CBSE Sample Paper-01
SUMMATIVE ASSESSMENT –I
Class IX MATHEMATICS

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

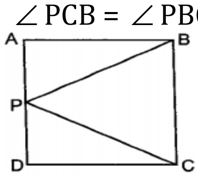
a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

Section A

1. Find the value of $0.\overline{23} + 0.\overline{22}$.
2. Find the value of $f(x) = 2x^2 + 7x + 3$ at $x = -2$.
3. It is given that $\Delta ABC \cong \Delta DEF$. Is it true to say that AB = EF? Justify your answer.
4. Find the point which lies on the line y = -3x having abscissa 3.

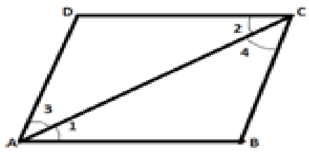
Section B

5. Find the value of $32^{\frac{1}{5}}$
6. Find the value of k, if y + 3 is a factor of $3y^2 + ky + 6$.
7. Prove that a triangle must have at least two acute angles.
8. Two supplementary angles are in the ratio $4:5$. Find the angles.
9. If the bisectors of a pair of corresponding angles formed by a transversal with two given lines are parallel, prove that the given lines are parallel.
10. In given figure ABCD is a square and P is the midpoint of AD. BP and CP are joined. Prove that $\angle$ PCB = $\angle$ PBC.



Section C

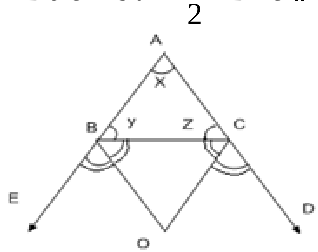
11. Prove that $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{a-c}+x^{b-c}} = 1$
12. Represent $\sqrt{3}$ on number line.
13. If x – 2k is a factor of $f(x) = x^4 - 4k^2k^2 + 2x + 3k + 3$ find k.
14. Find the remainder when $f(x) = 9x^3 - 3x^2 + 14x - 3$ is divided by $3x - 1 = 0$.
15. In the given figure, it is given that $\angle 1 = \angle 4$ and $\angle 3 = \angle 2$. By which Euclid's axiom, it can be shown that if $\angle 2 = \angle 4$ then $\angle 1 = \angle 3$.



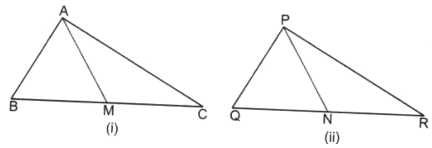
16. Show that in a quadrilateral ABCD, AB + BC + CD + DA > AC + BD.
17. Let OA, OB, OC and OD be the rays in the anti-clock wise direction starting from OA such that $\angle AOB = \angle COD = 100°, \angle BOC = 82°$ and $\angle AOD = 78°$. Is it true that AOC and BOD are straight lines? Justify your answer.
18. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.
19. Locate the points (5, 0), (0, 5), (2, 5), (5, 2), (-3, 5), (-3, -5) and (6, 1) in the Cartesian plane.
20. In a parallelogram PQRS. The Altitude corresponding to sides PQ and PS are respectively. 7 cm and 8 cm find PS, if PQ=10 cm.

Section D

21. Represent $\sqrt{5}$ on number line.
22. Rationalize the denominator of $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{10}}$.
23. Gita told her classmate Radha that "$\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is an irrational number." Radha replied that "you are wrong" and further claimed that "If there is a number x such that x^3 is an irrational number, then x^5 is also irrational." Gita said, No Radha, you are wrong". Radha took some time and after verification accepted her mistakes and thanked Gita for pointing out the mistakes.
    Read the above passage and answer the following questions:
    Justify both the statements.
    What value is depicted from this question?
24. Factorize by using factor theorem: $y^3 - 7y + 6$
25. Factorize: $x^3 + \frac{1}{x^3} - 2$
26. In given figure the side AB and AC of ΔABC are produced to point E and D respectively. If bisector BO and CO of $\angle CBE$ and $\angle BCD$ respectively meet at point O, then prove that $\angle BOC = 90° - \frac{1}{2}\angle BAC$ ..



27. In given figure, two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of PQR. Show that ABC PQR.



28. Two plane mirrors are placed perpendicular to each other, as shown in the figure. An incident ray AB to the first mirror is first reflected in the direction of BC and then reflected by the second mirror in the direction of CD. Prove that AB $\parallel$ CD.
29. In the figure, it is given that $\angle A = \angle C$ and AB = BC. Prove that $\Delta ABD \cong \Delta CBE$.
30. If $y^3 + ay^2 + by + 6$ is divisible by y – 2 and leaves remainder 3 when divided by y – 3, find the values of a and b.
31. The side of a square exceeds the side of another square by 4 cm and the sum of the areas of the two squares is 400 sq. cm. Find the dimensions of the squares.

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ANSWER KEY

1. $0.\overline{23} = 0.232323...$

   $0.\overline{22} = 0.222222...$

   $0.\overline{23} + 0.\overline{22} = 0.454545...$

   $= 0.\overline{45}$

2. $f(x) = 2x^2 + 7x + 3$

   $f(-2) = 2(-2)^2 + 7(-2) + 3$
   $= 8 - 14 + 3 = 11 - 14 = -3$

3. No, AB and EF are not corresponding sides in triangles ABC and DEF. Here, AB corresponds to DE.

4. When x = 3 then y = -9, Thus the point is (3, -9).

5. We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

   We conclude that $32^{\frac{1}{5}}$ can also be written as $\sqrt[5]{32} = \sqrt[2]{2\times2\times2\times2\times2}$

   $\sqrt[5]{32} = \sqrt[2]{2\times2\times2\times2\times2} = 2$

   Therefore the value of $32^{\frac{1}{5}}$ will be2.

6. Let $p(y) = 3y^2 + ky + 6$

   As y + 3 is a factor of p(y), so p(-3) = 0

   i.e., $3(-3)^2 + k(-3) + 6 = 0$
   $\Rightarrow \quad 27 - 3k + 6 = 0 \quad \Rightarrow \quad 33 - 3k = 0$
   $\Rightarrow \quad -3k = -33 \qquad \Rightarrow \quad k = 11$

7. Let us assume a triangle ABC which has only one acute angle (say $\angle A$) Then we have the following three cases:

   (i) The other two angle ($\angle B$ and $\angle C$) are right angles.

   Then $\angle A + \angle B + \angle C = \angle A + 90° + 90° = \angle A + 180° > 180°$ which is not possible.

   (ii) Then other two angles ($\angle B$ and $\angle C$) are obtuse angles.

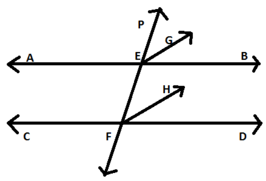
   Then $\angle A + \angle B + \angle C > 180°$ which is not possible.

   (iii) One angle (say $\angle B$) is right and the other angle($\angle C$) is obtuse.

Then $\angle$A+ $\angle$b + $\angle$C>180° which is not possible as we know that sum of the three angles of a triangle is 180° by angle sum property of a triangle. Thus, a triangle must have atleast two acute angles.

8. Two supplementary angles are $80°, 100°$.

9. **Given**: AB and CD are two lines where as PQ is a transversal line which intersect AB at E and CD at F point, $EG \parallel FH$.



To prove: $AB \parallel CD$

Proof: $EG \parallel FH$

$\Rightarrow \angle PEG = \angle EFH$ (corresponding angles)

$\Rightarrow \angle GEB = \angle HFD$

$\Rightarrow 2\angle GEB = 2\angle HFD$

$\Rightarrow \angle PEB = \angle EFD$ ($\therefore \angle GEB = \frac{1}{2}\angle PEB$ and $\angle HFD = \frac{1}{2}\angle EFD$)

But, these are corresponding angles where AB and CD are intersected by the transversal PQ.

$\because AB \parallel CD$ (corresponding angles axiom)

10. In triangles PAB and PDC,

PA = PD (given)

AB = CD (side of square)

$\angle$ PAB = $\angle$ PDC = 90° (By RHS, $\Delta PAB \cong \Delta PDC$)

$\therefore$ PC = PB $\Rightarrow$ $\angle$ PCB = $\angle$ PBC

11. $$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}$$

$$\frac{1}{1+x^{b}.x^{-a}+x^{c}.x^{-a}} + \frac{1}{1+x^{a}.x^{-b}+x^{c}.x^{-b}} + \frac{1}{1+x^{a}.x^{-c}+x^{b}.x^{-c}}$$

$$\frac{1}{x^{-a}.x^{a}+x^{b}.x^{-a}+x^{c}.x^{-a}}+\frac{1}{x^{b}.x^{-b}+x^{a}.x^{-b}+x^{c}.x^{-b}}+\frac{1}{x^{c}.x^{-c}+x^{a}.x^{-c}+x^{b}.x^{-c}}$$

$$\frac{1}{x^{-a}\left(x^{a}+x^{b}+x^{c}\right)}+\frac{1}{x^{-b}\left(x^{a}+x^{b}+x^{c}\right)}+\frac{1}{x^{-c}\left(x^{a}+x^{b}+x^{c}\right)}$$

$$\frac{x^{a}}{\left(x^{a}+x^{b}+x^{c}\right)}+\frac{x^{b}}{\left(x^{a}+x^{b}+x^{c}\right)}+\frac{x^{c}}{\left(x^{a}+x^{b}+x^{c}\right)}$$

$$=\frac{x^{a}+x^{b}+x^{c}}{x^{a}+x^{b}+x^{c}}=1$$

12. *Take* OA=AB=1 unit in same line

and $\angle A = 90^0$

In Δ OAB, $OB^2=1^2+1^2$

$OB^2=2$

$OB=\sqrt{2}$

$\therefore OB=$ OA=$\sqrt{2}=1.41\sqrt{2}$, BD=1 and ∠OBD=90^0

$OD^2=OB^2+BD^2$

$OD^2=\left(\sqrt{2}\right)^2+(1)^2$

$OD^2=2+1=3$

$OD=\sqrt{3}$

13. Here, $f(x)=x^4-4k^2k^2+2x+3k+3$

Since (x + 2k) is a factor of *f (x)* so by factor theorem,

$f(-2k)=0$

$(-2k)^4-4k^2(-2k)^2+2(-2k)+3k+3=0$

$16k^4-16k^4-4k+3k+3=0$

$\Rightarrow -k+3=0$

$\Rightarrow -k=-3$

$\Rightarrow k=3$

14. Taking g(x) = 0 we have,

$3x-1=0 \qquad \Rightarrow \qquad x=\frac{1}{3}$

By remainder theorem when *f(x)* is divided by *g(x)*, the remainder is equal to $f\left(\frac{1}{3}\right)$

Now, $f(x)=9x^3-3x^2+14x-3$

$$f\left(\frac{1}{3}\right)=9\left(\frac{1}{3}\right)^3-3\left(\frac{1}{3}\right)^2+14\left(\frac{1}{3}\right)-3$$

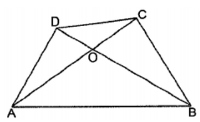
$$=9\times\frac{1}{27}-3\times\frac{1}{9}+\frac{14}{3}-3=\frac{1}{3}-\frac{1}{3}+\frac{14}{3}-3$$

$f\left(\frac{1}{3}\right)=\frac{5}{3}$

Hence, required remainder $=\frac{5}{3}$

15. By Euclid's 1axiom, which states that " things which are equal to the same thing are equal to one another ". Prove this statement youself.

16. Since the sum of any two sides of a triangle is greater than the third side.



Therefore, in ΔABC, we have

AB + BC > AC ...(i)

In ΔBCD, we have

BC + CD > BD ...(ii)

In ΔCDA, we have

CD + DA > AC ...(iii)

InΔDAB, we have

DA + AB > BD ...(iv)

Adding: (i) ,(ii) (iii) and (iv), we get

2Ab + 2BC + 2CD + 2AD > 2AC + 2BD

⇒2(AB + BC + CD + DA) > 2(AC + BD)

⇒AB + BC + CD + DA > AC + BD

17. Draw the figure

Given: OA, OB, OC and OD are rays in the anticlockwise direction such that

$\angle AOB=\angle COD=100°$ and $\angle BOC=82°, \angle AOD=78°$

AOC is not a line.

Because, $\angle AOB+\angle BOC=100°+82°=182°$, which is not equal to $180°$.

Similarly, BOD is not a line.

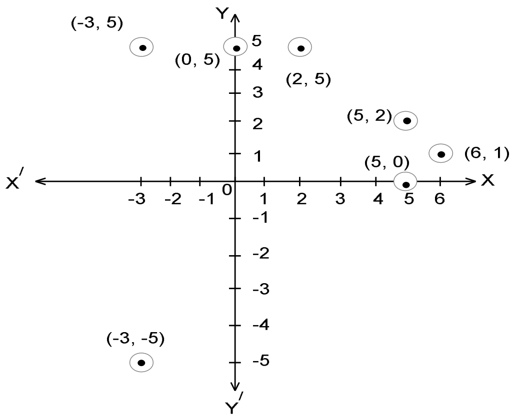
Because, $\angle COD+\angle AOD=78°+100°=178°$, which is not equal to $180°$.

18. **Given:** A ΔABC in which AD is the bisector of ∠A which meets BC in D such that BD = DC

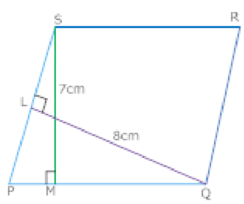
To prove: AB = AC
Construction: produced AD to E such that AD = DE and then join CE.
Proof: In ΔABD and ΔECD, we have
BD = CD (Given)
AD = ED (By construction)

and ∠ADB = ∠EDC (Vertically opposite angles)
Therefore, ΔABD≅ ΔECD (SAS congruence criterion)
So, AB = EC (CPCT) ...(i)
and ∠BAD = ∠CED (CPCT) ...(ii)
Also, ∠BAD = ∠CAD (Given) ...(iii)
Therefore, from (ii) and (iii)
∠CAD = ∠CED
So, AC = EC (Sides opposite to equal angles) ...(iv)
From (i) and (iv), we get
AB = AC

19.



20.



Area of ||gram PQRS = =PQ×SM

=10×7

=70 square cm………….. (i)

Area of Parallelogram PQRS

= PS×QL

= (AD×8) square cm……………(ii)

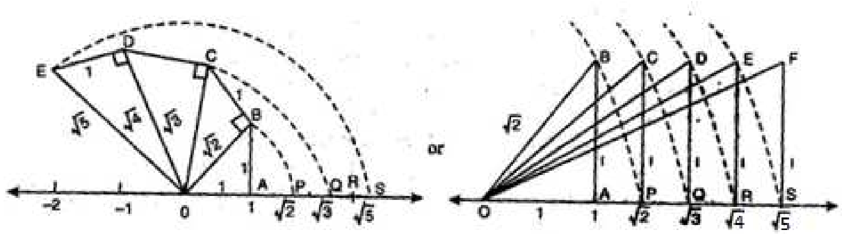
From (i) and (ii)

PS×8=70

PS=$\frac{70}{8}$

= 8.75 cm

21.



22. $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{10}}.\times\frac{\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{10}}{\left(\sqrt{2}+\sqrt{3}\right)-\sqrt{10}}$

$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{10}}{\left(\sqrt{2}+\sqrt{3}\right)^2-\left(\sqrt{10}\right)^2} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{10}}{2\sqrt{6}-5}$

$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{10}}{2\sqrt{6}-5}\times\frac{2\sqrt{6}+5}{2\sqrt{6}+5} = \frac{\left(\sqrt{2}+\sqrt{3}-\sqrt{10}\right)\left(2\sqrt{6}+5\right)}{\left(2\sqrt{6}\right)^2-\left(5\right)^2}$

$= \frac{2\sqrt{12}+5\sqrt{2}+2\sqrt{18}+5\sqrt{3}-2\sqrt{60}-5\sqrt{10}}{24-25} = -4\sqrt{3}-5\sqrt{2}-6\sqrt{2}-5\sqrt{3}+4\sqrt{15}+5\sqrt{10}$

$= -11\sqrt{2}-9\sqrt{3}+5\sqrt{0}+4\sqrt{15}$

23. (a) $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is an irrational number.

$= \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}} = \sqrt{\frac{\left(\sqrt{2}-1\right)^2}{2-1}} = \sqrt{2}-1$

which is an irrational number.

Let there is a number x such that x^3 is an irrational but x^5 is a rational number.

Let $x=\sqrt[5]{7}$ is any number

$\Rightarrow x^3 = \left(\sqrt[5]{7}\right)^3 = \left(7^{3/5}\right)$ is an irrational number.

$\Rightarrow x^5 = \left(\sqrt[5]{7}\right)^5 = \left(7^{5/5}\right) = 7$ is a rational number.

(b) Accepting own mistakes gracefully, co-operative learning among the classmates.

24. Let $f\left(y\right)=y^3-7y+6$

The constant term in $f\left(y\right)$ is 6 and its factors are $\pm1,\pm2,\pm3,\pm6.$

On putting $y=-1$ in given expression, we get,

$f\left(-1\right)=\left(-1\right)^3-7\left(-1\right)+6=-1+7+6\neq0$

$f\left(+1\right)=\left(1\right)^3-7\left(1\right)+6=0$

So $(y-1)$ is a factor of $f(y)$.

Now we divide $f(y)=y^3-7y+6$ by $y-1$ to get other factors.

$$
\begin{array}{r|l}
 & y^2+y-6 \\
\hline
y-1 & y^3-7y+6 \\
 & y^3-y^2 \\
 & -\quad + \\
\hline
 & \quad y^2-7y+6 \\
 & \quad y^2-y \\
 & \quad -\quad + \\
\hline
 & \qquad -6y+6 \\
 & \qquad -6y+6 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\therefore\ y^3-7y+6=(y-1)(y^2+y-6)$

$=(y-1)(y^2+3y-2y-6)$

$=(y-1)[y(y+3)-2(y+3)]$

$=(y-1)(y+3)(y-2)$

25. $x^3+\frac{1}{x^3}-2 = x^3+\left(\frac{1}{x}\right)^3+1-3$

$=x^3+\left(\frac{1}{x}\right)^3+(1)^3-3\times x\times\frac{1}{x}\times 1$

$=\left(x+\frac{1}{x}+1\right)\left[x^2+\left(\frac{1}{x}\right)^2+1-x\times\frac{1}{x}-\frac{1}{x}\times 1-1\times x\right]$

$=\left(x+\frac{1}{x}+1\right)\left(x^2+\left(\frac{1}{x}\right)^2+1-1-\frac{1}{x}-x\right)$

$=\left(x+\frac{1}{x}+1\right)\left(x^2+\frac{1}{x^2}-\frac{1}{x}-x\right)$

26. Ray BO bisects $\angle CBE$

$\therefore \angle CBO=\frac{1}{2}\angle CBE$

$=\frac{1}{2}(180^\circ-y)(\because \angle CBE+y=180^\circ)$

$=90^\circ-\frac{y}{2}$ .... (i)

Similarly ray CO bisects $\angle BCD$

$\angle BCO=\frac{1}{2}\angle BCD$

$= \frac{1}{2}(180° - Z)$

$= 90° - \frac{Z}{2}$ .... (ii)

In ΔBOC

$\angle BOC + \angle BCO + \angle CBO = 180°$

$\angle BOC + 90° - \frac{Z}{2} + 90° - \frac{y}{2} = 180°$ [From eq (i) and (ii)]

$\angle BOC = \frac{1}{2}(y + z)$

But x +y + z = $180°$

y + z = $180° - x$

$\angle BOC = \frac{1}{2}(180° - x)$

$= 90° - \frac{x}{2}$

$\angle BOC = 90° - \frac{1}{2}\angle BAC$

27. In ΔABC and ΔPQR,

BC = QR (Given)

$\Rightarrow \frac{1}{2}$ BC = $\frac{1}{2}$ QR

$\Rightarrow$ BM = QN

In triangles ABM and PQN, we have

AB = PQ (Given)

BM = QN (proved above)

AM = PN (Given)

$\therefore \Delta ABM \cong \Delta PQN$ (SSS congruence criterion)

$\Rightarrow \angle$ B = $\angle$ Q (CPCT)

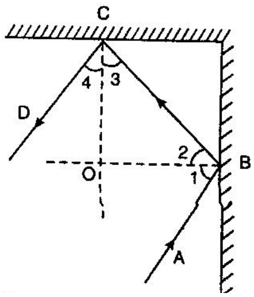
Now, in triangles ABC and PQR (Given)

$\angle$ B = $\angle$ Q (Proved above)

BC = QR (Given)

$\therefore \Delta ABC \cong \Delta PQR$ (SAS congruence criterion)

28. Let BO and CO be the normals to the mirrors. As mirrors are perpendicular to each other. SO their normals BO and CO are perpendicular.



$\therefore \angle BOC = 90^\circ$

In right angled triangle OBC, $\angle 2 + \angle 3 = 90^\circ$ .... (i)

$\angle 1 = \angle 2$

[Angle of incident = Angle of reflection]

$\angle 3 = \angle 4$

[Angle of incident = Angle of reflection]

On adding, $\angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 + \angle 4 = 90^\circ$ .... (ii)

On adding eq. (i) and (ii), we get,

$\angle 2 + \angle 3 + \angle 1 + \angle 4 = 180^\circ$

$\angle ABC + \angle BCD = 180^\circ$

But $\angle ABC$ and $\angle BCD$ are consecutive interior angles formed when the transversal BC intersect AB and CD.

$\therefore$ $AB \parallel CD$

29. In Δ s AOE and COD,

$\angle A = \angle C$ and $\angle AOE = \angle COD$ [Vertically opposite angles]

$\therefore \angle A + \angle AOE = \angle C + \angle COD$

$\Rightarrow 180^\circ - \angle AEO = 180^\circ - \angle CDO$ [$\because \angle A + \angle AEO = 180^\circ$ and $\angle C + \angle COD + \angle CDO = 180^\circ$

$\Rightarrow \angle AEO = \angle CDO$ .... (i)

Now, $\angle AEO + \angle OEB = 180^\circ$ [Angles of a linear pair]

and $\angle CDO + \angle ODB = 180^\circ$ [Angles of a linear pair]

$\Rightarrow \angle AEO + \angle OEB = \angle CDO + \angle ODB$

$\Rightarrow \angle OEB = \angle ODB$

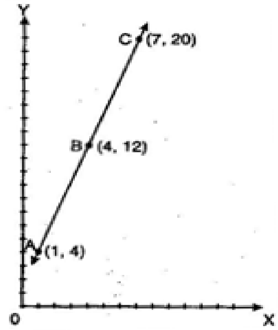
$\Rightarrow \angle CEB = \angle ADB$ .... (ii) [$\because \angle OEB = \angle CEB$ and $\angle ODB = \angle ADB$]

In Δ ADB and Δ CBE, $\angle A = \angle C$ [Given]

$\angle ADB = \angle CEB$ [From eq. (ii)]

And AB = BC [Given]

Δ ADB $\cong$ Δ CBE [By AAS]



30. Let $p(y) = y^3 + ay^2 + by + 6$

$p(y)$ is divisible by $y - 2$

Then $P(2) = 0$

$2^3 + a\times 2^2 + b\times 2 + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$4a+2b = -14$

$2a+b = -7$ (i)

If p (y) is divided by $y-3$ remainder is 3

$\therefore$ p (3)=3

$3^3 + a\times 3^2 + b\times 3 + 6 = 3$

$9a + 3b = -30$

$3a + b = -10$ (ii)

Subtracting (i) from (ii)

$-a = 3$ and $a = -3$

Put $a = -3$ in eq (i)

$2\times -3 + b = -7$

$-6 + b = -7$

$b = -7 + 6$

$b = -1$

31. Let S_1 and S_2 be the two squares. Let the side of the square S_2 be x cm in length.

Then the side of square S_1 is $(x+4)$ cm.

$\therefore$ Area of square S_1 = $(x+4)^2$

and Area of square S_2 = x^2

We are given that, Area of square S_1 + Area of square S_2 = 400 cm^2

$\Rightarrow (x+4)^2 + x^2 = 400$

$\Rightarrow x^2 + 8x + 16 + x^2 = 400$

$\Rightarrow 2x^2 + 8x - 384 = 0$

$\Rightarrow x^2 + 4x - 192 = 0$

$\Rightarrow x^2 + 16x - 12x - 192 = 0$

$\Rightarrow x(x+16) - 12(x+16) = 0$

$\Rightarrow (x+16)(x-12) = 0$

$\Rightarrow x = -16, 12$

As the length of the side of a square cannot be negative, therefore $x = 12$

$\therefore$ Side of square S_1 = $x + 4$ = 12 + 4 = 16 cm and side of square S_2 = 12 cm.