CBSE Test Paper 04

Chapter 8 Application of Integrals

1. The area bounded by the parabola y = x^2 + 1 and the straight line x + y = 3 is given by

   a. none of these
   b. $\frac{25}{4}$
   c. $\frac{45}{7}$
   d. $\frac{9}{2}$

2. The area bounded by x = sin t and y = cos t +3 , where -2008 < t < 2008 , is equal to

   1. 2π
   2. 3π
   3. none of these
   4. π

3. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the line x = y$\sqrt{3}$ is

   a. $\frac{\pi}{2}$
   b. none of these
   c. $\frac{\pi}{3}$
   d. π

4. Area of the region bounded by the curves y = e^x, x = a , x = b and the x- axis is given by

   a. e^b - e^a
   b. e^b - a
   c. none of these
   d. e^b + e^a

5. The area common to the circle $x^2 + y^2 = 16a^2$ and the parabola y^2 = 6ax is

a. $\frac{4a^2}{3}(4\pi + \sqrt{3})$
b. $\frac{4a^2}{3}(8\pi + \sqrt{3})$
c. none of these
d. $\frac{4a^2}{3}(4\pi - \sqrt{3})$

6. Show that $e^x < 1 + x \ \forall \ x \in (0, 1)$.

7. If $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^{2n}$ be a polynomial in $x \in R$ with $0 < a_1 < a_2 \ldots < a_n$, then show that P(x) has a minimum at x = 0 only.

8. Find the equation of the tangent to the curve $y = x + \frac{4}{x^2}$ which is parallel to the X-axis.

9. Find the area of the region $\{(x, y) : x^2 \leqslant y \leqslant x\}$.

10. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

11. Calculate the area of the region bounded by the two parabolas y = x^2 and x = y^2.

12. Find the area of the region:$\{(x, y) : x^2 + y^2 \leqslant 1 \leqslant x + y\}$. **4**

13. Find the area under the given curves and given lines: y = x^2, x = 1, x = 2 and x - axis.

14. Find the value of $\lim_{a \to \infty} \frac{1}{a} \sum_{v=1}^{a} sin^{2p} \frac{v\pi}{2a}$.

15. Using integration, find the area of the region given below:
$\{(x, y) : 0 \leqslant y \leqslant x^2 + 1, 0 \leqslant y \leqslant x + 1, 0 \leqslant x \leqslant 2\}$.

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Solution

1. (d) $\frac{9}{2}$

   Explanation: The two curves parabola and the line meet where,

   3 - x =$x^2+1 \Rightarrow x^2+x-2$ = 0 $\Rightarrow x=-2,1.$

   Required area: $\int\limits_{-2}^{1}\{3-x-(x^2+1)\}dx$ = $\left[2x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-2}^{1}$ = $\frac{9}{2}$

2. (d) π

   Explanation: We have : sin t = x , cos t = y – 3 ., therefore , sin2t + cos2t = 1,

   $\Rightarrow x^2+(y-3)^2=1$ It is a circle with centre (0, 3) and radius is 1. Therefore , area = $\pi r^2=\pi\times 1=\pi.$

3. (c) $\frac{\pi}{3}$

   Explanation: The given circle is $x^2+y^2=4$....(1) Its centre is (0,0) and radius = 2. The given line is x = $\sqrt{3}$ y ....(2) Therefore , we have When y = 1 , x = $\sqrt{3}$.

   Required area is:

   $\int\limits_{0}^{\sqrt{3}}\frac{x}{\sqrt{3}}dx+\int\limits_{\sqrt{3}}^{2}\sqrt{4-x^2}dx$ = $\frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}}$ + $\left[\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$ = $\frac{\pi}{3}$

4. (a) e^b - e^a

   Explanation: Required area = $\int\limits_a^b e^x dx=[e^x]_a^b=e^b-e^a$

5. (a) $\frac{4a^2}{3}(4\pi+\sqrt{3})$

   Explanation: Required area: $=2\left[\int\limits_0^{2a}\sqrt{6ax}\,dx+\int\limits_{2a}^{4a}\sqrt{16a^2-x^2}dx\right]$

   $$=2\sqrt{6a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{2a}+2\left[\frac{x\sqrt{16a^2-x^2}}{2}+\frac{16a^2}{2}\sin^{-1}\frac{x}{4a}\right]_{2a}^{4a}$$

   $=\frac{4a^2}{3}(4\pi+\sqrt{3})$

6. Let f(x) = e^x - x - 1, then

$f'(x) = e^x - 1 > 0, \forall\, x \in (0,1)$

(since e^x is an ncreasing function, ex > 0)

$\sin ce\ e^x\ is\ an\ increa\sin g\ function\ ,\ e^x > 0)$

$\Rightarrow$ f(x) is increasing in (0,1)

$when\ x = 0,\ f(\ 0)\ = e^0 - 0 - 1 = 1 - 1 = 0. Since\ f(x) is > 0, we\ have$

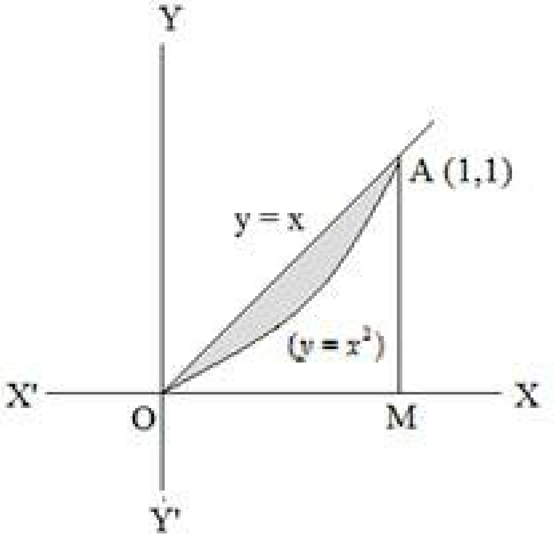
$\Rightarrow$ f(x) > f(0) for 0 < x < 1

$\Rightarrow$ e^x -1 - x > 0

$\Rightarrow$ e^x > 1 + x for 0 < x < 1

7. We have, $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^{2n}$
   $P'(x) = 2a_1x + 4a_2x^3 + \ldots + 2na_n\, x^{2n-1}$
   For maximum or minimum, $P'(x) = 0$
   $2a_1x + 4a_2x^3 + \ldots + 2na_n\, x^{2n-1} = 0$
   $x(2a_1 + 4a_2x^2 + \ldots + 2na_n\, x^{2n-2}) = 0$
   $x = 0\ [each\ a_i > 0\ and\ powers\ of\ x\ are\ even\ ]$
   Now, P''(x) = $2a_1 + 12a_2x^2 + \ldots + 2n(2n-1)a_n\, x^{2n-2}$
   $\therefore P''(x)]_{x=0} = 2a_1 > 0\ i.e\ P(x)$ has a minimum at x=0 only.
8. We have, $y = x + \frac{4}{x^2}$ ....(1)
   $\Rightarrow y\ =\ x\ +\ 4\, x^{-2}$
   On differentiating w.r.t x, we get,
   $\frac{dy}{dx} =\ 1\ +\ 4\ \times(\ -2\, x^{-3})\ =\ 1\ -\ 8\, x^{-3} = 1\ -\ \frac{8}{x^3}$
   $\frac{dy}{dx} = 1 - \frac{8}{x^3}$
   Since the tangent is parallel to X-axis, therefore
   $\frac{dy}{dx} = 0$
   $\Rightarrow\ 1 - \frac{8}{x^3} = 0$
   $\Rightarrow\ x^3 = 8$
   $\Rightarrow\ x = 2$
   From (1), when x = 2 , we get , y=$2 + \frac{4}{4} = 2 + 1 = 3$
   Therefore, y=3 is required equation.

9.



y = x^2

y = x

$\Rightarrow x=0, y=0$

x = 1, y = 1

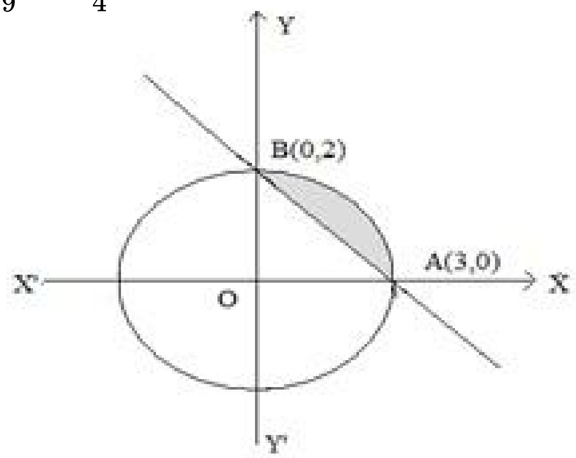
Area $=\int_0^1 x d x-\int_0^1 x^2 d x .$

$=\left[\frac{x^2}{2}\right]_0^1-\left[\frac{x^3}{3}\right]_0^1$

$=\frac{1}{2}-\frac{1}{3}$

$=\frac{1}{6}$ sq. units

10. $\frac{x^2}{9}+\frac{y^2}{4}=1$



$\frac{x}{3}+\frac{y}{2}=1$

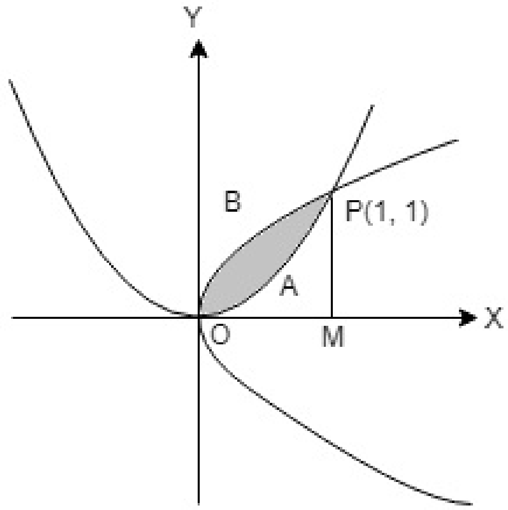
$\Rightarrow \frac{x^2}{(3)^2}+\frac{y^2}{(2)^2}=1$ is the equation of ellipse and

$\frac{x}{3}+\frac{y}{2}=1$ is the equation of intercept form

Area $=\frac{2}{3} \int_0^3 \sqrt{9-x^2} d x-\frac{2}{3} \int_0^3(3-x) d x$

$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}sin^{-1}(\frac{x}{3})-3x+\frac{x^2}{2}\right]_0^3$
$= \frac{2}{3}\left[\frac{9}{2}\frac{\pi}{2}-9+\frac{9}{2}\right]$
$= \frac{2}{3}\left[\frac{9\pi}{4}-\frac{9}{2}\right]$
$= \frac{3}{2}(\pi-2)$ sq units

11. The equations of parabolas are
 $y^2 = x$ ....(1)
 and $x^2 = y$ ....(2)



From (2), $y = x^2$ ....(3)

putting this value of y in (1),we get,

$x^4 = x \ or \ x(x^3-1)=0$

x=0,1

From (3) ,y=0,1

Therefore, parabolas (1) and (2) intersect in O(0,0),P(1,1).

From P, draw PM $\perp$ x-axis.

Required area

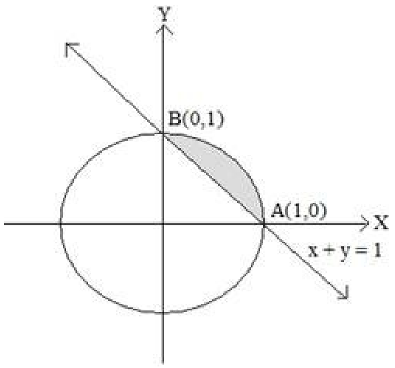
= Area of region OAPB=Area of region OBPM-area of region OAPM

= $\int_0^1 \sqrt{x}\,dx - \int_0^1 x^2 dx$

$= \left[\frac{x^{3/2}}{3/2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$

$= \frac{2}{3}[x^{3/2}]_0^1 - \frac{1}{3}\left[x^3\right]_0^1$

$= \frac{2}{3}[1-0]-\frac{1}{3}[1-0] = \frac{2}{3}-\frac{1}{3} = \frac{1}{3}$ sq.units

12. 

$x^2 + y^2 = 1$

Area $= \int\limits_0^1 \sqrt{1-x^2}dx - \int_0^1 (1-x)\,dx$

$= \int_0^1 [\sqrt{1-x^2} - (1-x)]dx$

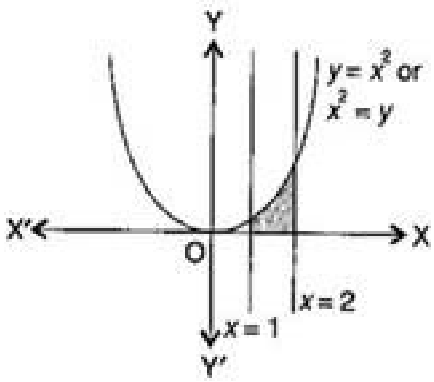
$= \left[\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}sin^{-1}(\frac{x}{1}) - x + \frac{x^2}{2}\right]_0^1$

$= [0 + \frac{1}{2}sin^{-1}1 - 1 + \frac{1}{2}] - 0$

$= \frac{\pi}{4} - \frac{1}{2}$

13. Equation of the curve (parabola) is

$y = x^2$ ...(i)



Required area bounded by curve (i), vertical line x = 1, x = 2 and x - axis

$= \left|\int\limits_1^2 ydx\right|$

$= \left|\int\limits_1^2 x^2dx\right|$

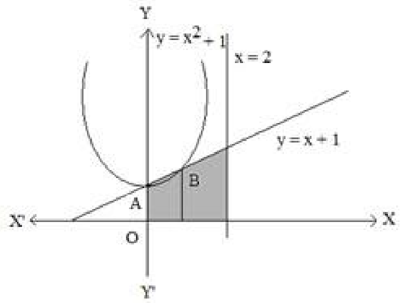
$= \left(\frac{x^3}{3}\right)_1^2$

$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$ sq units

14. $\lim_{a\to\infty} \frac{1}{a} \sum_{v=1}^{a} sin^{2p}\ \frac{v\pi}{2a}$

$= \int_0^1 sin^{2p}\ \frac{\pi y}{2}dy$

$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} sin^{2p}t\ dt\ \left(putting\ \frac{\pi y}{2} = t\ \Rightarrow dy = \frac{2}{\pi}dt\right)$
$= \frac{2}{\pi} \cdot \frac{(2p-1)(2p-3)...1}{2p(2p-2)...2} \cdot \frac{\pi}{2}$
$= \frac{[(2p-1)(2p-3)(2p-5)...1][2p.(2p-2)...2]}{2^p[p(p-1)(p-2)....1][2p.(2p-2)...2]}$
$= \frac{2p(2p-1)(2p-2)(2p-3)...2.1}{2^p[p(p-1)(p-2)...1].2^p[p(p-1)(p-2)....1]}$
$= \frac{(2p)!}{2^{2p}.(p!)^2}$

15. 

$y = x^2 + 1$.....(1)

y = x + 1.......(2)

Solving (1) and(2),we get, x = 1 and y = 2.

Area $= \int_0^1 \left(x^2+1\right)dx + \int_1^2 (x+1)dx$
$= [\frac{x^3}{3} + x]_0^1 + [\frac{x^2}{2} + x]_1^2$
$= [(\frac{1}{3}+1) - 0] + [(2+2) - (\frac{1}{2}+1)]$
$= \frac{23}{6}$