1. $\int\sqrt{1+x^2}\,dx$ = ...

(A) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right|+c$

(B) $\frac{2}{3}(1+x^2)^{\frac{3}{2}}+c$

(C) $\frac{2}{3}x\,(1+x^2)^{\frac{3}{2}}+c$

(D) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+c$

ઉકેલ (A) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|\left(x+\sqrt{1+x^2}\right)\right|+c$

➥ $I=\int\sqrt{1+x^2}\,dx$

$=\int\sqrt{(1)^2+x^2}\,dx$

$=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+c$

(∵ સૂત્રનો ઉપયોગ કરતાં)

∴ વિકલ્પ (A) આવે,

2. $\int\sqrt{x^2-8x+7}\,dx$ = ...

(A) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+c$

(B) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+c$

(C) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+c$

(D) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+c$

ઉકેલ (D) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+c$

➥ $I=\int\sqrt{x^2-8x+7}\,dx \quad =\int\sqrt{x^2-8x+16-9}\,dx$

$=\int\sqrt{(x-4)^2-(3)^2}\,dx$

$=\frac{x-4}{2}\sqrt{(x-4)^2-(3)^2}-\frac{(3)^2}{2}\log\left|(x-4)+\sqrt{(x-4)^2-(3)^2}\right|+c$

$=\frac{x-4}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+c$

∴ વિકલ્પ (D) આવે.

3. વિધેયનું સંકલન મેળવો : $\int \sqrt{4x^2 - 5}\, dx$

➠ $x\sqrt{x^2 - \frac{5}{4}} - \frac{5}{4} \log \left| x + \sqrt{x^2 - \frac{5}{4}} \right| + c$

4. વિધેયનું સંકલન મેળવો : $\int \sqrt{3 + 8x - 3x^2}\, dx$

➠ $\frac{3x - 4}{6} \sqrt{3 + 8x - 3x^2} + \frac{25}{18} \sin^{-1} \left(\frac{3x - 4}{5} \right) + c$

5. વિધેયનું સંકલન મેળવો : $\int x^2 \sqrt{8 - x^6}\, dx$

➠ $\frac{1}{3} \left[\frac{x^3}{2} \sqrt{8 - x^6} + 4\sin^{-1} \frac{x^3}{2\sqrt{2}} \right] + c$

6. વિધેયનું સંકલન મેળવો : $\int \sqrt{2x^2 + 3x + 4}\, dx$

➠ $\frac{4x + 3}{8} \sqrt{2x^2 + 3x + 4} + \frac{23}{16\sqrt{2}} \log \left[\frac{4x + 3}{4} + \sqrt{x^2 + \frac{3}{2}x + 2} \right] + c$

7. વિધેયનું સંકલન મેળવો : $\int \sqrt{2ax - x^2}\, dx$

➠ $\frac{1}{2}(x-a)\sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + c$

8. વિધેયનું સંકલન મેળવો : $\int \sqrt{3 - 2x - x^2}\, dx$

➠ $\frac{1}{2}(x+1)\sqrt{3 - 2x - x^2} + 2\sin^{-1} \left(\frac{x+1}{2} \right) + c$

9. વિધેયનું સંકલન મેળવો : $\int (2x - 5) \sqrt{x^2 - 4x + 3}\, dx$

➠ $\frac{2}{3}(x^2 - 4x + 3)^{\frac{3}{2}} - \frac{1}{2}(x-2)\sqrt{x^2 - 4x + 3} + \frac{1}{2} \log \left| x - 2 + \sqrt{x^2 - 4x + 3} \right| + c$

10. વિધેયનું સંકલન મેળવો : $\int \sqrt{2x^2 + 3x + 4}\, dx$

➠ $\frac{4x + 3}{8} \sqrt{2x^2\ 3x + 4} + \frac{23\sqrt{2}}{32} \log \left| x + \frac{3}{4} + \sqrt{x^2 + \frac{3}{2}x + 2} \right| + c$

11. વિધેયનું સંકલન મેળવો : $\int x\sqrt{x^4 + 1}\, dx$

➠ $\frac{x^2}{4} \sqrt{x^4 + 1} + \frac{1}{4} \log \left| x^2 + \sqrt{x^4 + 1} \right| + c$

12. વિધેયનું સંકલન મેળવો : $\int \cos x \sqrt{9 - \sin^2 x}\, dx$

➠ $\frac{\sin x}{2} \sqrt{9 - \sin^2 x} + \frac{9}{2} \sin^{-1} \left(\frac{\sin x}{3} \right) + c$

13. વિધેયના સંકલિત મેળવો : $\sqrt{4 - x^2}$

➥ $I = \int \sqrt{4 - x^2}\ dx$

$= \int \sqrt{(2)^2 - x^2}\ dx$

સૂત્ર : $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

નો ઉપયોગ કરતાં,

$I = \int \sqrt{(2)^2 - x^2}\ dx$

$= \frac{x}{2}\sqrt{(2)^2 - x^2} + \frac{(2)^2}{2}\sin^{-1}\left(\frac{x}{2}\right) + c$

$= \frac{x}{2}\sqrt{4 - x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right) + c$

14. વિધેયના સંકલિત મેળવો : $\sqrt{1 - 4x^2}$

➥ $I = \int \sqrt{1 - 4x^2}\ dx$

$= \int 2\sqrt{\frac{1}{4} - x^2}\ dx$

$= 2\int \sqrt{\left(\frac{1}{2}\right)^2 - x^2}\ dx$

$= 2\left[\frac{x}{2^2}\sqrt{\left(\frac{1}{2}\right)^2 - x^2} + \frac{\left(\frac{1}{2}\right)^2}{2}\sin^{-1}\left(\frac{x}{\frac{1}{2}}\right)\right] + c$ $\left\{\because \int \sqrt{a^2 - x^2}\ dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c \text{ નો ઉપયોગ કરતાં}\right.$

$= 2\left[\frac{x}{2}\sqrt{\frac{1}{4} - x^2} + \frac{1}{8}\sin^{-1}(2x)\right] + c$

$= \frac{x}{2}\sqrt{1 - 4x^2} + \frac{1}{8}\sin^{-1}(2x) + c$

15. વિધેયના સંકલિત મેળવો : $\sqrt{x^2 + 4x + 6}$

➥ $I = \int \sqrt{x^2 + 4x + 6}\ dx$

$= \int \sqrt{(x^2 + 4x + 4) + 2}\ dx$

$= \int \sqrt{(x + 2)^2 + \left(\sqrt{2}\right)^2}\ dx$

સૂત્ર : $\int \sqrt{x^2 + a^2}\ dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$ નો ઉપયોગ કરતાં,

$I = \int \sqrt{(x + 2)^2 + \left(\sqrt{2}\right)^2}\ dx$

$= \frac{x + 2}{2}\sqrt{(x + 2)^2 + \left(\sqrt{2}\right)^2} + \frac{\left(\sqrt{2}\right)^2}{2}$

$\log\left|(x + 2) + \sqrt{(x + 2)^2 + \left(\sqrt{2}\right)^2}\right| + c$

$= \frac{x+2}{2}\sqrt{x^2+4x+6}$

$\log\left|(x+2)+\sqrt{x^2+4x+6}\right| + c$

16. વિધેયના સંકલિત મેળવો : $\sqrt{x^2+4x+1}$

➥ $I = \int\sqrt{x^2+4x+1}\,dx$

$= \int\sqrt{x^2+4x+4-3}\,dx$

$= \int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx$

સૂત્ર : $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$ નો ઉપયોગ કરતાં,

$I = \int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx = \frac{x+2}{2}\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2} - \frac{\left(\sqrt{3}\right)^2}{2}\log\left|(x+2)+\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\right| + c = \frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + c$

17. વિધેયના સંકલિત મેળવો : $\sqrt{1-4x-x^2}$

➥ $I = \int\sqrt{1-4x-x^2}\,dx$

$= \int\sqrt{1+4-4-4x-x^2}\,dx$

$= \int\sqrt{5-(x^2+4x+4)}\,dx$

$= \int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$

સૂત્ર : $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

$I = \int\sqrt{(\sqrt{5})^2-(x+2)^2}\,dx$

$= \frac{x+2}{2}\sqrt{(\sqrt{5})^2-(x+2)^2} + \frac{(\sqrt{5})^2}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + c$

$= \frac{x+2}{2}\sqrt{1-4x-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + c$

18. વિધેયના સંકલિત મેળવો : $\sqrt{x^2+4x-5}$

➥ $I = \int\sqrt{x^2+4x-5}\,dx$

$= \int\sqrt{x^2+4x+4-9}\,dx$

$= \int\sqrt{(x+2)^2-(3)^2}\,dx$

સૂત્ર : $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$ નો ઉપયોગ કરતાં,

$I = \int\sqrt{(x+2)^2-(3)^2}\,dx$

$= \frac{x+2}{2}\sqrt{(x+2)^2-(3)^2} - \frac{(3)^2}{2}$

$\log\left|(x+2)+\sqrt{(x+2)^2-(3)^2}\right| + c$

$= \frac{x+2}{2}\sqrt{x^2+4x-5} - \frac{9}{2}$

$\log\left|(x+2)+\sqrt{x^2+4x-5}\right| + c$

19. વિધેયના સંકલિત મેળવો : $\sqrt{x^2+3x}$

➥ $I = \int\sqrt{x^2+3x}$

$= \int\sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\,dx$

$= \int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$

સૂત્ર : $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$ નો ઉપયોગ કરતાં,

$I = \int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$

$= \frac{x+\frac{3}{2}}{2}\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2} - \frac{\left(\frac{3}{2}\right)^2}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\right| + c$

$I = \frac{2x+3}{4}\sqrt{x^2+3x} - \frac{9}{8}\log\left|x+\frac{3}{2}+\sqrt{x^2+3x}\right| + c$

20. વિધેયના સંકલિત મેળવો : $\sqrt{1+\frac{x^2}{9}}$

➥ $I = \int\sqrt{1+\frac{x^2}{9}}\,dx$

$= \int\frac{1}{3}\sqrt{9+x^2}\,dx$

$= \frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$

સૂત્ર : $\int\sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right| + c$

$I = \frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$

$= \frac{1}{3}\left[\frac{x}{2}\sqrt{(3)^2+x^2} + \frac{(3)^2}{2}\log\left|x+\sqrt{(3)^2+x^2}\right|\right] + c$

$= \frac{x}{6}\sqrt{9+x^2} + \frac{3}{2}\log\left|x+\sqrt{9+x^2}\right| + c$

21. વિધેયના સંકલિત મેળવો : $\sqrt{1 + 3x - x^2}$

➥ $I = \int \sqrt{1 + 3x - x^2}\, dx$

$= \int \sqrt{1 - (x^2 - 3x)}\, dx$

$= \int \sqrt{1 + \frac{9}{4} - \left(x^2 - 3x + \frac{9}{4}\right)}\, dx$ (∵ પૂર્ણ વર્ગ બનાવતાં)

$= \int \sqrt{\frac{13}{4} - \left(x - \frac{3}{2}\right)^2}\, dx$

$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\, dx$

સૂત્ર : $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$ નો ઉપયોગ કરતાં,

$I = \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\, dx$

$= \frac{x - \frac{3}{2}}{2}\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2} + \frac{\left(\frac{\sqrt{13}}{2}\right)^2}{2}$

$\sin^{-1}\frac{\left(x - \frac{3}{2}\right)}{\frac{\sqrt{13}}{2}} + c$

$= \frac{2x - 3}{2}\sqrt{1 + 3x - x^2} + \frac{13}{8}$

$\sin^{-1}\left(\frac{2x - 3}{\sqrt{13}}\right) + c$

22. $x\sqrt{x + x^2}$

➥ $I = \int x\sqrt{x + x^2}\, dx$

$x = A\frac{d}{dx}(x + x^2) + B$

$\therefore\ x = A(1 + 2x) + B$

$\therefore\ x = 2Ax + A + B$

હવે x નાં સહગુણકો અને અચળ પદ સરખાવતાં,

$2A = 1 \Rightarrow A = \frac{1}{2}$ તથા $A + B = 0 \Rightarrow B = -A = -\frac{1}{2}$

$\therefore\ x = \frac{1}{2}(1 + 2x) - \frac{1}{2}$

$\therefore\ I = \int\left[\frac{1}{2}(1 + 2x) - \frac{1}{2}\right]\sqrt{x + x^2}\, dx$

$= \frac{1}{2}\int(1 + 2x)\sqrt{x + x^2}\, dx - \frac{1}{2}\int\sqrt{x + x^2}\, dx$

$$= \frac{1}{2}\int(1+2x)\sqrt{x+x^2}\,dx - \frac{1}{2}\int\sqrt{x^2+x+\frac{1}{4}-\frac{1}{4}}\,dx$$

$$= \frac{1}{2}(x+x^2)^{\frac{3}{2}}\cdot\frac{2}{3} - \frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\,dx$$

$$\left(\because \int f'(x)[f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1}\right.$$

$$= \frac{1}{3}(x+x^2)^{\frac{3}{2}} - \frac{1}{2}\left[\frac{x+\frac{1}{2}}{2}\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2} - \frac{1}{4\times 2}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right|\right] + c$$

$$\therefore I = \frac{1}{3}(x^2+x)^{\frac{3}{2}} - \frac{(2x+1)}{8}\sqrt{x^2+x} + \frac{1}{16}\log\left|x+\frac{1}{2}+\sqrt{x^2+x}\right| + c$$

23. $(x+1)\sqrt{2x^2+3}$

➥ $I = \int(x+1)\sqrt{2x^2+3}\,dx$

$$x+1 = A\cdot\frac{d}{dx}(2x^2+3) + B$$

$\therefore\ x+1 = A(4x) + B$

બંને બાજુ x નાં સહગુણકો અને અચળપદ સરખાવતાં,

$4A = 1 \Rightarrow A = \frac{1}{4}$ તથા $B = 1$

$$I = \int\left[\frac{1}{4}(4x)+1\right]\sqrt{2x^2+3}\,dx$$

$$= \frac{1}{4}\int 4x\sqrt{2x^2+3}\,dx + \int\sqrt{2x^2+3}\,dx$$

$$= \frac{1}{4}\int 4x\sqrt{2x^2+3}\,dx + \int\sqrt{2}\sqrt{x^2+\frac{3}{2}}\,dx$$

$$= \frac{1}{4}\int 4x\sqrt{2x^2+3}\,dx + \sqrt{2}\int\sqrt{x^2+\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2}\,dx$$

$$= \frac{1}{4}(2x^2+3)^{\frac{3}{2}}\cdot\frac{2}{3} + \sqrt{2}\left[\frac{x}{2}\sqrt{x^2+\frac{3}{2}} + \frac{3}{2\times 2}\log\left|x+\sqrt{x^2+\frac{3}{2}}\right|\right] + c$$

$$\therefore I = \frac{1}{6}(2x^2+3)^2 + \frac{x}{2}\sqrt{2x^2+3} + \frac{3\sqrt{2}}{4}\log\left|x+\sqrt{x^2+\frac{3}{2}}\right| + c$$

24. $(x+3)\sqrt{3-4x-x^2}$

➥ $I = \int(x+3)\sqrt{3-4x-x^2}\,dx$

$$x+3 = A\cdot\frac{d}{dx}(3-4x-x^2) + B$$

$\therefore\ x+3 = A(-4-2x) + B$

બંને બાજુ x નાં સહગુણકો અને અચળ પદ સરખાવતાં

$-2A = 1 \Rightarrow A = -\frac{1}{2}$ તથા $-4A + B = 3$

$\therefore\ B = 3 + 4A$

$= 3 - 2$

$= 1$

$$\therefore (x+3) = -\frac{1}{2}(-4-2x)+1$$

$$\therefore I = \int\left[-\frac{1}{2}(-4-2x)+1\right]\sqrt{3-4x-x^2}\,dx$$

$$= -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\,dx + \int\sqrt{3-4x-x^2}\,dx$$

$$= -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\,dx + \int\sqrt{7-(x^2+4x+4)}\,dx$$

$$= -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\,dx + \int\sqrt{(\sqrt{7})^2-(x+2)^2}\,dx$$

$$= -\frac{1}{2}(3-4x-x^2)^{\frac{3}{2}}\cdot\frac{2}{3} + \frac{x+2}{2}\sqrt{(\sqrt{7})^2-(x+2)^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right)+c$$

$$= -\frac{1}{3}(3-4x-x^2)^{\frac{3}{2}} + \frac{x+2}{2}$$

$$\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right)+c$$