
CBSE Sample Paper-03
Mathematics
Class – XII

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- d) Use of calculators is not permitted.

Section A

- 1. Is R defined on the set $A=\{1,2,3,4,5,6\}$ as $R=\{(x,y): y \text{ is divisible by } x\}$ symmetric.
- 2. Calculate the direction cosines of the vector $\vec{a} = 3i - 2j - 5k$.
- 3. What is the principal value branch of $\cos^{-1} x$?
- 4. Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$, $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.
- 5. Evaluate without expanding $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$.
- 6. Find $A' = [-2 \ 4 \ 5]$, $B' = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$ find $(AB)'$.

Section B

- 7. Using properties of determinants prove that
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = abc + bc + ca + ab$$
-

-
-
8. Find the equations of all lines having slope 0 and that are tangent to the curve $y = \frac{1}{x^2 - 2x + 2}$.
9. If $f(x) = \frac{x-1}{x+1}$, ($x \neq 1, -1$), show that $f \circ f^{-1}$ is an identity function.
10. Find $\frac{dy}{dx}$ if $\log(xy) = x^2 + y^2$.
11. Prove that $\tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$, $|x| < \frac{1}{\sqrt{3}}$
12. If A, B, C have the co-ordinates (2,0,0) , (0,1,0) , (0,0,2) , then show that ABC is an isosceles triangle.
13. (a) If A and B are two events defined on a sample space s.t.
 $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$, $P(B^c) = \frac{1}{3}$, find $P(A)$.
- (b) If A and B are two events defined on a sample space s.t.
 $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \text{ and } B) = \frac{1}{8}$, find $P(\text{not } A \text{ and not } B)$.
14. Find all intervals on which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is (a) strictly increasing (b) strictly decreasing.
15. Solve the differential equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$
16. Show that $(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}) \cdot (|\vec{a}|\vec{b} - |\vec{b}|\vec{a}) = 0$
17. Integrate $\int \log(1+x^2) dx$.
18. Find the shortest distance between the lines l_1 and l_2 given by :
 $\vec{r} = (i + 2j + k) + \lambda(i - j + k)$
 $\vec{r} = (2i - j - k) + \mu(2i + j + 2k)$
19. Find the vector equation of the plane passing through the points $i+j-k$ and $2i+6j+k$ and parallel to the line $\vec{r} = (3i - 5j + k) + \lambda(i - 2j + k)$
-

Section C

20. A factory can hire two tailors A and B in order to stitch pants and shirts. Tailor A can stitch 6 shirts and 4 pants in a day. Tailor B can stitch 10 shirts and 4 pants in a day. Tailor A charges 15 per day and tailor B charges 20 per day. The factory has to produce minimum 60 shirts and 32 pants. State as a linear programming problem and minimize the labour cost.
21. Find the area of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$
22. A, Band C play a game and the chances of winning in it in an attempt are $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{4}$ respectively. A has the first chance, followed by Band C. The cycle is repeated till one of them wins the game. Find their respective chances of winning the game.
23. If $x = a \sec^3 \theta$, $y = a \tan^3 \theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.
24. Integrate $\int \frac{x^2 dx}{(x+3)\sqrt{3x+4}}$
25. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
26. Solve the following system of equations using matrix method
- $$\begin{aligned}x + y + z &= 4 \\2x - y + z &= -1 \\2x + y - 3z &= -9\end{aligned}$$
-

CBSE Sample Paper-03
Mathematics
Class – XII

Time allowed: 3 hours

Answers

Maximum Marks: 100

Section A

1. Solution:

No. $(2, 4) \in R$ but $(4, 2) \notin R$

2. Solution:

$$|\vec{a}| = \sqrt{(3)^2 + (-2)^2 + (-5)^2} = \sqrt{38}$$

$$\therefore l = \frac{3}{\sqrt{38}}, m = \frac{-2}{\sqrt{38}}, n = \frac{-5}{\sqrt{38}}$$

3. Solution: $[0, \pi]$

4. Solution:

$$2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

5. Solution:

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6(17) & 6(18) & 6(6) \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0 (\because R_1 = R_3)$$

6. Solution:

$$(AB)' = B' A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

Section B

7. Solution:

Let $x = \tan \theta$

$$\begin{aligned} R.H.S &= \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) = \tan^{-1} \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) = \tan^{-1} (\tan 3\theta) = 3\theta = 3 \tan^{-1} x \\ &= 2 \tan^{-1} x + \tan^{-1} x = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1 - x^2} \right) \end{aligned}$$

$$\text{If } x = \tan y, \tan^{-1} \left(\frac{2x}{1 - x^2} \right) = \tan^{-1} \left(\frac{2 \tan y}{1 - \tan^2 y} \right) = \tan^{-1} (\tan 2y) = 2y = 2 \tan^{-1} x$$

8. Solution:

The position vectors of A, B, C are $2\mathbf{i}$, $\mathbf{j}$, $2\mathbf{k}$ respectively.

$$\therefore \overrightarrow{AB} = \text{p.v of } \vec{B} - \text{p.v of } \vec{A} = \mathbf{j} - 2\mathbf{i}$$

$$\therefore \overrightarrow{BC} = \text{p.v of } \vec{C} - \text{p.v of } \vec{B} = 2\mathbf{k} - \mathbf{j}$$

$$\therefore \overrightarrow{CA} = \text{p.v of } \vec{A} - \text{p.v of } \vec{C} = 2\mathbf{i} - 2\mathbf{k}$$

$$|\overrightarrow{AB}|^2 = (2)^2 + (1)^2 = 5 \Rightarrow |\overrightarrow{AB}| = \sqrt{5}$$

$$|\overrightarrow{BC}|^2 = (1)^2 + (2)^2 = 5 \Rightarrow |\overrightarrow{BC}| = \sqrt{5}$$

$$|\overrightarrow{CA}|^2 = (2)^2 + (2)^2 = 8 \Rightarrow |\overrightarrow{CA}| = \sqrt{8}$$

$$|\overrightarrow{AB}| = |\overrightarrow{BC}| \neq |\overrightarrow{CA}|$$

Thus, A, B, C form the vertices of an isosceles triangle.

9. Solution:

(a)

$$P(B) = 1 - P(B^c) = 1 - 1/3 = 2/3$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore \frac{5}{6} = P(A) + \frac{2}{3} - \frac{1}{3} \Rightarrow P(A) = \frac{1}{2}$$

(b)

$$P(\text{not } A \text{ and not } B) = P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)] = 1 - [1/4 + 1/2 - 1/8] = 3/8$$

10. Solution:

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

$$f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2) = -6(x+1)(x+2)$$

$$f'(x) = 0 \Rightarrow x = -1, -2$$

We study the sign of $f'(x)$ on the intervals $(-\infty, -2)$, $(-2, -1)$ and $(-1, \infty)$.

If $x < -2$, $f'(x)$ is negative, i.e $f(x)$ is strictly decreasing.

If $-2 < x < -1$, $f'(x)$ is positive, i.e $f(x)$ is strictly increasing.

If $x > -1$, $f'(x)$ is negative, i.e $f(x)$ is strictly decreasing.

11. Solution:

$$\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$$

$$\text{Let } z = x + y \Rightarrow \frac{dz}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dz}{dx} - 1 = \sin z + \cos z$$

$$\therefore \frac{dz}{\sin z + \cos z + 1} = dx$$

Integrating both sides, we get

$$\int \frac{dz}{\sin z + \cos z + 1} = \int dx$$

$$\int \frac{dz}{\frac{2 \tan z / 2}{1 + \tan^2 z / 2} + \frac{1 - \tan^2 z / 2}{1 + \tan^2 z / 2} + 1} = x + c$$

$$\int \frac{\sec^2 z / 2}{2 \tan z / 2 + 2} = x + c$$

$$\text{Let } \tan z / 2 = t \Rightarrow \frac{1}{2} \sec^2 z / 2 = dt$$

$$\therefore \int \frac{dt}{t+1} = x + c$$

$$\therefore \log|t+1| = x + c$$

$$\log|\tan(x+y)/2 + 1| = x + c$$

12. Solution:

$$\begin{aligned} \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} &= abc \begin{vmatrix} 1/a+1 & 1/a & 1/a \\ 1/b & 1/b+1 & 1/b \\ 1/c & 1/c & 1/c+1 \end{vmatrix} \quad (\text{taking } a, b, c \text{ common from } R_1, R_2, R_3) \\ &= \begin{vmatrix} 1/a+1/b+1/c+1 & 1/a+1/b+1/c+1 & 1/a+1/b+1/c+1 \\ 1/b & 1/b+1 & 1/b \\ 1/c & 1/c & 1/c+1 \end{vmatrix} \quad (R_1 \rightarrow R_1 + R_2 + R_3) \end{aligned}$$

$$\begin{aligned}
 &= abc(1/a + 1/b + 1/c + 1) \begin{vmatrix} 1 & 1 & 1 \\ 1/b & 1/b + 1 & 1/b \\ 1/c & 1/c & 1/c + 1 \end{vmatrix} \\
 &= abc(1/a + 1/b + 1/c + 1) \begin{vmatrix} 1 & 0 & 0 \\ 1/b & 1 & 0 \\ 1/c & 0 & 1 \end{vmatrix} = abc(1/a + 1/b + 1/c + 1)
 \end{aligned}$$

13. Solution:

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{-1}{(x^2 - 2x + 2)^2} (2x - 2) \\
 \therefore \text{slope} = 0 &\Rightarrow \frac{dy}{dx} = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1 \\
 \therefore x = 1, y &= \frac{1}{1 - 2 + 2} = 1 \\
 \therefore \text{the line is tangent at the point } (1, 1) \\
 \therefore \text{equation of required line is } y - 1 &= 0 \cdot (x - 1) \Rightarrow y = 1
 \end{aligned}$$

14. Solution:

1-1:

Let $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - 1}{x_1 + 1} = \frac{x_2 - 1}{x_2 + 1} \Rightarrow x_1 x_2 + x_1 - x_2 - 1 = x_1 x_2 + x_2 - x_1 - 1 \Rightarrow x_1 = x_2$$

$\therefore f$ is 1-1.

Range:

$$\text{Let } f(x) = k \Rightarrow \frac{x-1}{x+1} = k \Rightarrow x = \frac{1+k}{1-k}$$

Thus, x is not defined when $k=1$.

Also $f(1)=0$

$\therefore \text{Range} = \mathbb{R} \sim \{0, 1\}$

Thus f is invertible if $\text{range} = \mathbb{R} \sim \{0, 1\}$

Inverse:

Let $x \in \mathbb{R} \sim \{0, 1\}$.

$$\text{Let } f^{-1}(x) = k \Rightarrow x = f(k) \Rightarrow \frac{k-1}{k+1} = x \Rightarrow k = \frac{1+x}{1-x}$$

$$\therefore f^{-1}(x) = \frac{1+x}{1-x}, x \in \mathbb{R} \sim \{0,1\}$$

Thus,

$$f \circ f^{-1} = f\left(\frac{1+x}{1-x}\right) = \frac{\frac{1+x}{1-x} - 1}{\frac{1+x}{1-x} + 1} = \frac{2x}{2} = x$$

15. Solution:

$$\log(xy) = x^2 + y^2$$

$$\therefore \log(x) + \log(y) = x^2 + y^2$$

Differentiating both sides w.r.t x, we get

$$\frac{1}{x} + \frac{1}{y} \frac{dy}{dx} = 2x + 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(\frac{1}{y} - 2y \right) = 2x - \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y(2x^2 - 1)}{x(1 - 2y^2)}$$

16. Solution:

$$\begin{aligned} (|\vec{a}| \vec{b} + |\vec{b}| \vec{a}) \cdot (|\vec{a}| \vec{b} - |\vec{b}| \vec{a}) &= |\vec{a}| |\vec{a}| \vec{b} \cdot \vec{b} + |\vec{b}| |\vec{a}| \vec{a} \cdot \vec{b} - |\vec{a}| |\vec{b}| \vec{b} \cdot \vec{a} - |\vec{b}| |\vec{b}| \vec{a} \cdot \vec{a} \\ &= \cancel{|\vec{a}|^2 |\vec{b}|^2} + \cancel{|\vec{b}| |\vec{a}| \vec{a} \cdot \vec{b}} - \cancel{|\vec{a}| |\vec{b}| \vec{b} \cdot \vec{a}} - \cancel{|\vec{b}|^2 |\vec{a}|^2} = 0 (\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}) \end{aligned}$$

17. Solution:

$$I = \int \log(1+x^2) dx = \int 1 \cdot \log(1+x^2) dx$$

Integrating by parts,

$$I = \log(1+x^2) \int 1 \cdot dx - \int \left(\frac{d}{dx} \log(1+x^2) \right) \int 1 \cdot dx$$

$$= \log(1+x^2) x - \int \frac{2x}{1+x^2} (x) dx$$

$$= x \log(1+x^2) - 2 \int \left(1 - \frac{1}{1+x^2} \right) dx$$

$$= x \log(1+x^2) - 2x + 2 \tan^{-1} x + c$$

18. Solution:

$$Distance = \frac{\left| (\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) \right|}{\left| (\vec{b_1} \times \vec{b_2}) \right|}$$

$$\vec{a_1} = i + 2j + k, \vec{a_2} = 2i - j - k, \vec{b_1} = i - j + k, \vec{b_2} = 2i + j + 2k,$$

$$\therefore \vec{b_1} \times \vec{b_2} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3i + 3k$$

$$\therefore \left| \vec{b_1} \times \vec{b_2} \right| = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2}$$

$$\vec{a_2} - \vec{a_1} = i - 3j - 2k$$

$$\therefore d = \frac{\left| (-3i + 3k) \cdot (i - 3j - 2k) \right|}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$$

19. Solution:

$$\vec{a} = i + j - k, \vec{b} = 2i + 6j + k, \vec{c} = i - 2j + k$$

Equation of plane:

$$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times \vec{c}] = 0$$

$$(\vec{b} - \vec{a}) = i + 5j + 2k$$

$$(\vec{b} - \vec{a}) \times \vec{c} = \begin{vmatrix} i & j & k \\ 1 & 5 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 9i + j - 7k$$

$$\therefore [\vec{r} - (i + j - k)] \cdot (9i + j - 7k) = 0$$

$$\Rightarrow \vec{r} \cdot (9i + j - 7k) = 17$$

Section C

20. Solution:

Suppose tailor A works for x days and tailor B works for y days.

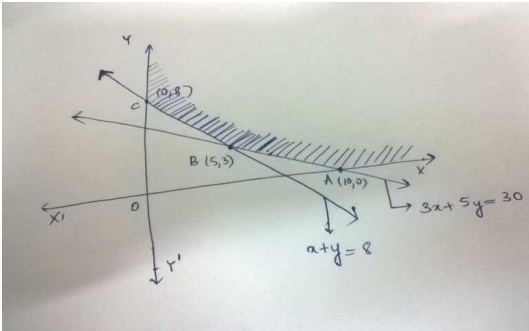
Then, Cost Z = 15x + 20y

	Tailor A	Tailor B	Min Requirement
Shirts	6	10	60
Pants	4	4	32
Cost per day	15	20	

The mathematical formulation of the problem is as follows:

$$\text{Min } Z = 15x + 20y$$

$$\begin{aligned}
 6x + 10y &\geq 60 \Rightarrow 3x + 5y \geq 30 \\
 \text{s.t } 4x + 4y &\geq 32 \Rightarrow x + y \geq 8 \\
 x &\geq 0, y \geq 0
 \end{aligned}$$



We graph the above inequalities. The feasible region is as shown in the figure. We observe the feasible region is unbounded and the corner points are A,B and C. The co-ordinates of the corner points are (10,0), (5,3),(0,8).

Corner Point	$Z=15x + 20y$
(10,0)	150
(5,3)	135
(0,8)	160

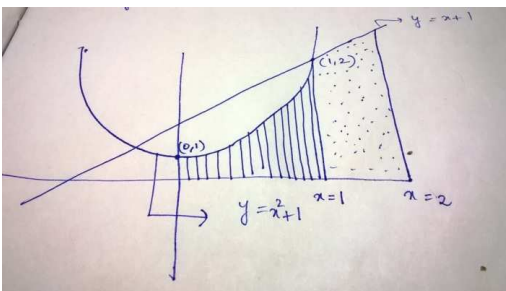
Thus cost is minimized by hiring A for 5 days and hiring B for 3 days.

21. Solution:

The curves are $x=2$, $y=x+1$, $y= x^2+1$.

The point of intersection of the curves $y=x+1$, $y=x^2+1$:

$$\begin{aligned}
 x^2 + 1 &= x + 1 \\
 \Rightarrow x(1 - x) &= 0 \Rightarrow x = 0, x = 1
 \end{aligned}$$



The shaded area is the required area.

$$\text{Area} = \int_0^1 y_1 dx + \int_1^2 y_2 dx$$

$$\begin{aligned}
&= \int_0^1 (x^2 + 1)dx + \int_1^2 (x + 1)dx \\
&= \frac{x^3}{3} + x \Big|_0^1 + \frac{x^2}{2} + x \Big|_1^2 = \frac{23}{6}
\end{aligned}$$

22. Solution:

Let A_i, B_i, C_i denote the events of winning A, B, C in their respective i th attempt.

$$P(A_i) = 2/3, P(B_i) = 1/2, P(C_i) = 1/4$$

$$P(\bar{A}_i) = \frac{1}{3}, P(\bar{B}_i) = \frac{1}{2}, P(\bar{C}_i) = \frac{3}{4}$$

$$\begin{aligned}
P(A \text{ wins}) &= P(A_1 \text{ or } \bar{A}_1 \bar{B}_1 \bar{C}_1 A_2 \text{ or } \bar{A}_1 \bar{B}_1 \bar{C}_1 \bar{A}_2 B_2 C_2 A_3 \dots) \\
&= P(A_1) + P(\bar{A}_1)P(\bar{B}_1)P(\bar{C}_1)P(A_2) + P(\bar{A}_1)P(\bar{B}_1)P(\bar{C}_1)P(\bar{A}_2)P(B_2)P(\bar{C}_2)P(A_3) + \dots \\
&= \frac{2}{3} + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) + \dots \\
&= \frac{2}{3} \left[1 + \frac{1}{8} + \left(\frac{1}{8}\right)^2 + \dots \right] = \frac{16}{21}
\end{aligned}$$

$$\begin{aligned}
P(B \text{ wins}) &= P(\bar{A}_1 \bar{B}_1 \text{ or } \bar{A}_1 \bar{B}_1 \bar{C}_1 \bar{A}_2 B_2 \text{ or } \bar{A}_1 \bar{B}_1 \bar{C}_1 \bar{A}_2 \bar{B}_2 C_2 A_3 B_3 \dots) \\
&= P(\bar{A}_1)P(B_1) + P(\bar{A}_1)P(\bar{B}_1)P(\bar{C}_1)P(\bar{A}_2)P(B_2) + P(\bar{A}_1)P(\bar{B}_1)P(\bar{C}_1)P(\bar{A}_2)P(\bar{B}_2)P(\bar{C}_2)P(\bar{A}_3)P(B_3) + \dots \\
&= \frac{1}{6} \left[1 + \frac{1}{8} + \left(\frac{1}{8}\right)^2 + \dots \right] = \frac{4}{21}
\end{aligned}$$

$$P(C \text{ wins}) = 1 - P(A \text{ wins}) - P(B \text{ wins}) = 1 - \frac{16}{21} - \frac{4}{21} = \frac{1}{21}$$

23. Solution:

$$\begin{aligned}
x &= a \sec^3 \theta, y = a \tan^3 \theta \\
\frac{dx}{d\theta} &= 3a \sec^2 \theta (\sec \theta \tan \theta) = 3a \sec^3 \theta \tan \theta \Rightarrow \frac{d\theta}{dx} = \frac{1}{3a \sec^3 \theta \tan \theta} \\
\frac{dy}{d\theta} &= 3a \tan^2 \theta (\sec^2 \theta) \\
\frac{dy}{dx} &= \left(\frac{dy}{d\theta}\right) \left(\frac{d\theta}{dx}\right) = 3a \tan^2 \theta \sec^2 \theta \frac{1}{3a \sec^3 \theta \tan \theta} = \frac{\tan \theta}{\sec \theta} = \sin \theta \\
\frac{d^2 y}{dx^2} &= \frac{d}{d\theta} \left(\frac{dy}{dx}\right) \frac{d\theta}{dx} = \frac{d}{d\theta} (\sin \theta) \frac{d\theta}{dx} = \cos \theta \frac{1}{3a \sec^3 \theta \tan \theta} = \frac{1}{3a} \frac{\cos^5 \theta}{\sin \theta} \\
\frac{d^2 y}{dx^2} \Big|_{\theta = \frac{\pi}{4}} &= \frac{1}{12a}
\end{aligned}$$

24. Solution:

$$I = \int \frac{x^2 dx}{(x+3)\sqrt{3x+4}}$$

$$\text{Let } z = \sqrt{3x+4} \quad \therefore x = \frac{z^2-4}{3} \Rightarrow dx = \frac{2z}{3} dz$$

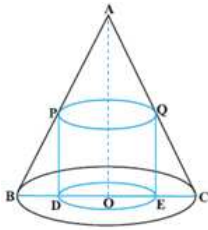
$$\begin{aligned} \therefore I &= \int \frac{\left(\frac{z^2-4}{3}\right)^2 \frac{2z}{3}}{\left(\frac{z^2-4}{3}+3\right)z} dz \\ &= \frac{2}{9} \int \frac{(z^2-4)^2}{z^2+5} dz = \frac{2}{9} \int \left(z^2 - 13 + \frac{81}{z^2+5}\right) dz \\ &= \frac{2}{9} \left[\frac{z^3}{3} - 13z + \frac{81}{\sqrt{5}} \tan^{-1} \frac{z}{\sqrt{5}} \right] + c \\ &= \frac{2}{27} (3x+4)^{3/2} - \frac{26}{9} \sqrt{3x+4} + \frac{18}{\sqrt{5}} \tan^{-1} \sqrt{\frac{3x+4}{5}} + c \end{aligned}$$

25. Solution:

Let $OC=r$ be the radius of the cone and $OA=h$ be its height.

Let a cylinder with radius $OE = x$ and height h' be inscribed in the cone.

Surface Area = $2 \pi x h'$



$$\because \triangle QEC \sim \triangle AOC,$$

$$\frac{QE}{AO} = \frac{CE}{CO} \Rightarrow \frac{h'}{h} = \frac{r-x}{r} \Rightarrow h' = h \left(\frac{r-x}{r} \right)$$

$$\therefore S = S(x) = 2\pi x h' = 2\pi x h \left(\frac{r-x}{r} \right) = \frac{2\pi h}{r} (rx - x^2)$$

$$S'(x) = \frac{2\pi h}{r} (r - 2x)$$

$$S''(x) = \frac{2\pi h}{r} (-2)$$

$$S'(x) = 0 \Rightarrow x = r/2$$

$$\text{Also, } S''(r/2) = \frac{-4\pi h}{r} < 0$$

Hence, $x=r/2$ is a point of maxima.

Thus, the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

26. Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}, b = \begin{bmatrix} 4 \\ -1 \\ 9 \end{bmatrix}$$

$$|A| = 14 \neq 0, A^{-1} = \frac{1}{14} \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ -4 & 1 & -3 \end{bmatrix}, X = A^{-1}b = \frac{1}{14} \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ -4 & 1 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 9 \end{bmatrix} = \frac{1}{14} \begin{bmatrix} -14 \\ 28 \\ 42 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore x = -1, y = 2, z = 3$$
