CBSE Test Paper 03

Chapter 8 Gravitation

1. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the sun = 2 $\times$ 10^{30} kg; mass of mars = 6.4 $\times$ 10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28 $\times$ 10^{8} km; G = 6.67 $\times$ 10^{-11} N m^2 kg^{-2}. **1**
   a. 5.5 $\times$ 10^{11} J
   b. 5.66 $\times$ 10^{11} J
   c. 5.96 $\times$ 10^{11} J
   d. 5.86 $\times$ 10^{11} J
2. For a satellite to be in a circular orbit 780 km above the surface of the earth, what is the period of the orbit (in hours)? **1**
   a. 1.98 hr
   b. 1.65 hr
   c. 1.78 hr
   d. 1.88 hr
3. A body of mass m is taken from earth surface to the height h equal to radius of earth, the increase in potential energy will be **1**
   a. $\frac{1}{2}mgR$
   b. mgR
   c. $\frac{1}{4}mgR$
   d. 2mgR
4. Titania, the largest moon of the planet Uranus, has $\frac{1}{8}$ the radius of the earth and $\frac{1}{1700}$ the mass of the earth. What is the average density of Titania? Data: G = 6.67$\times10^{-11}$ N m^2/kg^2, R_E = 6.38 $\times$ 10^{6} m, M_E = 5.97 $\times$ 10^{24} kg. **1**
   a. 2300 kg/m^3
   b. 1900 kg/m^3

c. 2700 kg/m^3
d. 1700 kg/m^3

5. If the radius of earth reduces by 4% and density remains same then escape velocity will **1**

   a. Increase by 2%
   b. Reduce by 4%
   c. Increase by 4%
   d. Reduce by 2%

6. Two satellites are at different heights. Which would have greater velocity? **1**

7. By which law is the Kepler's law of areas identical? **1**

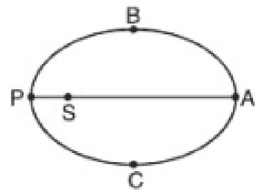
8. Where does a body weight more; at the surface of the earth or in the mine? **1**

9. A uniform ring of mass m and radius a is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is at a distance 33 a from the centre of the sphere. Find the gravitational force exerted by the sphere on the ring. **2**

10. State and briefly explain the law of areas. **2**

11. A body hanging from a spring stretches it by 1 cm at the earth's surface. How much will the same body stretch at a place 1600 klm above the earth's surface? Radius of earth 6400 km. **2**

12. Let the speed of the planet at perihelion P in fig be v_p and Sun planet distance SP be r_p. Relate (r_p, v_p) to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to traverse BAC and CPB? **3**

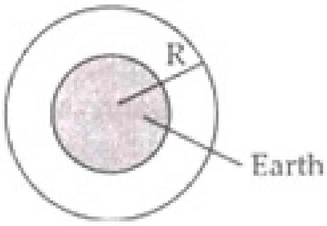


13. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence?

Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; G = $6.67 \times 10^{-11} \mathrm{Nm^2kg^{-2}}$. **3**

14. A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun = 2×10^{30} kg). **3**

15. Show the nature of the following graph for a satellite orbiting the earth. **5**



    i. K.E. *versus* orbital radius *R*.
    ii. P.E. *versus* orbital radius *R*.
    iii. T.E. *versus* orbital radius *R*.

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Answer

1. c. 5.96×10^{11} J

Explanation: Mass of the spaceship, m_s = 1000 kg

Mass of the Sun, M = 2 $\times$ 10^{30}kg

Mass of Mars, M_m = 6.4 $\times$ 10^{23}kg

Orbital radius of Mars, r = 3395 km = 3.395 $\times$ 10^6m

Universal gravitational constant, G = 6.67 $\times$ 10^{-11} Nm^2kg^{-2}

Potential energy of the spaceship due to gravitational attraction of the Sun = -$\frac{GMm_s}{R}$

Potential energy of the spaceship due to gravitational attraction of the Mars = -$\frac{GM_m m_s}{r}$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.

Total energy of the spaceship = $-\frac{GMm_s}{R}$ - $\frac{GM_m m_s}{r}$

$= -Gm_s\left[\frac{M}{R} + \left(\frac{m_m}{r}\right)\right]$

The negative sign indicates that the system is in bound state.

Energy required for launching the spaceship out of the solar system

= - Total energy of the spaceship = $Gm_s\left[\frac{M}{R} + \left(\frac{m_m}{r}\right)\right]$

$= 6.67 \times 10^{-11} \times 10^3 \times \left(\frac{2\times10^{30}}{2.28\times10^{11}} + \frac{6.4\times10^{23}}{3.395\times10^6}\right)$

= 6.67 $\times$ 10^{-8} (87.72 $\times$ 10^{17} + 1.88 $\times$ 10^{17})

= 6.67 $\times$ 10^{-8} $\times$ 89.50 $\times$ 10^{17}

= 596.96 $\times$ 10^9

= 5.96 $\times$ 10^{11} J

2. b. 1.65 hr

Explanation: Mass of the Earth, M_e = 6.0 $\times$ 10^{24}kg

Radius of the Earth, $\text{Re} = 6.4 \times 10^6 m$

Universal gravitational constan t, G = $6.67 \times 10^{-11}\ N\, m^2\ kg^{-2}$

Height of the satellite, h = 780 km = 780 $\times$ 10^3m = 0.78 $\times$ 10^6m

Time Period of the satellite, T = $2\pi\sqrt{\frac{(R_e+h)^3}{GM_e}}$

$$= 2 \times \frac{22}{7} \times \sqrt{\frac{(6.4\times10^6+0.78\times10^6)^3}{6.67\times10^{-11}\times6.0\times10^{24}}}$$
$$= 2 \times \frac{22}{7} \times \sqrt{\frac{(7.18\times10^6)^3}{40\times10^{13}}}$$
$$= 2 \times \frac{22}{7} \times \sqrt{9 \times 10^5}$$
$$= 2 \times \frac{22}{7} \times 948$$
$$= 2 \times 22 \times 135.42$$

= 5958.85 sec = 1.65 hr

3. a. $\frac{1}{2}mgR$

Explanation: Work done$= \frac{mgh}{1+\frac{h}{R}}$

If h = R then,

$\Rightarrow Work\ done = \frac{mgh}{1+\frac{R}{R}} = \frac{1}{2}mgh$

4. d. 1700 kg/m^3

Explanation: Density of Titania = $\frac{\text{Mass Of Titania}}{\text{Volume Of Titania}} = \frac{\frac{M_E}{1700}}{\frac{4}{3}\pi\left(\frac{R_E}{8}\right)^3}$

Here, mass of earth (M_E = 5.97 $\times$ 10^{24}kg) and radius of earth(R_E = 6.38 $\times$ 10^6m)

$$= \frac{\frac{5.97\times10^{24}}{1700}kg}{\frac{4}{3}\pi\left(\frac{6.38\times10^6}{8}m\right)^3}$$
$$= \frac{3.5\times10^{21}kg}{\frac{4}{3}\times3.14\times(0.79\times10^6 m)^3}$$
$$= \frac{3.5\times10^{21}kg}{2.064\times10^{18}m^3}$$

= 1.7 $\times$ 10^3kg/m^3 = 1700 kg/m^3

5. b. Reduce by 4%

Explanation: Escape velocity $v_e \alpha R\sqrt{\rho}$ and if density remains constant

$\Rightarrow v_e\ \alpha$R

So if the radius reduces by 4% then escape velocity also reduces by 4%

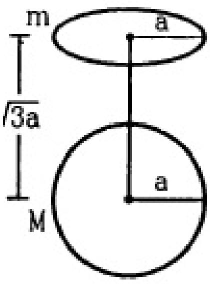
6. $v_a \propto \frac{1}{\sqrt{r}}$. The velocity is inversely proportional to the height. Therefore the satellite at smaller height would possess greater velocity.

7. The Kepler's law of areas is identical to the law of conservation of angular momentum.

8. The value of g in mine is less than that on the surface of the earth. Therefore weight will be more on the surface of earth as compared to the mines.

9. The gravitational field at any point on the ring due to the sphere is equal to the field due to a single particle of mass M placed at the centre of the sphere. Thus, the force on the ring due to the sphere is also equal to the force on it by a particle of mass M placed at this point. By Newton's third law it is equal to the force on the particle by the ring. Now the gravitational field due to the ring at a distance d = $\sqrt{3}$ a on its axis is

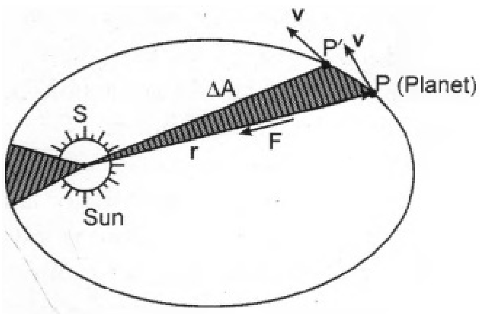
   E = $\frac{Gmd}{\left(a^2+d^2\right)^{3/2}}$ = $\frac{\sqrt{3}Gm}{8a^2}$

   The force on a particle of mass M placed here is F = ME

   = $\frac{\sqrt{3}GMm}{8a^2}$

   This is also the force due to the sphere on the ring.

10. 

   The law of areas or Kepler's second law of planetary motion states that the line joining Sun and the planet sweeps out equal areas in equal intervals of time, howsoever small these intervals may be. Alternately, we can say that the areal velocity of a planet remains constant.

In Figure, if a planet moves from P to P' in a small time interval At, then area swept

ΔA = area SPP'

$= \frac{1}{2}(\vec{v}\Delta t) \times \vec{r}$

and $\frac{\Delta A}{\Delta t} = \frac{1}{2}(\vec{v} \times \vec{r})$ = a constant quantity

11. In equilibrium mg = kg, $g = \frac{GM}{R^2}$

At height h mg' = kx', $g' = \frac{GM}{(R+h)^2}$

$\frac{g'}{g} = \frac{x'}{x} = \frac{R^2}{(R+h)^2}$

$\frac{x'}{x} = \frac{(6400)^2}{(6400+1600)^2} = \frac{6}{25}$

$\therefore x = \frac{16}{25} \times 1cm = 0.64cm$

12. The magnitude of angular momentum at P, $L_p = m_p r_p v_p$

Similarly, magnitude of angular momentum at P is $L_A = m_A r_A$vA

From conservation of angular momentum

$m_p r_p v_p = m_A r_A v_A$ $(m_p = m_A)$

$\frac{v_p}{v_A} = \frac{r_A}{r_p}$

as $r_A > r_p \Rightarrow v_p > v_A$

The magnitude of angular momentum at P, Lp = $m_p r_p v_p$

Similarly, magnitude of angular momentum at A is $L_A = m_A r_A v_A$

From conservation of angular momentum

$m_p r_p v_p = m_A r_A v_A$ $(m_p = m_A)$

$\frac{v_p}{v_A} = \frac{r_A}{r_p}$

as $r_A > r_p \Rightarrow v_p > v_A$

area bounded by SB & SC are (SBAC > SBPC)

$\because$ By 2nd law equal areas are swept in equal intervals of time. Time taken to transverse BAC > time taken to traverse CPB.

13. $Me = 6.0 \times 10^{24}$kg

Mass of the satellite, m = 200 kg

$Re = 6.4 \times 10^6 m$

$G = 6.67 \times 10^{-11}$Nm^{2}kg^{-2}

Height of the satellite, h = 400 km $= 4 \times 10^5 m = 0.4 \times 10^6 m$

Total energy of the satellite at height h =Kinectic energy + Potential energy

Total energy of the satellite at height h$= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e+h}\right)$

Orbital velocity of the satellite, $v = \sqrt{\frac{GM_e}{R_x+h}}$

Total energy=$\frac{1}{2}m\left(\frac{GM_e}{R_e+h}\right) - \frac{GM_e m}{R_e+h} = -\frac{1}{2}\left(\frac{GM_e m}{R_e+h}\right)$

Energy required to send the satellite out of its orbit = -(Bound energy)

$= \frac{1}{2}\frac{GM_e m}{(R_e+h)}$

$= \frac{1}{2} \times \frac{6.67\times10^{-11}\times6.0\times10^{24}\times200}{(6.4\times10^6+0.4\times10^6)}$

$= \frac{1}{2} \times \frac{6.67\times10^{-11}\times12\times10^{26}}{(6.8\times10^6)}$

$= \frac{1}{2} \times \frac{6.67\times12\times10^9}{6.8\times10^6} = 5.9 \times 10^9 J$

14. For the object to remain stuck to the star, the gravitational force of the star must be equal to or greater than the centripetal force. Under this condition, the centrifugal force does not overcome the gravitational force and not fly off the object. It means

mg > mv^2/r

g > v^2/r

g > a_c

Where a_c=v^2/r

Centripital acceleration

for the object to remain stuck, the acceleration due to gravity (g) on the star must be > centripetal acceleration

Gravitational force, $f_g = \frac{GIm}{R^2}$

Where,

M = Mass of the star = $2.5 \times 2 \times 10^{30} = 5 \times 10^{30}$kg

m = Mass of the body

R = Radius of the star = 12 km = 1.2 ×104 m

$\therefore f_z = \frac{6.67\times10^{-11}\times5\times10^{30}\times m}{(1.2\times10^4)^2} = 2.3110^{11} mN$

Centrifugal force,$f_c = mr\omega^2$

ω = Angular speed = 2πν

v = Angular frequency = 1.2 rev s^{-1}

$f_c = mR(2\pi v)^2 = m \times \left(1.2 \times 10^4\right) \times 4 \times (3.14)^2 \times (1.2)^2 = 1.7 \times 10^5 mN$

Since $f_\varepsilon > f_\varepsilon$, the body will remain stuck to the surface of the star.

15. Mass of earth $= M_e$

    Radius of orbit of satellite= R

    Mass of satellite = m

    Orbital Velocity $\nu_0 = \sqrt{\frac{GM}{R}}$

    a. E_K versus R:

       $KE_k = \frac{1}{2}mv_0^2 = \frac{1}{2}m \cdot \frac{GM}{R} = \frac{GMm}{2R}$

       $E_K \propto \frac{1}{R}$ we can say that kinetic energy is inversely proportional to R. It means the KE decreases exponentially with radius. The graph will be a rectangular hyperbola. Hence, the variation of kinetic energy versus orbital radius is shown in graph. $i.e.$, E_k decreases exponentially with R.

    b. E_p versus R: potential energy is twice of kinetic energy and negative sign implies that graph is downward hyperbola. $E_p = \frac{-GMm}{R}$ $E_p \propto -\frac{1}{R}$

    c. T.E. versus R: Negative total energy, E signifies that earth and the satellite is a bounded system.

       If E ≥ U, the satellite will be free from earth's gravity