CBSE Test Paper 01

Chapter 8 Introduction to Trigonometry

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1. Given that $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\cos\beta = \frac{1}{\sqrt{2}}$, then the value of $(\alpha + \beta)$ is **(1)**
   a. 90°
   b. 45°
   c. 60°
   d. 30°
2. cot A tan A **(1)**
   a. tan A
   b. sec A
   c. 1
   d. cot A
3. If A and B are acute angles and sin A = cos B, then the value of (A + B) is **(1)**
   a. 0°
   b. 90°
   c. 30°
   d. 60°
4. Which of the following statement is true: **(1)**
   a. $\frac{\cos ecA}{\sin A} = \cos A$
   b. $\frac{\cos A}{\sin A} = \sec A$
   c. $\frac{\sin A}{\cos ecA} = \cot A$
   d. $\frac{\sin A}{\cos A} = \tan A$
5. Choose the correct option and justify your choice:sin 2A = 2 sin A is true when A = **(1)**
   a. 45°
   b. 0°
   c. 30°
   d. 60°

6. Express $\cos 83^\circ$ - $\sec 76^\circ$ in terms of trigonometric ratios of angles between 0° and 45°. **(1)**

7. Prove the trigonometric identity:

$\cos^4 A - \cos^2 A = \sin^4 A - \sin^2 A$ **(1)**

8. Prove that cos1° cos2° cos3°...cos180° = 0. **(1)**

9. If $\sin 5\theta = \cos 4\theta$, where 5θ and 4θ are acute angles, find the value of θ. **(1)**

10. If $\sec 5A = cosec(A + 30^o)$ where 5A is an acute angle, then find the value of A. **(1)**

11. Prove that sin (50° + θ) - cos (40° - θ) + tan 1° tan10° tan 20° tan70° tan 80° tan 89° =1. **(2)**

12. Prove the following identity : $\sin^6 A + \cos^6 A = 1 - 3\sin^2 A \cos^2 A$ **(2)**

13. Verify that, $\cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ = \frac{1}{2}$. **(2)**

14. If sin 3θ = cos (θ - 6°), where 3θ and θ - 6 ° are both acute angles, find the value of θ. **(3)**

15. Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$, using identity $sec^2\theta = 1 + tan^2\theta$. **(3)**

16. Prove the identity:

    $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$ **(3)**

17. Find the value of the following without using trigonometric tables:

    $\frac{\cos 50^\circ}{2\sin 40^\circ} + \frac{4(\text{cosec}^2 59^\circ - \tan^2 31^\circ)}{3\tan^2 45^\circ} - \frac{2}{3}$ tan 12° tan 78°. sin 90° **(3)**

18. Evaluate : $\frac{\cos 65^\circ}{\sin 25^\circ} - \frac{\tan 20^\circ}{\cot 70^\circ}$ - sin 90° + tan 5° tan 35° tan 60° tan 55° tan 85°. **(4)**

19. If tanθ = $\frac{1}{\sqrt{5}}$

    i. Evaluate : $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$

    ii. Verify the identity : $\sin^2\theta + \cos^2\theta = 1$. **(4)**

20. Given the value of $\sec\theta = \frac{13}{12}$, Calculate all other trigonometric ratios. **(4)**

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Solution

1. a. 90°

   Explanation: Given: $\sin\alpha = \frac{1}{\sqrt{2}}$

   $\Rightarrow \sin\alpha = \sin 45^\circ$

   $\Rightarrow \alpha = 45^\circ$

   And $\cos\beta = \frac{1}{\sqrt{2}}$

   $\Rightarrow \cos\beta = \cos 45^\circ$

   $\Rightarrow \beta = 45^\circ$

   $\therefore \alpha + \beta = 45^\circ + 45^\circ = 90^\circ$

2. c. 1

   Explanation: cot A tan A $= \cot \mathrm{A} \times \frac{1}{\cot \mathrm{A}} = 1 \times 1$ = 1

3. b. 90°

   Explanation: Given: $\sin \mathrm{A} = \cos \mathrm{B}$

   Since A and B are acute angles, then

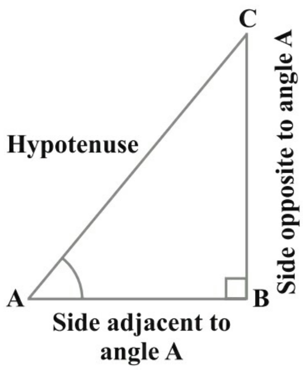
   $\sin \mathrm{A} = \sin(90^\circ - \mathrm{B})$

   $\Rightarrow \mathrm{A} = 90^\circ - \mathrm{B}$

   $\Rightarrow \mathrm{A} + \mathrm{B} = 90^\circ$

4. d. $\frac{\sin A}{\cos A} = \tan A$

   Explanation: In right Ld triangle ABC rt Ld at B



   Sin A $= \frac{BC}{AC}$

   Cos A $= \frac{AB}{AC}$

Then $\frac{SinA}{CosA} = \frac{\frac{BC}{AC}}{\frac{AB}{AC}}$

$\frac{BC}{AC} \times \frac{AC}{AB} = \frac{BC}{AB}$

$L.H.S = \frac{\sin A}{\cos A} = \frac{BC}{AB}$

$R.H.S = tanA$

= $\frac{sinA}{cosA} = \frac{BC}{AB}$ = $\frac{BC}{AB}$

L.H.S = R.H.S

Therefore $\frac{sinA}{cosA}$ = tan A is true.

5. b. 0°

Explanation: When A = 0^o

$\sin 2A = \sin 2\left(0^{\circ}\right) = 0$

$2\sin A = 2\sin(0^{\circ}) = 2(0) = 0$

$\therefore \sin 2A = 2\sin 2A$

Hence, the correct option is (A) = 0^o

6. cos 83^o - sec 76^o = cos (90^o - 7^o) - sec (90^o - 14^o)

= (sin 7^o - cosec 14^o).

7. We have,

LHS = $\cos^4$A - $\cos^2$A

$\Rightarrow$ LHS = $\cos^2$A ($\cos^2$A - 1)

$\Rightarrow$ LHS = -$\cos^2$A (1 - $\cos^2$A)

$\Rightarrow$ LHS = -$\cos^2$A $\sin^2$A = -(1 - $\sin^2$A)$\sin^2$ A = -$\sin^2$A + $\sin^4$A

$\Rightarrow$ LHS = $\sin^4$A - $\sin^2$A = RHS

8. We have,

LHS = cos 1° cos2° cos3°...cos 180°

= cos1° cos2° cos3°... cos89° cos90° cos 91°... cos180°

= cos1°cos2°cos3°... $\times$ cos 89° $\times$ 0 $\times$ cos 91° $\times$ ...cos180° = 0 = RHS $[\because \cos 90^{\circ} = 0]$

9. We have,

$\sin 5\theta = \cos 4\theta$

$\Rightarrow \quad \sin 5\theta = \sin(90^{\circ} - 4\theta) [\because Sin(90^{\circ} - \theta) = Cos\theta]$

$\Rightarrow \quad 5\theta = 90^\circ - 4\theta$

$\Rightarrow \quad 9\theta = 90^\circ$

$\Rightarrow \quad \theta = 10^\circ$

10. Given, $\sec 5A = cosec(A + 30^o)$

 cosec(90^o - 5A) = cosec(A + 30^o) [$\because cosec(90^o - \theta) = sec\theta$]

 $90^\circ - 5A = A + 30^\circ$

 $90^\circ - 30^\circ = A + 5A$

 $\Rightarrow 6A = 60^o$

 Therefore,$A = 10^o$

11. **LHS** = sin(50^o + θ) - cos (40^o - θ) + tan 1^o tan 10^o tan 20^o tan 70^o tan 80^o tan 89^o

 = [sin (90^o - (40 - θ))] - cos (40 - θ) + tan (90^o - 89^o) tan (90^o - 80^o) tan (90^o - 70^o) $\times$ tan 70^o tan 80^o tan 89^o

 = cos (40^o - θ) - cos (40^o - θ) + cot 89^o $\times$ cot 80^o $\times$ cot 70^o $\times \frac{1}{\cot 70^\circ} \times \frac{1}{\cot 80^\circ} \times \frac{1}{\cot 89^\circ}$

 $= 0 + 1$

 $= 1 = RHS$

12. To prove : $\sin^6$ A + $\cos^6$ A = 1 - 3 $\sin^2$ A $\cos^2$ A

 LHS = $\sin^6$ A + $\cos^6$ A

 = $(\sin^2 A)^3 + (\cos^2 A)^3$

 = $(\sin^2 A + \cos^2 A)^3$ - 3$\sin^2$ A . $\cos^2$ A($\sin^2$ A + $\cos^2$ A) [$\because a^3 + b^3 = (a + b)^3 - 3ab(a + b)$]

 = 1^3 - 3$\sin^2$ A . $\cos^2$ A $\times$ 1

 = 1 - 3$\sin^2$ A . $\cos^2$ A

 = RHS.

 Hence proved.

13. We have,

 $\cos 30^\circ = \frac{\sqrt{3}}{2}, \ \sin 30^\circ = \frac{1}{2}$

 therefore,

 R.H.S = $\cos^2 30^\circ - \sin^2 30^\circ$

 $= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

and, L.H.S = $\cos 60^{\circ} = \frac{1}{2}$

∴ L.H.S = R.H.S

i.e, $\cos 60^{\circ} = \cos^2 30^{\circ} - \sin^2 30^{\circ} = \frac{1}{2}$

14. Given,

    sin 3θ = cos (θ - 6°)

    cos (90° - 3θ) = cos (θ - 6°)

    90° - 3θ = θ - 6 °

    4θ = 90° + 6° = 96°

    $\therefore \quad \theta = \frac{96^{\circ}}{4} = 24^{\circ}$

15. We have to prove that, $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ using identity $sec^2\theta = 1 + tan^2\theta$

    LHS = $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ [dividing the numerator and denominator by $\cos\theta$.]

    $= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} = \frac{\{(\tan\theta + \sec\theta) - 1\}(tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$ [Multiplying and dividing by $(\tan\theta - \sec\theta)$]

    $= \frac{(\tan^2\theta - \sec^2\theta) - (tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$ $[\because (a-b)(a+b) = a^2 - b^2]$

    $= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$ $[\because \tan^2\theta - \sec^2\theta = -1]$

    $= \frac{-(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} = \frac{-1}{\tan\theta - \sec\theta}$

    $= \frac{1}{\sec\theta - \tan\theta}$ = RHS

    Hence Proved.

16. Take ,

    LHS = 2($\sin^6\theta$ + $\cos^6\theta$) - 3 ($\sin^4\theta$ + $\cos^4\theta$) + 1

    $\Rightarrow$ LHS = 2 {$(\sin^2\theta)^3$ + $(\cos^2\theta)^3$} - 3 {$(\sin^2\theta)^2$+ $(\cos^2\theta)^2$} + 1

    Using $a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)$ and $a^2 + b^2 = (a + b)^2 - 2ab$ in above expression,

    where a = $\sin^2\theta$ & b =$\cos^2\theta$; we get :-

    LHS = 2 {$(\sin^2\theta + \cos^2\theta)^3$ - 3 $\sin^2\theta \cos^2\theta$ ($\sin^2\theta + \cos^2\theta$)} -3 {$(\sin^2\theta + \cos^2\theta)^2$ - 2 $\sin^2\theta$ $\cos^2\theta$ } + 1

    $\Rightarrow$ LHS = 2 (1 - 3$\sin^2\theta \cos^2\theta$) - 3 (1 - 2$\sin^2\theta \cos^2\theta$) + 1 [Since, $\sin^2 A + \cos^2 A = 1$]

    $\Rightarrow$ LHS = 2 - 6 $\sin^2\theta \cos^2\theta$ - 3 + 6 $\sin^2\theta \cos^2\theta$ + 1

    Hence, L.H.S. = 0 = R.H.S.

Hence, proved.

17. We know that,

$\cos 50° = \cos(90°-40)° = \sin 40°$

$\text{cosec}^2\ 59° = \text{cosec}^2\ (90° - 31°) = \sec^2\ 31°$

and $\tan 78° = \tan(90° - 12°) = \cot 12°$

Now,

$= \frac{\cos 50°}{2\sin 40°} + \frac{4\left(\text{cosec}^2 59° - \tan^2 31°\right)}{3\tan^2 45°} - \frac{2}{3} tan 12^o tan 78^o sin 90^o$

$= \frac{\sin 40°}{2\sin 40°} + \frac{4\left(\sec^2 31° - \tan^2 31°\right)}{3\times(1)^2} - \frac{2}{3}\tan 12° \cot 12° \times 1$

$= \frac{1}{2} + \frac{4}{3} - \frac{2}{3} = \frac{7}{6}$

18. First, we need to solve given equation in parts

$\Rightarrow \frac{\cos 65°}{\sin 25°} = \frac{\cos 65°}{\sin(90° - 65°)} = \frac{\cos 65°}{\cos 65°} = 1$

$\Rightarrow \frac{\tan 20°}{\cot 70°} = \frac{\tan(90° - 70°)}{\cot 70°} = \frac{\cot 70°}{\cot 70°} = 1$

$\Rightarrow \sin 90° = 1$

$\Rightarrow \tan 5° \tan 35° \tan 60° \tan 55° \tan 85°$

$= \tan(90° - 85°) \tan(90° - 55°) \tan 55° \tan 60° \tan 85°$

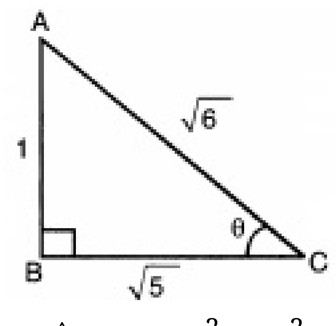
$= \cot 85° \tan 85° \cot 55° \tan 55°.\sqrt{3}$

$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$

$\therefore$ Given Expression = 1 - 1 - 1+ $\sqrt{3}$

Therefore, $\sqrt{3}$ - 1 is the answer.

19. Given, $\tan\theta = \frac{P}{B} = \frac{AB}{BC} = \frac{1}{\sqrt{5}}$



In $\triangle$ABC, $AC^2 = AB^2 + BC^2 = 1 + 5 = 6$

$\therefore$ AC = $\sqrt{6}$

i. $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$

$$= \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$$
$$= \frac{\cot^2\theta-\tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$
$$= \frac{(\sqrt{5})^2-\left(\frac{1}{\sqrt{5}}\right)^2}{2+(\sqrt{5})^2+\left(\frac{1}{\sqrt{5}}\right)^2}$$
$$= \frac{5-\frac{1}{5}}{2+5+\frac{1}{5}}$$
$$= \frac{25-1}{35+1}$$
$$= \frac{24}{36} = \frac{2}{3}$$

ii. LHS = $\sin^2\theta + \cos^2\theta$
$$= \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$$
$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6}$ = 1 = R.H.S

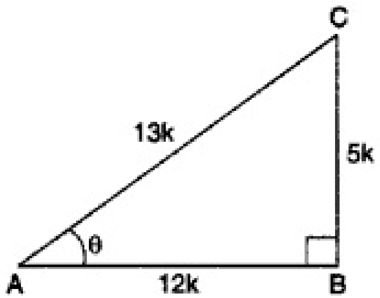
20. Let us draw a right triangle ABC in which $\angle BAC = \theta$

$\sec\theta = \frac{13}{12}, \ldots,$ Given

$\Rightarrow \frac{AC}{AB} = \frac{13}{12} \Rightarrow \frac{AC}{13} = \frac{AB}{12} = k(say)$

where k is a positive number.

$\Rightarrow AC = 13k, AB = 12k$



By using the Pythagoras theorem, we have

AC^2 = AB^2 + BC^2

$\Rightarrow (13\text{k})^2 = (12\text{k})^2 + \text{BC}^2$

$\Rightarrow 169\text{k}^2 = 144\text{k}^2 + \text{BC}^2$

$\Rightarrow BC^2 = 169k^2 - 144k^2$

$\Rightarrow BC^2 = 25k^2 \Rightarrow BC = \sqrt{25}\,k^2$

$\Rightarrow BC = 5k$

Therefore, $\sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$$\left.\begin{aligned}\cos\theta &= \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13} \\ \cos\theta &= \frac{1}{\sec\theta} = \frac{12}{13}\end{aligned}\right\}$$

$$\left.\begin{aligned}\tan\theta &= \frac{\mathrm{BC}}{\mathrm{AB}} = \frac{\mathrm{5k}}{\mathrm{12k}} = \frac{5}{12} \\ \tan\theta &= \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{13}}{\frac{13}{13}} = \frac{5}{12}\end{aligned}\right\}$$

$$\left.\begin{aligned}cosec\theta &= \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5} \\ cosec\theta &= \frac{1}{\sin\theta} = \frac{13}{5}\end{aligned}\right\}$$

$$\cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5} \text{ or } \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{5}{12}} = \frac{12}{5}$$