CHAPTER 9 Conic Sections

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is....... *(1994 - 2 Marks)*

2. An ellipse has eccentricity $\frac{1}{2}$ and one focus at the point $P\left(\frac{1}{2}, 1\right)$. Its one directrix is the common tangent, nearer to the point P, to the circle $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$. The equation of the ellipse, in the standard form, is............ *(1996 - 2 Marks)*

C MCQs with One Correct Answer

1. The equation $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1, \quad r > 1$ represents *(1981 - 2 Marks)*
   (a) an ellipse (b) a hyperbola
   (c) a circle (d) none of these

2. Each of the four inequalties given below defines a region in the xy plane. One of these four regions does not have the following property. For any two points (x_1, y_1) and (x_2, y_2) in the region, the point $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is also in the region. The inequality defining this region is *(1981 - 2 Marks)*
   (a) $x^2 + 2y^2 \le 1$ (b) Max $\{|x|, |y|\} \le 1$
   (c) $x^2 - y^2 \le 1$ (d) $y^2 - x \le 0$

3. The equation $2x^2 + 3y^2 - 8x - 18y + 35 = k$ represents *(1994)*
   (a) no locus if $k > 0$ (b) an ellipse if $k < 0$
   (c) a point if $k = 0$ (d) a hyperbola if $k > 0$

4. Let E be the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and C be the circle $x^2 + y^2 = 9$. Let P and Q be the points (1, 2) and (2, 1) respectively. Then *(1994)*
   (a) Q lies inside C but outside E
   (b) Q lies outside both C and E
   (c) P lies inside both C and E
   (d) P lies inside C but outside E

5. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and parabola is *(1995S)*
   (a) $\left(\frac{p}{2}, p\right)$ or $\left(\frac{p}{2}, -p\right)$ (b) $\left(\frac{p}{2}, -\frac{p}{2}\right)$
   (c) $\left(-\frac{p}{2}, p\right)$ (d) $\left(-\frac{p}{2}, -\frac{p}{2}\right)$

6. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having its centre at (0, 3) is *(1995S)*
   (a) 4 (b) 3
   (c) $\sqrt{\frac{1}{2}}$ (d) $\frac{7}{2}$

7. Let P (a sec θ, b tanθ) and Q (a sec ϕ, b tan ϕ), where $\theta + \phi = \pi/2$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P and Q, then k is equal to *(1999 - 2 Marks)*
   (a) $\frac{a^2 + b^2}{a}$ (b) $-\left(\frac{a^2 + b^2}{a}\right)$
   (c) $\frac{a^2 + b^2}{b}$ (d) $-\left(\frac{a^2 + b^2}{b}\right)$

8. If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents is *(1999 - 2 Marks)*
   (a) $9x^2 - 8y^2 + 18x - 9 = 0$ (b) $9x^2 - 8y^2 - 18x + 9 = 0$
   (c) $9x^2 - 8y^2 - 18x - 9 = 0$ (d) $9x^2 - 8y^2 + 18x + 9 = 0$

9. The curve described parametrically by $x = t^2 + t + 1$, $y = t^2 - t + 1$ represents *(1999 - 2 Marks)*
   (a) a pair of straight lines (b) an ellipse
   (c) a parabola (d) a hyperbola

10. If $x+y=k$ is normal to $y^2=12x$, then k is *(2000S)*
(a) 3 (b) 9 (c) –9 (d) –3

11. If the line $x-1=0$ is the directrix of the parabola $y^2-kx+8=0$, then one of the values of k is *(2000S)*
(a) 1/8 (b) 8 (c) 4 (d) 1/4

12. The equation of the common tangent touching the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$ above the x-axis is *(2001S)*
(a) $\sqrt{3}y=3x+1$ (b) $\sqrt{3}y=-(x+3)$
(c) $\sqrt{3}y=x+3$ (d) $\sqrt{3}y=-(3x+1)$

13. The equation of the directrix of the parabola $y^2+4y+4x+2=0$ is *(2001S)*
(a) $x=-1$ (b) $x=1$ (c) $x=-3/2$ (d) $x=3/2$

14. If $a>2b>0$ then the positive value of m for which $y=mx-b\sqrt{1+m^2}$ is a common tangent to $x^2+y^2=b^2$ and $(x-a)^2+y^2=b^2$ is *(2002S)*
(a) $\frac{2b}{\sqrt{a^2-4b^2}}$ (b) $\frac{\sqrt{a^2-4b^2}}{2b}$
(c) $\frac{2b}{a-2b}$ (d) $\frac{b}{a-2b}$

15. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2=4ax$ is another parabola with directrix *(2002S)*
(a) $x=-a$ (b) $x=-a/2$ (c) $x=0$ (d) $x=a/2$

16. The equation of the common tangent to the curves $y^2=8x$ and $xy=-1$ is *(2002S)*
(a) $3y=9x+2$ (b) $y=2x+1$
(c) $2y=x+8$ (d) $y=x+2$

17. The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is *(2003S)*
(a) 27/4 sq. units (b) 9 sq. units
(c) 27/2 sq. units (d) 27 sq. units

18. The focal chord to $y^2=16x$ is tangent to $(x-6)^2+y^2=2$, then the possible values of the slope of this chord, are *(2003S)*
(a) $\{-1,1\}$ (b) $\{-2,2\}$
(c) $\{-2,-1/2\}$ (d) $\{2,-1/2\}$

19. For hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$ which of the following remains constant with change in 'α' *(2003S)*
(a) abscissae of vertices (b) abscissae of foci
(c) eccentricity (d) directrix

20. If tangents are drawn to the ellipse $x^2+2y^2=2$, then the locus of the mid-point of the intercept made by the tangents between the coordinate axes is *(2004S)*
(a) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (b) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$
(c) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (d) $\frac{x^2}{4}+\frac{y^2}{2}=1$

21. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2=4x$ is *(2004S)*
(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

22. If the line $2x+\sqrt{6}\,y=2$ touches the hyperbola $x^2-2y^2=4$, then the point of contact is *(2004S)*
(a) $(-2,\sqrt{6})$ (b) $(-5,2\sqrt{6})$
(c) $\left(\frac{1}{2},\frac{1}{\sqrt{6}}\right)$ (d) $(4,-\sqrt{6})$

23. The minimum area of triangle formed by the tangent to the $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & coordinate axes is *(2005S)*
(a) ab sq. units (b) $\frac{a^2+b^2}{2}$ sq. units
(c) $\frac{(a+b)^2}{2}$ sq. units (d) $\frac{a^2+ab+b^2}{3}$ sq. units

24. Tangent to the curve $y=x^2+6$ at a point (1, 7) touches the circle $x^2+y^2+16x+12y+c=0$ at a point Q. Then the coordinates of Q are *(2005S)*
(a) (–6, –11) (b) (–9, –13)
(c) (–10, –15) (d) (–6, –7)

25. The axis of a parabola is along the line $y=x$ and the distances of its vertex and focus from origin are $\sqrt{2}$ and $2\sqrt{2}$ respectively. If vertex and focus both lie in the first quadrant, then the equation of the parabola is *(2006 - 3M, –1)*
(a) $(x+y)^2=(x-y-2)$ (b) $(x-y)^2=(x+y-2)$
(c) $(x-y)^2=4(x+y-2)$ (d) $(x-y)^2=8(x+y-2)$

26. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is *(2007 - 3 marks)*
(a) $x^2\operatorname{cosec}^2\theta-y^2\sec^2\theta=1$ (b) $x^2\sec^2\theta-y^2\operatorname{cosec}^2\theta=1$
(c) $x^2\sin^2\theta-y^2\cos^2\theta=1$ (d) $x^2\cos^2\theta-y^2\sin^2\theta=1$

27. Let a and b be non-zero real numbers. Then, the equation $(ax^2+by^2+c)(x^2-5xy+6y^2)=0$ represents *(2008)*
(a) four straight lines, when $c=0$ and a, b are of the same sign.
(b) two straight lines and a circle, when $a=b$, and c is of sign opposite to that of a
(c) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a
(d) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

28. Consider a branch of the hyperbola

$$x^2-2y^2-2\sqrt{2}x-4\sqrt{2}y-6=0$$

with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is *(2008)*
(a) $1-\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{3}{2}}-1$ (c) $1+\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{2}}+1$

29. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse

$$x^2 + 9y^2 = 9$$

meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is ***(2009)***

(a) $\frac{31}{10}$ (b) $\frac{29}{10}$ (c) $\frac{21}{10}$ (d) $\frac{27}{10}$

30. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x - axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points ***(2009)***

(a) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (b) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\sqrt{\frac{19}{4}}\right)$

(c) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (d) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

31. The locus of the orthocentre of the triangle formed by the lines

$$(1+p)x - py + p(1+p) = 0,$$
$$(1+q)x - qy + q(1+q) = 0,$$

and $y = 0$, where $p \neq q$, is ***(2009)***

(a) a hyperbola (b) a parabola
(c) an ellipse (d) a straight line

32. Let $P(6, 3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at $(9, 0)$, then the eccentricity of the hyperbola is ***(2011)***

(a) $\sqrt{\frac{5}{2}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

33. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0, 0)$ to (x, y) in the ratio $1 : 3$. Then the locus of P is ***(2011)***

(a) $x^2 = y$ (b) $y^2 = 2x$ (c) $y^2 = x$ (d) $x^2 = 2y$

34. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0, 4)$ circumscribes the rectangle R. The eccentricity of the ellipse E_2 is ***(2012)***

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

35. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral $PQRS$ is

(JEE Adv. 2014)

(a) 3 (b) 6 (c) 9 (d) 15

D MCQs with One or More than One Correct

1. The number of values of c such that the straight line $y = 4x + c$ touches the curve $(x^2/4) + y^2 = 1$ is ***(1998 - 2 Marks)***

(a) 0 (b) 1 (c) 2 (d) infinite.

2. If $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals ***(1998 - 2 Marks)***

(a) 8 (b) 6 (c) 10 (d) 12

3. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are ***(1999 - 3 Marks)***

(a) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$

(c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (d) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

4. The equations of the common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are ***(2006 - 5M, –1)***

(a) $y = 4(x-1)$ (b) $y = 0$
(c) $y = -4(x-1)$ (d) $y = -30x - 50$

5. Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then ***(2006 - 5M, –1)***

(a) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(b) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(c) focus of hyperbola is $(5, 0)$

(d) vertex of hyperbola is $(5\sqrt{3}, 0)$

6. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are ***(2008)***

(a) $x^2 + 2\sqrt{3}\,y = 3 + \sqrt{3}$ (b) $x^2 - 2\sqrt{3}\,y = 3 + \sqrt{3}$

(c) $x^2 + 2\sqrt{3}\,y = 3 - \sqrt{3}$ (d) $x^2 - 2\sqrt{3}\,y = 3 - \sqrt{3}$

7. In a triangle ABC with fixed base BC, the vertex A moves such that

$$\cos B + \cos C = 4\sin^2\frac{A}{2}.$$

If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, then ***(2009)***

(a) $b + c = 4a$
(b) $b + c = 2a$
(c) locus of point A is an ellipse
(d) locus of point A is a pair of straight lines

8. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose *(2009)*

(a) vertex is $\left(\frac{2a}{3}, 0\right)$ (b) directrix is $x = 0$

(c) latus rectum is $\frac{2a}{3}$ (d) focus is $(a, 0)$

9. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then *(2009)*

(a) equation of ellipse is $x^2 + 2y^2 = 2$

(b) the foci of ellipse are $(\pm 1, 0)$

(c) equation of ellipse is $x^2 + 2y^2 = 4$

(d) the foci of ellipse are $(\pm\sqrt{2}, 0)$

10. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be *(2010)*

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ (c) $\frac{2}{r}$ (d) $-\frac{2}{r}$

11. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then *(2011)*

(a) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(b) a focus of the hyperbola is (2, 0)

(c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

12. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by *(2011)*

(a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$

(c) $y + x - 15 = 0$ (d) $y - 2x + 12 = 0$

13. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are *(2012)*

(a) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (b) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(c) $\left(3\sqrt{3}, -2\sqrt{2}\right)$ (d) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

14. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P? *(JEE Adv. 2015)*

(a) $(4, 2\sqrt{2})$ (b) $(9, 3\sqrt{2})$

(c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (d) $(1, \sqrt{2})$

15. Let E_1 and E_2 be two ellipses whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y - 1)^2 = 2$. The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is (are) *(JEE Adv. 2015)*

(a) $e_1^2 + e_2^2 = \frac{43}{40}$ (b) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

(c) $\left|e_1^2 - e_2^2\right| = \frac{5}{8}$ (d) $e_1 e_2 = \frac{\sqrt{3}}{4}$

16. Consider the hyperbola $H : x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle PMN, then the correct expression(s) is(are) *(JEE Adv. 2015)*

(a) $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$

(b) $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

(c) $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$

(d) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

17. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1, at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y–axis, then *(JEE Adv. 2016)*

(a) $Q_2Q_3 = 12$

(b) $R_2R_3 = 4\sqrt{6}$

(c) area of the triangle OR_2R_3 is $6\sqrt{2}$

(d) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

18. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then *(JEE Adv. 2016)*

(a) $SP = 2\sqrt{5}$

(b) $SQ : QP = (\sqrt{5}+1) : 2$

(c) the x-intercept of the normal to the parabola at P is 6

(d) the slope of the tangent to the circle at Q is $\frac{1}{2}$

E Subjective Problems

1. Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point (h, k). Show that $h > 2$. *(1981 - 4 Marks)*

2. A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B. If AB subtends a right angle at the vertex of the parabola. find the slope of AB. *(1982 - 5 Marks)*

3. Three normals are drawn from the point (c, 0) to the curve $y^2 = x$. Show that c must be greater than 1/2. One normal is always the x-axis. Find c for which the other two normals are perpendicular to each other. *(1991 - 4 Marks)*

4. Through the vertex O of parabola $y^2 = 4x$, chords OP and OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ. *(1994 - 4 Marks)*

5. Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1: 2 is a parabola. Find the vertex of this parabola. *(1995 - 5 Marks)*

6. Let 'd' be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent drawn at a point P on the ellipse. If F_1 and F_2 are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2\left(1-\frac{b^2}{d^2}\right)$. *(1995 - 5 Marks)*

7. Points A, B and C lie on the parabola $y^2 = 4ax$. The tangents to the parabola at A, B and C, taken in pairs, intersect at points P, Q and R. Determine the ratio of the areas of the triangles ABC and PQR. *(1996 - 3 Marks)*

8. From a point A common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ and parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola. *(1996 - 2 Marks)*

9. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. Prove that the tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles. *(1997 - 5 Marks)*

10. The angle between a pair of tangents drawn from a point P to the parabola $y^2 = 4ax$ is 45°. Show that the locus of the point P is a hyperbola. *(1998 - 8 Marks)*

11. Consider the family of circles $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common taingent to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A and B, then find the equation of the locus of the mid-point of AB. *(1999 - 10 Marks)*

12. Find the co-ordinates of all the points P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, for which the area of the triangle PON is maximum, where O denotes the origin and N, the foot of the perpendicular from O to the tangent at P. *(1999 - 10 Marks)*

13. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a > b)$ meets the ellipse respectively, at P, Q, R. so that P, Q, R lie on the same side of the major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P, Q and R are concurrent. *(2000 - 7 Marks)*

14. Let C_1 and C_2 be respectively, the parabolas $x^2 = y - 1$ and $y^2 = x - 1$. Let P be any point on C_1 and Q be any point on C_2. Let P_1 and Q_1 be the reflections of P and Q, respectively, with respect to the line $y = x$. Prove that P_1 lies on C_2, Q_1 lies on C_1 and $PQ \geq \min\{PP_1, QQ_1\}$. Hence or otherwise determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (P,Q) with P on C_1 and Q on C_2. *(2000 - 10 Marks)*

15. Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, 0 < b < a$. Let the line parallel to y–axis passing through P meet the circle $x^2 + y^2 = a^2$ at the point Q such that P and Q are on the same side of x–axis. For two positive real numbers r and s, find the locus of the point R on PQ such that $PR : RQ = r : s$ as P varies over the ellipse. *(2001 - 4 Marks)*

16. Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. *(2002 - 5 Marks)*

17. Normals are drawn from the point P with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_1 m_2 = \alpha$ is a part of the parabola itself then find α. *(2003 - 4 Marks)*

18. Tangent is drawn to parabola $y^2 - 2y - 4x + 5 = 0$ at a point P which cuts the directrix at the point Q. A point R is such that it divides QP externally in the ratio 1/2 : 1. Find the locus of point R. *(2004 - 4 Marks)*

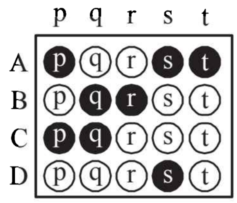
19. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid-point of the chord of contact. *(2005 - 4 Marks)*

20. Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$. Also find the length of the intercept of the tangent between the coordinate axes. *(2005 - 4 Marks)*

F Match the Following

DIRECTIONS (Q. 1-3) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Match the following : (3, 0) is the pt. from which three normals are drawn to the parabola $y^2 = 4x$ which meet the parabola in the points P, Q and R. Then ***(2006 - 6M)***

Column I	*Column II*
(A) Area of ΔPQR	(p) 2
(B) Radius of circumcircle of ΔPQR	(q) 5/2
(C) Centroid of ΔPQR	(r) (5/2, 0)
(D) Circumcentre of ΔPQR	(s) (2/3, 0)

2. Match the statements in Column I with the properties in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS. ***(2007 -6 marks)***

Column I	*Column II*
(A) Two intersecting circles	(p) have a common tangent
(B) Two mutually external circles	(q) have a common normal
(C) Two circles, one strictly inside the other	(r) do not have a common tangent
(D) Two branches of a hyperbola	(s) do not have a common normal

3. Match the conics in **Column I** with the statements/expressions in **Column II**. ***(2009)***

Column I	*Column II*
(A) Circle	(p) The locus of the point (h,k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
(B) Parabola	(q) Points z in the complex plane satisfying $\lvert z+2\rvert - \lvert z-2\rvert = \pm 3$
(C) Ellipse	(r) Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right),\quad y = \frac{2t}{1+t^2}$
(D) Hyperbola	(s) The eccentricity of the conic lies in the interval $1 \le x < \infty$
	(t) Points z in the complex plane satisfying $\text{Re}\,(z+1)^2 = \lvert z\rvert^2 + 1$

DIRECTIONS (Q. 4) : Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

4. A line $L : y = mx + 3$ meets y – axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x, 0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum. ***(JEE Adv. 2013)***

Match List I with List II and select the correct answer using the code given below the lists :

	List I		**List II**
P.	$m =$	1.	$\frac{1}{2}$
Q.	Maximum area of ΔEFG is	2.	4
R.	$y_0 =$	3.	2
S.	$y_1 =$	4.	1

Codes:

	P	**Q**	**R**	**S**
(a)	4	1	2	3
(b)	3	4	1	2
(c)	1	3	2	4
(d)	1	3	4	2

G Comprehension Based Questions

PASSAGE 1

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the curcle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S. ***(2007 -4 marks)***

1. The ratio of the areas of the triangles PQS and PQR is

(a) $1:\sqrt{2}$ (b) $1:2$ (c) $1:4$ (d) $1:8$

2. The radius of the circumcircle of the triangle PRS is ***(2007 -4 marks)***

(a) 5 (b) $3\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

3. The radius of the incircle of the triangle PQR is ***(2007 -4 marks)***

(a) 4 (b) 3 (c) $\frac{8}{3}$ (d) 2

PASSAGE 2

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B. ***(2010)***

4. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(a) $2x - \sqrt{5}y - 20 = 0$ (b) $2x - \sqrt{5}y + 4 = 0$

(c) $3x - 4y + 8 = 0$ (d) $4x - 3y + 4 = 0$

5. Equation of the circle with AB as its diameter is

(a) $x^2 + y^2 - 12x + 24 = 0$ (b) $x^2 + y^2 + 12x + 24 = 0$

(c) $x^2 + y^2 + 24x - 12 = 0$ (d) $x^2 + y^2 - 24x - 12 = 0$

PASSAGE 3

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B. ***(2010)***

6. The coordinates of A and B are

(a) (3, 0) and (0, 2)

(b) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(c) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $(0, 2)$

(d) $(3, 0)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

7. The orthocenter of the triangle PAB is

(a) $\left(5, \frac{8}{7}\right)$ (b) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (c) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (d) $\left(\frac{8}{25}, \frac{7}{5}\right)$

8. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(a) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(b) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(c) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(d) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

PASSAGE 4

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

9. Length of chord PQ is ***(JEE Adv. 2013)***

(a) $7a$ (b) $5a$ (c) $2a$ (d) $3a$

10. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$ ***(JEE Adv. 2013)***

(a) $\frac{2}{3}\sqrt{7}$ (b) $\frac{-2}{3}\sqrt{7}$ (c) $\frac{2}{3}\sqrt{5}$ (d) $\frac{-2}{3}\sqrt{5}$

PASSAGE 5

Let a, r, s, t be nonzero real numbers. Let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point $(2a, 0)$ ***(JEE Adv. 2014)***

11. The value of r is

(a) $-\frac{1}{t}$ (b) $\frac{t^2+1}{t}$ (c) $\frac{1}{t}$ (d) $\frac{t^2-1}{t}$

12. If $st = 1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(a) $\frac{(t^2+1)^2}{2t^3}$ (b) $\frac{a(t^2+1)^2}{2t^3}$

(c) $\frac{a(t^2+1)^2}{t^3}$ (d) $\frac{a(t^2+2)^2}{t^3}$

PASSAGE 6

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

13. The orthocentre of the triangle F_1MN is ***(JEE Adv. 2016)***

(a) $\left(-\frac{9}{10}, 0\right)$ (b) $\left(\frac{2}{3}, 0\right)$

(c) $\left(\frac{9}{10}, 0\right)$ (d) $\left(\frac{2}{3}, \sqrt{6}\right)$

14. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is ***(JEE Adv. 2016)***

(a) $3:4$ (b) $4:5$

(c) $5:8$ (d) $2:3$

H Assertion & Reason Type Questions

1. **STATEMENT-1 :** The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$. because
STATEMENT-2 : A parabola is symmetric about its axis. ***(2007 -3 marks)***
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

I Integer Value Correct Type

1. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is ***(2010)***

2. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is ***(2011)***

3. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is ***(2012)***

4. A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ ***(JEE Adv. 2013)***
(a) $g(x)$ is continuous but not differentiable at a
(b) $g(x)$ is differentiable on R
(c) $g(x)$ is continuous but not differentiable at b
(d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

5. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is ***(JEE Adv. 2015)***

6. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is ***(JEE Adv. 2015)***

7. Suppose that the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2} + m_2^2\right)$ is ***(JEE Adv. 2015)***

Section-B JEE Main / AIEEE

1. Two common tangents to the circle $x^2+y^2=2a^2$ and parabola $y^2=8ax$ are **[2002]**
(a) $x=\pm(y+2a)$ (b) $y=\pm(x+2a)$
(c) $x=\pm(y+a)$ (d) $y=\pm(x+a)$

2. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then
(a) $t_2=t_1+\frac{2}{t_1}$ (b) $t_2=-t_1-\frac{2}{t_1}$ **[2003]**
(c) $t_2=-t_1+\frac{2}{t_1}$ (d) $t_2=t_1-\frac{2}{t_1}$

3. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is **[2003[**
(a) 9 (b) 1 (c) 5 (d) 7

4. If $a\neq 0$ and the line $2bx+3cy+4d=0$ passes through the points of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$, then **[2004[**
(a) $d^2+(3b-2c)^2=0$ (b) $d^2+(3b+2c)^2=0$
(c) $d^2+(2b-3c)^2=0$ (d) $d^2+(2b+3c)^2=0$

5. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x=4$, then the equation of the ellipse is: **[2004]**
(a) $4x^2+3y^2=1$ (b) $3x^2+4y^2=12$
(c) $4x^2+3y^2=12$ (d) $3x^2+4y^2=1$

6. Let P be the point $(1, 0)$ and Q a point on the locus $y^2=8x$. The locus of mid point of PQ is **[2005]**
(a) $y^2-4x+2=0$ (b) $y^2+4x+2=0$
(c) $x^2+4y+2=0$ (d) $x^2-4y+2=0$

7. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y=\alpha x+\beta$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is **[2005]**
(a) an ellipse (b) a circle
(c) a parabola (d) a hyperbola

8. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is **[2005]**
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

9. The locus of the vertices of the family of parabolas $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$ is **[2006]**
(a) $xy=\frac{105}{64}$ (b) $xy=\frac{3}{4}$ (c) $xy=\frac{35}{16}$ (d) $xy=\frac{64}{105}$

10. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is **[2006]**
(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{\sqrt{5}}$

11. Angle between the tangents to the curve $y=x^2-5x+6$ at the points (2, 0) and (3, 0) is **[2006]**
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

12. For the Hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$, which of the following remains constant when α varies = ? **[2007]**
(a) abscissae of vertices (b) abscissae of foci
(c) eccentricity (d) directrix.

13. The equation of a tangent to the parabola $y^2=8x$ is $y=x+2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is **[2007]**
(a) (2,4) (b) (–2,0) (c) (–1, 1) (d) (0,2)

14. The normal to a curve at $P(x, y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a **[2007]**
(a) circle (b) hyperbola (c) ellipse (d) parabola.

15. A focus of an ellipse is at the origin. The directrix is the line $x=4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is **[2008]**
(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

16. A parabola has the origin as its focus and the line $x=2$ as the directrix. Then the vertex of the parabola is at **[2008]**
(a) (0,2) (b) (1, 0) (c) (0, 1) (d) (2, 0)

17. The ellipse $x^2+4y^2=4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is : **[2009]**

(a) $x^2+12y^2=16$ (b) $4x^2+48y^2=48$

(c) $4x^2+64y^2=48$ (d) $x^2+16y^2=16$

18. If two tangents drawn from a point P to the parabola $y^2=4x$ are at right angles, then the locus of P is **[2010]**

(a) $2x+1=0$ (b) $x=-1$

(c) $2x-1=0$ (d) $x=1$

19. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is **[2011]**

(a) $5x^2+3y^2-48=0$ (b) $3x^2+5y^2-15=0$

(c) $5x^2+3y^2-32=0$ (d) $3x^2+5y^2-32=0$

20. **Statement-1 :** An equation of a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2+y^2=4$ is $y=2x+2\sqrt{3}$

Statement-2 : If the line $y=mx+\frac{4\sqrt{3}}{m},(m\neq 0)$ is a common tangent to the parabola $y^2=16\sqrt{3}x$ and the ellipse $2x^2+y^2=4$, then m satisfies $m^4+2m^2=24$ **[2012]**

(a) Statement-1 is false, Statement-2 is true.

(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(d) Statement-1 is true, statement-2 is false.

21. An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ is semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[2012]**

(a) $4x^2+y^2=4$ (b) $x^2+4y^2=8$

(c) $4x^2+y^2=8$ (d) $x^2+4y^2=16$

22. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is **[JEE M 2013]**

(a) $x^2+y^2-6y-7=0$ (b) $x^2+y^2-6y+7=0$

(c) $x^2+y^2-6y-5=0$ (d) $x^2+y^2-6y+5=0$

23. **Given :** A circle, $2x^2+2y^2=5$ and a parabola, $y^2=4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y=x+\sqrt{5}$.

Statement-2 : If the line, $y=mx+\frac{\sqrt{5}}{m}$ $(m\neq 0)$ is their common tangent, then m satisfies $m^4-3m^2+2=0$. **[JEE M 2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

24. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2+3y^2=6$ on any tangent to it is **[JEE M 2014]**

(a) $\left(x^2+y^2\right)^2=6x^2+2y^2$ (b) $\left(x^2+y^2\right)^2=6x^2-2y^2$

(c) $\left(x^2-y^2\right)^2=6x^2+2y^2$ (d) $\left(x^2-y^2\right)^2=6x^2-2y^2$

25. The slope of the line touching both the parabolas $y^2=4x$ and $x^2=-32y$ is **[JEE M 2014]**

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

26. Let O be the vertex and Q be any point on the parabola, $x^2=8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then locus of P is : **[JEE M 2015]**

(a) $y^2=2x$ (b) $x^2=2y$ (c) $x^2=y$ (d) $y^2=x$

27. The normal to the curve, $x^2+2xy-3y^2=0$, at (1, 1) **[JEE M 2015]**

(a) meets the curve again in the third quadrant.

(b) meets the curve again in the fourth quadrant.

(c) does not meet the curve again.

(d) meets the curve again in the second quadrant.

28. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is : **[JEE M 2015]**

(a) $\frac{27}{2}$ (b) 27 (c) $\frac{27}{4}$ (d) 18

29. Let P be the point on the parabola, $y^2=8x$ which is at a minimum distance from the centre C of the circle, $x^2+(y+6)^2=1$. Then the equation of the circle, passing through C and having its centre at P is: **[JEE M 2016]**

(a) $x^2+y^2-\frac{x}{4}+2y-24=0$

(b) $x^2+y^2-4x+9y+18=0$

(c) $x^2+y^2-4x+8y+12=0$

(d) $x^2+y^2-x+4y-12=0$

30. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[JEE M 2016]**

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{4}{3}$ (d) $\frac{4}{\sqrt{3}}$

9 Conic Sections

Section-A : JEE Advanced/ IIT-JEE

A **1.** (–1, 0) **2.** $\dfrac{\left(x-\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)^2}+\dfrac{(y-1)^2}{\left(\frac{1}{2\sqrt{3}}\right)^2}=1$

C

1. (d)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (a)	**6.** (a)	**7.** (d)
8. (b)	**9.** (c)	**10.** (b)	**11.** (c)	**12.** (c)	**13.** (d)	**14.** (a)
15. (c)	**16.** (d)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (a)	**21.** (c)
22. (d)	**23.** (a)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (b)
29. (d)	**30.** (c)	**31.** (d)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (d)

D

1. (c)	**2.** (c)	**3.** (b, d)	**4.** (a, b)	**5.** (a, c)	**6.** (b, c)	**7.** (b, c)
8. (a, d)	**9.** (a, b)	**10.** (c, d)	**11.** (b, d)	**12.** (a, b, d)	**13.** (a, b)	**14.** (a, d)
15. (a, b)	**16.** (a, b, d)	**17.** (a, b, c)	**18.** (a, c, d)			

E **2.** $m=\pm\sqrt{2}$ **3.** $c=\dfrac{3}{4}$ **4.** $y^2=2(x-4)$ **5.** $\left(\dfrac{2}{9},\dfrac{8}{9}\right)$

7. $2:1$ **8.** $\dfrac{15a^2}{4}$ **11.** $\dfrac{25}{x^2}+\dfrac{4}{y^2}=4$

12. $\left(\dfrac{a^2}{\sqrt{a^2+b^2}},\dfrac{b^2}{\sqrt{a^2+b^2}}\right)$ **15.** $\dfrac{x^2}{a^2}+\dfrac{y^2(r+s)^2}{(bs+ar)^2}=1$ **17.** $\alpha=2$

18. $(x-1)(y-1)^2+4=0$ **19.** $\dfrac{x^2}{9}-\dfrac{y^2}{4}=\left(\dfrac{x^2+y^2}{9}\right)^2$ **20.** $y=-\dfrac{2}{\sqrt{3}}x+4\sqrt{\dfrac{7}{3}}\,;\ \dfrac{14}{\sqrt{3}}$

F **1.** (A)-(p), (B)-(q), (C)-(s), (D)-(r) **2.** (A)-p, q; (B)-p, q ; (C)-q, r ; (D)-q, r

3. (A)-p; (B)-s, t ; (C)-r ; (D)-q, s **4.** (a)

G **1.** (c) **2.** (b) **3.** (d) **4.** (b) **5.** (a) **6.** (d) **7.** (c) **8.** (a) **9.** (b) **10.** (d)

11. (d) **12.** (b) **13.** (a) **14.** (c)

H **1.** (a)

I **1.** 2 **2.** 2 **3.** 4 **4.** 9 **5.** 2 **6.** 4 **7.** 4

Section-B : JEE Main/ AIEEE

1. (b)	**2.** (b)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (a)	**7.** (d)
8. (a)	**9.** (a)	**10.** (a)	**11.** (b)	**12.** (b)	**13.** (b)	**14.** (b)
15. (a)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (b)	**21.** (d)
22. (a)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (b)
29. (c)	**30.** (a)					

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Given parabola is $y^2 = 4x$; $a = 1$

Extremities of latus rectum are $(1, 2)$ and $(1, -2)$ tangent to $y^2 = 4x$ at $(1, 2)$ is $y.2 = 2(x + 1)$ i.e. $y = x + 1$ ...(1)

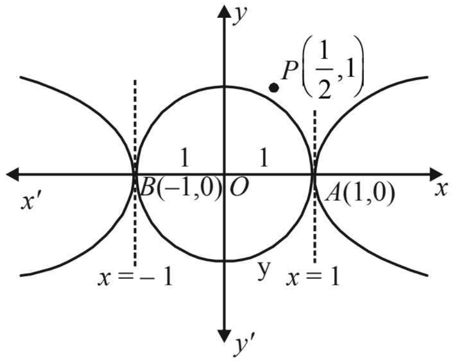
Similarly tangent at $(1, -2)$ is, $y = -x - 1$ ...(2)

Intersection pt. of these tangents can be obtained by solving (1) and (2), which is $(-1, 0)$.

2. Rough graph of $x^2 + y^2 = 1$ (circle) ...(1)

and $x^2 - y^2 = 1$ (hyperbola) ...(2)

is as shown below.



It is clear from graph that there are two common tangents to the curves (1) and (2) namely $x = 1$ and $x = -1$ out of which $x = 1$ is nearer to pt. P.

Hence directrix of required ellipse is $x - 1 = 0$

Also $e = 1/2$, focus $(1/2, 1)$ then equation of ellipse is given by

$$(x - 1/2)^2 + (y - 1)^2 = \frac{1}{4}(x - 1)^2$$

$$\Rightarrow \frac{(x - 1/3)^2}{(1/3)^2} + \frac{(y - 1)^2}{(1/2\sqrt{3})^2} = 1$$

which is the standard equation of the ellipse.

C. MCQs with ONE Correct Answer

1. (d) Given that $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1,\ r > 1$

As $r > 1$

$\therefore$ $1 - r < 0$ and $1 + r > 0$

$\therefore$ Let $1 - r = -a^2$, $1 + r = b^2$, then we get

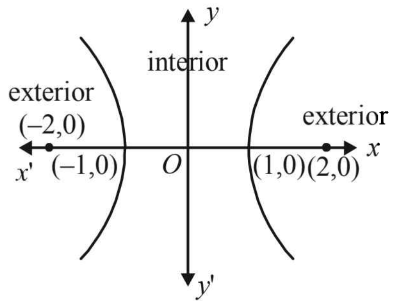
$$\frac{x^2}{-a^2} - \frac{y^2}{b^2} = 1 \quad \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$$

which is not possible for any real values of x and y.

2. (c) (a) $x^2 + 2y^2 \le 1$ represents interior region of an ellipse where on taking any two pts the mid pt of that segment will also lie inside that ellipse

(b) Max $\{|x|, |y|\} \le 1$
$\Rightarrow |x| \le 1, |y| \le 1 \Rightarrow -1 \le x \le 1$ and $-1 \le y \le 1$
which represents the interior region of a square with its sides $x = \pm 1$ and $y = \pm 1$ in which for any two pts, their mid pt also lies inside the region.

(c) $x^2 - y^2 \ge 1$ represents the exterior region of hyperbola in which if we take two points $(2, 0)$ and $(-2, 0)$ then their mid pt $(0, 0)$ does not lie in the same region (as shown in the figure.)



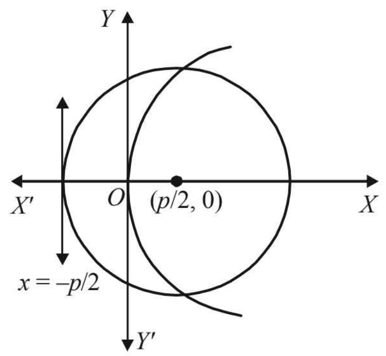
(d) $y^2 \le x$ represents interior region of parabola in which for any two pts, their mid point also lie inside the region.

3. (c) We have $2x^2 + 3y^2 - 8x - 18y + 35 = k$
$\Rightarrow$ $2(x - 2)^2 + 3(y - 3)^2 = k$
For $k = 0$, we get $2(x - 2)^2 + 3(y - 3)^2 = 0$ which represents the point $(2, 3)$.

4. (d) Since $1^2 + 2^2 = 5 < 9$ and $2^2 + 1^1 = 5 < 9$ both P and Q lie inside C.

Also $\frac{1^2}{9} + \frac{2^2}{4} = \frac{1}{9} + 1 > 1$ and $\frac{2^2}{9} + \frac{1}{4} = \frac{25}{36} < 1$, P lies outside E and Q lies inside E. Thus P lies inside C but outside E.

5. (a) The focus of parabola $y^2 = 2px$ is $\left(\frac{p}{2}, 0\right)$ and directrix $x = -p/2$



In the figure, we have supposed that $p > 0$]

$\therefore$ Centre of circle is $\left(\frac{p}{2}, 0\right)$ and radius $= \frac{p}{2} + \frac{p}{2} = p$

$\therefore$ Equation of circle is $\left(x - \frac{p}{2}\right)^2 + y^2 = p^2$

For pts of intersection of $y^2 = 2px$ ...(i)
and $4x^2 + 4y^2 - 4px - 3p^2 = 0$ ...(ii)
can be obtained by solving (i) and (ii) as follows

$4x^2 + 8px - 4px - 3p^2 = 0 \Rightarrow (2x + 3p)(2x - p) = 0$

$\Rightarrow$ $x = \frac{-3p}{2}, \frac{p}{2}$

$\Rightarrow$ $y^2 = -3p^2$ (not possible), $p^2 \Rightarrow y = \pm p$

$\therefore$ Required pts are $(p/2, p)$, $(p/2, -p)$

6. **(a)** For ellipse $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$, $a=4, b=3$

$\Rightarrow \quad e=\sqrt{1-\left(\frac{3}{4}\right)^2}=\frac{\sqrt{7}}{4}$

$\therefore$ Foci are $(\sqrt{7},0)$ and $(-\sqrt{7},0)$

Centre of circle is at (0, 3) and it passes through $(\pm\sqrt{7},0)$, therefore radius of circle $=\sqrt{(\sqrt{7})^2+(3)^2}=4$

7. **(d)** **KEY CONCEPT:**

Equation of the normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $(a\sec\alpha, b\tan\alpha)$ is given by $ax\cos\alpha+by\cot\alpha=a^2+b^2$

Normals at θ, ϕ are $\begin{cases} ax\cos\theta+by\cot\theta=a^2+b^2 \\ ax\cos\theta+by\cot\phi=a^2+b^2 \end{cases}$

where $\phi=\frac{\pi}{2}-\theta$ and these pass through (h, k)

$\therefore \quad ah\cos\theta+bk\cot\theta=a^2+b^2$

$ah\sin\theta+bk\tan\theta=a^2+b^2$

Eliminating h, $bk(\cot\theta\ \sin\theta-\tan\cos\theta)$

$=(a^2+b^2)(\sin\theta-\cos\theta)$ or $k=-(a^2+b^2)/b$

8. **(b)** Chord $x=9$ meets $x^2-y^2=9$ at $(9,6\sqrt{2})$ and $(9,-6\sqrt{2})$ at which tangents are

$9x-6\sqrt{2}y=9$ and $9x+6\sqrt{2}y=9$

or $\quad 3x-2\sqrt{2}y-3=0$ and $3x+2\sqrt{2}y-3=0$

$\therefore$ Combined equation of tangents is

$(3x-2\sqrt{2}y-3)(3x+2\sqrt{2}y-3)=0$

or $\quad 9x^2-8y^2-18x+9=0$

9. **(c)** **KEY CONCEPT**

The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a parabola if $\Delta\neq 0$ and $h^2=ab$ where $\Delta=abc+2fgh-af^2-bg^2-ch^2$

Now we have $x=t^2+t+1$ and $y=t^2-t+1$

$\frac{x+y}{2}=t^2+1, \frac{x-y}{2}=t$ (Adding and subtracting values of x and y)

Eliminating t, $2(x+y)=(x-y)^2+4$ .... (1)

$\Rightarrow\ x^2-2xy+y^2-2x-2y+4=0$ .... (2)

Here, $a=1, h=-1, b=1, g=-1, f=-1, c=4$

$\therefore\ \Delta\neq 0.$ and $h^2=ab$

Hence the given curve represents a parabola.

10. **(b)** $y=mx+c$ is normal to the parabola $y^2=4ax$ if $c=-2am-am^3$

Here $m=-1$, $c=k$ and $a=3$

$\therefore\quad c=k=-2(3)(-1)-3(-1)^3=9$

11. **(c)** **KEY CONCEPT :** The directrix of the parabola $y^2=4a(x-x_1)$ is given by $x=x_1-a$.

$y^2=kx-8 \Rightarrow y^2=k\left(x-\frac{8}{k}\right)$

Directrix of parabola is $x=\frac{8}{k}-\frac{k}{4}$;

Now, $x=1$ also coincides with $x=\frac{8}{k}-\frac{k}{4}$

On comparision, $\frac{8}{k}-\frac{k}{4}=1$, or $k^2-4k-32=0$

On solving we get $k=4$

12. **(c)** Let the equation of tangent to $y^2=4x$ be $y=mx+\frac{1}{m}$ where m is the slope of the tangent.

If it is tangent to the circle $(x-3)^2+y^2=9$ then length of perpendicular to tangent from centre (3, 0) should be equal to the radius 3.

$\therefore \quad \frac{3m+\frac{1}{m}}{\sqrt{m^2+1}}=3$

$\Rightarrow \quad 9m^2+\frac{1}{m^2}+6=9m^2+9 \Rightarrow m=\pm\frac{1}{\sqrt{3}}$

$\therefore$ Tangents are $x-y\sqrt{3}+3=0$ and $x+y\sqrt{3}+3=0$ out of which $x-y\sqrt{3}+3=0$ meets the parabola at $(3,2\sqrt{3})$ i.e., above x-axis.

13. **(d)** $y^2+4y+4x+2=0$

$y^2+4y+4=-4x+2$

$(y+2)^2=-4(x-1/2)$

It is of the form $Y^2=-4AX$

whose directrix is given by $X=A$

$\therefore$ Req. equation is $x-1/2=1 \Rightarrow x=3/2$.

14. **(a)** Given that $a>2b>0$ and $m>0$

Also $\quad y=mx-b\sqrt{1+m^2}$ ...(1)

is tangent to $\quad x^2+y^2=b^2$ ...(2)

as well as to $\quad (x-a)^2+y^2=b^2$ ...(3)

$\because$ (1) is tangent to (3)

$\therefore \quad \left|\frac{am-b\sqrt{1+m^2}}{\sqrt{m^2+1}}\right|=b$

[length of perpendicular from $(a, 0)$ to (1) = radius b]

$\Rightarrow \quad am-b\sqrt{1+m^2}=\pm b\sqrt{1+m^2} \Rightarrow am-2b\sqrt{1+m^2}=0$

or $\quad am=0$ (not possible as $a, m>0$)

$\Rightarrow \quad a^2m^2=4b^2(1+m^2) \Rightarrow m^2=\frac{4b^2}{a^2-4b^2}$

$\Rightarrow \quad m=\frac{2b}{\sqrt{a^2-4b^2}} \qquad (\because m>0)$

15. **(c)** If (h, k) is the mid point of line joining focus $(a, 0)$ and $Q(at^2, 2at)$ on parabola then $h=\frac{a+at^2}{2}, k=at$

Eliminating t, we get $2h=a+a\left(\frac{k^2}{a^2}\right)$

$\Rightarrow \quad k^2=a(2h-a) \quad \Rightarrow k^2=2a(h-a/2)$

$\therefore$ Locus of (h, k) is $y^2=2a(x-a/2)$

whose directrix is $(x-a/2)=-\frac{a}{2}$

$\Rightarrow \quad x=0$

16. **(d)** The given curves are

$y^2 = 8x$ ...(1)

and $xy = -1$ ...(2)

If m is the slope of tangent to (1), then eq^n of tangent is

$y = mx + 2/m$

If this tangent is also a tangent to (2), then putting value of y in curve (2)

$$x\left(mx + \frac{2}{m}\right) = -1$$

$$\Rightarrow mx^2 + \frac{2}{m}x + 1 = 0 \Rightarrow m^2x^2 + 2x + m = 0$$

We should get repeated roots for the eq^n (condition of tangency)

$\Rightarrow D = 0$

$\therefore (2)^2 - 4m^2.m = 0$

$\Rightarrow m^3 = 1 \quad \Rightarrow m = 1$

Hence required tangent is $y = x + 2$

17. **(d)** The given ellipse is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

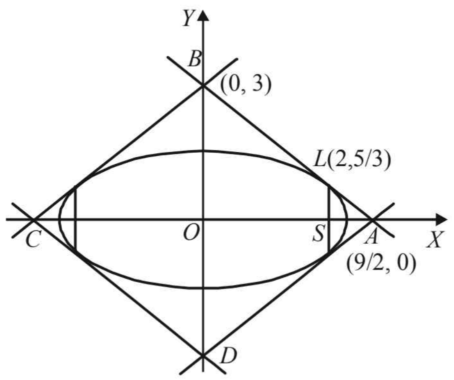
Then $a^2 = 9, b^2 = 5 \Rightarrow e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$

$\therefore$ end point of latus rectum in first quadrant is $L(2, 5/3)$

Equation of tangent at L is $\frac{2x}{9} + \frac{y}{3} = 1$

It meets x-axis at A (9/2, 0) and y-axis at B (0, 3)

$\therefore$ Area of ΔOAB $= \frac{1}{2} \times \frac{9}{2} \times 3 = \frac{27}{4}$



By symmetry area of quadrilateral

$= 4 \times (\text{Area } \Delta OAB) = 4 \times \frac{27}{4} = 27$ sq. units.

18. **(a)** For parabola $y^2 = 16x$, focus $\equiv$ (4, 0). Let m be the slope of focal chord then eq^n is

$y = m(x - 4)$ ...(1)

But given that above is a tangent to the circle

$(x - 6)^2 + y^2 = 2$

With Centre, $C(6, 0)$, r $= \sqrt{2}$

$\therefore$ Length of $\perp^{lar}$ from (6, 0) to (1) = r

$$\Rightarrow \frac{6m - 4m}{\sqrt{m^2 + 1}} = \sqrt{2} \Rightarrow 2m = \sqrt{2(m^2 + 1)}$$

$\Rightarrow 2m^2 = m^2 + 1 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

19. **(b)** The given eq^n of hyperbola is

$$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

$\Rightarrow a = \cos\alpha, b = \sin\alpha$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \tan^2\alpha} = \sec\alpha$$

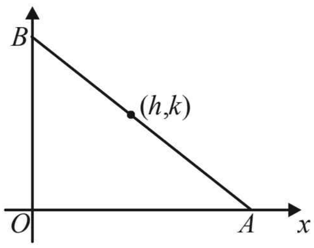
$\Rightarrow ae = 1$

$\therefore$ foci $(\pm 1, 0)$

$\therefore$ foci remain constant with respect to α.

20. **(a)** Any tangent to ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ is

$$\frac{x\cos\theta}{\sqrt{2}} + y\sin\theta = 1$$



$\therefore A(\sqrt{2}\sec\theta, 0)$; B (0, cosec θ)

$\Rightarrow 2h = \sqrt{2}\sec\theta$ and $2k = \text{cosec}\,\theta$

(Using mid pt. formula)

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}h} \text{ and } \sin\theta = \frac{1}{2k}$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}h}\right)^2 + \left(\frac{1}{2k}\right)^2 = 1 \Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$$

Required locus,

$$\frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

21. **(c)** $y = mx + 1/m$

Above tangent passes through (1, 4)

$\Rightarrow 4 = m + 1/m \Rightarrow m^2 - 4m + 1 = 0$

Now angle between the lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1m_2}}{1 + m_1m_2}$$

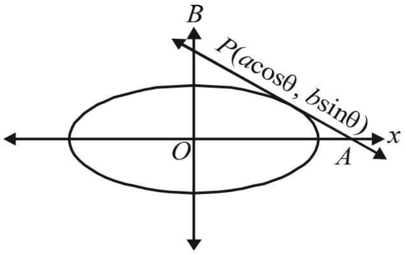
$$= \frac{\sqrt{16 - 4}}{1 + 1} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$$

22. **(d)** Equation of tangent to hyperbola $x^2 - 2y^2 = 4$ at any point (x_1, y_1) is $xx_1 - 2yy_1 = 4$

Comparing with $2x + \sqrt{6}y = 2$ or $4x + 2\sqrt{6}y = 4$

$\Rightarrow x_1 = 4$ and $-2y_1 = 2\sqrt{6} \Rightarrow (4, -\sqrt{6})$ is the required point.

23. (a) Any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(a\cos\theta, b\sin\theta)$ is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$



It meets co-ordinate axes at $A(a\sec\theta, 0)$ and $B(0, b\,\text{cosec}\,\theta)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\times a\sec\theta\times b\,\text{cosec}\,\theta$

$\Rightarrow \Delta = \frac{ab}{\sin 2\theta}$

For Δ to be min, $\sin 2\theta$ should be max. and we know max value of $\sin 2\theta = 1$

$\therefore \Delta_{max} = ab$ sq. units.

24. (d) The given curve is $y = x^2+6$

Equation of tangent at (1, 7) is

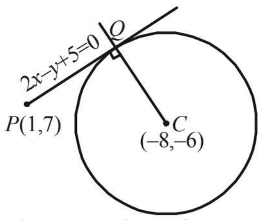
$\frac{1}{2}(y+7) = x.1+6$

$\Rightarrow 2x-y+5=0$ ...(1)

As given this tangent (1) touches the circle

$x^2+y^2+16x+12y+c=0$ at Q

Centre of circle = (– 8, – 6).



Then equation of CQ which is perpendicular to (1) and passes through (– 8, – 6) is $y+6 = -\frac{1}{2}(x+8)$

$\Rightarrow x+2y+20=0$ ...(2)

Now Q is pt. of intersection of (1) and (2)

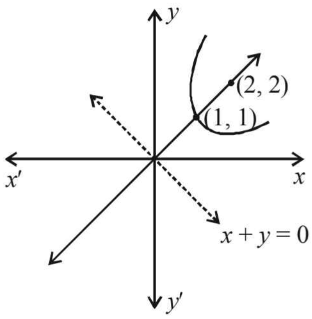
$\therefore$ Solving eq^n (1) & (2) we get

$x=-6, y=-7$

$\therefore$ Req. pt. is (– 6, – 7).

25. (d) Since, distance of vertex from origin is $\sqrt{2}$ and focus is $2\sqrt{2}$

$\therefore$ Vertex is (1, 1) and focus is (2, 2), directrix $x+y=0$



$\therefore$ Equation of parabola is

$$(x-2)^2+(y-2)^2 = \left(\frac{x+y}{\sqrt{2}}\right)^2$$

$\Rightarrow 2(x^2-4x+4)+2(y^2-4y+4) = x^2+y^2+2xy$

$\Rightarrow x^2+y^2-2xy = 8(x+y-2)$

$\Rightarrow (x-y)^2 = 8(x+y-2)$

26. (a) The length of transverse axis $= 2\sin\theta = 2a$

$\Rightarrow a = \sin\theta$

Also for ellipse $3x^2+4y^2=12$

or $\frac{x^2}{4}+\frac{y^2}{3}=1$, $a^2=4, b^2=3$

$$e=\sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{3}{4}} = \frac{1}{2}$$

$\therefore$ Focus of ellipse $= \left(2\times\frac{1}{2}, 0\right) \Rightarrow (1, 0)$

As hyperbola is confocal with ellipse, focus of hyperbola = (1, 0) $\Rightarrow ae = 1 \Rightarrow \sin\theta \times e = 1$

$\Rightarrow e = \text{cosec}\,\theta$

$\therefore b^2 = a^2(e^2-1) = \sin^2\theta(\text{cosec}^2\theta - 1) = \cos^2\theta$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1$$

or, $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$

27. (b) $x^2-5xy+6y^2=0$ represents a pair of straight lines given by $x-3y=0$ and $x-2y=0$.

Also $ax^2+by^2+c=0$ will represent a circle if $a=b$ and c is of sign opposite to that of a.

28. (b) The given hyperbola is

$x^2-2y^2-2\sqrt{2}x-4\sqrt{2}y-6=0$

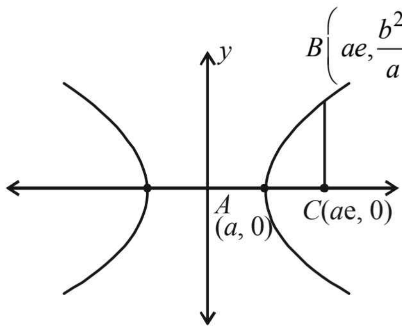
$\Rightarrow (x^2-2\sqrt{2}x+2)-2(y^2+2\sqrt{2}y+2) = 6+2-4$

$\Rightarrow (x-\sqrt{2})^2-2(y+\sqrt{2})^2 = 4$

$$\Rightarrow \frac{(x-\sqrt{2})^2}{2^2}-\frac{(y+\sqrt{2})^2}{(\sqrt{2})^2}=1$$

$\therefore a=2, b=\sqrt{2} \Rightarrow e=\sqrt{1+\frac{2}{4}}=\sqrt{\frac{3}{2}}$

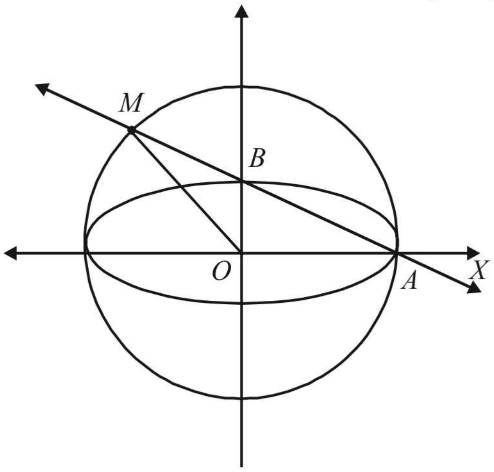
Clearly ΔABC is a right triangle.



$\therefore$ Ar (ΔABC) $= \frac{1}{2} \times AC \times BC = \frac{1}{2}(ae - a) \times \frac{b^2}{a}$

$= \frac{1}{2}(e-1) \times b^2 = \frac{1}{2}\left(\sqrt{\frac{3}{2}} - 1\right) \times 2 = \sqrt{\frac{3}{2}} - 1$

29. **(d)** The given ellipse is $x^2 + 9y^2 = 9$ or $\frac{x^2}{3^2} + \frac{y^2}{1^2} = 1$



So, that $A(3,0)$ and $B(0,1)$

$\therefore$ Equation of AB is $\frac{x}{3} + \frac{y}{1} = 1$

or $x + 3y - 3 = 0$ (1)

Also auxillary circle of given ellipse is

$x^2 + y^2 = 9$ (2)

Solving equation (1) and (2), we get the point M where line AB meets the auxillary circle.

Putting $x = 3 - 3y$ from eq^n(1) in eq^n(2)

we get $(3-3y)^2 + y^2 = 9$

$\Rightarrow 9 - 18y + 9y^2 + y^2 = 9 \Rightarrow 10y^2 - 18y = 0$

$\Rightarrow y = 0, \frac{9}{5} \Rightarrow x = 3, \frac{-12}{5}$

Clearly M $\left(\frac{-12}{5}, \frac{9}{5}\right)$

$\therefore$ Area of $\Delta OAM = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ \frac{-12}{5} & \frac{9}{5} & 1 \end{vmatrix} = \frac{27}{10}$

30. **(c)** The given ellipse is $\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1$

such that $a^2 = 16$ and $b^2 = 4$

$\therefore$ $e^2 = 1 - \frac{4}{16} = \frac{3}{4} \Rightarrow e = \frac{\sqrt{3}}{2}$

Let $P(4\cos\theta, 2\sin\theta)$ be any point on the ellipse, then equation of normal at P is

$4x\sin\theta - 2y\cos\theta = 12\sin\theta\cos\theta$

$\Rightarrow \frac{x}{3\cos\theta} - \frac{y}{6\sin\theta} = 1$

$\therefore$ Q, the point where normal at P meets x–axis, has coordinates $(3\cos\theta, 0)$

$\therefore$ Mid point of PQ is $M\left(\frac{7\cos\theta}{2}, \sin\theta\right)$

For locus of point M we consider

$x = \frac{7\cos\theta}{2}$ and $y = \sin\theta$

$\Rightarrow \cos\theta = \frac{2x}{7}$ and $\sin\theta = y$

$\Rightarrow \frac{4x^2}{49} + y^2 = 1$ ...(1)

Also the latus rectum of given ellipse is

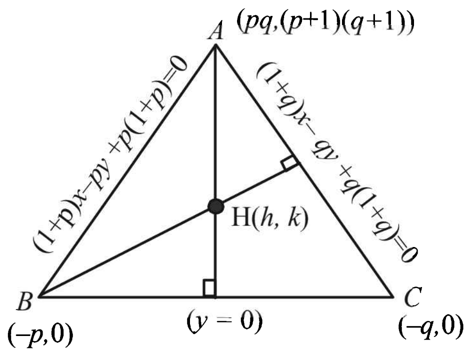
$x = \pm ae = \pm 4 \times \frac{\sqrt{3}}{2} = \pm 2\sqrt{3}$ or $x = \pm 2\sqrt{3}$ ...(2)

Solving equations (1) and (2), we get

$\frac{4 \times 12}{49} + y^2 = 1 \Rightarrow y^2 = \frac{1}{49}$ or $y = \pm\frac{1}{7}$

$\therefore$ The required points are $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$.

31. **(d)** The triangle is formed by the lines



$AB : (1+p)x - py + p(1+p) = 0$

$AC : (1+q)x - qy + q(1+q) = 0$

$BC : y = 0$

So that the vertices are

$A(pq, (p+1)(q+1)),\ B(-p, 0), C(-q, 0)$

Let $H(h,k)$ be the orthocentre of ΔABC. Then as $AH \perp BC$ and passes through $A(pq, (p+1)(q+1))$

The eq^n of AH is $x = pq$

$\therefore$ $h = pq$ ...(1)

Also BH is perpendicular to AC

$\therefore$ $m_1 m_2 = -1 \Rightarrow \frac{k-0}{h+p} \times \frac{1+q}{q} = -1$

$\Rightarrow \quad \dfrac{k}{pq+p} \times \dfrac{1+q}{q} = -1$ (using eq^n (1))

$\Rightarrow \quad k = -pq$ ...(2)

From (1) and (2) we observe $h + k = 0$

$\therefore$ Locus of (*h*, *k*) is $x + y = 0$ which is a straight line.

32. (b) For hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, we have

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$$

$\therefore$ Slope of normal at P (6, 3)

$$= -\frac{1}{\left(\frac{dy}{dx}\right)_{(6,3)}} = -\frac{3a^2}{6b^2}$$

$\therefore$ Equation of normal is

$$\frac{y-3}{x-6} = -\frac{3a^2}{6b^2}$$

As it intersects x-axis at (9, 0)

$$\therefore \frac{0-3}{9-6} = \frac{-3a^2}{6b^2} \Rightarrow a^2 = 2b^2 \quad ...(1)$$

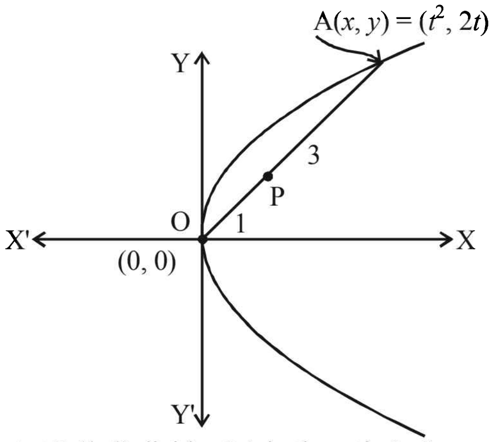
Also for hyperbola, $b^2 = a^2 (e^2 - 1)$

Using $a^2 = 2b^2$; we get

$b^2 = 2b^2 (e^2 - 1)$

$$\frac{1}{2} = e^2 - 1 \text{ or } e^2 = \frac{3}{2} \text{ or } e = \sqrt{\frac{3}{2}}$$

33. (c) Let A $(x, y) = (t^2, 2t)$ be any point on parabola $y^2 = 4x$.



Let P (*h, k*) divides OA in the ratio 1 : 3

Then $(h, k) = \left(\dfrac{t^2}{4}, \dfrac{2t}{4}\right)$

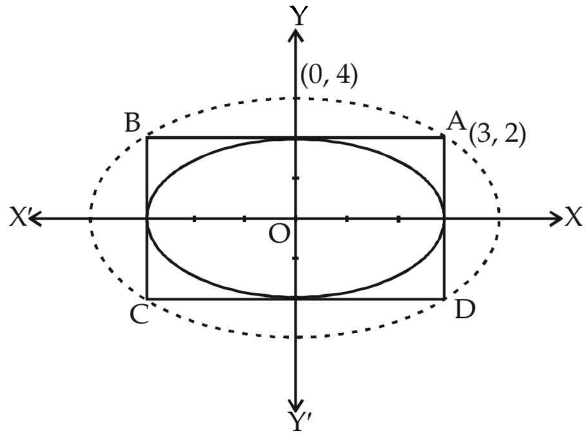
$$\Rightarrow h = \frac{t^2}{4} \text{ and } k = \frac{t}{2} \Rightarrow h = k^2$$

$\therefore$ locus of P (*h, k*) is $x = y^2$.

34. (c) As rectangle *ABCD* circumscribed the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$\therefore \quad A = (3, 2)$



Let the ellipse circumscribing the rectangle *ABCD* is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Given that it passes through (*a*, 4)

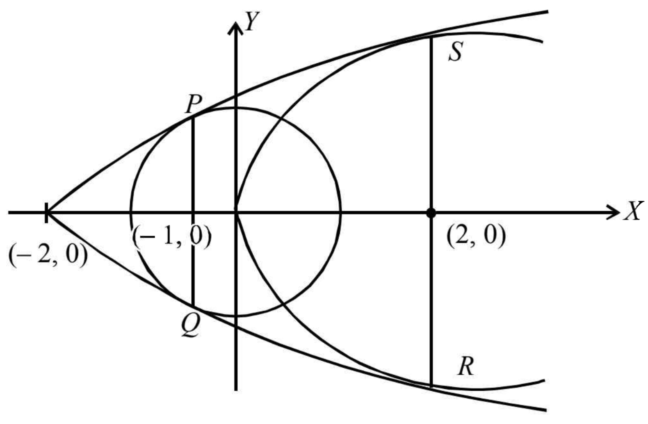
$\therefore \; b^2 = 16$

Also it passes through *A* (3, 2)

$$\therefore \quad \frac{9}{a^2} + \frac{4}{16} = 1 \Rightarrow a^2 = 12$$

$$\therefore \quad e = \sqrt{1 - \frac{12}{16}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

35. (d)



Let the tangent to $y^2 = 8x$ be $y = mx + \dfrac{2}{m}$

If it is common tangent to parabola and circle, then

$y = mx + \dfrac{2}{m}$ is a tangent to $x^2 + y^2 = 2$

$$\therefore \quad \left|\frac{\frac{2}{m}}{\sqrt{m^2+1}}\right| = \sqrt{2} \Rightarrow \frac{4}{m^2(1+m^2)} = 2$$

$\Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow (m^2 + 2)(m^2 - 1) = 0$

$\Rightarrow m = 1$ or -1

$\therefore$ Required tangents are $y = x + 2$ and $y = -x - 2$

Their common point is $(-2, 0)$

$\therefore$ Tangents are drawn from $(-2, 0)$

$\therefore$ Chord of contact PQ to circle is

$x.(-2) + y.0 = 2$ or $x = -1$

and Chord of contact RS to parabola is

$y.0 = 4(x-2)$ or $x = 2$

Hence coordinates of P and Q are $(-1, 1)$ and $(-1, -1)$

Also coordinates of R and S are $(2, -4)$ and $(2, 4)$

$\therefore$ Area of trapezium $PQRS$ is $\frac{1}{2}(2+8)\times 3 = 15$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(c)** The given curve is $\frac{x^2}{4}+\frac{y^2}{1}=1$ (an ellipse) and given line is $y = 4x + c$.

We know that $y = mx + c$ touches the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $c = \pm\sqrt{a^2m^2+b^2}$

Here $c = \pm\sqrt{4\times 16+1} = \pm\sqrt{65}$

$\therefore$ two values of c exist

2. **(c)** The ellipse can be written as, $\frac{x^2}{25}+\frac{y^2}{16}=1$

Here $a^2 = 25, b^2 = 16$, but $b^2 = a^2(1-e^2)$

$\Rightarrow$ $16/25 = 1 - e^2$

$\Rightarrow$ $e^2 = 1 - 16/25 = 9/25 \Rightarrow e = 3/5$

Foci of the ellipse are $(+ae, 0) = (\pm 3, 0)$, i.e., F_1 and F_2

$\therefore$ We have $PF_1 + PF_2 = 2a = 10$ for every point P on the ellipse.

3. **(b, d)** Let $y = \frac{8}{9}x + C$ be the tangent to $\frac{x^2}{1/4}+\frac{y^2}{1/9}=1$

where $C = \pm\sqrt{a^2m^2+b^2} = \pm\sqrt{\frac{1}{4}\times\frac{64}{81}+\frac{1}{9}} = \pm\frac{5}{9}$

and pts of contact are $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right) = \left(\frac{2}{5}, \frac{-1}{5}\right)$

or $\left(\frac{-2}{5}, \frac{1}{5}\right)$

4. **(a, b)** if $y = mx + c$ is tangent to $y = x^2$ then $x^2 - mx - c = 0$ has equal roots

$\Rightarrow$ $m^2 + 4c = 0 \Rightarrow c = -\frac{m^2}{4}$

$\therefore$ $y = mx - \frac{m^2}{4}$ is tangent to $y = x^2$

$\therefore$ This is also tangent to $y = -(x-2)^2$

$\Rightarrow$ $mx - \frac{m^2}{4} = -x^2 + 4x - 4$

$\Rightarrow$ $x^2 + (m-4)x + \left(4 - \frac{m^2}{4}\right) = 0$ has equal roots

$\Rightarrow$ $m^2 - 8m + 16 = -m^2 + 16 \Rightarrow m = 0, 4$

$\Rightarrow$ $y = 0$ or $y = 4x - 4$ are the tangents.

5. **(a,c)** For the given ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1 \Rightarrow e = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

$\Rightarrow$ Eccentricity of hyperbola $= \frac{5}{3}$

Let the hyperbola be $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$ then

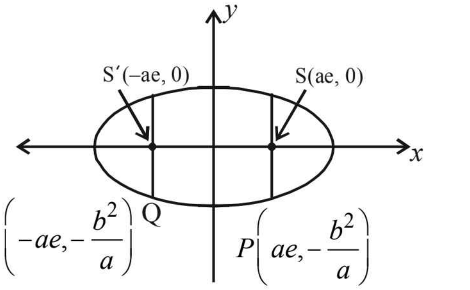
$B^2 = A^2\left(\frac{25}{9}-1\right) = \frac{16}{9}A^2$

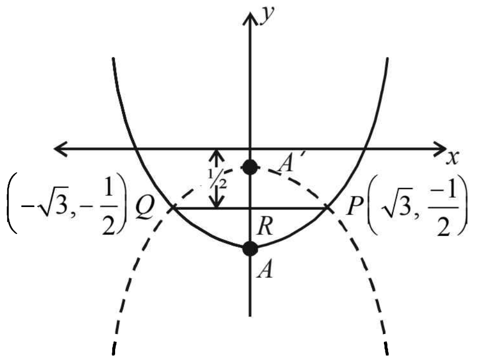
$\therefore$ $\frac{x^2}{A^2}-\frac{9y^2}{16A^2}=1$ As it passes through focus of ellipse i.e. $(3, 0)$

$\therefore$ we get $A^2 = 9 \Rightarrow B^2 = 16$

$\therefore$ Equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$, focus of hyperbola is $(5, 0)$, vertex of hyperbola is $(3, 0)$.

6. **(b,c)**





Given ellipse is $x^2 + 4y^2 = 4$

or $\frac{x^2}{2^2}+\frac{y^2}{1}=1 \Rightarrow a = 2, b = 1$

$\therefore$ $e = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$ $\therefore$ $ae = \sqrt{3}$

As per question $P \equiv (ae, -b^2/a) = \left(\sqrt{3}, -\frac{1}{2}\right)$

$Q \equiv (-ae, -b^2/a) = \left(-\sqrt{3}, -\frac{1}{2}\right)$

$\therefore$ $PQ = 2\sqrt{3}$

Now if PQ is the length of latus rectum to be found,

then $PQ = 4a = 2\sqrt{3} \Rightarrow a = \frac{\sqrt{3}}{2}$

Also as PQ is horizontal, parabola with PQ as latus rectum can be upward parabola (with vertex at A) or down ward parabola (with vertex at A').
For upward parabola,

$$AR = a = \frac{\sqrt{3}}{2} \therefore \text{ Coordinates of } A = \left(0, -\left(\frac{\sqrt{3}+1}{2}\right)\right)$$

So equation of upward parabola is given by

$$x^2 = 2\sqrt{3}\left(y + \frac{\sqrt{3}+1}{2}\right) \text{ or } x^2 - 2\sqrt{3}y = 3 + \sqrt{3} \quad ...(1)$$

For downward parabola $A'R = a = \frac{\sqrt{3}}{2}$

$\therefore$ Coordinates of $A' = \left(0, -\left(\frac{1-\sqrt{3}}{2}\right)\right)$

So equation of downward parabola is given by

$$x^2 = -2\sqrt{3}\left(y + \frac{1-\sqrt{3}}{2}\right) \text{ or } x^2 + 2\sqrt{3}y = 3 - \sqrt{3} \quad ...(2)$$

$\therefore$ the equation of required parabola is given by equation (1) or (2).

7. **(b,c)** In ΔABC, given that

$$\cos B + \cos C = 4\sin^2\frac{A}{2}$$

$$\Rightarrow 2\cos\frac{B+C}{2}\cos\frac{B-C}{2} - 4\sin^2\frac{A}{2} = 0$$

$$\Rightarrow 2\sin\frac{A}{2}\left[\cos\frac{B-C}{2} - 2\sin\frac{A}{2}\right] = 0$$

$$\Rightarrow \sin\frac{A}{2} = 0 \text{ or } \cos\frac{B-C}{2} - 2\cos\frac{B+C}{2} = 0$$

But in a triangle $\sin\frac{A}{2} \neq 0$

$$\therefore \cos\frac{B-C}{2} - 2\cos\frac{B+C}{2} = 0 \Rightarrow \frac{\cos\left(\frac{B+C}{2}\right)}{\cos\left(\frac{B-C}{2}\right)} = \frac{1}{2}$$

Applying componendo and dividendo, we get

$$\frac{\cos\left(\frac{B+C}{2}\right) + \cos\left(\frac{B-C}{2}\right)}{\cos\left(\frac{B+C}{2}\right) - \cos\left(\frac{B-C}{2}\right)} = \frac{1+2}{1-2} = -3$$

$$\Rightarrow \frac{2\cos\frac{B}{2}\cos\frac{C}{2}}{-2\sin\frac{B}{2}\sin\frac{C}{2}} = -3 \Rightarrow \tan\frac{B}{2}\tan\frac{C}{2} = \frac{1}{3}$$

$$\Rightarrow \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{1}{3}$$

$$\Rightarrow \frac{s-a}{s} = \frac{1}{3} \text{ or } 2s = 3a$$

$$\Rightarrow a + b + c = 3a \quad \text{or} \quad b + c = 2a$$

i.e. $AC + AB$ = constant

($\because$ Base $BC = a$ is given to be constant)

$\Rightarrow$ A moves on an ellipse.

8. **(a, d)** Let $P(at^2, 2at)$ be any point on the parabola

$$y^2 = 4ax.$$

Then tangent to parabola at P is $y = \frac{x}{t} + at$
which meets the axis of parabola *i.e* x-axis *at* $T(-at^2, 0)$.

Also normal to parabola at P is $tx + y = 2at + at^3$
which meets the axis of parabola at $N(2a + at^2, 0)$

Let $G(x, y)$ be the centriod of ΔPTN, then

$$x = \frac{at^2 - at^2 + 2a + at^2}{3} \text{ and } y = \frac{2at}{3}$$

$$\Rightarrow x = \frac{2a + at^2}{3} \text{ and } y = \frac{2at}{3}$$

Eliminating t from above, we get the locus of centriod G as

$$3x = 2a + a\left(\frac{3y}{2a}\right)^2 \Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2}{3}a\right)$$

which is a parabola with vertex $\left(\frac{2a}{3}, 0\right)$, directrix as

$x - \frac{2a}{3} = -\frac{a}{3}$ or $x = \frac{a}{3}$, latus rectum as $\frac{4a}{3}$ and focus as $(a, 0)$.

9. **(a, b)** The given hyperbola is

$$x^2 - y^2 = \frac{1}{2} \quad ...(1)$$

which is a rectangular hyperbola (i.e. $a = b$)

$\therefore \quad e = \sqrt{2}$.

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Its eccentricity $= \frac{1}{\sqrt{2}}$

$$\therefore \quad b^2 = a^2\left(1 - \frac{1}{2}\right) \Rightarrow b^2 = \frac{a^2}{2}$$

So, the equation of ellipse becomes

$$x^2 + 2y^2 = a^2 \quad ...(2)$$

Let the hyperbola (1) and ellipse (2) intersect each other at $P(x_1, y_1)$.

Then slope of hyperbola (1) at P is given by

$$m_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{x_1}{y_1}$$

and that of ellipse (2) at P is

$$m_2 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{x_1}{2y_1}$$

As the two curves intersect orthogonally,

$\therefore \quad m_1 m_2 = -1$

$\Rightarrow \quad \frac{x_1}{y_1}.\left(-\frac{x_1}{2y_1}\right) = -1 \Rightarrow \quad x_1^2 = 2y_1^2 \qquad$ ...(i)

Also $P(x_1, y_1)$ lies on $x^2 - y^2 = \frac{1}{2}$

$\therefore \quad x_1^2 - y_1^2 = \frac{1}{2} \qquad$ ...(ii)

Solving (i) and (ii), we get $y_1^2 = \frac{1}{2}$ and $x_1^2 = 1$

Also $P(x_1, y_1)$ lies on ellipse $x^2 + 2y^2 = a^2$

$\therefore \quad x_1^2 + 2y_1^2 = a^2 \Rightarrow 1 + 1 = a^2$ or $a^2 = 2$

$\therefore$ The required ellipse is $x^2 + 2y^2 = 2$ whose foci are $(\pm ae, 0) = \left(\pm\sqrt{2}\times\frac{1}{\sqrt{2}}, 0\right) = (\pm 1, 0)$

10. (c,d) Given parabola $y^2 = 4x$

Let $A(t_1^2, 2t_1)$ and $B(t_2^2, 2t_2)$

Then centre of circle drawn with AB as diameter is

$$\left(\frac{t_1^2 + t_2^2}{2}, t_1 + t_2\right)$$

As circle touches x-axis

$\therefore r = |t_1 + t_2| \Rightarrow t_1 + t_2 = \pm r$

Also slope of AB $= \frac{2(t_2 - t_1)}{t_2^2 - t_1^2} = \frac{2}{t_2 + t_1} = \pm\frac{2}{r}$

11. (b, d) For $x^2 + 4y^2 = 4$ or $\frac{x^2}{4} + \frac{y^2}{1} = 1$

$$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

As per question, $\sqrt{1 + \frac{b^2}{a^2}} = \frac{2}{\sqrt{3}} \Rightarrow \frac{b}{a} = \frac{1}{\sqrt{3}}$

focus of ellipse is $(\pm\sqrt{3}, 0)$

As hyperbola passes through $(\pm\sqrt{3}, 0)$

$\therefore \frac{3}{a^2} = 1$ or $a = \sqrt{3}$

$\therefore b = 1$ and focus of hyperbola $(\pm 2, 0)$

$\therefore$ Equation of hyperbola is $\frac{x^2}{3} - \frac{y^2}{1} = 1$

or $x^2 - 3y^2 = 3$

12. (a, b, d)

The equation of normal to $y^2 = 4x$ is $y = mx - 2m - m^3$

As it passes through (9, 6)

$\therefore 6 = 9m - 2m - m^3$

$\Rightarrow m^3 - 7m + 6 = 0 \Rightarrow (m-1)(m^2 + m - 6) = 0$

$\Rightarrow (m-1)(m+3)(m-2) = 0 \Rightarrow m = 1, 2, -3$

$\therefore$ Normal is $y = x - 3$ or $y = 2x - 12$ or $y = -3x + 33$

$\therefore$ a, b, d are the correct option.

13. (a, b)

KEY CONCEPT : If slope of tangent is m, then equations of tangents to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $y = mx \pm \sqrt{a^2m^2 - b^2}$ with the points of contact

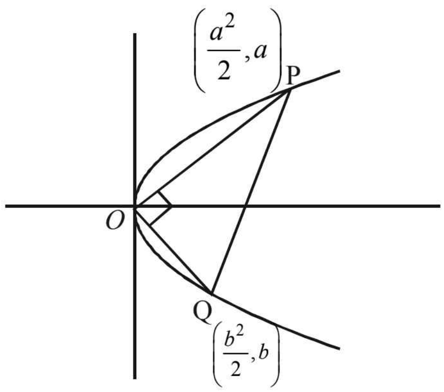
$$\left(\frac{\pm a^2 m}{\sqrt{a^2m^2 - b^2}}, \frac{\pm b^2}{\sqrt{a^2m^2 - b^2}}\right)$$

$\because$ Tangent to hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is parallel to $2x - y = 1$, therefore slope of tangent = 2

$\therefore$ Points of contact are $\left(\frac{\pm 9\times 2}{\sqrt{9\times 4 - 4}}, \frac{\pm 4}{\sqrt{9\times 4 - 4}}\right)$

i.e. $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

14. (a, d) Let point P in first quadrant, lying on parabola $y^2 = 2x$ be $\left(\frac{a^2}{2}, a\right)$. Let Q be the point $\left(\frac{b^2}{2}, b\right)$. Clearly $a > 0$.



$\because \quad PQ$ is the diameter of circle through P, O, Q

$\therefore \quad \angle POQ = 90° \Rightarrow \frac{a}{a^2/2} \times \frac{b}{b^2/2} = -1 \Rightarrow ab = -4$

$\Rightarrow \quad b$ is negative.

Also ar. $\Delta POQ = 3\sqrt{2}$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{a^2}{2} & a & 1 \\ \frac{b^2}{2} & b & 1 \end{vmatrix} = 3\sqrt{2}$$

$\Rightarrow \frac{1}{4}ab(a-b) = \pm 3\sqrt{2}$

$\Rightarrow a-b = \pm 3\sqrt{2}$ (using $ab=-4$)

As a is positive and b is negative, we have $a-b = 3\sqrt{2}$

$a + \frac{4}{a} = 3\sqrt{2}$ (using $ab = -4$)

$\Rightarrow a^2 - 3\sqrt{2}\,a + 4 = 0 \Rightarrow a^2 - 2\sqrt{2}\,a - \sqrt{2}\,a + 4 = 0$

$\Rightarrow (a-2\sqrt{2})(a-\sqrt{2}) = 0 \Rightarrow a = 2\sqrt{2}, \sqrt{2}$

$\therefore$ Point P can be $\left(\frac{(2\sqrt{2})^2}{2}, 2\sqrt{2}\right)$ or $\left(\frac{(\sqrt{2})^2}{2}, \sqrt{2}\right)$

i.e. $(4, 2\sqrt{2})$ or $(1, \sqrt{2})$

15. (a, b)

Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$

and $E_2 : \frac{x^2}{c^2} + \frac{y^2}{d^2} = 1$ where $c < d$

Also $S : x^2 + (y-1)^2 = 2$

Tangent at $P(x_1, y_1)$ to S is $x + y = 3$

To find point of contact put $x = 3 - y$ in S. We get P(1, 2)

Writing eq^n of tangent in parametric form

$$\frac{x-1}{\frac{-1}{\sqrt{2}}} = \frac{y-2}{\frac{1}{\sqrt{2}}} = \pm\frac{2\sqrt{2}}{3}$$

$x = \frac{-2}{3} + 1$ or $\frac{2}{3} + 1$ and $y = \frac{2}{3} + 2$ or $\frac{-2}{3} + 2$

$\Rightarrow x = \frac{1}{3}$ or $\frac{5}{3}$ and $y = \frac{8}{3}$ or $\frac{4}{3}$

$\therefore Q\left(\frac{5}{3}, \frac{4}{3}\right)$ and $R\left(\frac{1}{3}, \frac{8}{3}\right)$

eq^n of tangent to E_1 at Q is

$\frac{5x}{3a^2} + \frac{4y}{3b^2} = 1$ which is identical to $\frac{x}{3} + \frac{y}{3} = 1$

$\Rightarrow a^2 = 5$ and $b^2 = 4 \Rightarrow e_1^2 = 1 - \frac{4}{5} = \frac{1}{5}$

eq^n of tangent to E_2 at R is

$\frac{x}{3c^2} + \frac{8y}{3d^2} = 1$ identical to $\frac{x}{3} + \frac{y}{3} = 1$

$\Rightarrow c^2 = 1, d^2 = 8 \Rightarrow e_2^2 = 1 - \frac{1}{8} = \frac{7}{8}$

$\therefore e_1^2 + e_2^2 = \frac{43}{40}, e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}}, \left|e_1^2 - e_2^2\right| = \frac{27}{40}$

16. (a, b, d) $H : x^2 - y^2 = 1$ S : Circle with centre $N(x_2, 0)$

Common tangent to H and S at $P(x_1, y_1)$ is

$xx_1 - yy_1 = 1 \Rightarrow m_1 = \frac{x_1}{y_1}$

Also radius of circle S with centre $N(x_2, 0)$ through point of contact (x_1, y_1) is perpendicular to tangent

$\therefore m_1m_2 = -1 \Rightarrow \frac{x_1}{y_1} \times \frac{0-y_1}{x_2-x_1} = -1$

$\Rightarrow x_1 = x_2 - x_1$ or $x_2 = 2x_1$

M is the point of intersection of tangent at P and x-axis

$\therefore M\left(\frac{1}{x_1}, 0\right)$

$\because$ Centroid of ΔPMN is (ℓ, m)

$\therefore x_1 + \frac{1}{x_1} + x_2 = 3\ell$ and $y_1 = 3m$

Using $x_2 = 2x_1$,

$\Rightarrow \frac{1}{3}\left(3x_1 + \frac{1}{x_1}\right) = l$ and $\frac{y_1}{3} = m$

$\therefore \frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}, \frac{dm}{dy_1} = \frac{1}{3}$

Also (x_1, y_1) lies on H, $\therefore x_1^2 - y_1^2 = 1$ or $y_1 = \sqrt{x_1^2 - 1}$

$\therefore m = \frac{1}{3}\sqrt{x_1^2 - 1} \quad \therefore \frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$

17. (a, b, c) $C_1 : x^2 + y^2 = 3$ ...(i)

parabola : $x^2 = 2y$ ...(ii)

Intersection point of (i) and (ii) in first quadrant

$y^2 + 2y - 3 = 0 \Rightarrow y = 1$ ($\because y \neq -3$)

$\therefore x = \sqrt{2}$

$P(\sqrt{2}, 1)$

Equation of tangent to circle C_1 at P is $\sqrt{2}x + y - 3 = 0$

Let centre of circle C_2 be $(0, k)$; $r = 2\sqrt{3}$

$\therefore \left|\frac{k-3}{\sqrt{3}}\right| = 2\sqrt{3} \Rightarrow k = 9$ or -3

$\therefore Q_2(0, 9), Q_3(0, -3)$

(a) $Q_2Q_3 = 12$

(b) R_2R_3 = length of transverse common tangent

$= \sqrt{(Q_2Q_3)^2 - (r_1 + r_2)^2}$

$= \sqrt{(12)^2 - (4\sqrt{3})^2} = 4\sqrt{6}$

(c) Area $(\Delta OR_2R_3) = \frac{1}{2} \times R_2R_3 \times$ length of $\perp$ from origin to tangent

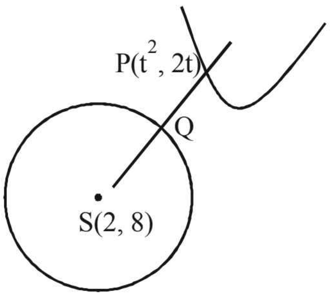
$= \frac{1}{2} \times 4\sqrt{6} \times \sqrt{3} = 6\sqrt{2}$

(d) Area $(\Delta PQ_2Q_3) = \frac{1}{2} \times Q_2Q_3 \perp$ distance of P from y-axis $= \frac{1}{2} \times 12 \times \sqrt{2} = 6\sqrt{2}$

18. (a, c, d)

Let point P on parabola $y^2 = 4x$ be $(t^2, 2t)$

$\because$ PS is shortest distance, therefore PS should be the normal to parabola.



Equation of normal to $y^2 = 4x$ at P $(t^2, 2t)$ is

$y - 2t = -t(x - t^2)$

It passes through S(2, 8)

$\therefore\quad 8 - 2t = -t(2 - t^2) \Rightarrow t^3 = 8$ or $t = 2$

$\therefore\quad P(4, 4)$

Also slope of tangent to circle at $Q = \dfrac{-1}{\text{Slope of PS}} = \dfrac{1}{2}$

Equation of normal at $t = 2$ is $2x + y = 12$

Clearly x-intercept = 6

$SP = 2\sqrt{5}$ and $SQ = r = 2$

$\therefore\quad$ Q divides SP in the ratio SP : PQ

$= 2 : 2(\sqrt{5} - 1)$ or $(\sqrt{5} + 1) : 4$

Hence a, c , d are the correct options.

E. Subjective Problems

1. The equation of a normal to the parabola $y^2 = 4ax$ in its slope form is given by

$y = mx - 2am - am^3$

$\therefore\quad$ Eq. of normal to $y^2 = 4x$, is

$y = mx - 2m - m^3$ ...(1)

Since the normal drawn at three different points on the parabola pass through (h, k), it must satisfy the equation (1)

$\therefore\quad k = mh - 2m - m^3$

$\Rightarrow\quad m^3 - (h - 2)m + k = 0$

This cubic eq. in m has three different roots say m_1, m_2, m_3

$\therefore\quad m_1 + m_2 + m_3 = 0$ ...(2)

$m_1m_2 + m_2m_3 + m_3m_1 = -(h - 2)$ ...(3)

Now, $(m_1 + m_2 + m_3)^2 = 0$ [Squaring (2)]

$\Rightarrow\quad m_1^2 + m_2^2 + m_3^2 = -2(m_1m_2 + m_2m_3 + m_3m_1)$

$\Rightarrow\quad m_1^2 + m_2^2 + m_3^2 = 2(h - 2)$ [Using (3)]

Since LHS of this equation is he sum of perfect squares, therefore it is + ve

$\therefore\quad h - 2 > 0 \Rightarrow h > 2$ Proved

2. Parabola $y^2 = 4ax$.

Let at any pt A equation of normal is

$y = mx - 2am - am^3$. ...(1)

Combined equation of OA and OB can be obtained by making equation of parabola homogeneous with the help of normal.

$\therefore\quad$ Combined eq. of OA and OB is

$$y^2 = 4ax\left(\frac{mx - y}{2am + am^3}\right)$$

[From eqn. (1) using $\dfrac{mx - y}{2am + am^3} = 1$]

$$y^2 = \frac{4x(mx - y)}{2m + m^3}$$

$\Rightarrow\quad 4mx^2 - 4xy - (2m + m^3)y^2 = 0$

But angle between the lines represented by this pair is 90°.

$\Rightarrow\quad$ coeff. of x^2 + coeff of $y^2 = 0 \Rightarrow 4m - 2m - m^3 = 0$

$\Rightarrow\quad m^3 - 2m = 0 \Rightarrow m = 0, \sqrt{2}, -\sqrt{2}$

But for $m = 0$ eq. of normal becomes $y = 0$ which does not intersect the parabola at any other point.

$\therefore\quad m = \pm\sqrt{2}$

3. Given parabola is $y^2 = x$.

Normal is $y = mx - \dfrac{m}{2} - \dfrac{m^3}{4}$

As per question this normal passes through $(c, 0)$ therefore, we get

$$mc - \frac{m}{2} - \frac{m^3}{4} = 0 \quad ...(1)$$

$\Rightarrow\quad m\left[c - \dfrac{1}{2} - \dfrac{m^2}{4}\right] = 0 \Rightarrow\quad m = 0$ or $m^2 = 4\left(c - \dfrac{1}{2}\right)$

$m = 0$ shows normal is $y = 0$ i.e. x-axis is always a normal.

Also $m^2 \geq 0 \Rightarrow 4\left(c - \dfrac{1}{2}\right) \geq 0 \Rightarrow c \geq 1/2$

At $c = \dfrac{1}{2}$, from (1) $m = 0$

$\therefore\quad$ for other real values of m, $c > 1/2$

Now for other two normals to be perpendicular to each other, we must have $m_1.m_2 = -1$

Or in other words, if m_1, m_2 are roots of $\dfrac{m^2}{4} + \dfrac{1}{2} - c = 0$, then product of roots = – 1

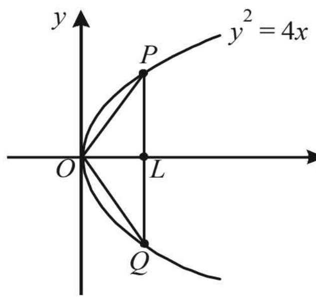
$\Rightarrow\quad \dfrac{\left(\frac{1}{2} - c\right)}{1/4} = -1 \Rightarrow \dfrac{1}{2} - c = -\dfrac{1}{4} \Rightarrow\quad c = 3/4$

4. Let the equation of chord OP be $y = mx$.

Then eq^n of chord OQ will be $y = -\dfrac{1}{m}x$ $[\because OQ \perp OP]$

P is pt. of intersection of $y = mx$ and $y^2 = 4x$.

Solving the two we get $P\left(\dfrac{4}{m^2}, \dfrac{4}{m}\right)$



Q is pt. of intersection of $y = -\dfrac{1}{m}x$ and $y^2 = 4x$.

Solving the two we get $Q(4m^2, -4m)$
Now eq. of PQ is,

$$y+4m=\frac{\frac{4}{m}+4m}{\frac{4}{m^2}-4m^2}(x-4m^2)$$

$\Rightarrow\ y+4m=\frac{m}{1-m^2}(x-4m^2)$

$\Rightarrow\ (1-m^2)y+4m-4m^3=mx-4m^3$

$\Rightarrow\ mx-(1-m^2)y-4m=0$

This line meets x-axis where $y=0$ i.e. $x=4$

$\Rightarrow\ OL=4$, which is constant as independent of m. Again let (h, k) be the mid pt. of PQ, then

$$h=\frac{4m^2+\frac{4}{m^2}}{2} \text{ and } k=\frac{\frac{4}{m}-4m}{2}$$

$\Rightarrow\ h=2\left(m^2+\frac{1}{m^2}\right)$ and $k=2\left(\frac{1}{m}-m\right)$

$\Rightarrow\ h=2\left[\left(\frac{1}{m}-m\right)+2\right]\ \Rightarrow\ h=2\left[\frac{k^2}{4}+2\right]$

$\Rightarrow\ 2h=k^2+8\ \Rightarrow\ k^2=2(h-4)$

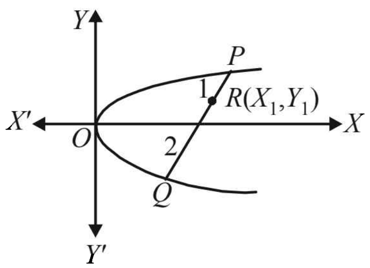
$\therefore$ Locus of (h, k) is $y^2=2(x-4)$

5. Let $P(t_1^2, 2t_1)$ and $Q(t_2^2, 2t_2)$ be the ends of the chord PQ of the parabola

$y^2=4x$ ...(1)

$\therefore$ Slope of chord $PQ=\frac{2t_2-2t_1}{t_2^2-t_1^2}=2$

$\Rightarrow\ t_2+t_1=1$ ...(2)



If $R(x_1, y_1)$ is a point dividing PQ internally in the ratio 1 : 2, then

$$x_1=\frac{1.t_2^2+2.t_1^2}{1+2},\ y_1=\frac{1.2t_2+2.2t_1}{1+2}$$

$\Rightarrow\ t_2^2+2t_1^2=3x_1$ ...(3)

and $t_2+2t_1=(3y_1)/2$ ...(4)

From (2) and (4), we get

$$t_1=\frac{3}{2}y_1-1, t_2=2-\frac{3}{2}y_1$$

Substituting in (3), we get

$$\left(2-\frac{3}{2}y_1\right)^2+2\left(\frac{3}{2}y_1-1\right)^2=3x_1$$

$\Rightarrow\ (9/4)y_1^2-4y_1=x_1-2$

$$\left(y_1-\frac{8}{9}\right)^2=\left(\frac{4}{9}\right)\left(x_1-\frac{2}{9}\right)$$

$\therefore$ Locus of the point $R(x_1, y_1)$ is
$(y-8/9)^2=(4/9)(x-2/9)$
which is a parabola having vertex at the point (2/9, 8/9).

6. Equation to the tangent at the point $P(a\cos\theta, b\sin\theta)$ on $x^2/a^2+y^2/b^2=1$ is

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ ...(1)

$\therefore$ d = perpendicular distance of (1) from the centre (0, 0) of the ellipse

$$=\frac{1}{\sqrt{\frac{1}{a^2}\cos^2\theta+\frac{1}{b^2}\sin^2\theta}}=\frac{(ab)}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$$

$\therefore\ 4a^2\left(1-\frac{b^2}{d^2}\right)=4a^2\left\{1-\frac{b^2\cos^2\theta+a^2\sin^2\theta}{a^2}\right\}$

$=4(a^2-b^2)\cos^2\theta=4a^2e^2\cos^2\theta$ ....(2)

The coordinates of focii F_1 and F_2 are
$F_1=(ae, 0)$ and $F_2=(-ae, 0)$

$\therefore\ PF_1=\sqrt{[(a\cos\theta-ae)^2+(b\sin\theta)^2]}$

$=\sqrt{[(a^2(\cos\theta-e)^2+(b\sin\theta)^2]}$

$=\sqrt{[(a^2(\cos\theta-e)^2+a^2(1-e^2)\sin^2\theta)]}$

[Using $b^2=a^2(1-e^2)$]

$=a\sqrt{[1+e^2(1-\sin^2\theta)-2e\cos\theta]}$

$=a(1-e\cos\theta)$

Similarly, $PF_2=a(1+e\cos\theta)$

$\therefore\ (PF_1-PF_2)^2=4a^2e^2\cos^2\theta$ ...(3)

Hence from (2) and (3), we have

$$(PF_1-PF_2)^2=4a^2\left(1-\frac{b^2}{a^2}\right)$$

7. Let the three points on the parabola $y^2=4ax$ be
$A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$ and $C(at_3^2, 2at_3)$.
Then using the fact that equation of tangent to $y^2=4ax$ at $(at^2, 2at)$ is $y=\frac{x}{t}+at$, we get equations of tangents at A, B and C as folllows

$y=\frac{x}{t_1}+at_1$ ...(1)

$y=\frac{x}{t_2}+at_2$ ...(2)

$y=\frac{x}{t_3}+at_3$ ...(3)

Solving the above equations pair wise we get the pts.

$P\,(at_1t_2, a\,(t_1+t_2))$
$Q\,(at_2t_3, a\,(t_2+t_3))$
$R\,(at_3t_1, a\,(t_3+t_1))$

$$\text{Now, area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & at_1^2 & 2at_1 \\ 1 & at_2^2 & 2at_2 \\ 1 & at_3^2 & 2at_3 \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{vmatrix}$$

$$= \left| a^2(t_1-t_2)(t_2-t_3)(t_3-t_1) \right| \quad \text{...(4)}$$

$$\text{Also area of } \Delta PQR = \frac{1}{2}\begin{vmatrix} 1 & at_1t_2 & a(t_1+t_2) \\ 1 & at_2t_3 & a(t_2+t_3) \\ 1 & at_3t_1 & a(t_3+t_1) \end{vmatrix}$$

$$= \frac{a^2}{2}\begin{vmatrix} 1 & t_1t_2 & t_1+t_2 \\ 1 & t_2t_3 & t_2+t_3 \\ 1 & t_3t_1 & t_3+t_1 \end{vmatrix}$$

$$= \frac{a^2}{2}\begin{vmatrix} 0 & (t_1-t_3)t_2 & t_1-t_3 \\ 0 & (t_2-t_1)t_3 & t_2-t_1 \\ 1 & t_3t_1 & t_3+t_1 \end{vmatrix}$$

$[R_1 \to R_1 - R_3 \text{ and } R_2 \to R_2 - R_3]$

Expanding along C_1,

$$= \left|\frac{a^2}{2}(t_1-t_3)(t_2-t_1)(t_2-t_3)\right|$$

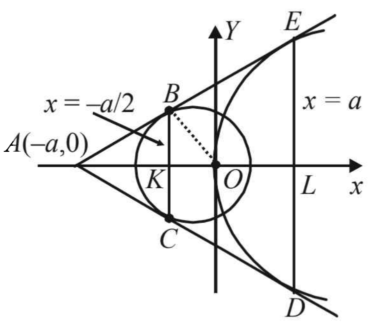
$$= \left|\frac{a^2}{2}(t_1-t_2)(t_2-t_3)(t_3-t_1)\right| \quad \text{...(5)}$$

From equations (4) and (5), we get

$$\frac{Ar(\Delta ABC)}{Ar(\Delta PQR)} = \frac{a^2\left|(t_1-t_2)(t_2-t_3)(t_3-t_1)\right|}{\frac{a^2}{2}\left|(t_1-t_2)(t_2-t_3)(t_3-t_1)\right|} = \frac{2}{1}$$

$\therefore$ The required ratio is 2 : 1

8.



This line will touch the circle $x^2 + y^2 = a^2/2$

if $\dfrac{a}{m} = \pm\dfrac{a}{\sqrt{2}}\sqrt{m^2+1}$ $\quad [c = \pm r\sqrt{1+m^2}]$

$\Rightarrow \dfrac{a^2}{m^2} = \dfrac{a^2}{2}(m^2+1)$

$\Rightarrow 2 = m^4 + m^2 \Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow (m^2+2)(m^2-1) = 0 \Rightarrow m = 1, -1$

Thus the two tangents (common one) are

$y = x + a$ and $y = -x - a$

These two intersect each other at $(-a, 0)$

The chord of contact at $A\,(-a, 0)$ for the circle $x^2 + y^2 = a^2/2$ is $(-a.x) + 0.y = a^2/2$ i.e., $x = -a/2$

and the chord of contact at $A\,(-a,0)$ for the parabola $y^2 = 4ax$ is

$0.y = 2a\,(x - a)$ i.e., $x = a$

Note that DE is latus recturn of parabola $y^2 = 4ax$, therefore its lengths is $4a$.

Chords of contact are clearly parallel to each other, so req. quadrilateral is a trapezium.

$$Ar\,(\text{trap } BCDE) = \frac{1}{2}(BC + DE) \times KL$$

$$= \frac{1}{2}(a + 4a)\left(\frac{3a}{2}\right) = \frac{15a^2}{4}$$

9. The given ellipses are

$$\frac{x^2}{4} + \frac{y^2}{1} = 1 \quad \text{...(1)}$$

and $$\frac{x^2}{6} + \frac{y^2}{3} = 1 \quad \text{...(2)}$$

Then the equation of tangent to (1) at any point T $(2\cos\theta, \sin\theta)$ is given by

$$\frac{x.2\cos\theta}{4} + \frac{y.\sin\theta}{1} = 1$$

or $$\frac{x\cos\theta}{2} + y\sin\theta = 1 \quad \text{...(3)}$$

Let this tangent meet the ellipse (2) at P and Q.

Let the tangents drawn to ellipse (2) at P and Q meet each other at $R\,(x_1, y_1)$

Then PQ is chord of contact of ellipse (2) with respect to the pt $R\,(x_1, y_1)$ and is given by

$$\frac{xx_1}{6} + \frac{yy_1}{3} = 1 \quad \text{...(4)}$$

Clearly equations (3) and (4) represent the same lines and hence should be identical. Therefore comparing the cofficients, we get

$$\frac{\frac{\cos\theta}{2}}{\frac{x_1}{6}} = \frac{\sin\theta}{\frac{y_1}{3}} = \frac{1}{1}$$

$\Rightarrow x_1 = 3\cos\theta, y_1 = 3\sin\theta \Rightarrow x_1^2 + y_1^2 = 9$

$\Rightarrow$ Locus of (x_1, y_1) is $x^2 + y^2 = 9$

which is the director circle of the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ and

Thus tangents at P and Q are at right $\angle$'s.

KEY CONCEPT : We know that the director circle is the locus of intersection point of the tangents which are at right $\angle$.

10. Let $P(e, f)$ be any point on the locus. Equation of pair of tangents from $P(e, f)$ to the parabola $y^2 = 4ax$ is

$$[fy-2a(x+e)]^2=(f^2-4ae)(y^2-4ax) \quad [T^2=SS_1]$$

Here, a = coefficient of $x^2 = 4a^2$ ...(1)

$2h$ = coefficient of $xy = -4af$ ...(2)

and b = coefficient of $y^2 = f^2-(f^2-4ae) = 4ae$ ...(3)

If they include an angle 45°, then

$$1=\tan 45° = \frac{2\sqrt{h^2-ab}}{a+b}$$

or, $(a+b)^2 = 4(h^2-ab)$

or, $(4a^2+4ae)^2 = 4[4a^2f^2-(4a^2)(4ae)]$

or, $(a+e)^2 = f^2-4ae$ or $e^2+6ae+a^2-f^2=0$

or $(e+3a)^2-f^2=8a^2$

Hence the required locus is $(x+3a)^2-y^2=8a^2$, which is a hyperbola.

11. Let any point P on ellipse $4x^2+25y^2=100$ be $(5\cos\theta, 2\sin\theta)$. So equation of tangent to the ellipse at P will be

$$\frac{x\cos\theta}{5}+\frac{y\sin\theta}{2}=1$$

Tangent (1) also touches the circle $x^2+y^2=r^2$, so its distance from origin must be r.

Tangent (2) intersects the coordinate axes at $A\left(\frac{5}{\cos\theta},0\right)$

and $B\left(0,\frac{2}{\sin\theta}\right)$ respectively. Let $M(h, k)$ be the midpoint of line segment AB. Then by mid point formula

$$h=\frac{5}{2\cos\theta},\ k=\frac{1}{\sin\theta} \Rightarrow \cos\theta=\frac{5}{2h},\ \sin\theta=\frac{1}{k}$$

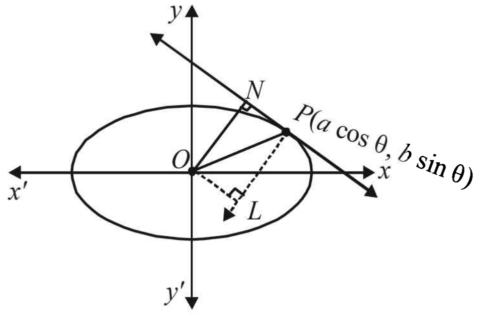
$$\Rightarrow \cos^2\theta+\sin^2\theta=\frac{25}{4h^2}+\frac{1}{k^2}$$

Hence locus of $M(h, k)$ is $\frac{25}{x^2}+\frac{4}{y^2}=4$

Locus is independent of r.

12. The ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(1)

Since this ellipse is symmetrical in all four quadrants, either there exists no such P or four points, one in each qudrant. Without loss of generality we can assume that $a > b$ and P lies in first quadrant.



Let $P(a\cos\theta, b\sin\theta)$ then equation of tangent is

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

$$\therefore \quad ON=\frac{ab}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$$

Equation of ON is, $\frac{x}{b}\sin\theta-\frac{y}{a}\cos\theta=0$

Equation of normal at P is

$ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2-b^2$

$$\therefore OL=\frac{a^2-b^2}{\sqrt{a^2\sec^2\theta+b^2\mathrm{cosec}^2\theta}}=\frac{(a^2-b^2)\sin\theta\cos\theta}{\sqrt{a^2\sin^2+b^2\cos^2\theta}}$$

and $NP = OL$

$$\therefore \quad NP=\frac{(a^2-b^2)\sin\theta\cos\theta}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}$$

$$\therefore \quad Z = \text{Area of } OPN = \frac{1}{2}\times ON\times NP$$

$$=\frac{1}{2}ab(a^2-b^2)\ \frac{\sin\theta\cos\theta}{a^2\sin^2\theta+b^2\cos^2\theta}$$

Let $u=\frac{a^2\sin^2\theta+b^2\cos^2\theta}{\sin\theta\cos\theta}=a^2\tan\theta+b^2\cot\theta$

$$\frac{du}{d\theta}=a^2\sec^2\theta-b^2\mathrm{cosec}^2\theta=0 \Rightarrow \tan\theta=b/a$$

$\left(\frac{d^2u}{d\theta^2}\right)_{\tan^{-1}b/a}>0$, u is minimum at $\theta=\tan^{-1}b/a$

So Z is maximum at $\theta=\tan^{-1}b/a$

$$\therefore \quad P \text{ is } \left(\frac{a^2}{\sqrt{a^2+b^2}},\frac{b^2}{\sqrt{a^2+b^2}}\right)$$

By symmetry, we have four such points

$$\left(\pm\frac{a^2}{\sqrt{a^2+b^2}},\pm\frac{b^2}{\sqrt{a^2+b^2}}\right)$$

13. Let A, B, C be the point on circle whose coordinates are $A = [a\cos\theta, a\sin\theta]$

$$B = \left[a\cos\left(\theta+\frac{2\pi}{3}\right), a\sin\left(\theta+\frac{2\pi}{3}\right)\right]$$

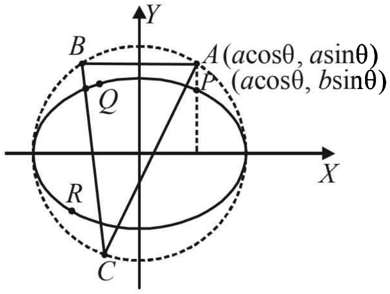
and $C = \left[a\cos\left(\theta+\frac{4\pi}{3}\right), a\sin\left(\theta+\frac{4\pi}{3}\right)\right]$

Further, $P[a\cos\theta, b\sin\theta]$ (Given)

$$Q = \left[a\cos\left(\theta+\frac{2\pi}{3}\right), b\sin\left(\theta+\frac{2\pi}{3}\right)\right]$$

and $R = \left[a\cos\left(\theta+\frac{4\pi}{3}\right), b\sin\left(\theta+\frac{4\pi}{3}\right)\right]$

It is given that P, Q, R are on the same side of x-axis as A, B, C.



So required normals to the ellipse are

$ax\sec\theta - by\,\text{cosec}\,\theta = a^2 - b^2$ ...(1)

$$ax\sec\left(\theta+\frac{2\pi}{3}\right) - by\,\text{cosec}\left(\theta+\frac{2\pi}{3}\right) = a^2 - b^2 \quad ...(2)$$

$$ax\sec\left(\theta+\frac{4\pi}{3}\right) - by\,\text{cosec}\left(\theta+\frac{4\pi}{3}\right) = a^2 - b^2 \quad ...(3)$$

Now, above three normals are concurrent

$\Rightarrow \quad \Delta = 0$

$$\text{where } \Delta = \begin{vmatrix} \sec\theta & \text{cosec}\,\theta & 1 \\ \sec\left(\theta+\frac{2\pi}{3}\right) & \text{cosec}\left(\theta+\frac{2\pi}{3}\right) & 1 \\ \sec\left(\theta+\frac{4\pi}{3}\right) & \text{cosec}\left(\theta+\frac{4\pi}{3}\right) & 1 \end{vmatrix}$$

Multiplying and dividing the different rows R_1, R_2 and R_3 by $\sin\theta\cos\theta$,

$$\sin\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{2\pi}{3}\right)$$

and $\sin\left(\theta+\frac{4\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)$ respectively, we get

$$\Delta = \frac{1}{\sin\theta\cos\theta\sin\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{2\pi}{3}\right)\sin\left(\theta+\frac{4\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)} \times$$

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} = 0$$

[Operating $R_2 \to R_2 + R_3$ and simplyfing R_2 we get $R_2 \equiv R_1$]

14. Given that

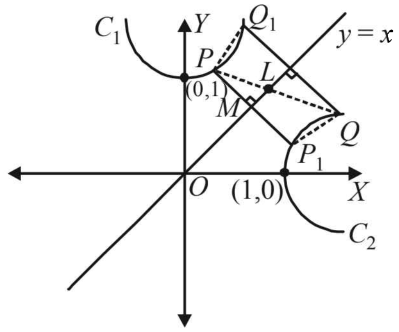
$C_1 : x^2 = y - 1$; $\quad C_2 : y^2 = x - 1$

Let $P\ (x_1, x_1^2 + 1)$ on C_1 and $Q\ (y_2^2 + 1, y_2)$ on C_2.

Now the reflection of pt P in the line $y = x$ can be obtained by interchanging the values of abscissa and ordiante.

Thus reflection of pt. $P\ (x_1, x_1^2 + 1)$ is $P_1(x_1^2 + 1, x_1)$

and reflection of pt. $Q\ (y_2^2 + 1, y_2)$ is $Q_1(y_2, y_2^2 + 1)$



It can be seen clearly that P_1 lies on C_2 and Q_1 on C_1.

Now PP_1 and QQ_1 both are perpendicular to mirror line $y = x$. Also M is mid pt. of PP_1 ($\because P_1$ is morror image of P in $y = x$)

$\therefore \quad PM = \frac{1}{2}PP_1$

In rt ΔPML,

$PL > PM \Rightarrow PL > \frac{1}{2}PP_1$ ...(i)

Similarly,

$LQ > \frac{1}{2}QQ_1$ ...(ii)

Adding (i) and (ii) we get

$$PL + LQ > \frac{1}{2}(PP_1 + QQ_1)$$

$\Rightarrow \quad PQ > \frac{1}{2}(PP_1 + QQ_1)$

$\Rightarrow \quad PQ$ is more than the mean of PP_1 and QQ_1

$\Rightarrow \quad PQ \geq \min(PP_1, QQ_1)$

Let $\min(PP_1, QQ_1) = PP_1$

then $PQ^2 \geq PP_1^2 = (x_1^2 + 1 - x_1)^2 + (x_1^2 + 1 - x_1)^2$

$= 2(x_1^2 + 1 - x_1)^2 = f(x_1)$

$\Rightarrow \quad f'(x_1) = 4(x_1^2 + 1 - x_1)(2x_1 - 1)$

$$= 4\left(\left(x_1 - \frac{1}{2}\right)^2 + \frac{3}{4}\right)(2x_1 - 1)$$

$\therefore \quad f'(x_1) = 0$ when $x_1 = \frac{1}{2}$

Also $f'(x_1) < 0$ if $x_1 < \frac{1}{2}$ and $f'(x_1) > 0$ if $x_1 > \frac{1}{2}$

$\Rightarrow \quad f(x_1)$ is min when $x_1 = \frac{1}{2}$

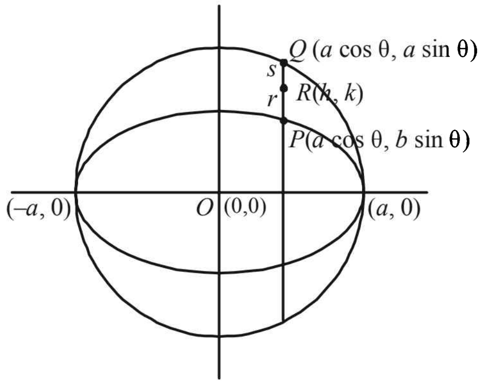
Thus if at $x_1 = \frac{1}{2}$ pt P is P_0 on C_1

$$P_0\left(\frac{1}{2}, \left(\frac{1}{2}\right)^2 + 1\right) = \left(\frac{1}{2}, \frac{5}{4}\right)$$

Similarly Q_0 on C_2 will be image of P_0 with respect to $y = x$

$\therefore \quad Q_0\left(\frac{5}{4}, \frac{1}{2}\right)$

15. Let the co-ordinates of P be $(a\cos\theta, b\sin\theta)$ then co-ordinates of Q are $(a\cos\theta, a\sin\theta)$



As $R(h, k)$ divides PQ in the ratio $r : s$, then

$$h = \frac{s.(a\cos\theta) + r(a\cos\theta)}{(r+s)} = a\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{h}{a}$$

$$k = \frac{s(b\sin\theta) + r(a\sin\theta)}{(r+s)} = \frac{\sin\theta(bs + ar)}{(r+s)}$$

$$\Rightarrow \quad \sin\theta = \frac{k(r+s)}{(bs+ar)} \qquad \because \cos^2\theta + \sin^2\theta = 1$$

$$\therefore \quad \frac{h^2}{a^2} + \frac{k^2(r+s)^2}{(bs+ar)^2} = 1.$$

Hence locus of R is $\frac{x^2}{a^2} + \frac{y^2(r+s)^2}{(bs+ar)^2} = 1$ which is equation of an ellipse.

16. Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and O be the centre.

Tangent at $P(x_1, y_1)$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0$ whose

slope $= -\frac{b^2 x_1}{a^2 y_1}$, Focus is $S(ae, 0)$.

Equation of the line perpendicular to tangent at P is

$$y = \frac{a^2 y_1}{b^2 x_1}(x - ae) \qquad \text{....(1)}$$

Equation of OP is $y = \frac{y_1}{x_1}x$ ...(2)

(1) and (2) intersect $\Rightarrow \frac{y_1}{x_1}x = \frac{a^2 y_1}{b^2 x_1}(x - ae)$

$\Rightarrow \quad x(a^2 - b^2) = a^3 e \Rightarrow x.\, a^2e^2 = a^3 e$

$\Rightarrow \quad x = a/e$

Which is the corresponding directrix.

17. Let P be the pt. (h, k). Then eqn of normal to parabola $y^2 = 4x$ from point (h, k), if m is the slope of normal, is

$y = mx - 2m - m^3$

As it passes through (h, k), therefore

$mh - k - 2m - m^3 = 0$

or, $\quad m^3 + (2 - h) + k = 0$ ...(1)

Whcih is cubic in m, giving three values of m say m_1, m_2 and m_3. Then $m_1 m_2 m_3 = -k$ (from eq^n) but given that $m_1 m_2 = \alpha$

$\therefore \quad$ We get $m_3 = -\frac{k}{\alpha}$

But m_3 must satisfy eq^n (1)

$$\therefore \quad \frac{-k^3}{\alpha^3} + (2-h)\left(\frac{-k}{\alpha}\right) + k = 0$$

$\Rightarrow \quad k^2 - 2\alpha^2 - h\alpha^2 - \alpha^3 = 0$

$\therefore \quad$ Locus of $P(h, k)$ is $y^2 = \alpha^2 x + (\alpha^3 - 2\alpha^2)$

But ATQ, locus of P is a part of parabola $y^2 = 4x$, therefore comparing the two, we get $\alpha^2 = 4$ and $\alpha^3 - 2\alpha^2 = 0 \Rightarrow \alpha = 2$

18. The given eq^n of parabola is

$y^2 - 2y - 4x + 5 = 0$ ...(1)

$\Rightarrow \quad (y-1)^2 = 4(x-1)$

Any parametric point on this parabola is

$P(t^2 + 1, 2t + 1)$

Differentiating (1) w.r. to x, we get

$$2y\frac{dy}{dx} - 2\frac{dy}{dx} - 4 = 0 \Rightarrow \frac{dy}{dx} = \frac{2}{y-1}$$

$\therefore \quad$ Slope of tangent to (1) at point P $(t^2 + 1, 2t + 1)$ is

$$m = \frac{2}{2t} = \frac{1}{t}$$

$\therefore \quad$ Eq^n of tangent at P $(t^2 + 1, 2t + 1)$ is

$$y - (2t + 1) = \frac{1}{t}(x - t^2 - 1)$$

$\Rightarrow \quad yt - 2t^2 - t = x - t^2 = 1$

$\Rightarrow \quad x - yt + (t^2 + t - 1) = 0$ ...(2)

Now directrix of given parabola is

$(x - 1) = -1 \Rightarrow x = 0$

Tangent (2) meets directix at $Q\left(0, \frac{t^2 + t - 1}{t}\right)$

Let pt. R be (h, k)

ATQ, *R* divides the line joing *QP* in the ratio $\frac{1}{2}:1$ i.e., 1 : 2 externally.

$$\therefore \quad (h, k) = \left(\frac{1(1+t^2)-0}{-1}, \frac{t+2t^2-2t^2-2t+2}{-t}\right)$$

$\Rightarrow \quad h = -(1+t^2)$ and $k = \frac{t-2}{t}$

$\Rightarrow \quad t^2 = -1-h$ and $t = \frac{2}{1-k}$

Eliminating *t*, we get $\left(\frac{2}{1-k}\right)^2 = -1-h$

$\Rightarrow \quad 4 = -(1-k)^2(1-h)$

$\Rightarrow \quad (h-1)(k-1)^2+4=0$

$\therefore$ Locus of R (h, k) is $(x-1)(y-1)^2+4=0$

19. Any pt on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ is $(3\sec\theta, 2\tan\theta)$

Then, equation of chord of contact to the circle $x^2+y^2=9$, with respect to the pt. $(3\sec\theta, 2\tan\theta)$ is

$(3\sec\theta)x+(2\tan\theta)y=9$ ...(i)

If (h, k) be the mid point of chord of contact then equation of chord of contact will be

$hx+ky-9=h^2+k^2-9$ $(T=S_1)$

or, $hx+ky=h^2+k^2$ ...(ii)

But equations (i) and (ii) represent the same st. line and hence should be identical, therefore, we get

$$\frac{3\sec\theta}{h}=\frac{2\tan\theta}{k}=\frac{9}{h^2+k^2}$$

$$\Rightarrow \quad \sec\theta=\frac{3h}{h^2+k^2}, \tan\theta=\frac{9k}{2(h^2+k^2)}$$

$\sec^2\theta-\tan^2\theta=1$

$$\Rightarrow \quad \frac{9h^2}{(h^2+k^2)^2}-\frac{81k^2}{4(h^2+k^2)^2}=1$$

$$\Rightarrow \quad 4h^2-9k^2=\frac{4}{9}(h^2+k^2)^2$$

$$\text{or,} \quad \frac{h^2}{9}-\frac{k^2}{4}=\left(\frac{h^2+k^2}{9}\right)^2$$

$$\therefore \quad \text{Locus of } (h, k) \text{ is } \frac{x^2}{9}-\frac{y^2}{4}=\left(\frac{x^2+y^2}{9}\right)^2$$

20. Let the common tangent to circle $x^2+y^2=16$ and ellipse $x^2/25+y^2/4=1$ be

$y=mx+\sqrt{25m^2+4}$ ..(i)

As it is tangent to circle $x^2+y^2=16$, we should have

$$\frac{\sqrt{25m^2+4}}{\sqrt{m^2+1}}=4$$

[Using : length of perpendicular from (0,0) to (1) = 4]

$\Rightarrow \quad 25m^2+4=16m^2+16 \Rightarrow 9m^2=12$

$\Rightarrow \quad m=\frac{-2}{\sqrt{3}}$

[Leaving + ve sign to consider tangent in I quadrant]

$\therefore$ Equation of common tangent is

$$y=-\frac{2}{\sqrt{3}}x+\sqrt{25.\frac{4}{3}+4} \Rightarrow y=-\frac{2}{\sqrt{3}}x+4\sqrt{\frac{7}{3}}$$

This tangent meets the axes at $A(2\sqrt{7},0)$ and $B\left(0,4\sqrt{\frac{7}{3}}\right)$

$\therefore$ Length of intercepted portion of tangent between axes

$$= AB = \sqrt{(2\sqrt{7})+\left(4\sqrt{\frac{7}{3}}\right)^2} = 14/\sqrt{3}$$

F. Match the Following

1. Let $y=mx-2m-m^3$ be the equation of normal to $y^2=4x$. As it passes through (3, 0), we get $m = 0, 1, -1$

Then three points on parabola are given by $(m^2, -2m)$ for m = 0, 1, – 1

$\therefore \quad P(0,0), Q(1,2), R(1,-2)$

$$\therefore \quad \text{Area of } \Delta PQR = \frac{1}{2}\times\begin{vmatrix}0 & 0 & 1\\ 1 & 2 & 1\\ 1 & -2 & 1\end{vmatrix} = 2 \text{ sq. units}$$

Radius of circum-circle,

$$R=\frac{abc}{4\Delta}=\frac{\sqrt{5}\times\sqrt{5}\times4}{4\times2}=\frac{5}{2}$$ **NOTE THIS STEP**

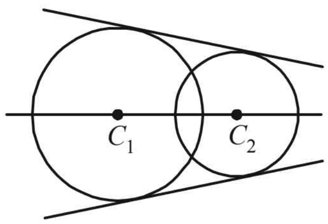
(where, *a*, *b*, *c* are the sides of ΔPQR)

Centroid of $\Delta PQR = \left(\frac{2}{3},0\right)$

Circumcentre $=\left(\frac{5}{2},0\right)$

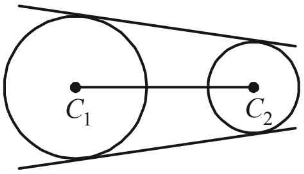
Thus, **(A) - (p); (B) - (q); (C) – (s); (D) – (r)**

2. (A) - p, q



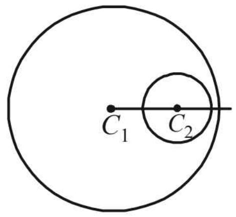
It is clear from the figure that two intersecting circles have a common tangent and a common normal joining the centres

(B) - p, q



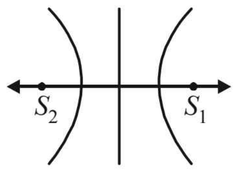
(C) – q, r

Two circle when one is completely inside the other have a common normal C_1C_2 but no common tangent.



(D) – q, r

Two branches of hyperbola have no common tangent but have a common normal joining S_1S_2.



Matrix Match

(A) – p, q; (B) – p, q; (C) – q, r; (D) – q, r

3. $A \to p; B \to s,t; C \to r; D \to q,s$

(p) As the line $hx + ky = 1$, touches the circle $x^2 + y^2 = 4$

$\therefore$ Length of perpendicular from centre (0 , 0) of circle to line = radius of the circle

$$\Rightarrow \quad \frac{1}{\sqrt{h^2+k^2}} = 2 \Rightarrow \quad h^2 + k^2 = \frac{1}{4}$$

$\therefore$ Locus of (*h, k*) is $x^2 + y^2 = \frac{1}{4}$, which is a circle.

(q) We know that if $|z - z_1| - |z - z_2| = k$

where $|k| < |z_1 - z_2|$

then *z* traces a hyperbola.

Here $|z+2| - |z-2| = \pm 3$

$\therefore$ Locus of *z* is a hyperbola.

(r) We have $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right),\ y = \frac{2t}{1+t^2}$

$$\Rightarrow \quad \frac{x}{\sqrt{3}} = \frac{1-t^2}{1+t^2} \quad \text{and } y = \frac{2t}{1+t^2}$$

On squaring and adding, we get

$$\frac{x^2}{3} + y^2 = \frac{(1-t^2)^2 + 4t^2}{(1+t^2)^2} = 1 \quad \text{or} \quad \frac{x^2}{3} + \frac{y^2}{1} = 1$$

which is the equation of an ellipse.

(s) We know eccentricity for a parabola = 1
for an ellipse < 1
for a hyperbola > 1

$\therefore$ The conics whose eccentricity lies in $1 \le x < \infty$ are parabola and hyperbola.

(t) Let $z = x + iy$ then

$$\text{Re } [(x+1)+iy]^2 = x^2 + y^2 + 1$$

$$\Rightarrow \quad (x+1)^2 - y^2 = x^2 + y^2 + 1$$

$$\Rightarrow \quad y^2 = x \text{, which is a parabola.}$$

4. (a) Equation of tangent to $y^2 = 16x$ at F (x_0, y_0)

$yy_0 = 8(x + x_0)$

$$\Rightarrow \quad G\left(0, \frac{8x_0}{y_0}\right)$$

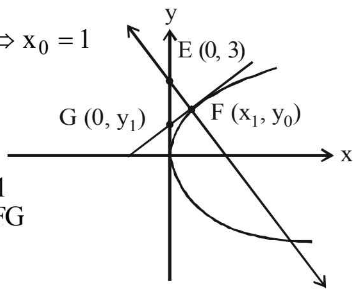
$$\text{Area of } \Delta EFG = \frac{1}{2} \times (3 - y_1) \times x_0$$

$$A = \frac{1}{2}x_0\left(3 - \frac{8x_0}{y_0}\right)$$

$$A = \frac{1}{2} \times \frac{y_0^2}{16}\left(3 - \frac{y_0}{2}\right) = \frac{1}{32}\left(3y_0^2 - \frac{y_0^3}{2}\right)$$

$$\frac{dA}{dy_0} = \frac{1}{32}\left(6y_0 - \frac{3y_0^2}{2}\right)$$

$$\frac{dA}{dy_0} = 0 \Rightarrow y_0 = 4 \Rightarrow x_0 = 1$$



$$\therefore \quad y_1 = \frac{8 \times 1}{4} = 2$$

Also $y_0 = mx_0 + 3$

$\therefore$ $4 = m + 3$ or $m = 1$

maximum area of ΔEFG

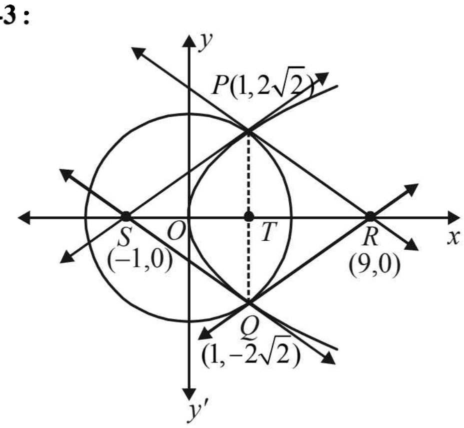
$$= \frac{1}{32}\left[3 \times 4^2 - \frac{4^3}{2}\right]$$

$$= \frac{1}{32}[48 - 32] = \frac{1}{2}$$

$\therefore$ (P) $\to$ (4), (Q) $\to$ (1), (R) $\to$ (2), (S) $\to$ (3)

G. Comprehension Based Questions

For Qs. 1-3 :



1. (c) $$\frac{Ar\,\Delta PQS}{Ar\,\Delta PQR} = \frac{\frac{1}{2}PQ \times ST}{\frac{1}{2}PQ \times TR} = \frac{ST}{TR} = \frac{2}{8} = \frac{1}{4}a$$

2. (b) For ΔPRS,

$$Ar\,(\Delta PRS) = \Delta = \frac{1}{2} \times SR \times PT = \frac{1}{2} \times 10 \times 2\sqrt{2}$$

$\therefore$ $\Delta = 10\sqrt{2},\ a = PS = 2\sqrt{3}$

$b = PR = 6\sqrt{2},\ c = SR = 10$

$\therefore$ Radius of circumference

$$= R = \frac{abc}{4\Delta} = \frac{2\sqrt{3} \times 6\sqrt{2} \times 10}{4 \times 10\sqrt{2}} = 3\sqrt{3}$$

3. (d) Radius of incircle

$$= \frac{\text{area of } \Delta PQR}{\text{semi perimeter of } \Delta PQR} = \frac{\Delta}{s}$$

We have $a = PR = 6\sqrt{2}$, $b = QP = PR = 6\sqrt{2}$

$c = PQ = 4\sqrt{2}$

and $\Delta = \frac{1}{2} \times PQ \times TR = 16\sqrt{2}$

$$\therefore \quad s = \frac{6\sqrt{2}+6\sqrt{2}+4\sqrt{2}}{2} = 8\sqrt{2} \quad \therefore\ r = \frac{16\sqrt{2}}{8\sqrt{2}} = 2$$

4. (b) Any tangent to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is $\frac{x\sec\alpha}{3} - \frac{y\tan\alpha}{2} = 1$

It touches circle with center (4,0) and radius = 4

$$\therefore \frac{\frac{4\sec\alpha - 3}{3}}{\sqrt{\frac{\sec^2\alpha}{9} + \frac{\tan^2\alpha}{4}}} = 4$$

$$\Rightarrow 16\sec^2\alpha - 24\sec\alpha + 9 = 144\left(\frac{\sec^2\alpha}{9} + \frac{\tan^2\alpha}{4}\right)$$

$$\Rightarrow 12\sec^2\alpha + 8\sec - 15 = 0 \Rightarrow \sec\alpha = \frac{5}{6} \text{ or } \frac{-3}{2}$$

but $\sec\alpha = \frac{5}{6} < 1$ is not possible

$$\therefore \sec\alpha = -3/2 \Rightarrow \tan\alpha = \pm\frac{\sqrt{5}}{2}$$

$$\therefore \text{slope of tangent} = \frac{2\sec\alpha}{3\tan\alpha} = \frac{2(-3/2)}{3(-\sqrt{5}/2)}$$

(for +ve value of m)

$$= \frac{2}{\sqrt{5}}$$

$$\therefore \text{ Equation of tangent is } \frac{-x}{2} + \frac{y\sqrt{5}}{4} = 1$$

or $2x - \sqrt{5}y + 4 = 0$

5. (a) The intersection points of given circle

$x^2 + y^2 - 8x = 0$ ...(1)

and hyperbola $4x^2 - 9y^2 - 36 = 0$ ...(2)

can be obtained by solving these equations Substituting value of y^2 from eqn (1) in eqn (2), we get

$$4x^2 - 9(8x - x^2) = 36 \Rightarrow 13x^2 - 72x - 36 = 0$$

$$\Rightarrow x = 6, \frac{-6}{13} \Rightarrow y^2 = 12, \frac{-48}{13}\frac{-36}{169} \ (\textit{not possible})$$

$\therefore \left(6, 2\sqrt{3}\right)$ and $\left(6, -2\sqrt{3}\right)$ are points of intersection.

So eqn of required circle is

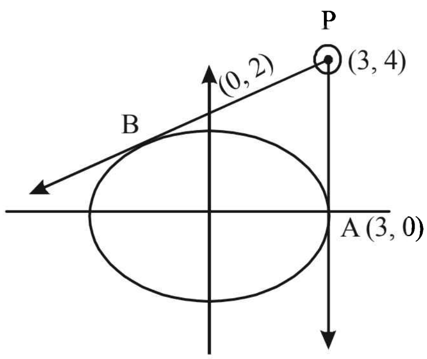
$$(x-6)(x-6) + \left(y - 2\sqrt{3}\right)\left(y + 2\sqrt{3}\right) = 0$$

$$\Rightarrow x^2 + 36 - 12x + y^2 - 12 = 0$$

$$\Rightarrow x^2 + y^2 - 12x + 24 = 0$$

6. (d) Tangent to $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ at the point $(3\cos\theta, 2\sin\theta)$ is

$$\frac{x\cos\theta}{3} + \frac{y\sin\theta}{2} = 1$$



As it passes throught (3,4), we get

$\cos\theta + 2\sin\theta = 1$

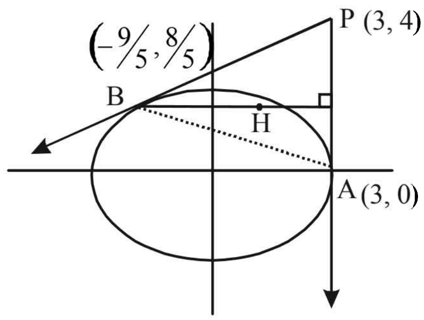
$$\Rightarrow 4\sin^2\theta = 1 + \cos^2\theta - 2\cos\theta$$

$$\Rightarrow 5\cos^2\theta - 2\cos\theta - 3 = 0$$

$$\Rightarrow \cos\theta = 1, -\frac{3}{5} \Rightarrow \sin\theta = 0, \frac{4}{5}$$

$\therefore$ Required points are A (3,0) and $B\left(-\frac{9}{5}, \frac{8}{5}\right)$

7. (c) Let H be the orthocentre of ΔPAB, then as $BH \perp AP$, BH is a horizontal line through B.



$\therefore$ y- cordinate of B = 8/5

Let H has coordinater $(\alpha, 8/5)$

$$\text{Then slope of PH} = \frac{\frac{8}{5} - 4}{\alpha - 3} = \frac{-12}{5(\alpha - 3)}$$

$$\text{and slope of AB} = \frac{\frac{8}{5} - 0}{\frac{-9}{5} - 3} = \frac{8}{-24} = \frac{-1}{3}$$

But PH ⊥ AB $\Rightarrow \dfrac{-12}{5(\alpha-3)}\times\left(\dfrac{-1}{3}\right)=-1$

$\Rightarrow\ 4=-5\alpha+15$ or $\alpha=11/5$

Hence $H\left(11/5, 8/5\right)$.

8. (a) Clearly the moving point traces a parabola with focus at $P(3,4)$ and directrix as

$AB:\dfrac{y-0}{x-3}=\dfrac{-1}{3}$ or $x+3y-3=0$

$\therefore$ Equation of parabola is

$$(x-3)^2+(y-4)^2=\frac{(x+3y-3)^2}{10}$$

or $9x^2+y^2-6xy-54x-62y+241=0$

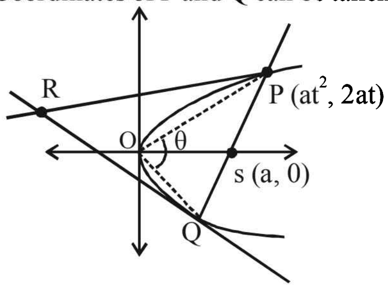
9. (b) $PQ=\sqrt{\left(at^2-\dfrac{a}{t^2}\right)^2+\left(2at+\dfrac{2a}{t}\right)^2}$

$=a\sqrt{\left(t+\dfrac{1}{t}\right)^2\left(t-\dfrac{1}{t}\right)^2+4\left(t+\dfrac{1}{t}\right)^2}$

$=a\left(t+\dfrac{1}{t}\right)\sqrt{\left(t-\dfrac{1}{t}\right)^2+4}\ =a\left(t+\dfrac{1}{t}\right)^2\ =5a$

10. (d) As PQ is the focal chord of $y^2=4ax$

$\therefore$ Coordinates of P and Q can be taken as



$\left(\dfrac{a}{t^2},\dfrac{-2a}{t}\right)$

$P(at^2, 2at)$ and $Q\left(\dfrac{a}{t^2},\dfrac{-2a}{t}\right)$

Tangents at P and Q are

$y=\dfrac{x}{t}+at$ and $y=-xt-\dfrac{a}{t}$

which intersect each other at $R\left(-a, a\left(t-\dfrac{1}{t}\right)\right)$

As R lies on the $y=2x+a,\ a>0$

$\therefore\ a\left(t-\dfrac{1}{t}\right)=-2a+a\ \Rightarrow\ t-\dfrac{1}{t}=-1\ \Rightarrow\ t+\dfrac{1}{t}=\sqrt{5}$

Now, $m_{OP}=\dfrac{2}{t}$ and $m_{OQ}=-2t$

$\therefore\ \tan\theta=\dfrac{\dfrac{2}{t}+2t}{1-4}=\dfrac{2\left(t+\dfrac{1}{t}\right)}{-3}=\dfrac{2\sqrt{5}}{-3}$

11. (d) $\because$ PQ is a focal chord, $Q\left(\dfrac{a}{t^2},\dfrac{-2a}{t}\right)$

Also $QR \parallel PK \Rightarrow m_{QR}=m_{PK}$

$\Rightarrow \dfrac{-\dfrac{2a}{t}-2ar}{\dfrac{a}{t^2}-ar^2}=\dfrac{0-2at}{2a-at^2}$

$\Rightarrow \dfrac{-2a\left(\dfrac{1}{t}+r\right)}{a\left(\dfrac{1}{t}+r\right)\left(\dfrac{1}{t}-r\right)}=\dfrac{-2at}{a\left(2-t^2\right)}$

$\Rightarrow 2-t^2=t\left(\dfrac{1}{t}-r\right)$

$\left[\because r\neq-\dfrac{1}{t}\text{ otherwise } Q \text{ will coincide with } R\right]$

$\Rightarrow\ 2-t^2=1-tr\ \Rightarrow\ r=\dfrac{t^2-1}{t}$

12. (b) Tangent at P is

$ty=x+at^2$ ....(i)

Normal at S

$sx+y=2as+as^3$ ....(ii)

But given $st=1 \Rightarrow s=\dfrac{1}{t}$

$\therefore\ \dfrac{x}{t}+y=\dfrac{2a}{t}+\dfrac{a}{t^3}$

$\Rightarrow xt^2=yt^3=2at^2+a$

Putting value of x from equation (i) in above equation we get

$\Rightarrow t^2\left(ty-at^2\right)+yt^3=2at^2+a$

$\Rightarrow \left(t^3+t^3\right)y-at^4=2at^2+a$

$\Rightarrow 2t^3y=a\left(t^4+2t^2+1\right)$

$y=\dfrac{a\left(t^4+2t^2+1\right)}{2t^3}=\dfrac{a\left(t^2+1\right)^2}{2t^3}$

For (Q. 13 and 14)

For ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{8}=1$, $e=\sqrt{1-\dfrac{8}{9}}=\dfrac{1}{3}$

$\therefore$ $F_1(-1,0)$ and $F_2(1,0)$

Parabola with vertex at (0, 0) and focus at $F_2(1,0)$ is $y^2=4x$.

Intersection points of ellipse and parabola are $M\left(\dfrac{3}{2},\sqrt{6}\right)$ and $N\left(\dfrac{3}{2},-\sqrt{6}\right)$

13. (a) For orthocentre of ΔF_1MN, clearly one altitude is x-axis i.e. y = 0 and altitude from M to F_1N is

$$y-\sqrt{6} = \frac{5}{2\sqrt{6}}\left(x-\frac{3}{2}\right)$$

Putting y = 0 in above equation, we get

$$x = -\frac{9}{10}$$

$\therefore$ Orthocentre $\left(-\frac{9}{10}, 0\right)$

14. (c) Tangents to ellipse at M and N are

$$\frac{x}{6}+\frac{y\sqrt{6}}{8}=1 \text{ and } \frac{x}{6}-\frac{y\sqrt{6}}{8}=1$$

Their intersection point is R (6, 0)

Also normal to parabola at $M\left(\frac{3}{2}, \sqrt{6}\right)$ is

$$y-\sqrt{6} = -\frac{\sqrt{6}}{2}\left(x-\frac{3}{2}\right)$$

Its intersection with x-axis is $Q\left(\frac{7}{2}, 0\right)$

Now ar $(\Delta MQR) = \frac{1}{2}\times\frac{5}{2}\times\sqrt{6} = \frac{5\sqrt{6}}{4}$

Also area $(MF_1NF_2) = 2 \times \text{Ar}(F_1MF_2)$

$$= 2\times\frac{1}{2}\times 2\times\sqrt{6} = 2\sqrt{6}$$

$$\therefore \frac{\text{ar}(\Delta MQR)}{\text{ar}(MF_1NF_2)} = \frac{5\sqrt{6}}{4\times 2\sqrt{6}} = 5:8$$

H. Assertion & Reason Type Questions

1. (a) The given curve is $y = -\frac{x^2}{2}+x+1$

or $(x-1)^2 = -2(y-3/2)$

which is a parabola, so should be symmetric with respect to its axis $x-1=0$

$\therefore$ Both the statements are true and statement 2 is a correct explanation for statement 1.

I. Integer Value Correct Type

1. (2) Intersection point of nearest directrix $x = \frac{a}{e}$ and x-axis is $\left(\frac{a}{e}, 0\right)$

As $2x+y=1$ passes through $\left(\frac{a}{e}, 0\right)$

$$\therefore \frac{2a}{e} = 1 \Rightarrow a = \frac{e}{2}$$

Also $y=-2x+1$ is a tangent to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$\therefore 1 = a^2(-2)^2 - b^2 \Rightarrow 4a^2-b^2=1$$

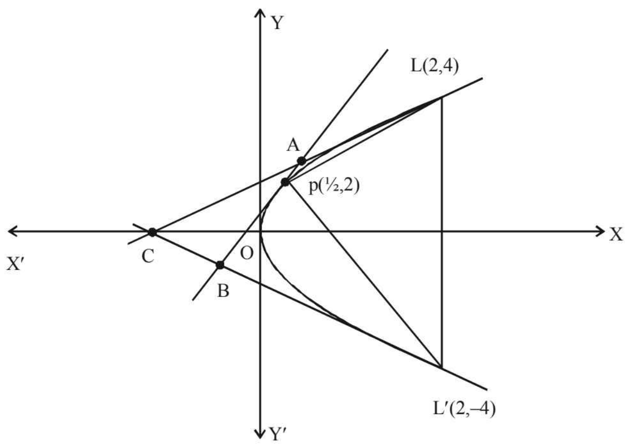
$$\Rightarrow 4a^2 - a^2\left(e^2-1\right)=1 \Rightarrow 4\times\frac{e^2}{4}-\frac{e^2}{4}\left(e^2-1\right)=1$$

$$\Rightarrow 4e^2 - e^4 + e^2 = 4 \Rightarrow e^4 - 5e^4 + 4 = 0$$

$\Rightarrow e^2 = 4$ *as* $e > 1$ for hyperbola. $\Rightarrow e = 2$

2. (2)

$$\Delta_1 = \text{Area of } \Delta PLL' = \frac{1}{2}\times 8\times\frac{3}{2} = 6$$



Equation of AB, $y = 2x+1$ Equation of AC, $y = x+2$

Equation of BC, $-y = x+2$ Solving above equations we get A (1, 3), B (–1, –1), C (–2, 0)

$$\therefore \Delta_2 = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ -1 & -1 & 1 \\ -2 & 0 & 1 \end{vmatrix} = 3 \;\therefore \frac{\Delta_1}{\Delta_2} = 2$$

3. (4) We observe both parabola $y^2 = 8x$ and circle $x^2+y^2-2x-4y=0$ pass through origin

$\therefore$ One end of common chord PQ is origin. Say P(0, 0)

Let Q be the point $(2t^2, 4t)$, then it will satisfy the equation of circle.

$\therefore \quad 4t^4 + 16t^2 - 4t^2 - 16t = 0$

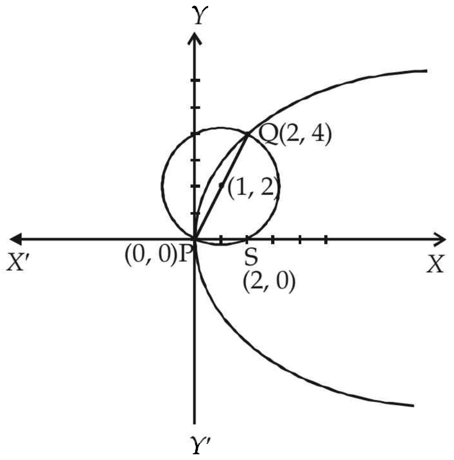
$\Rightarrow \quad t^4 + 3t^2 - 4t = 0 \Rightarrow t(t^3+3t-4) = 0$

$\Rightarrow \quad t(t-1)(t^2+t-4) = 0 \Rightarrow t = 0$ or 1

For $t = 0$, we get point *P*, therefore $t = 1$ gives point *Q* as (2, 4).

We also observe here that P(0, 0) and Q(2, 4) are end points of diameter of the given circle and focus of the parabola is the point S(2, 0).

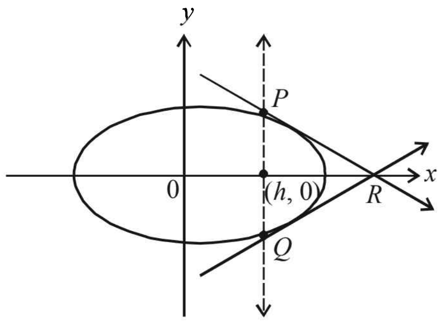
$\therefore$ Area of $\Delta PQS = \frac{1}{2} \times PS \times QS = \frac{1}{2} \times 2 \times 4 = 4$ sq. units



4. **(9)** Vertical line $x = h$, meets the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at

$P\left(h, \frac{\sqrt{3}}{2}\sqrt{4-h^2}\right)$ and $Q\left(h, \frac{-\sqrt{3}}{2}\sqrt{4-h^2}\right)$

By symmetry, tangents at P and Q will meet each other at x-axis.



Tangent at P is $\frac{xh}{4} + \frac{y\sqrt{3}}{6}\sqrt{4-h^2} = 1$

which meets x-axis at $R\left(\frac{4}{h}, 0\right)$

Area of $\Delta PQR = \frac{1}{2} \times \sqrt{3}\sqrt{4-h^2} \times \left(\frac{4}{h} - h\right)$

i.e., $\Delta(h) = \frac{\sqrt{3}}{2}\frac{(4-h^2)^{3/2}}{h}$

$\frac{d\Delta}{dh} = -\sqrt{3}\left[\frac{\sqrt{4-h^2}(h^2+2)}{h^2}\right] < 0$

$\therefore$ $\Delta(h)$ is a decreasing function.

$\therefore$ $\frac{1}{2} \le h \le 1 \Rightarrow \Delta_{\max} = \Delta\left(\frac{1}{2}\right)$ and $\Delta_{\min} = \Delta(1)$

$\therefore$ $\Delta_1 = \frac{\sqrt{3}}{2}\frac{\left(4-\frac{1}{4}\right)^{3/2}}{\frac{1}{2}} = \frac{45}{8}\sqrt{5}$

$\Delta_2 = \frac{\sqrt{3}}{2}\frac{3\sqrt{3}}{1} = \frac{9}{2}$ $\therefore$ $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = 45 - 36 = 9$

5. **(2)** End points of latus rectum of $y^2 = 4x$ are $(1, \pm 2)$

Equation of normal to $y^2 = 4x$ at $(1, 2)$ is

$y - 2 = -1(x-1)$ or $x + y - 3 = 0$

As it is tangent to circle $(x-3)^2 + (y+2)^2 = r^2$

$\therefore$ $\left|\frac{3+(-2)-3}{\sqrt{2}}\right| = r \Rightarrow r^2 = 2$

6. **(4)** Let $(t^2, 2t)$ be any point on $y^2 = 4x$. Let (h, k) be the image of $(t^2, 2t)$ in the line $x + y + 4 = 0$. Then

$\frac{h-t^2}{1} = \frac{k-2t}{1} = \frac{-2(t^2+2t+4)}{2}$

$\Rightarrow$ $h = -(2t+4)$ and $k = -(t^2+4)$

For its intersection with, $y = -5$, we have

$-(t^2+4) = -5 \quad \Rightarrow t = \pm 1$

$\therefore$ $A(-6, -5)$ and $B(-2, -5)$

$\therefore$ $AB = 4$.

7. **(4)** Ellipse: $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$\Rightarrow$ $a = 3, b = \sqrt{5}$ and $e = \frac{2}{3}$

$\therefore$ $f_1 = 2$ and $f_2 = -2$

$P_1 : y^2 = 8x$ and $P_2 : y^2 = -16x$

$T_1 : y = m_1 x + \frac{2}{m_1}$

It passes through $(-4, 0)$,

$0 = -4m_1 + \frac{2}{m_1} \Rightarrow m_1^2 = \frac{1}{2}$

$T_2 : y = m_2 x - \frac{4}{m_2}$

It passes through $(2, 0)$

$0 = 2m_2 - \frac{4}{m_2} \Rightarrow m_2^2 = 2$

$\therefore$ $\frac{1}{m_1^2} + m_2^2 = 4$

Section-B JEE Main/ AIEEE

1. (b) Any tangent to the parabola $y^2 = 8ax$ is

$$y = mx + \frac{2a}{m} \quad \text{...(i)}$$

If (i) is a tangent to the circle, $x^2 + y^2 = 2a^2$ then,

$$\sqrt{2}a = \pm\frac{2a}{m\sqrt{m^2+1}}$$

$\Rightarrow m^2(1+m^2) = 2 \Rightarrow (m^2+2)(m^2-1) = 0 \Rightarrow m = \pm 1.$

So from (i), $y = \pm(x+2a)$.

2. (b) Equation of the normal to a parabola $y^2 = 4bx$ at point $\left(bt_1^2, 2bt_1\right)$ is $y = -t_1x + 2bt_1 + bt_1^3$

As given, it also passes through $\left(bt_2^2, 2bt_2\right)$ then

$$2bt_2 = -t_1 bt_2^2 + 2bt_1 + bt_1^3$$

$$2t_2 - 2t_1 = -t_1\left(t_2^2 - t_1^2\right) = -t_1(t_2+t_1)(t_2-t_1)$$

$$\Rightarrow 2 = -t_1(t_2+t_1) \Rightarrow t_2 + t_1 = -\frac{2}{t_1}$$

$$\Rightarrow t_2 = -t_1 - \frac{2}{t_1}$$

3. (d) $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$a = \sqrt{\frac{144}{25}}, b = \sqrt{\frac{81}{25}}, \quad e = \sqrt{1+\frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$$

$\therefore$ Foci $= (\pm 3, 0)$

$\therefore$ foci of ellipse = foci of hyperbola

$\therefore$ for ellipse $ae = 3$ but $a = 4$,

$\therefore$ $e = \frac{3}{4}$

Then $b^2 = a^2(1-e^2) \quad \Rightarrow b^2 = 16\left(1-\frac{9}{16}\right) = 7$

4. (d) Solving equations of parabolas

$y^2 = 4ax$ and $x^2 = 4ay$

we get (0, 0) and $(4a, 4a)$

Substituting in the given equation of line

$2bx + 3cy + 4d = 0,$

we get $d = 0$ and $2b + 3c = 0 \Rightarrow d^2 + (2b+3c)^2 = 0$

5. (b) $e = \frac{1}{2}$. Directrix, $x = \frac{a}{e} = 4$

$$\therefore a = 4 \times \frac{1}{2} = 2 \qquad \therefore b = 2\sqrt{1-\frac{1}{4}} = \sqrt{3}$$

Equation of elhipe is $\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$

6. (a) $P = (1, 0)$ $Q = (h, k)$ Such that $K^2 = 8h$

Let (α, β) be the midpoint of PQ

$$\alpha = \frac{h+1}{2}, \qquad \beta = \frac{k+0}{2}$$

$$2\alpha - 1 = h \qquad 2\beta = k.$$

$$(2\beta)^2 = 8(2\alpha - 1) \Rightarrow \beta^2 = 4\alpha - 2$$

$$\Rightarrow y^2 - 4x + 2 = 0.$$

7. (d) Tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Given that $y = \alpha x + \beta$ is the tangent of hyperbola

$\Rightarrow \quad m = \alpha$ and $a^2m^2 - b^2 = \beta^2$

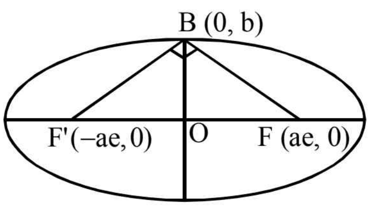
$\therefore \quad a^2\alpha^2 - b^2 = \beta^2$

Locus is $a^2x^2 - y^2 = b^2$ which is hyperbola.

8. (a) $\because \angle FBF' = 90° \Rightarrow FB^2 + F'B^2 = FF'^2$

$$\therefore \left(\sqrt{a^2e^2+b^2}\right)^2 + \left(\sqrt{a^2e^2+b^2}\right)^2 = (2ae)^2$$

$$\Rightarrow 2(a^2e^2+b^2) = 4a^2e^2 \Rightarrow e^2 = \frac{b^2}{a^2}$$



Also $e^2 = 1 - b^2/a^2 = 1 - e^2$

$$\Rightarrow 2e^2 = 1, \; e = \frac{1}{\sqrt{2}}.$$

9. **(a)** Given parabola is $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$

$\Rightarrow y = \frac{a^3}{3}\left(x^3 + \frac{3}{2a}x + \frac{9}{16a^2}\right) - \frac{3a}{16} - 2a$

$\Rightarrow y + \frac{35a}{16} = \frac{a^3}{3}\left(x + \frac{3}{4a}\right)^2$

$\therefore$ Vertex of parabola is $\left(\frac{-3}{4a}, \frac{-35a}{16}\right)$

To find locus of this vertex,

$x = \frac{-3}{4a}$ and $y = \frac{-35a}{16}$

$\Rightarrow a = \frac{-3}{4x}$ and $a = -\frac{16y}{35}$

$\Rightarrow \frac{-3}{4x} = \frac{-16y}{35} \Rightarrow 64xy = 105$

$\Rightarrow xy = \frac{105}{64}$ which is the required locus.

10. **(a)** $2ae = 6 \Rightarrow ae = 3$; $2b = 8 \Rightarrow b = 4$

$b^2 = a^2(1-e^2)$; $16 = a^2 - a^2e^2 \Rightarrow a^2 = 16 + 9 = 25$

$\Rightarrow a = 5 \qquad \therefore e = \frac{3}{a} = \frac{3}{5}$

11. **(b)** $\frac{dy}{dx} = 2x - 5 \;\therefore\; m_1 = (2x-5)_{(2,0)} = -1$,

$m_2 = (2x-5)_{(3,0)} = 1 \Rightarrow m_1m_2 = -1$

i.e. the tangents are perpendicular to each other.

12. **(b)** Given, equation of hyperbola is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$

We know that the equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Here, $a^2 = \cos^2\alpha$ and $b^2 = \sin^2\alpha$

We know that, $b^2 = a^2(e^2 - 1)$

$\Rightarrow \sin^2\alpha = \cos^2\alpha(e^2 - 1)$

$\Rightarrow \sin^2\alpha + \cos^2\alpha = \cos^2\alpha . e^2$

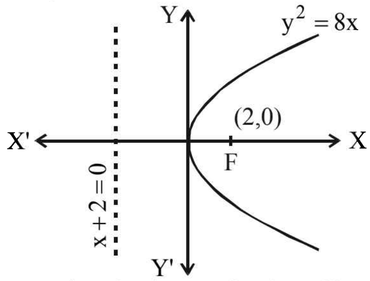
$\Rightarrow e^2 = 1 + \tan^2\alpha = \sec^2\alpha \Rightarrow e = \sec\alpha$

$\therefore ae = \cos\alpha . \frac{1}{\cos\alpha} = 1$

Co-ordinates of foci are $(\pm ae, 0)$ i.e. $(\pm 1, 0)$

Hence, abscissae of foci remain constant when α varies.

13. **(b)** Parabola $y^2 = 8x$



We know that the locus of point of intersection of two perpendicular tangents to a parabola is its directrix. Point must be on the directrix of parabola

$\because$ equation of directrix $x + 2 = 0 \Rightarrow x = -2$

Hence the point is (–2, 0)

14. **(b)** Equation of normal at $P(x, y)$ is $Y - y = -\frac{dx}{dy}(X - x)$

Coordinate of G at X axis is $(X, 0)$ (let)

$\therefore 0 - y = -\frac{dx}{dy}(X - x) \Rightarrow y\frac{dy}{dx} = X - x$

$\Rightarrow X = x + y\frac{dy}{dx}$ $\therefore$ Co-ordinate of $G\left(x + y\frac{dy}{dx}, 0\right)$

Given distance of G from origin = twice of the abscissa of P.

$\because$ distance cannot be –ve, therefore abscissa x should be +ve

$\therefore x + y\frac{dy}{dx} = 2x \Rightarrow y\frac{dy}{dx} = x \Rightarrow ydy = xdx$

On Integrating $\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c_1 \Rightarrow x^2 - y^2 = -2c_1$

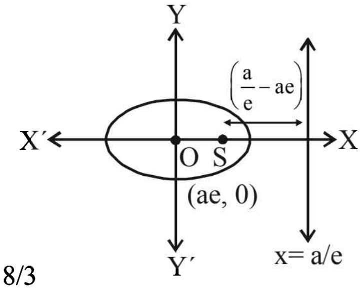
$\therefore$ the curve is a hyperbola

15. **(a)** Perpendicular distance of directrix from focus

$= \frac{a}{e} - ae = 4$

$\Rightarrow a\left(2 - \frac{1}{2}\right) = 4$

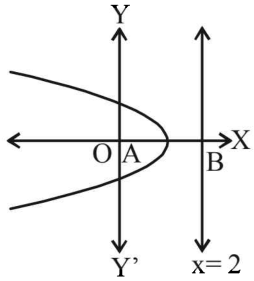
$\Rightarrow a = \frac{8}{3}$



$\therefore$ Semi major axis = 8/3

16. **(b)** Vertex of a parabola is the mid point of focus and the point where directrix meets the axis of the parabola.

Here focus is $O(0, 0)$ and directxix meets the axis at $B(2, 0)$



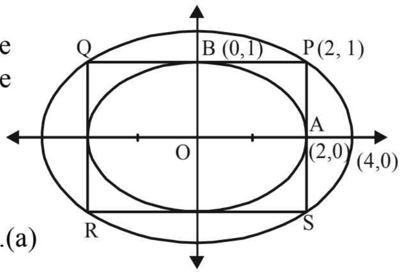
$\therefore$ Vertex of the parabola is (1, 0)

17. **(a)** The given ellipse is $\frac{x^2}{4}+\frac{y^2}{1}=1$

So $A=(2, 0)$ and $B=(0, 1)$

If $PQRS$ is the rectangle in which it is inscribed, then $P=(2, 1)$.

Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be the ellipse circumscribing the rectangle $PQRS$. Then it passes through $P\ (2,1)$



$\therefore\ \frac{4}{a^2}+\frac{1}{b^2}=1$ ....(a)

Also, given that, it passes through (4, 0)

$\therefore\ \frac{16}{a^2}+0=1 \Rightarrow a^2=16$

$\Rightarrow\ b^2=4/3$ [substituting $a^2=16$ in eq^n (a)]

$\therefore$ The required ellipse is $\frac{x^2}{16}+\frac{y^2}{4/3}=1$ or $x^2+12y^2=16$

18. **(b)** The locus of perpendicular tangents is directrix i.e., $x=-1$

19. **(d)** Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

It passes through $(-3, 1)$ so $\frac{9}{a^2}+\frac{1}{b^2}=1$ ...(i)

Also, $b^2=a^2(1-2/5)$

$\Rightarrow 5b^2=3a^2$ ...(ii)

Solving (i) and (ii) we get $a^2=\frac{32}{3}, b^2=\frac{32}{5}$

So, the equation of the ellipse is $3x^2+5y^2=32$

20. **(b)** Given equation of ellipse is $2x^2+y^2=4$

$\Rightarrow\ \frac{2x^2}{4}+\frac{y^2}{4}=1 \Rightarrow \frac{x^2}{2}+\frac{y^2}{4}=1$

Equation of tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ is

$y=mx\pm\sqrt{2m^2+4}$ ...(1)

($\because$ equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $y=mx+c$ where $c=\pm\sqrt{a^2m^2+b^2}$)

Now, Equation of tangent to the parabola $y^2=16\sqrt{3}x$ is $y=mx+\frac{4\sqrt{3}}{m}$ ...(2)

($\because$ equation of tangent to the parabola $y^2=4ax$ is $y=mx+\frac{a}{m}$)

On comparing (1) and (2), we get

$\frac{4\sqrt{3}}{m}=\pm\sqrt{2m^2+4}$

Squaring on both the sides, we get

$16(3)=(2m^2+4)m^2$

$\Rightarrow\ 48=m^2(2m^2+4) \Rightarrow\ 2m^4+4m^2-48=0$

$\Rightarrow\ m^4+2m^2-24=0 \Rightarrow\ (m^2+6)(m^2-4)=0$

$\Rightarrow\ m^2=4\ (\because m^2\neq-6) \Rightarrow\ m=\pm 2$

$\Rightarrow$ Equation of common tangents are $y=\pm 2x\pm 2\sqrt{3}$

Thus, statement-1 is true.

Statement-2 is obviously true.

21. **(d)** Equation of circle is $(x-1)^2+y^2=1$

$\Rightarrow$ radius $=1$ and diameter $=2$

$\therefore$ Length of semi-minor axis is 2.

Equation of circle is $x^2+(y-2)^2=4=(2)^2$

$\Rightarrow$ radius $=2$ and diameter $=4$

$\therefore$ Length of semi major axis is 4

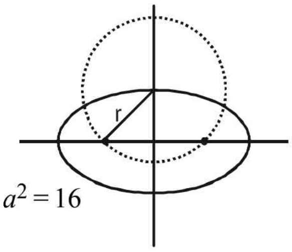
We know, equation of ellipse is given by

$$\frac{x^2}{(\text{Major axis})^2}+\frac{y^2}{(\text{Minor axis})^2}=1$$

$\Rightarrow\ \frac{x^2}{(4)^2}+\frac{y^2}{(2)^2}=1 \Rightarrow \frac{x^2}{16}+\frac{y^2}{4}=1 \Rightarrow x^2+4y^2=16$

22. **(a)** From the given equation of ellipse, we have

$a=4, b=3, e=\sqrt{1-\frac{9}{16}}$



$\Rightarrow\ e=\frac{\sqrt{7}}{4}$

Now, radius of this circle $=a^2=16$

$\Rightarrow$ Focii $=(\pm\sqrt{7}, 0)$

Now equation of circle is $(x-0)^2+(y-3)^2=16$

$x^2+y^2-6y-7=0$

23. **(b)** Let common tangent be

$y=mx+\frac{\sqrt{5}}{m}$

Since, perpendicular distance from centre of the circle to the common tangent is equal to radius of the circle,

therefore $\frac{\frac{\sqrt{5}}{m}}{\sqrt{1+m^2}}=\sqrt{\frac{5}{2}}$

On squaring both the side, we get

$m^2(1+m^2)=2$

$\Rightarrow\ m^4+m^2-2=0 \Rightarrow (m^2+2)(m^2-1)=0$

$\Rightarrow\ m=\pm 1$ $\quad(\because m\neq\pm\sqrt{2})$

$y=\pm(x+\sqrt{5})$, both statements are correct as $m=\pm 1$ satisfies the given equation of statement-2.

24. **(a)** Given eq^n of ellipse can be written as

$$\frac{x^2}{6}+\frac{y^2}{2}=1 \Rightarrow a^2=6, b^2=2$$

Now, equation of any variable tangent is

$$y=mx\pm\sqrt{a^2m^2+b^2} \quad ...(i)$$

where m is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$$y=\frac{-x}{m} \quad ...(ii)$$

Eliminating m, we get

$$(x^4+y^4+2x^2y^2)=a^2x^2+b^2y^2$$

$$\Rightarrow \quad (x^2+y^2)^2=a^2x^2+b^2y^2$$

$$\Rightarrow \quad \boxed{(x^2+y^2)^2=6x^2+2y^2}$$

25. **(c)** Let tangent to $y^2=4x$ be $y=mx+\frac{1}{m}$

Since this is also tangent to $x^2=-32y$

$$\therefore \quad x^2=-32\left(mx+\frac{1}{m}\right) \Rightarrow x^2+32mx+\frac{32}{m}=0$$

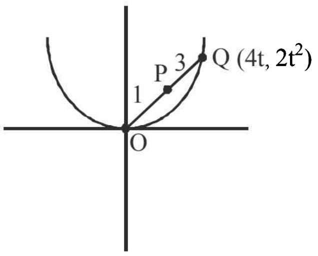
Now, D = 0

$$(32)^2-4\left(\frac{32}{m}\right)=0 \Rightarrow m^3=\frac{4}{32} \quad \Rightarrow \quad m=\frac{1}{2}$$

26. **(b)** Let P(h, k) divides OQ in the ratio 1 : 3

Let any point Q on $x^2=8y$ is $(4t, 2t^2)$.

Then by section formula



$$\Rightarrow \quad k=\frac{t^2}{2} \text{ and } h=t$$

$$\Rightarrow 2k=h^2$$

Required locus of P is $x^2=2y$

27. **(b)** Given curve is

$$x^2+2xy-3y^2=0 \quad ...(1)$$

Differentiate w.r.t. x, $2x+2x\frac{dy}{dx}+2y-6y\frac{dy}{dx}=0$

$$\left(\frac{dy}{dx}\right)_{(1,1)}=1$$

Equation of normal at (1, 1) is

$$y=2-x \quad ...(2)$$

Solving eq. (1) and (2), we get x = 1 , 3

Point of intersection (1, 1), (3, –1)

Normal cuts the curve again in 4th quadrant.

28. **(b)** The end point of latus rectum of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in first quadrant is $\left(ae, \frac{b^2}{a}\right)$ and the tangent at this point intersects x-axis at $\left(\frac{a}{e}, 0\right)$ and y-axis at (0, a).

The given ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

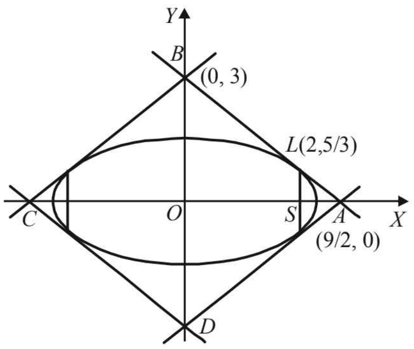
Then $a^2=9, b^2=5 \Rightarrow e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$

$\therefore$ end point of latus rectum in first quadrent is $L(2, 5/3)$

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$

It meets x-axis at A (9/2, 0) and y-axis at B (0, 3)

$$\therefore \quad \text{Area of } \Delta OAB=\frac{1}{2}\times\frac{9}{2}\times 3=\frac{27}{4}$$



By symmetry area of quadrilateral

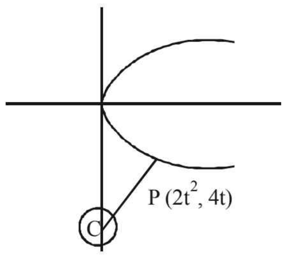
$$=4\times(\text{Area } \Delta OAB)=4\times\frac{27}{4}=27 \text{ sq. units.}$$

29. **(c)** Minimum distance ⇒ perpendicular distance

Eq^n of normal at $p(2t^2, 4t)$

$y=-tx+4t+2t^3$

It passes through C(0, –6) $\Rightarrow t^3+2t+3=0 \Rightarrow t=-1$



Centre of new circle = $P(2t^2, 4t)=P(2,-4)$

Radius = $PC=\sqrt{(2-0)^2+(-4+6)^2}=2\sqrt{2}$

$\therefore$ Equation of the circle is

$$(x-2)^2+(y+4)^2=(2\sqrt{2})^2$$

$$\Rightarrow \quad x^2+y^2-4x+8y+12=0$$

30. **(a)** $\frac{2b^2}{a}=8$ and $2b=\frac{1}{2}(2ae)$

$$\Rightarrow 4b^2=a^2e^2 \Rightarrow 4a^2(e^2-1)=a^2e^2 \Rightarrow 3e^2=4 \Rightarrow e=\frac{2}{\sqrt{3}}$$