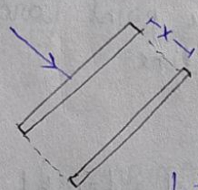
lecture - 10
5/04/19

$$P + \frac{1}{2}\rho v^2 + \rho g z = \text{const.}$$

P = Pressure energy per unit vol^m



Workdone = F.X = P(AX)

⇓ Energy

↓ volume

$\frac{1}{2}\rho v^2 \Rightarrow$ kinetic energy per unit vol^n

$$\left\{ \frac{\frac{1}{2} m v^2}{\forall} = \frac{1}{2}\rho . v^2 \right\}$$

$\rho . g . z \Rightarrow$ Potential energy per unit vol^m

$$\frac{mg(z)}{V} = \rho . g . z$$

Assumptions → Here

(1) Flow along a streamline
(or)
the points where $\vec{\nabla} \times \vec{V} = 0$ (irrotational) in the flow

(2) Inviscid region of fluid flow [$f_{vis} = 0$]

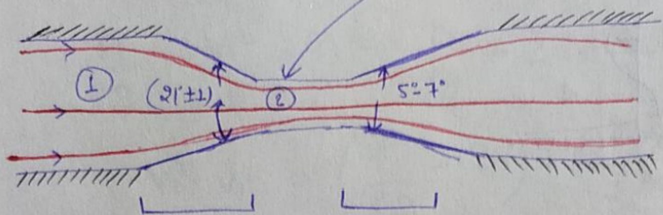
(3) steady flow

(4) incompressible

Bernaullie equ^n →
under the above assumption the summation of all the energies (pressure energy, kinetic energy and potential energy) per unit vol^m b/w any two point is constant.

- Constructional detail :

Throat (minimum C/S Area secti



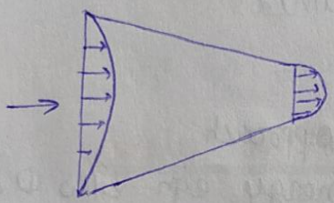
converging Diverging

$D_1 = \frac{D_2}{3}$ or $\frac{D_2}{2}$

Generally, $\frac{D_1}{2} \approx$

eg. 9J venturimeter 3$\cdots$

D_1



$A\downarrow \rightarrow \boxed{V\uparrow}$

acceleration flow

$\boxed{P\downarrow}$

Converging section

continuity eqn

$AV =$ constant

low ← → high

Energy Pressure

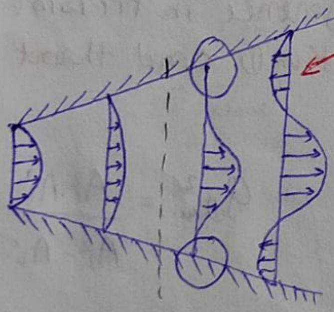
$\frac{P}{\rho g} + \frac{V^2}{2g} + Z =$ constant

↓ decrease ↓ High

$\frac{dP}{dx} < 0$ (favourable pressure gradien

② Diverging section:-

Flow seperation



In extreme condn

$A\uparrow = \boxed{V\downarrow}, \boxed{P\uparrow}$

(Deacceleration Flow)

Apply continuity

$A.V =$ const

↓ high ↓ Low

Energy eqn

$\frac{P}{\rho g} + \frac{V^2}{2g} + Z =$ const

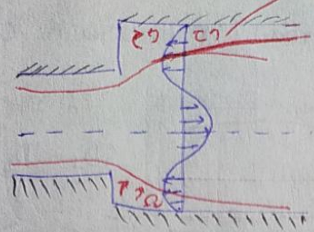
↓ High ↓ Low (energy eqn)

$\frac{dP}{dx} > 0$ (Adverse pressure gradient)

In general

Flow separation

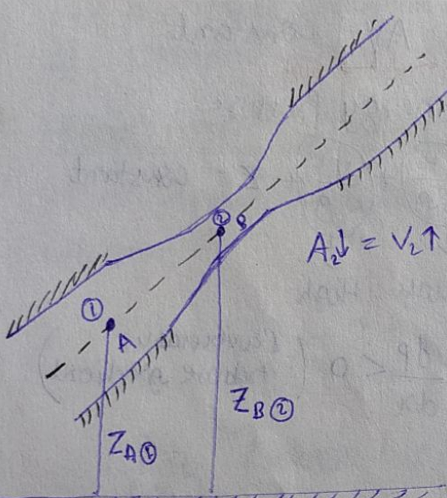
Vortex (Vorticity)



Installation:

May be Horizontal, Vertical, inclined

Incline installation:



Theortical approach

Apply energy eqn b/w ① and ②

$$E_1 = E_2$$

$$\left[\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2\right]$$

$$\frac{V_2^2 - V_1^2}{2g} = \underbrace{\left[\frac{P_1}{\rho g} + Z_1\right] - \left[\frac{P_2}{\rho g} + Z_2\right]}_{h}$$

h → difference in Piezometric head b/w section ① and throat section ②

$$V_2^2 - V_1^2 = 2gh$$

Applying continuity

$$Q_{theortical} = A_1 V_1 = A_2 V_2$$

$$Q_{theo.}^2 \left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right] = 2gh$$

$$Q_{theo}^2 \left[\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right] = 2gh$$

$$Q_{theo.}^2 = \frac{A_1^2 . A_2^2}{A_1^2 - A_2^2} . 2gh$$

$$\boxed{Q_{theo} = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

Actual approach:

Apply energy b/w ① and ②

$$E_1 = E_2 + h_L$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{V_2^2 - V_1^2}{2g} = \underbrace{\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right)}_{h} - h_L$$

$$V_2^2 - V_1^2 = 2g(h - h_L)$$

Apply continuity-

$$Q_{actual} = A_1V_1 = A_2V_2$$

$$Q_{actual}^2\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right] = 2g(h-h_L)$$

$$Q_{actual}^2\left[\frac{A_1^2 - A_2^2}{A_2^2 A_1^2}\right] = 2g(h-h_L)$$

$$Q_{actual}^2 = \left[\frac{A_1^2 \cdot A_2^2}{A_1^2 - A_2^2}\right] \cdot 2g(h-h_L)$$

$$Q_{actual} = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2g(h-h_L)}$$

$$= \underbrace{\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g \cdot h}}_{Q_{theo}} \; \underbrace{\sqrt{\frac{h-h_L}{h}}}_{C_d}$$

*** $$Q_{actual} = C_d \underbrace{\boxed{\frac{A_1A_2\sqrt{2g}}{\sqrt{A_1^2 - A_2^2}}}}_{\text{constant of venturimeter}} \sqrt{h}$$

Pb (35)

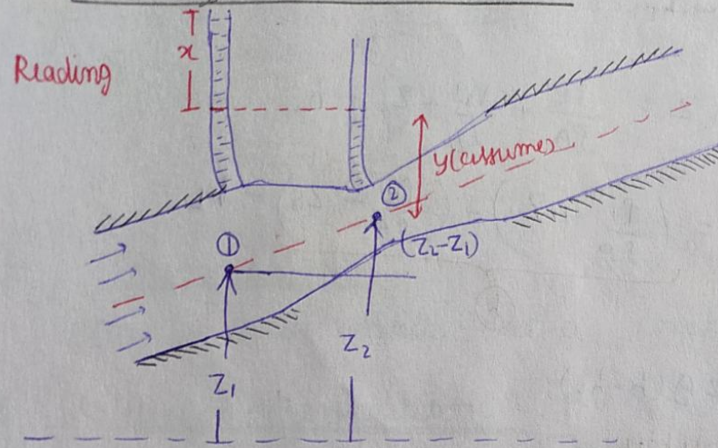
$$Q = 0.96 \times 0.3 \times \sqrt{0.12}$$

$$= 0.128 \ m^3/s$$

**

Unit: $$\frac{m^2 \cdot \cancel{m^2}}{\cancel{\sqrt{(m^2)^2}}}\sqrt{\frac{m}{s^2}} = \frac{m^{2.5}}{s}$$

Form differential Piezometer :



$$h = \left[\frac{P_1}{\rho g}+Z_1\right]-\left[\frac{P_2}{\rho g}+Z_2\right]$$

$$P_1 = \rho g(x+y+Z_2-Z_1)$$
$$P_2 = \rho g(y)$$

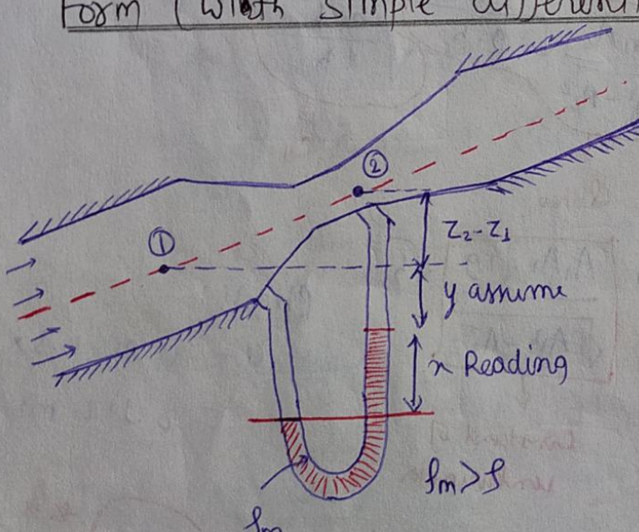
$$\left\{\frac{P_2}{\rho g}+Z_2 = y+Z_2\right\}$$

$$\frac{P_1}{\rho g}+Z_1 = x+y+Z_2$$

$$h = (x+y+Z_2)-(y+Z_2)$$

$$\boxed{h = x}$$

Form (with simple differential U-tube manometer)



$$P_1+\rho g(x+y) = P_2+\rho g(Z_2-Z_1+y) + \rho_m \cdot g x$$

Divide by ρg.

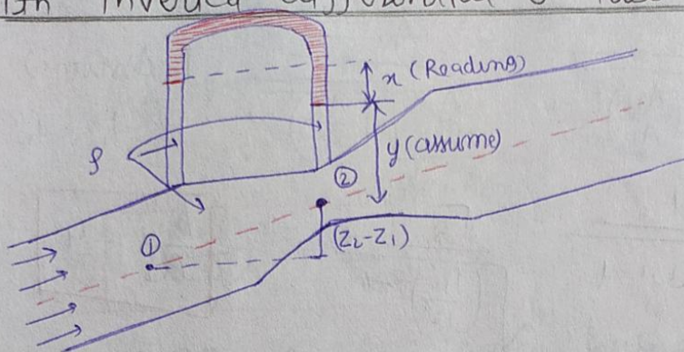
$$\frac{P_1}{\rho g}+x+y = \frac{P_2}{\rho g}+Z_2-Z_1+y+\frac{\rho_m}{\rho}x$$

$$\underbrace{\left[\frac{P_1}{\rho g}+Z_1\right]-\left[\frac{P_2}{\rho g}+Z_2\right]}_{h} = \frac{\rho_m}{\rho}\cdot x - x$$

[No effect of θ because manom is in vertical cond)]

$$\boxed{h = x\left[\frac{\rho_m}{\rho}-1\right]} \quad \rho_m > \rho$$

With inverted differential U-tube manometer :-



$$P_1 + \rho g(z_2 - z_1 + y + x) = P_2 - \rho g y - \rho_m g x$$

Divide by ρg

$$\frac{P_1}{\rho g} + (z_2 - z_1 + y + x) = \frac{P_2}{\rho g} - y - \frac{\rho_m g x}{\rho g}$$

$$\underbrace{\left[\frac{P_1}{\rho g} + z_1\right] - \left[\frac{P_2}{\rho g} + z_2\right]}_{h} = x - \frac{\rho_m}{\rho} x$$

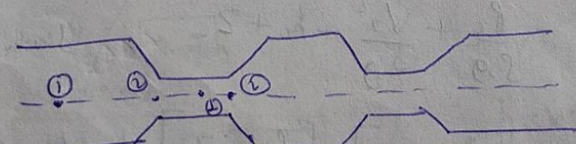
$$\boxed{h = x\left(1 - \frac{\rho_m}{\rho}\right)} \quad \rho_m < \rho$$

① $h = x$ (Piezometer)

② $h = x\left(\frac{\rho_m}{\rho} - 1\right)$ U tube manometer ($\rho_m > \rho$)

③ $h = x\left(1 - \frac{\rho_m}{\rho}\right)$ Inverted U tube manometer ($\rho_m < \rho$)

Pb - 13



$\frac{A_1}{A_2} = 2$, (h) $\quad \frac{A_1'}{A_2'} = 3.3$, $5h$

$$\cancel{C_{d_1}} \frac{\cancel{A_1} A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh} = \frac{C_{d_2} \cdot \cancel{A_1} A_2' \sqrt{2g(5h)}}{\sqrt{A_1'^2 - A_2'^2}}$$

$$\frac{A_2}{A_2\sqrt{\left(\frac{A_1}{A_2}\right)^2-1}} = \frac{A_2'}{A_2'\sqrt{\left(\frac{A_1'}{A_2'}\right)^2-1}}$$ (Assume) $Cd_1 = C.d_2$

$$\frac{1}{\sqrt{(2)^2-1}} = \frac{\sqrt{5}}{\sqrt{\left(\frac{A_1'}{A_2'}\right)^2-1}} = \boxed{\frac{A_1'}{A_2'} = 4}$$