2(0) ≱ 10 ∴ 0 ≱ 10 ∴ non-origin side



Shaded portion represents the graphical solution.

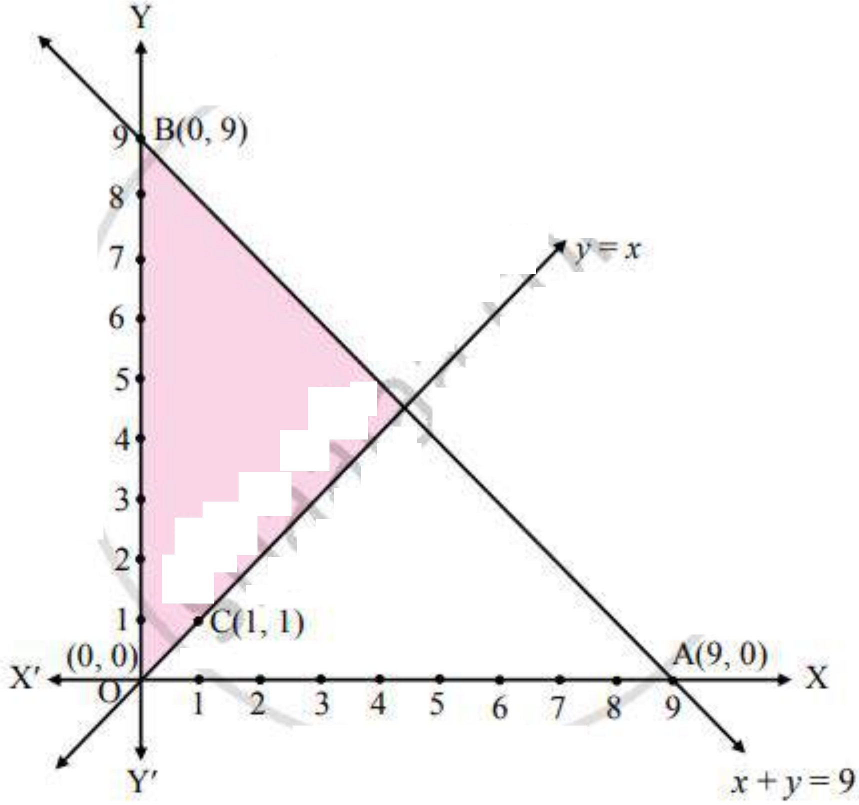
Miscellaneous Exercise 8 | Q 8 | Page 122

Solve the following system of inequalities graphically x + y ≤ 9, y > x, x ≥ 0

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
$x + y \le 9$	$x + y = 9$	$\frac{x}{9} + \frac{y}{9} = 1$	A (9, 0), B (0, 9)	$0 + 0 \le 9$ $\therefore 0 \le 9$ $\therefore$ origin side
$y \ge x$	$y = x$	–	0 (0, 0), C (1, 1)	$\therefore 0 \ge 0$ $\therefore$ origin side
$x \ge 0$	$x = 0$	–		R.H.S. of Y-axis



Shaded portion represents the graphical solution.

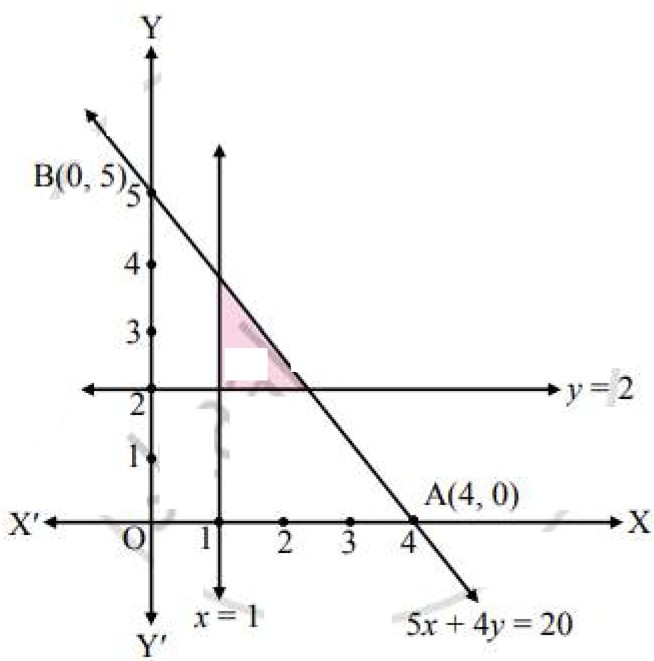
Miscellaneous Exercise 8 | Q 9 | Page 122

Solve the following system of inequalities graphically $5x + 4y \le 20$, $x \ge 1$, $y \ge 2$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
$5x + 4y \leq 20$	$5x + 4y = 20$	$\frac{x}{4} + \frac{y}{5} = 1$	A (4, 0), B (0, 5)	$5(0) + 4(0) \leq 20$ $\therefore 0 \leq 20$ $\therefore$ origin side
$x \geq 1$	$x = 1$	–	–	$\therefore 0 \not\geq 1$ $\therefore$ R.H.S. of line $x = 1$
$y \geq 2$	$y = 2$	–	–	$\therefore 0 \not\geq 2$ $\therefore$ above the line $y = 2$



Shaded portion represents the graphical solution.

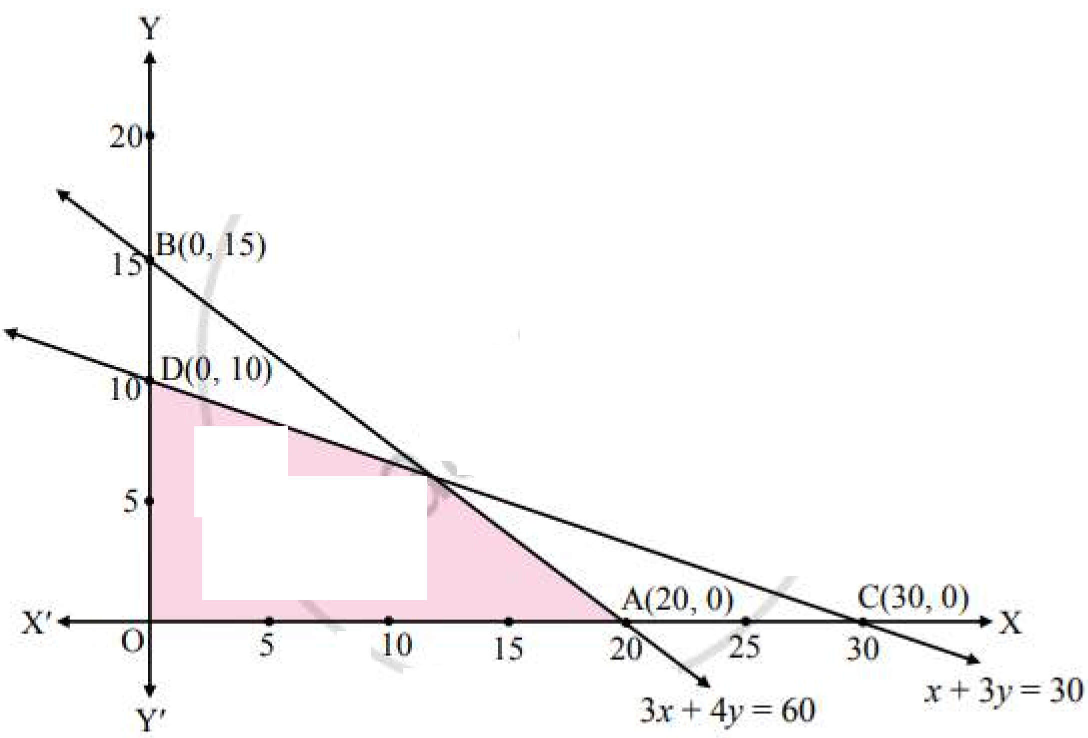
Miscellaneous Exercise 8 | Q 10 | Page 122

Solve the following system of inequalities graphically $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
3x + 4y ≤ 60	3x + 4y = 60	$\frac{x}{20}+\frac{y}{15}=1$	A (20, 0), B (0, 15)	5(0) + 4(0) ≤ 60 ∴ 0 ≤ 60 ∴ origin side
x + 3y ≤ 30	x + 3y = 30	$\frac{x}{30}+\frac{y}{10}=1$	C (30, 0), D (0, 10)	0 + 3 (0) ≤ 30 ∴ 0 ≤ 30 ∴ origin side
x ≥ 0	x = 0	–	–	R.H.S. of Y-axis
y ≥ 0	y = 0	–	–	above X-axis



Shaded portion represents the graphical solution.

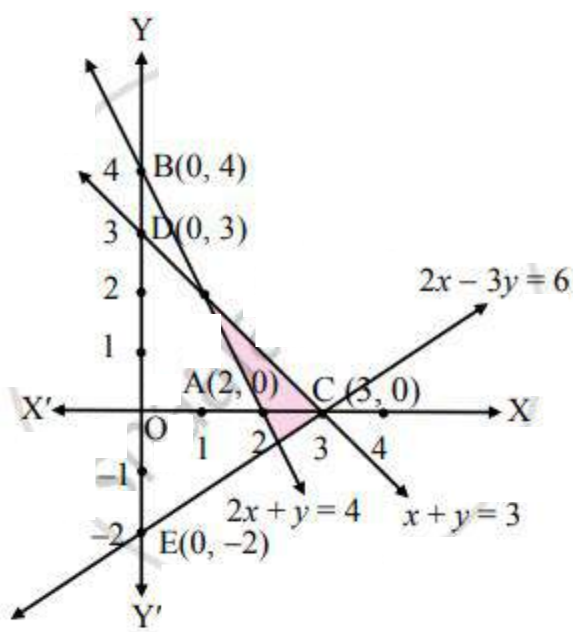
Miscellaneous Exercise 8 | Q 11 | Page 122

Solve the following system of inequalities graphically $2x + y \geq 4$, $x + y \leq 3$, $2x - 3y \leq 6$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
$2x + y \geq 4$	$2x + y = 4$	$\frac{x}{2} + \frac{y}{4} = 1$	A (2, 0), B (0, 4)	$2(0) + 0 \not\geq 4$ $\therefore 0 \not\geq 4$
$x + y \leq 3$	$x + y = 3$	$\frac{x}{3} + \frac{y}{3} = 1$	C (3, 0), D (0, 3)	$0 + 0 \leq 3$ $\therefore 0 \leq 3$ $\therefore$ origin side
$2x - 3y \leq 6$	$2x - 3y = 6$	$\frac{x}{3} - \frac{y}{2} = 1$ i.e., $\frac{x}{3} + \frac{y}{-2} = 1$	C (3, 0), E (0, – 2)	$2(0) - 3(0) \leq 6$ $\therefore 0 \leq 6$ $\therefore$ origin side



Shaded portion represents the graphical solution.

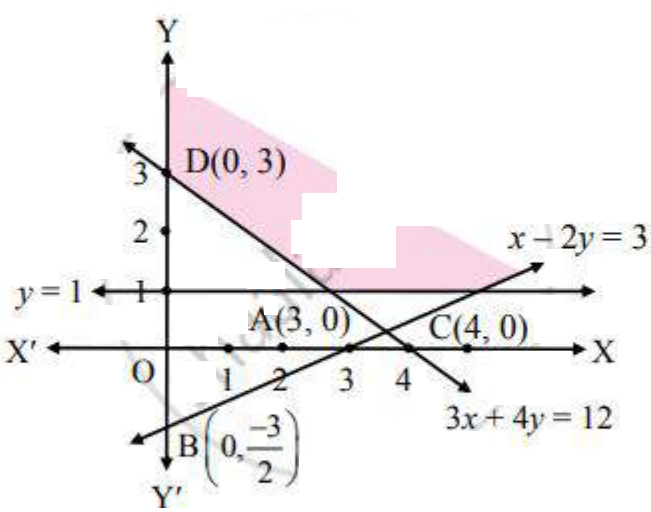
Miscellaneous Exercise 8 | Q 12 | Page 122

Solve the following system of inequalities graphically x – 2y ≤ 3, 3x + 4y ≥ 12, x ≥ 0, y ≥ 1

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
x – 2y ≤ 3	x – 2y = 3	$\frac{x}{3} - \frac{2y}{3} = 1$ i.e., $\frac{x}{3} + \frac{y}{\left(\frac{-3}{2}\right)} = 1$	A (3, 0), B $\left(0, \frac{-3}{2}\right)$	0 – 2(0) ≤ 3 ∴ 0 ≤ 3 ∴ origin side
3x + 4y ≥ 12	3x + 4y = 12	$\frac{x}{4} + \frac{y}{3} = 1$	C (4, 0), D (0, 3)	3(0) + 4(0) ≱ 12 ∴ 0 ≱ 12 ∴ non-origin side
x ≥ 0	x = 0	–	–	R.H.S. of Y-axis
y ≥ 1	y = 1	–	–	0 ≱ 1 Above the line y = 1



Shaded portion represents the graphical solution.

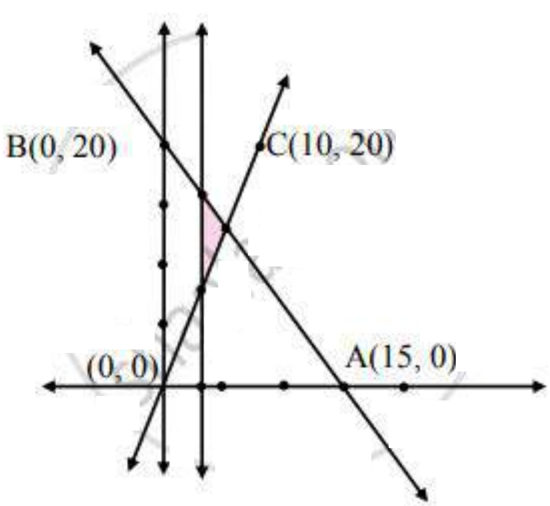
Miscellaneous Exercise 8 | Q 13 | Page 122

Solve the following system of inequalities graphically $4x + 3y \leq 60$, $y \geq 2x$, $x \geq 3$, $x, y \geq 0$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
$4x + 3y \leq 60$	$4x + 3y = 60$	$\frac{x}{15} + \frac{y}{20} = 1$	A (15, 0), B (0, 20)	$4(0) + 3(0) \leq 60$ $\therefore 0 \leq 60$ $\therefore$ origin side
$y \geq 2x$	$y = 2x$	–	–	$0 \geq 2(0)$ $\therefore 0 \geq 0$ $\therefore$ origin side
$x \geq 3$	$x = 3$	–	–	$0 \not\geq 3$ $\therefore$ R.H.S. of the line $x = 3$
$x \geq 0$	$x = 0$	–	–	R.H.S. of Y-axis
$y \geq 0$	$y = 0$	–	–	Above X-axis



Shaded portion represents the graphical solution.

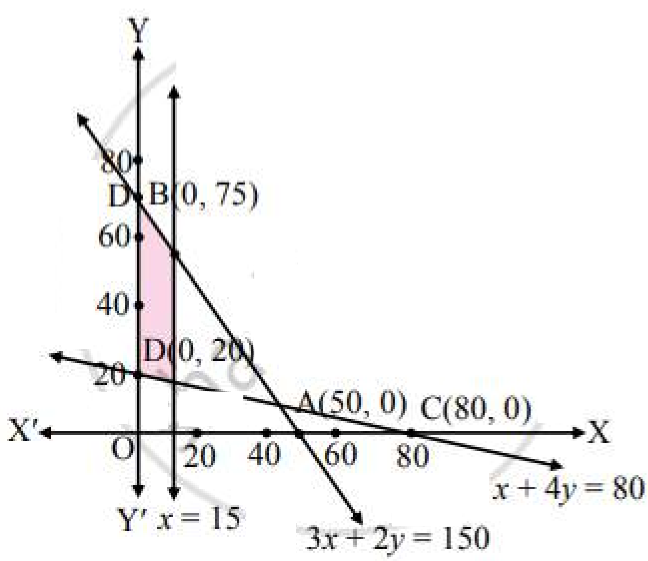
Miscellaneous Exercise 8 | Q 14 | Page 122

Solve the following system of inequalities graphically $3x + 2y \le 150$, $x + 4y \ge 80$, $x \le 15$, $y \ge 0$, $x \ge 0$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Doule Intercept form	Points (x, y)	Region
$3x + 2y \le 150$	$3x + 2y = 150$	$\frac{x}{5} + \frac{y}{75} = 1$	A (50, 0), B (0, 75)	$3(0) + 2(0) \le 150$ $\therefore 0 \le 150$ $\therefore$ origin side
$x + 4y \ge 80$	$x + 4y = 80$	$\frac{x}{80} + \frac{y}{20} = 1$	C (80, 0), D (0, 20)	$0 + 4(0) \not\ge 80$ $\therefore 0 \not\ge 80$ $\therefore$ non-origin side
$x \le 15$	$x = 15$	–	–	$0 \le 15$ $\therefore$ L.H.S. of the line $x = 15$
$x \ge 0$	$x = 0$	–	–	R.H.S. of Y-axis
$y \ge 0$	$y = 0$	–	–	Above X-axis



Shaded portion represents the graphical solution.

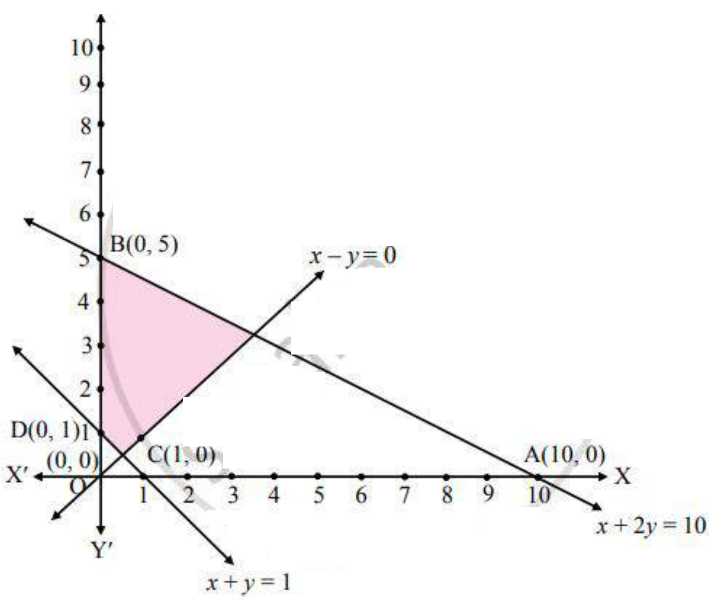
Miscellaneous Exercise 8 | Q 15 | Page 122

Solve the following system of inequalities graphically $x + 2y \le 10$, $x + y \ge 1$, $x - y \le 0$, $x \ge 0$, $y \ge 0$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	**Equation**	**Double Intercept form**	**Points (x, y)**	**Region**
$x + 2y \le 10$	$x + 2y = 10$	$\frac{x}{10} + \frac{y}{5} = 1$	A (10, 0), B (0, 5)	$0 + 2(0) \le 10$ $\therefore 0 \le 10$ $\therefore$ origin side
$x + y \ge 1$	$x + y = 1$	$\frac{x}{1} + \frac{y}{1} = 1$	C (1, 0), D (0, 1)	$0 + 0 \ngeq 1$ $\therefore 0 \ngeq 1$ $\therefore$ non-origin side
$x - y \le 0$	$x - y = 0$	–	0 (0, 0), E(1, 1)	$0 - 0 \le 0$ $\therefore 0 \le 0$ $\therefore$ origin side
$x \ge 0$	$x = 0$	–	–	R.H.S. of Y-axis
$y \ge 0$	$y = 0$	–	–	Above X-axis



Shaded portion represents the graphical solution.

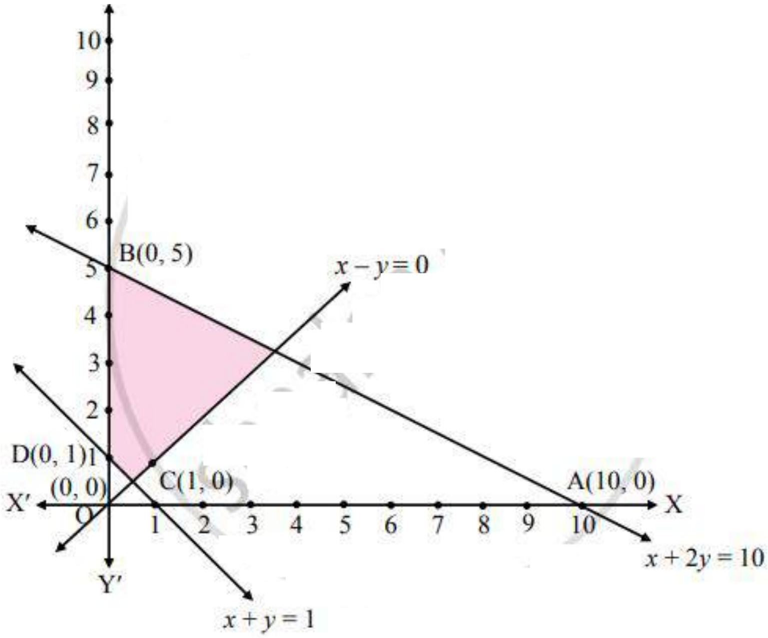
Miscellaneous Exercise 8 | Q 15 | Page 122

Solve the following system of inequalities graphically $x + 2y \le 10$, $x + y \ge 1$, $x - y \le 0$, $x \ge 0$, $y \ge 0$

SOLUTION

To find graphical solution, construct the table as follows:

Inequation	Equation	Double Intercept form	Points (x, y)	Region
$x + 2y \le 10$	$x + 2y = 10$	$\frac{x}{10} + \frac{y}{5} = 1$	A (10, 0), B (0, 5)	$0 + 2(0) \le 10$ $\therefore 0 \le 10$ $\therefore$ origin side
$x + y \ge 1$	$x + y = 1$	$\frac{x}{1} + \frac{y}{1} = 1$	C (1, 0), D (0, 1)	$0 + 0 \ngeq 1$ $\therefore 0 \ngeq 1$ $\therefore$ non-origin side
$x - y \le 0$	$x - y = 0$	–	0 (0, 0), E(1, 1)	$0 - 0 \le 0$ $\therefore 0 \le 0$ $\therefore$ origin side
$x \ge 0$	$x = 0$	–	–	R.H.S. of Y-axis
$y \ge 0$	$y = 0$	–	–	Above X-axis



Shaded portion represents the graphical solution.