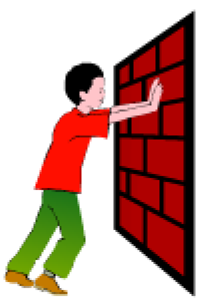
Energy and Power

Work

Consider the following day-to-day activities: reading, speaking, singing, writing, thinking, etc. We require energy to perform these activities, which we derive from the food we eat. Did you know that actually no work is involved in performing these activities? Do you think that a weightlifter does work while standing with weight over his head, as shown in the given figure?

Even if you push a wall with the maximum force that you can apply, the wall will not move. It will be interesting for you to note that even in this case, you are not doing any work at all! Do you know why?



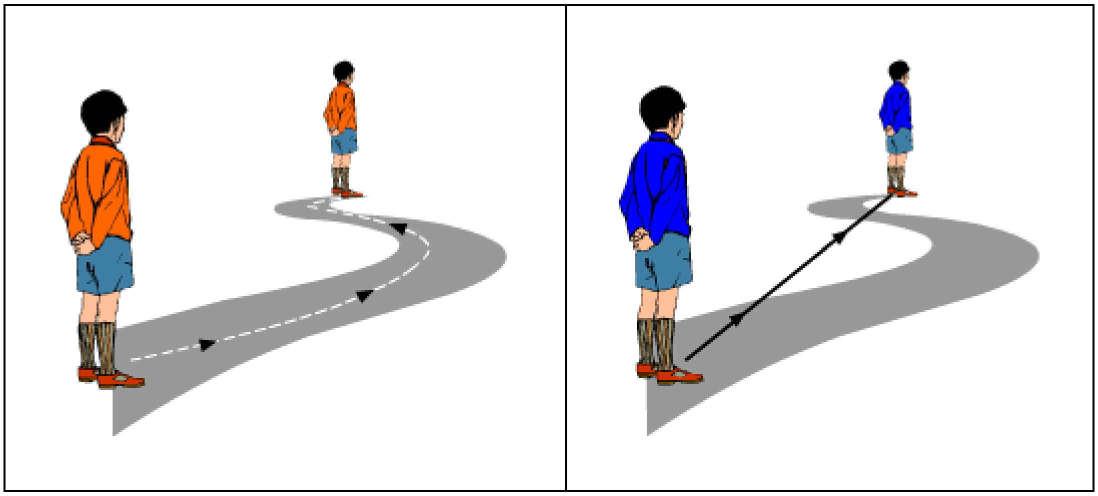
Work is not done in all the above activities because there is a basic difference between the term **work** and the term which we use for our daily activities.

Scientifically, work is defined as the work done by a force that causes a displacement in an object.

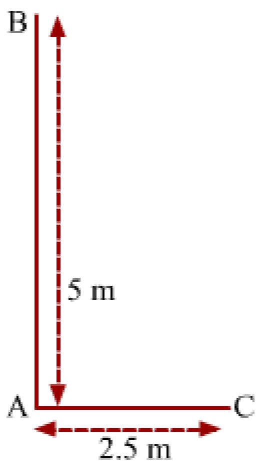
Whenever a body moves, it covers a **distance**. The straight line that joins the initial and final positions of the body is called its **displacement**.

Distance is the length of the path travelled by a body while moving from an initial position to a final position. It is a scalar quantity. Its SI unit is metre (m).

Displacement is the shortest distance between the initial and final positions of the body. It is a vector quantity. Its SI unit is also metre (m).



In displacement, the direction of motion is always directed from the initial position toward the final position.



In the science class, the teacher walks back and forth while discussing a problem in physics. He walks 5 m toward the students, turns around and then returns to his initial point. Then he walks 2.5 m toward his left and stops to answer a query from a student. What is the total distance covered by the teacher and his displacement from the point where he turns around?

The teacher walks from A to B, turns around and then walks back to A. Then, he walks from A to C.

So, total distance covered = AB + BA + AC = 5 m + 5 m + 2.5 m = 12.5 m

The teacher turns around at B. So, we need his displacement from B to C. BC is the hypotenuse of the right triangle BAC.

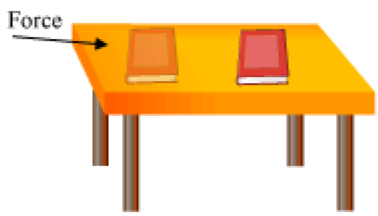
So: $BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = 5^2 + 2.5^2 \Rightarrow BC^2 = 31.25 \Rightarrow \therefore BC = 5.6$ m

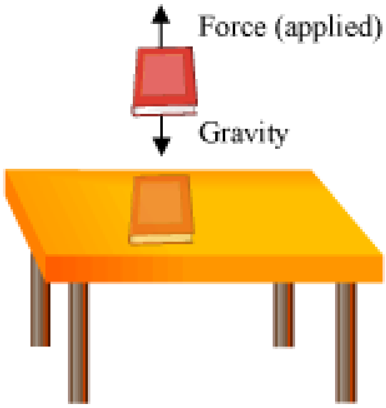
For a straight-line motion, the distance travelled and the displacement are equal in magnitude.

If you push a book placed on a table with a force, then it will move to a certain distance. Scientifically, we will say that some work has been done on the book. **Can you name the force against which work is done?**

In this case, work is done against frictional force, which exists between the book and the surface of the table.



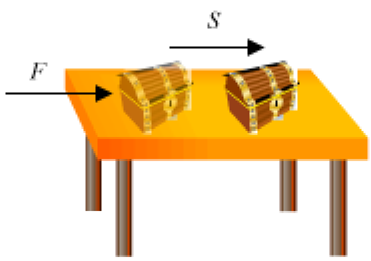
If you lift the book to a certain height, then a force is exerted against gravity, which displaces the book to a certain height. Hence, one can say that work is done on the book against the force of gravity.



If you push a trolley full of books, then it will move through a certain distance. In this case, the applied force causes a displacement in the trolley. **Do you think any work is done on the trolley?**

Work Done by a Constant Force

A wooden block is kept on a table. When a force of magnitude F acts on the block, it gets displaced through a distance S in the direction of the applied force, as shown in the given figure.



The magnitude of work done is given by the product of force (*F*) and displacement (*S*).

Let *W* be the work done on the block.

∴ Work = Force × Displacement

$W = F \times S$

Work has magnitude only. It has no direction.

Unit of Work

To obtain the unit of work, we substitute the SI units of force, i.e. N, and distance, i.e. m, in the equation of work.

W = N × m = N m

Hence, the unit of work is N m. In the honour of physicist James P. Joule, **the SI unit of work is written as Joule (J).**

Hence, 1 J = 1 N m

1 Joule is defined as the amount of work done by a unit force such that it displaces an object by a distance of 1 m.

Energy

Work and energy are the two terms used very often in our day-to-day lives. We often call someone very energetic, if the person is capable of doing a lot of work. In physics also, energy and work are very closely related and their meanings are not very different from the way we use them in our daily lives. What is the relation between work and energy?

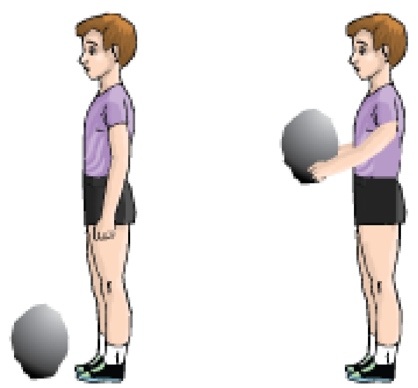
Energy is defined as the ability to do work.

You know what work is.

When a **force** displaces an object along its direction, we say the force does a work.

Therefore, a body has the ability to do work, if it possesses some energy. Without spending energy, a body cannot do any work. If we think about it a bit, then we will see that to apply a force, some amount of energy has to be spent. From this, we can also conclude that if work is done on a body, then some energy gets transferred to the body.

For example, if we lift a stone to a certain height, then a work is done on the stone to lift it against the force of gravity. For that, we use energy stored in our muscles. Again, the work is stored as energy in the stone by virtue of its position. Therefore, when the stone is released from the height, the stored energy gets released.



a. A stone lying on the ground　　b. Stone lifted up

Thus, we can conclude that **energy is required to do some work**. On the other hand, **if some work is done on a body, then the spent energy that gets stored in the body in turn becomes capable to do some work**. This is called work−energy relation.

Potential Energy

Potential Energy: Core Concepts

An object possesses potential energy by virtue of its position or height.

Take a rubber band and stretch it. When you release one end of the rubber band, it returns to its original position. The band had acquired energy in the stretched position. **How did it acquire this energy?**

Take a spring-loaded toy car and wind it using its key. When you release it on the ground, the toy car begins to move. **How did it get this energy to move?**

In the above case, the energy was stored in the objects because of the deformations in their configuration. When work was done to change their shape, energy got stored in them. This energy is also known as potential energy or **elastic potential energy**.

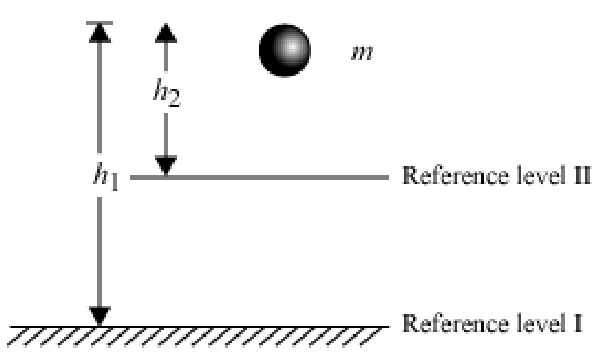
So, when you stretched the rubber band, you transferred energy to the rubber band. Similarly, when you wound the spring of the toy car, you transferred energy to the spring. Consequently, the rubber band and the spring retained potential energy by virtue of their configuration.

We can thus define potential energy as the energy stored in a body by virtue of its position or configuration.

Potential Energy of an Object at a Height

As the distance or height of a body from the ground changes, the potential energy possessed by the body also undergoes change. As it rises, its potential energy also increases. Its potential energy becomes zero when it is brought back to the ground level.

Suppose a body of mass m is initially at a height h_1 from the ground. It is then taken to a height h_2.



Note: Gravitational potential energy depends upon the reference level

Potential energy at height $h_1 = mgh_1$

Potential energy at height $h_2 = mgh_2$

Change in potential energy due to change in height = $mg\,(h_2 - h_1)$

Let us say that the body was originally on the ground and was then taken to a height h. In that case,

$h_1 = 0$

$h_2 = h$

So, change in potential energy = $mg\,(h_2 - h_1) = mg\,(h - 0) = mgh$

Whiz Kid

The potential energy of an object at a height depends upon a chosen reference level, called the zero level. It is so named because the potential energy of an object placed on this reference level is zero. In the given figure, the potential energy of the ball with respect to ‘Reference level I’ is greater than the energy it possesses with respect to ‘Reference level II’.

Solved Examples

Easy

Example 1:

What is the potential energy of a body of mass 2 kg kept at a height of 10 m above the zero level? (Take g = 9.8 m/s^2)

Solution:

The potential energy of the body is computed as:

$E_p = m \times g \times h$

Here, mass, m = 2 kg

Acceleration due to gravity, g = 9.8 m/s^2

Height, h = 10 m

$\therefore E_p = 2 \times 9.8 \times 10 = 196$ J

Hence, the potential energy of the body is 196 J.

Medium

Example 2:

The potential energy of an object of mass 10 kg increases by 5000 J when it is raised through a height *h.* What is the value of *h*? (Take g = 9.8 m/s^2)

Solution:

The potential energy of the object is given as:

$E_p = m \times g \times h$

Here, mass, m = 10 kg

Acceleration due to gravity, g = 9.8 m/s^2

Potential energy, E_p = 5000 J

$\Rightarrow 5000 = 10 \times 9.8 \times h$

$\Rightarrow \therefore h = \frac{5000}{98} = 51.02$ m

Hard

Example 3:

'*n*' books each of thickness 'd' and mass 'm' lie flat on a table. How much work is required to stack them one on top of another?

Solution:

No work is done to place the first book as it is already in position.

The second book must be moved upwards by a distance d and the force required is equal to its weight, mg.

The force and the displacement are in the same direction, so the work is mgd.

The third book will need to be moved a distance of 2d by the same size force, so the work is 2mgd.

Similarly, the work done to lift the n^{th} book is $(n - 1)\ mgd$.

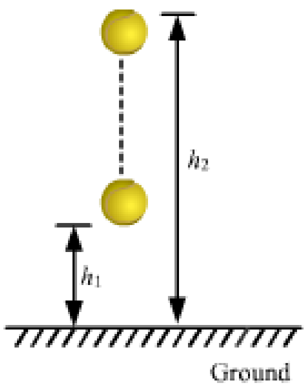
Thus the work done is, $W = mgd + 2mgd + 3mgd + \ldots\ldots\ldots\ldots + (n - 1)mgd$

$\Rightarrow W = \frac{1}{2}\, n(n - 1)mgd$

Potential Energy and Work

Work has to be done to raise the potential energy of a body. The work done in changing the position or configuration of a body is stored in the body as potential energy.

Suppose a body of mass m is initially at a height h_1 from the ground. It is then taken to a height h_2. A force equal to its weight, mg, is applied to increase its height and the body is moved with zero acceleration.



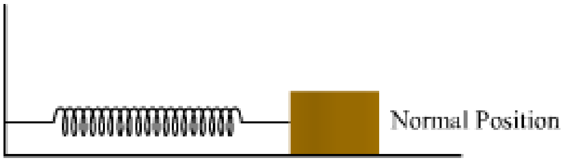
The work done by the force mg in displacing the body from height h_1 to height h_2 is given as:

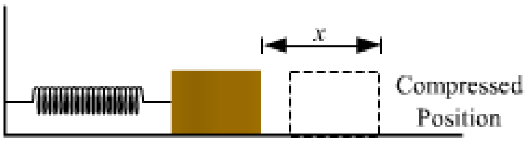
$W = mg\ (h_2 - h_1)$

This expression is the same as the change in the potential energy of the body. Thus, we can say that the work done in changing its height is stored in it as potential energy.

Energy Stored in a Compressed Spring

Suppose a spring of spring constant k is compressed by a length x.





The work done in compressing the spring is found to be:

$$W = \frac{1}{2}kx^2$$

This work done is stored in the spring as elastic potential energy.

∴ Elastic potential energy of a spring compressed by a length x = $\frac{1}{2}kx^2$

Kinetic Energy

Energy

The world requires a lot of energy. To satisfy this demand, we have natural energy sources such as the sun, wind, water at a height and tides. We also have artificial energy sources such as petroleum and natural gas.

Energy exists in various forms. Some of these are

- Light energy
- Sound energy
- Heat energy
- Mechanical energy
- Electrical energy
- Chemical energy
- Nuclear energy

Kinetic Energy

Mechanical energy is the energy possessed by an object having the potential to do work. It is associated with the motion or the position and configuration of the object. Mechanical energy is of the following two types.

- Kinetic energy (associated with the motion of an object)
- Potential energy (associated with the position and configuration of an object)

The amount of energy carried by a moving object is linked to its mass and speed. This energy is called kinetic energy. For example, a moving truck causes more damage than a small car travelling at the same speed. which implies that the truck carries more energy than the car.

The truck damages the house while the car damages only the wall.	The player gets hurt by the fast ball, and not much by the slower one.

Kinetic Energy

The energy of a body by virtue of its motion is called **kinetic energy**.

The SI unit of work is joule (J), named after the physicist James P. Joule.

Suppose a body of mass *m* is moving with a uniform velocity *u*. Let an external force be applied on it so that it gets displaced by distance *s* and its velocity becomes *v*. In this scenario, the kinetic energy of the moving body is equal to the work that was required to change its velocity from *u* to *v*.

Thus, we have the velocity−position relation as:

$v^2 = u^2 + 2as$

OR

$$s = \frac{v^2 - u^2}{2a} \qquad ...(i)$$

Where, *a* is the acceleration of the body during the change in its velocity

Now, the work done on the body by the external force is given by:

$W = F \times s$

$F = ma$ *…(ii)*

From equations (i) and (ii), we obtain:

$$W = ma \times \left(\frac{v^2 - u^2}{2a}\right) = \frac{1}{2} m (v^2 - u^2)$$

If the body was initially at rest (i.e., $u = 0$), then:

$$W = \frac{1}{2} m v^2$$

Since kinetic energy is equal to the work done on the body to change its velocity from 0 to *v*, we obtain:

$$\text{Kinetic energy, } E_k = \frac{1}{2} m v^2$$

The kinetic energy of a body is directly proportional to —

- Its mass (*m*)
- The square of its velocity (v^2)

It is the kinetic energy of the wind that is used for generating electricity through windmills.

Relationship between kinetic energy and momentum

$$K.E.,\ E_k = \frac{1}{2}mv^2 \ldots \ (1)$$

$$\text{Momentum},\ p = mv\ \ldots\ (2)$$

From (1) and (2)

$$E_k\ = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{1}{2}\frac{p}{m}^2$$

$$p^2\ = 2mE_k = \sqrt{2mE_k}$$

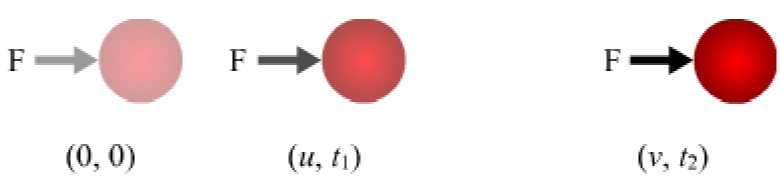
Work−Energy Theorem

Work can be done to induce motion in a body at rest. The moving body possesses kinetic energy. Thus, we can say that the work done on the body is stored in it as some form of energy.

The work−energy theorem states that the work done on a body is equal to the change in the kinetic energy of the body.

Suppose a body of mass *m* pushed by a force F has an acceleration *a*, due to which its velocity is *u* at time $t = t_1$ and its velocity becomes *v* at time $t = t_2$.

The force on the body is, $F = ma$



So, from the third equation of kinematics the distance s travelled is,

$$v^2 = u^2 + 2as$$

$$\Rightarrow mv^2 = mu^2 + 2mas$$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = (ma)s$$

$$\Rightarrow \text{F}s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \quad [s \text{ is the displacement in time } t_2 - t_1]$$

$\therefore$ Work done = Final Kinetic energy - Initial Kinetic energy

Thus, the work done by the force to increase the speed of the moving body is stored in the body as its increased kinetic energy.

Law of Conservation of Energy

Conservation of Energy − An Overview

The energy that you use to press the enter/return key on your keyboard has its source in the sun. Strange, isn't it? The red light at the bottom of the computer mouse glows using electrical energy generated in a thermal or hydel power station.

In a power station, different forms of energy get converted into electrical energy for our use. Nowhere in this universe is energy ever created. It is only converted from one form to another. Go through this lesson to understand the concept behind the law of conservation of energy.

Consider a system of bodies which neither receives energy from without nor gives up any. In such a system, the total amount of energy remains unchanged—regardless of the actions or changes that may take place within the system. This unchanging energy simply manifests itself in different forms (e.g., sound, heat, light, etc.). Our universe is such a system of bodies.

The law of conservation of energy states that the total energy of this universe is conserved or constant. Energy cannot be created or destroyed; however, it can be transformed from one form to another.

The sum of the kinetic energy and potential energy of a system is called **mechanical energy**.

E_{Mech} = Kinetic energy + Potential Energy Or, $E_{Mech} = K + U$

These two forms of energy change as they transform back and forth into each other; however, at any point, their sum remains constant.

$\Delta E_{Mech} = \Delta K + \Delta U$

The mechanical energy of a system is conserved only when the system does not gain or lose energy in any form.

Quick Questions

<u>Question 1:</u> So many forms of energy are observed in nature. How do we use these forms of energy in our daily life? The chemical energy stored in an electric cell can be used to power a bulb to produce light. Where does this light energy come from?

Solution: Various devices like generators, wind mill, solar panel etc convert one form of energy to another. Example: Wind mill converts wind energy to electrical energy which can be used to light a bulb or charge batteries.

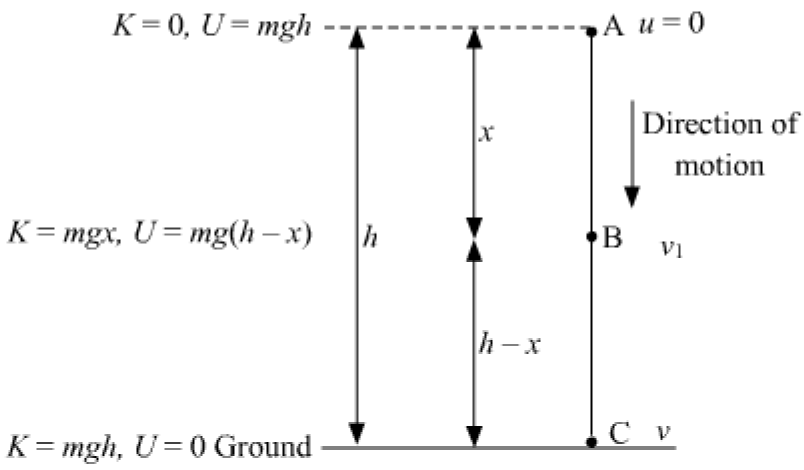
In an electric cell the chemical energy gets converted into electrical energy which heats the filament of a bulb. The hot filament produces light and some heat. Thus, we have chemical energy transformed into heat and light energy.

<u>Question 2:</u> **How the transformation of energy takes place in the wind-up toy car? What is the prime source of energy in the process? Do you know any more toys that work on the same principle?**

Solution: When you turn the key of the wind-up toy car the muscular energy from your body is stored in the coiled spring of the toy car. Which when released rotates the wheel of the toy car and hence the energy appears as the kinetic energy of the car.

The energy of the muscle comes from the food we take which is obtained from plants and animals. The prime source of the energy contained in the food molecules comes from the sun. Thus, sun is the prime source of energy in the process.

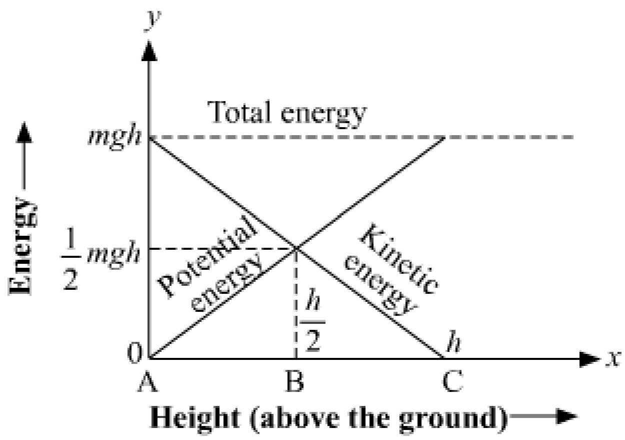
Total energy of a freely falling body is always constant. Let see this using a graphical representation.



Free fall of a body

A body of mass *m* is falling freely under the action of gravity from the height *h* above the ground. As this body falls down, its potential energy changes into the kinetic energy but at each point of motion the sum of potential and kinetic energy remains unchanged. Hence the mechanical energy remains conserved.

Curve Showing Conservation of Mechanical Energy of a Freely Falling Body:



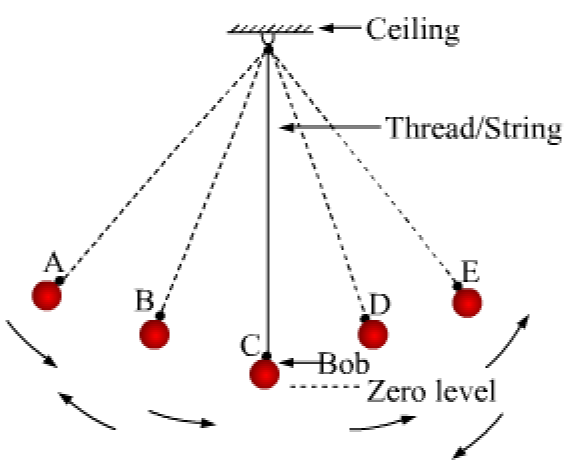
Similarly, when a body is thrown up with some initial velocity, its kinetic energy goes on decreasing whereas its potential energy goes on increasing with height. The motion is such that at each point of time mechanical energy remains conserved.

Thus, the initial kinetic energy of the body of mass *m* thrown upwards with say initial velocity *u* to reach certain height, say *h*, must be equal to the potential energy of the body at that height. So,

$$\frac{1}{2}mv^2 = mgh$$
$$\Rightarrow v = \sqrt{2gh}$$

Conservation of Mechanical Energy in a Simple Pendulum

A simple pendulum consists of a bob suspended from a string with a support. It works on the principle of alternative transformation of kinetic and potential energy. At any instant, the total energy of the bob remains the same.



Position	Energy of the bob
A	Potential energy only
B	Both potential and kinetic energies
C	Kinetic energy only
D	Both potential and kinetic energies
E	Potential energy only

At point A, the bob has potential energy, but zero kinetic energy. This is because it is at rest. When released from this point, its potential energy starts decreasing. At the same time, it gains kinetic energy. Consequently, at point B, it has both potential and kinetic energy.

Potential energy becomes zero as the bob passes through the zero level at point C. As it moves further, its kinetic energy starts decreasing. Simultaneously, it gains potential energy. Thus, at point D, the bob has both forms of energy. Finally, at point E, the bob is again at rest and has only potential energy (like when it was at point A).

Solved Examples

Easy

Example 1:

Two bodies of masses 6m and 12m are kept at a height of h and 2h from a reference level. What is the ratio of potential energy of the masses?

Solution:

Mass of A is 6m

Height of A from the ground is h

PE of A is = (6m)(g)(h) = 6mgh

Mass of B is 12m

Height of A from the ground is 2h

PE of A is = (12m)(g)(2h) = 24mgh

Therefore,

PE of A : PE of B = 6mgh : 24mgh = 1 : 4

Medium

Example 2:

A ball of mass 200 g is dropped from a height of 10 m. What will be its velocity when it hits the ground? (Take g = 9.8 m/s^2)

Solution:

By the law of conservation of energy:

Loss in the potential energy of the ball = Gain in the kinetic energy of the ball

Loss in potential energy = mgh

Here, m = Mass of the ball = 200 g

g = Acceleration due to gravity = 9.8 m/s^2

h = Height from which the ball falls = 10 m

Gain in kinetic energy = $\frac{1}{2}mv^2$

Here, v = Velocity of the ball just before it hits the ground

According to the law of conservation of energy:

$$mgh = \frac{1}{2}mv^2$$

⇒ $v^2 = 2gh$

⇒ $v^2 = 2 \times 9.8 \times 10 = 196\ (\text{m/s})^2$

⇒ $v = 14$ m/s

Therefore, when the ball hits the ground, its velocity will be 14 m/s.

Hard

Example 3:

Two objects of masses m_1 and m_2 have same kinetic energy. Both are stopped with the same retarding force F. If $m_1 > m_2$, then which mass will stop in shorter distance?

Solution:

Let the particles have mass m_1 and m_2 and $m_1 > m_2$.
The have velocities say u_1 and u_2.
$\therefore$ A/Q,

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_2u_2^2$$

$$\Rightarrow m_1u_1^2 = m_2u_2^2 \text{..................(1)}$$

To bring the particles to rest same force F is applied.
Let the retardation of particle 1 be a_1 and distance traveled be x_1.
So,

$$0^2 = u_1^2 - 2a_1x_1$$

$$\Rightarrow m_1u_1^2 = 2m_1a_1x_1$$

$$\Rightarrow m_1u_1^2 = 2Fx_1 \text{..................(2)}$$

Let the retardation of particle 2 be a_2 and distance traveled be x_2.
So,

$$0^2 = u_2^2 - 2a_2x_2$$

$$\Rightarrow m_2u_2^2 = 2m_2a_2x_2$$

$$\Rightarrow m_2u_2^2 = 2Fx_2 \text{..................(3)}$$

(1), (2) and (3) $\Rightarrow 2Fx_1 = 2Fx_2$

$$\Rightarrow x_1 = x_2$$

Thus, the particles come to rest after traveling same distance.

Power

Here, the tortoise and rabbit apply the same force to move the box through the same distance. The rabbit gets lazy, but the tortoise maintains its slow and steady pace. Undoubtedly, both do the same work, but the tortoise takes less time to complete the work. So, the tortoise proves to be more powerful.

Considering that the same force of magnitude is applied, the work done to raise a weight through a distance is the same as the work done to push another weight through the same distance. The time required to do the work determines the rate of working, but has nothing to do with the amount of work.

Power − Definition and Unit

A given amount of work may be done either in a short time or a long time. In commercial operations, the rate of working or the work done per second/per hour is an important consideration.

Power is defined as the rate of doing work. The SI unit of power is watt (W) which is joules per second.

This relation shows that for a given work, power is inversely proportional to the time taken. We can obtain a mathematical relation for power by dividing the work done by time taken.

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}} \quad \text{or } P = \frac{W}{t}$$

We know that energy is consumed when work is done. Therefore, we can also define power as the rate at which energy is consumed or utilised. Consequently, we can calculate power by dividing energy consumed by time taken.

$$\text{Power} = \frac{\text{Energy consumed}}{\text{Time taken}} \quad \text{or } P = \frac{E}{t}$$

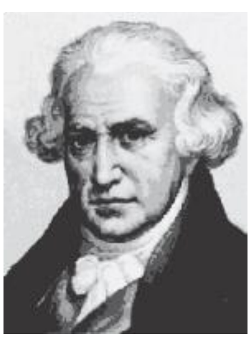
Since energy has only magnitude and no direction, power also has only magnitude and no direction.

Power is also defined as the product of force and average speed.

If a constant force *F* acts on a body and displaces it by distance *S* in the direction of force in time *t,* then

$$W = F \times S$$
$$P = \frac{W}{t} = \frac{F \times S}{t}, \text{ But } v = \frac{S}{t}$$
$$P = \text{Force} \times \text{average speed}$$
$$P = F \times v$$

Know Your Scientist
James Watt (1736−1819)



He was a Scottish inventor and mechanical engineer. Improving upon the Newcomen steam engine, he developed his own machine. Used for pumping water out from mines, it was four times more powerful than other machines based on Thomas Newcomen's design. Watt measured the power of his steam engine with a strong horse. This led him to conclude that a 'horsepower' equals 746 watts.

Power − Definition and Unit

1 Watt is the power of a device that does work at the rate of 1 joule per second. We can also say that **power is 1 W when the rate of consumption of energy is 1 Js^{-1}**. We express larger rates of energy transfer in terms of kilowatt (kW), with 1 kW = 1000 W.

Horse power: It is another unit of power, broadly used in mechanical engineering. 1 H.P. = 746 W = 0.746 kW

Solved Examples

Easy

Example 1:

A body does hundred joules of work in ten seconds. What is its power?

Solution:

Power can be calculated as follows:

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

Here, work done = 100 J

Time taken = 10 s

On putting these values in the formula, we get:

$$\text{Power} = \frac{100\text{ J}}{10\text{ s}} = 10\text{ W}$$

Hence, the power of the body is ten watts.

Medium

Example 2:

A pump lifts ten kilograms of water in two seconds to the top floor of a house from the ground. The height of the house is ten metres. What is the power of the pump?
(Take g = 9.8 m/s^2)

Solution:

First, we need to calculate the work done by the pump in lifting water against the force of gravity.

The work done against gravity is given as:

$W = mgh$

Here, mass (m) of the water lifted $= 10$ kg

Acceleration due to gravity, g $= 9.8$ m/s^2

Height (h) of the house $= 10$ m

Therefore, the work done by the pump against gravity is:

$W = 10 \times 9.8 \times 10 = 980$ J

Time taken (t) to lift the water $= 2$ s

$$\therefore \quad \text{Power, } P = \frac{W}{t} = \frac{980}{2} = 490 \text{ W}$$

Hence, the power of the pump is 490 W.

Hard

Example 3:

Water is to be pumped to fill a tank of volume 30 kL at a height 40 m from the ground in 15 minutes by a water pump on the ground floor. What is the electric power consumed by the water pump? Efficiency of the pump is 30%. Density of water is ρ = 1000 kg/m^3 and g = 9.8 m/s^2.

Solution:

Volume of water pumped up in the 15 min is = 30 kL = 30 m^3

So, mass of water pumped = $V\rho = 30\rho$ kg

Weight of water pumped = $30\rho g$ N

So, work done in pumping water to a height 40 m above ground = weight × height

$= (30\rho g)(40)$

$= 1200\rho g$

So, power required to lift this water in 15 min or 900 s is $= 1200\rho g/900 = 4\rho g/3$

Let, P be the power consumed by the pump. 30% of this power is only used to lift the water.

So, 30% of $P = 4\rho g/3$

$\Rightarrow 0.30P = 4(1000)(9.8)/3$

$\Rightarrow P = 43555.5$ W