Type 6: Polar Plot

For Concept, refer to Control Systems K-Notes, Frequency Response Analysis

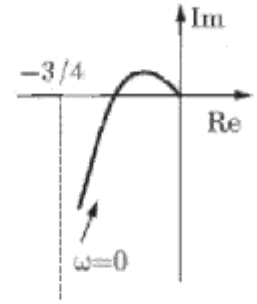
Common Mistake:
Remember the direct polar plot but also remember how to calculate the intersection with real axis as you may be given two options with same shape of polar plot but different intersection point.

Sample Problem 6:
The frequency response of

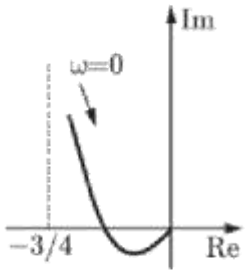
$$G(s)=\frac{1}{s(s+1)(s+2)}$$

plotted in the complex $G(j\omega)$ plane (for $0 < \omega < 3$) is

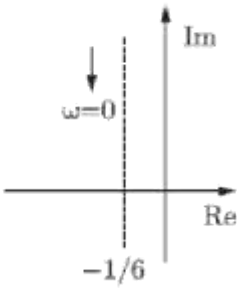
(A)



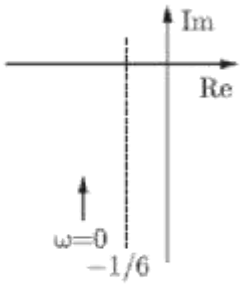
(B)



(C)



(D)



Solution: (A) is correct option

Given $G(s)=\frac{1}{s(s+1)(s+2)}$

$$G(s)=\frac{1}{j\omega(j\omega+1)(j\omega+2)}$$

$$|G(j\omega)|=\frac{1}{\omega\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

$$\angle G(j\omega)=-90^0-\tan^{-1}(\omega)-\tan^{-1}(\frac{\omega}{2})$$

In Nyquist plot

For $\omega=0$ $|G(j\omega)|=\infty$

$\angle G(j\omega)=-90^0$

For $\omega=\infty$ $|G(j\omega)|=0$

$\angle G(j\omega)=-90^0-90^0-90^0=-270^0$

Intersection at real axis

$$G(s)=\frac{1}{j\omega(j\omega+1)(j\omega+2)}$$

$$=\frac{1}{j\omega(-\omega^2+3j\omega+2)}$$

$$=\frac{-3\omega^2}{9\omega^4+\omega^2(2-\omega^2)^2}-\frac{j\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)^2}$$

At real axis

$$\text{Im}\left[G(j\omega)\right]=0$$

So, $\dfrac{\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)^2}=0$

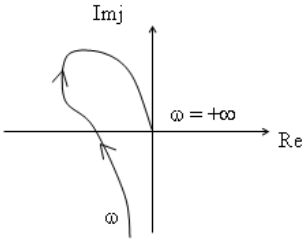
$(2-\omega^2)=0\Rightarrow\omega=\sqrt{2}$ rad/sec

At $\omega=\sqrt{2}$ rad / sec, magnitude response is

$$|G(j\omega)|_{\text{at }\omega=\sqrt{2}}=\frac{1}{\sqrt{2}\sqrt{2+1}\sqrt{2+4}}=\frac{1}{6}<\frac{3}{4}$$

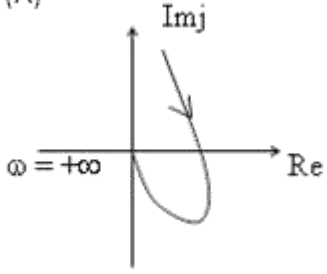
Unsolved Problems:

Q.1 The polar plot for a transfer function is shown below**:**

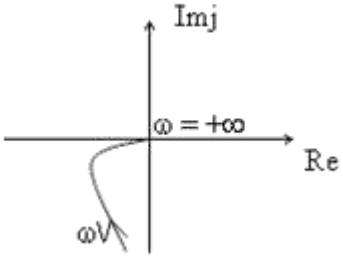


An addition of a zero will change the plot as

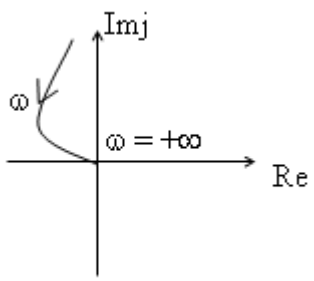
(A)



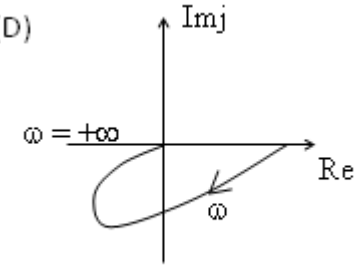
(B)



(C)



(D)



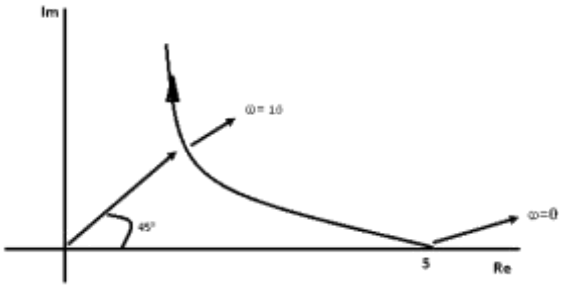
Q.2 Fig shows the polar plot of a system. The transfer function of the system is

(A) $5(1+0.1s)$

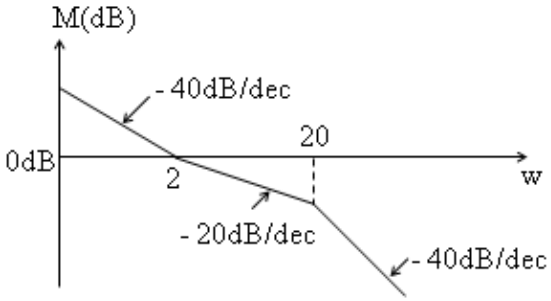
(B) $(1+0.5s)$

(C) $5(1+10s)$

(D) $5(1+s)$

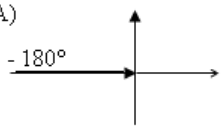


Q.3 The magnitude plot for minimum phase system is shown in figure

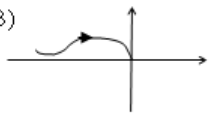


The polar plot of the above system is

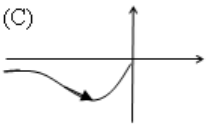
(A)



(B)



(C)



(D) None

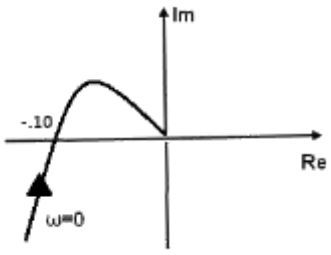
Q.4 The polar plot of G(s)H(s) for K=10 is given below . The range of 'K' for stability is?

(A) $0 < K < 10$

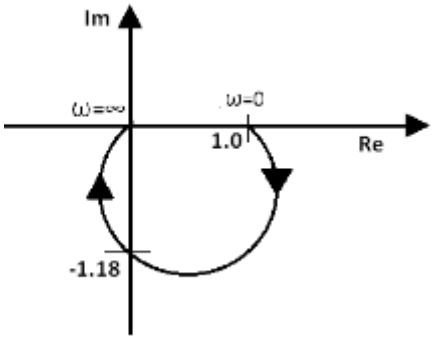
(B) $0 < K < 10^3$

(C) $0 < K < 10^2$

(D) $0 < K < 1$



Q.5 The polar plot of frequency response of a linear damped second order system is shown in the figure given. What is the transfer function of this system?



(A) $\frac{8}{s^2+10s+1}$

(B) $\frac{8}{s^2+8.48s+10}$

(C) $\frac{100}{s^2+8.48s+100}$

(D) $\frac{100}{s^2+10s+8.48}$