Exponents and Powers

- We use exponents to write very large numbers.

  For example, 1000000000 can be written as $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$
  It is read as ten raised to the power nine, where 10 is known as base and 9 as the exponent.
  The number 10^9 is known as exponential form of 1000000000.

- If the exponent of a negative base is odd, then the value of the exponential form is negative. However, if the exponent of a negative base is even, then the value of the exponential form is positive.

  For example, $(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$
  $(-1)^6 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1$

- There are certain rules which helps us in comparing exponential numbers.
  - An exponential form having a negative base and an even exponent will always be greater than an exponential form having a negative base and an odd exponent.
  - An exponential form having a positive base and any exponent will always be greater than an exponential form having a negative base and an odd exponent.
  - If two exponential forms have the same positive base, then the number with greater exponent will be greater.

  Example:
  Arrange 5^4, $(-4)^6$, and 6^3 in decreasing order.
  Solution:
  $5^4 = 5 \times 5 \times 5 \times 5 = 625$
  $(-4)^6 = (-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) = 4096$
  $6^3 = 6 \times 6 \times 6 = 216$
  Now, $4096 > 625 > 216$
  $\therefore (-4)^6 > 5^4 > 6^3$
  Thus, the given numbers can be arranged in descending order as $(-4)^6$, 5^4, 6^3.

- $a^m \times a^n = a^{m+n}$, where *a* is a non-zero integer and *m* and *n* are whole numbers.

  For example, $2^7 \times 2^3 = 2^{7+3}$

- $a^m \div a^n = a^{m-n}$

  Example:

Solve : $3^4 \div 3^5$

Solution:

$3^4 \div 3^5 = 3^{4-5} = 3^{-1}$

- $(a^m)^n = a^{mn}$

Example:

Solve : $\left(5^2\right)^3$

Solution:

$\left(5^2\right)^3 = 5^{2\times3} = 5^6$

- $a^m \times b^m = (ab)^m$

Example:

Solve : $2^3 \times 5^3$

Solution:

$2^3 \times 5^3 = \left(2 \times 5\right)^3 = 10^3$

- $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

Example: Simplify $\left[\left(3^4\right)^3 \div 3^5\right] \div 5^7$

Solution:

$$\left[\left(3^4\right)^3 \div 3^5\right] \div 5^7 = \left[(3)^{4\times3} \div 3^5\right] \div 5^7$$
$$= \left[\left(3\right)^{12} \div 3^5\right] \div 5^7$$
$$= \left[(3)^{12-5}\right] \div 5^7$$
$$= 3^7 \div 5^7$$
$$= \left(\frac{3}{5}\right)^7$$

- Any number with exponent 0 is equal to 1, that is, $2^0 = 3^0 = (-1)^0 = 1$
- $(-1)^{\text{odd number}} = -1$
- $(-1)^{\text{even number}} = 1$

- Rule for writing a number in standard form:

  Place a decimal point after the first digit from the left in the number. Next, count the number of digits in the number after the decimal point and remove all the zeroes, if any, which appear at the end of the number. This number of digits equals the exponent of 10 that has to be multiplied with the decimal.
  For example, 2943 can be written is standard form as:
  $2943 = 2.943 \times 1000 = 2.943 \times 10^3$

- When a number is written in standard form, its decimal part is greater than or equal to 1.0 and lesser than 10.0.
- Rule for identifying a number from its standard form:

  First write the exponent of 10 as a number and then multiply it with the decimal part to obtain the required number.
  For example, 9.005×10^6 can be written in normal form as:
  $9.005 \times 10^6 = 9.005 \times 1000000 = 9005000$