CBSE Test Paper 02

Chapter 11 Three Dimensional Geometry

1. Find the equation of the plane with intercept 3 on the y – axis and parallel to ZOX plane.
   a. y = 3
   b. y = 5
   c. y = 4
   d. y = 2
2. Equation of a plane perpendicular to a given line with direction ratios A, B, C and passing through a given point $(x_1,\ y_1,\ z_1)$ is
   a. A (x – x1) + B (y – y1) + C (z – z1) = 1
   b. A (x + x1) + B (y – y1) + C (z – z1) = 0
   c. A (x – x1) + B (y – y1) + C (z – z1) = 0
   d. A (x – x1) + B (y + y1) + C (z – z1) = 1
3. Determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.7x + 5y + 6z + 30 = 0 and 3x – y – 10z + 4 = 0
   a. $\cos^{-1}\left(\frac{2}{5}\right)$
   b. $\sin^{-1}\left(\frac{2}{5}\right)$
   c. $\cot^{-1}\left(\frac{2}{5}\right)$
   d. $\tan^{-1}\left(\frac{2}{5}\right)$
4. How do you find the shortest distance between two skew lines?
   a. The line segment from origin to both the lines
   b. The line segment parallel to both the lines
   c. The line segment at minimum angle to both the lines
   d. The line segment in space perpendicular to both the lines
5. In the following case, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them. 4x + 8y + z – 8 = 0 and y + z – 4 = 0.
   a. 43°
   b. 45°

c. 49°

d. 47°

6. The equations of x-axis in space are ________.
7. Vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is ________.
8. The cartesian equation of the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$
9. If a line has the direction ratios -18, 12, -4 then what are its direction cosines
10. Find the value of λ, such that the line $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{-4}$ is perpendicular to the plane 3x - y - 2z = 7.
11. Find the Cartesian equation of the plane $\overrightarrow{r}.[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}]=15.$
12. Find the angle between the line $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$.
13. Find the angle between the pair of line given by $\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ $\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$.
14. Find the distance of the point (–1, –5, – 10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5.$
15. Find the vector and cartesian equations of the line passing through the point (2,1,3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$.
16. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.
17. Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.
18. Find the vector and Cartesian equations of the plane which passes through the point (5,2,-4) and $\perp$ to the line with direction ratios (2,3,-1).

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Solution

1. a. y = 3

   Explanation: The required equation of plane is y = 3.

2. c. A (x – x1) + B (y – y1) + C (z – z1) = 0

   Explanation: In Cartesian co – ordinate system Equation of a plane perpendicular to a given line with direction ratios A, B, C and passing through a given point (x1, y1, z1) is given by : A (x – x1) + B (y – y1) + C (z – z1) = 0.

3. a. $\cos^{-1}\left(\frac{2}{5}\right)$

   Explanation: 7x + 5y + 6z + 30 = 0 and 3x - y - 10z + 4 = 0

   Let θ be the angle between the planes, then

   $$\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$$

   $$\Rightarrow \left|\frac{7\times3+5\times(-1)+6\times(-10)}{\sqrt{49+25+36}\sqrt{9+1+100}}\right| = \left|\frac{-44}{\sqrt{110}\sqrt{110}}\right| \Rightarrow \theta = \cos^{-1}\left(\frac{2}{5}\right)$$

4. d. The line segment in space perpendicular to both the lines

   Explanation: Shortest distance between two skew lines is the line segment in space perpendicular to both the lines.

5. b. 45°

   Explanation: We have, 4x + 8y + z – 8 = 0 and y + z – 4 = 0. Let be the angle between the planes, then

   $$\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+A_2^2+A_2^2}}\right|$$

   $$= \left|\frac{4\times0+8\times1+1\times1}{\sqrt{16+64+1}\sqrt{0+1+1}}\right| = \left|\frac{9}{\sqrt{18}\sqrt{2}}\right|$$

   $$\Rightarrow \cos\theta = \left(\frac{1}{\sqrt{2}}\right) \Rightarrow \theta = 45^o$$

6. y = 0, z = 0
7. $\vec{r} = \vec{a} + \lambda\vec{b}$
8. x + y - z = 2

9. a = -18, b = 12, c = - 4

   $a^2 + b^2 + c^2 = (-18)^2 + (12)^2 + (-4)^2$

   = 484

   Therefore, direction cosines are,

   $\left(-\frac{18}{484}, \frac{12}{484}, -\frac{4}{484}\right)$

10. According to the question, line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to plane $3x - y - 2z = 7$.

    $\therefore$ DR's of the line are proportional to the DR's of normal to the plane.

    $\frac{6}{3} = \frac{\lambda}{-1} = \frac{-4}{-2}$

    $\Rightarrow \quad 2 = -\lambda \quad \Rightarrow \quad \lambda = -2$

11. Given equation of plane is $\overrightarrow{r}.[(5-2t)\hat{i} + (3-t)\hat{j} + (25+t)\hat{k}] = 15$

    $\left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot \left[(5-2t)\hat{i} + (3-t)\hat{j} + (25+t)\hat{k}\right] = 15$

    (5 - 2t)x + (3 - t)y + (25 + t)z = 15

12. $\vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\hat{k}$

    $\vec{b}_2 = -\hat{i} + 8\hat{j} + 4\hat{k}$

    $\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right|$

    $= \left|\frac{2(-1)+5(8)+(-3)(4)}{\sqrt{4+25+9}\sqrt{1+64+16}}\right|$

    $= \frac{26}{9\sqrt{38}}$

    $\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

13. $\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$

    $\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

    $\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right| = \frac{19}{21}$

    $= \left|\frac{(\hat{i}+2\hat{j}+2\hat{k}).(3\hat{i}+2\hat{j}+6\hat{k})}{\sqrt{1+4+4}\sqrt{9+4+36}}\right|$

    $= \frac{3+4+12}{3\times7}$

    $= \frac{19}{21}$

    $\therefore \theta = cos^{-1}\frac{19}{21}$

14. We have $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5.$

Solving these two equations, we get

$\left[\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$[(3\lambda+2)\hat{i}+(4\lambda-1)\hat{j}+(2\lambda+2)\hat{]}.(\hat{i}-\hat{j}+\hat{k})$=5

$(3\lambda+2)-(4\lambda-1)+(2\lambda+2)=5$

$\lambda+5=5$

$\lambda=0$

$\therefore\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$

Therefore, the point of intersection of line and the plane is $(2,-1,2)$ and the other given point is $(-1,-5,-10)$. Hence the distance between these two points is

$\sqrt{[2-(-1)]^2+[-1+5]^2+[2-(-10)]^2}$ = 13

15. We have to find the vector and cartesian equations of the line passing through the point (2,1,3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$.

We know that any line through the point (2, 1, 3) can be written as

$\frac{x-2}{a}=\frac{y-1}{b}=\frac{z-3}{c}$........................(i)

where, a, b and c are the direction ratios of line (i).

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

and $\frac{x-0}{-3}=\frac{y-0}{2}=\frac{z-0}{5}$

Direction ratios of these two lines are (1,2,3) and (-3, 2,5), respectively.

We know that, if two lines are perpendicular,

then

$a_1a_2+b_1b_2+c_1c_2=0$

$\therefore\quad a+2b+3c=0$ ....(ii)

and $\quad -3a+2b+5c=0$ ....(iii)

In Equations. (ii) and (iii), by cross-multiplication,

we get

$\frac{a}{10-6}=\frac{b}{-9-5}=\frac{c}{2+6}$

$\Rightarrow\quad\frac{a}{4}=\frac{b}{-14}=\frac{c}{8}$

$\Rightarrow\quad\frac{a}{2}=\frac{b}{-7}=\frac{c}{4}=\lambda(\text{ say })$

$\therefore\quad a=2\lambda,b=-7\lambda$

and $c=4\lambda$

On substituting the values of a, band c in Equation(i),

we get ,

$\frac{x-2}{2\lambda} = \frac{y-1}{-7\lambda} = \frac{z-3}{4\lambda} \Rightarrow \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$

which is the required cartesian equation of the line.

The vector equation of line which passes through (2,1,3) and parallel to the vector $2\hat{i} - 7\hat{j} + 4\hat{k}$ is

$\vec{r} = 2\hat{i} + \hat{j} + 3\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$

which is the required vector equation of the line.

16. $\vec{n} = 3\hat{i} + 5\hat{j} - 6\hat{k}$

$\left|(\overrightarrow{n})\right| = \sqrt{70}$

$\hat{n} = \frac{\overrightarrow{n}}{\left|\overrightarrow{n}\right|}$

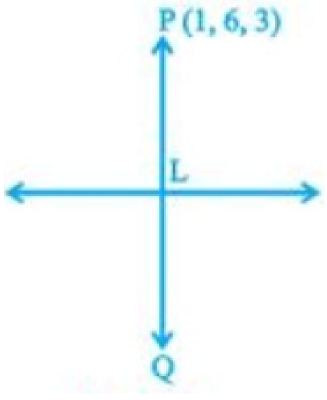
$= \frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}$

Equation of required plane is

$\overrightarrow{r} \cdot \hat{n} = 7$

$\overrightarrow{r} \cdot \left(\frac{3}{\sqrt{7}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}\right) = 7$

17. Let $P\,(1, 6, 3)$ be the given point and let L be the foot of perpendicular from P to the given line.



The coordinates of a general point on the given line are

$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$ i.e., $x = \lambda, y = 2\lambda + 1, z = 3\lambda + 2$

If the coordinates of L are $(\lambda, 2\lambda + 1, 3\lambda + 2)$, then the direction ratios of PL are $\lambda - 1, 2\lambda - 5, 3\lambda - 1.$

But the direction ratios of given line which is perpendicular to PL are 1,2,3. Therefore, $(\lambda - 1) + (2\lambda - 5)\,2 + (3\lambda - 1)\,3 = 0$which gives $\lambda = 1$. Hence coordinate of L are (1, 3, 5).

Let $Q\left(x_1, y_1, z_1\right)$ be the image of $P\left(1, 6, 3\right)$ in the given line. Then L is the mid-point of PQ. Therefore, $\frac{x_1+1}{2} = 1, \frac{y_1+6}{2} = 3, \frac{z_1+3}{2} = 5$

$\Rightarrow x_1 = 1, y_1 = 0, z_1 = 7$

Hence, the image of (1, 6, 3) in the given line is (1, 0, 7).

18. $\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{N} = 2\hat{i} + 3\hat{j} - \hat{k}$

Vector equation is

$(\vec{r} - \vec{a}) . \vec{N} = 0$

$\left[\vec{r} - \left(5\hat{i} + 2\hat{j} - 4\hat{k}\right)\right] . \left(2\hat{i} + 3\hat{j} - \hat{k}\right) = 0$

Cartesian equation is

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\left[x\hat{i} + y\hat{j} + z\hat{k} - 5\hat{i} - 2\hat{j} + 4\hat{k}\right] . \left[2\hat{i} + 3\hat{j} - \hat{k}\right] = 0$

$\left((x - 5)\hat{i} + (y - 2)\hat{j} + (z - 4)\hat{k}\right) . (2\hat{i} + 3\hat{j} - \hat{k}) = 0$

2(x - 5) + 3(y - 2) - (z + 4) = 0

2x - 10 + 3y - 6 - z - 4 = 0

2x + 3y - z = 20