*Continuity of a function

- A function is said to be continuous at $x=a$ if $\lim_{x \to a} f(x) = f(a)$ otherwise it is discontinuous.

<u>NOTE</u>: Every continuous function may or may not be differentiable but every differentiable function is continuous.

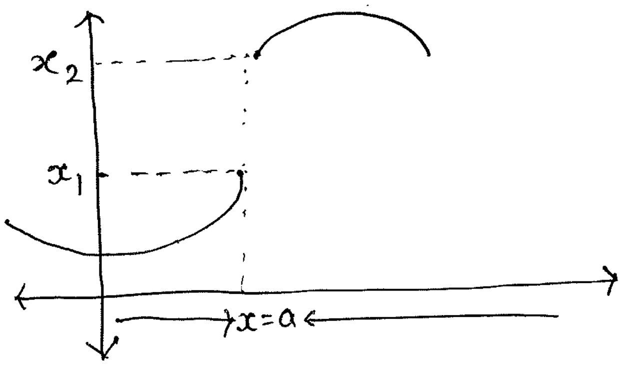
Eg:- $|x|$ → continuous
but $x=0$ not differentiable

$|3x-2|$ → continuous
but not diff. $x = 2/3$.

* Types of discontinuity

① Discontinuity of first kind/**Jumped discontinuous**

- A function is said to be discontinuous of first kind if $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$



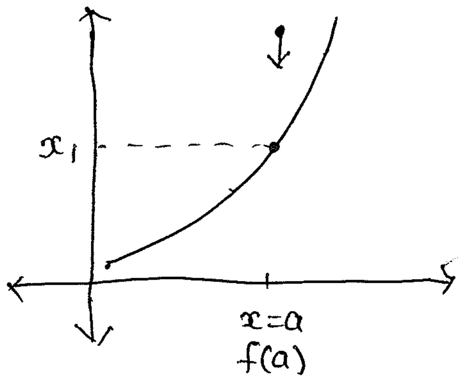
② Discontinuity of second kind/**Essential discontinuity**

(can't remove)

- If either of the limit or both doesn't exists i.e. discontinuity of 2^{nd} kind. Eg:- $\lim_{x\to\infty} \sin x$

③ Discontinuity of third kind/**Removable discontinuity**

- A function is said to have discontinuity of 3^{rd} kind if $\left[\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)\right] \neq f(a)$



we can remove discontinuity by any how but can't make it equal to f(a).

Standard continuous function

① x^n ; $n>0$
$n<0$
$n\neq 0$

② a^x & e^x , $\forall x$

③ $\log x$; $n>0$

④ $|x|$, $\forall x$

⑤ $\sin x$
$\cos x$, $\forall x$

⑥ $\tan x$
$\sec x$, $\forall x$ except $x=(2n+1)\frac{\pi}{2}$

⑦ $\cot x$
$\operatorname{cosec} x$, $\forall x$ except $x=n\pi$

Q:- Check whether the function is continuous or not?

$f(x) = \frac{1}{1+2^{1/x}}$ at $x=0$.

Solⁿ:- $\lim_{x\to 0} f(x)$

~~$L.H.L. = \lim_{x\to 0^-} \frac{1}{1+2^{1/x}} = 0$~~ ~~$R.H.L. = \lim_{x\to 0^+} \frac{1}{1+2^{1/x}}$~~ $= 0$

$f(0) = \frac{1}{1+2^{\infty}} = 0$

$L.H.L. = \lim_{x\to a^-} f(x)$

$= \lim_{h\to 0} f(a-h)$

$= \lim_{h\to 0} \frac{1}{1+2^{\frac{1}{0-h}}}$

$= \frac{1}{1+2^{-\infty}}$

$L.H.S. = 1$

$R.H.L. = \lim_{h\to 0} f(a+h)$

$= \lim_{h\to 0} \frac{1}{1+2^{1/h}}$

$= \frac{1}{1+2^{\infty}}$

$R.H.L. = 0$

$L.H.L \neq R.H.L.$

Discontinuity of 1st kind.

ME-2014

Q:- If a function is continuous at a point?

(a) The limit of the function may not exists at a point

(b) The function must be derivable at the point

(c) The limit of a function at a point tends to ∞

(d) ✓ The limit must exists at a point and the value of limit should be same as the value of the function at that point.

Q:- The value of x for which the function $\frac{x^2-3x+4}{x^2+3x-4} = f(x)$ is not continuous?

Sol^n:- $x^2+3x-4=0$ (denominator should be not zero)

$x^2+4x-x-4=0$

$x(x+4)-1(x+4)=0$

$\underline{x=-4}$ and $\underline{x=1}$

Q *Multipled valued function

Q:-A function $f(x)$ is defined by

$$f(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{2}-x & 0 < x < \frac{1}{2} \\ \frac{1}{x} & x = \frac{1}{2} \\ \frac{3}{2}-x & \frac{1}{2} < x < 1 \\ 1 & x > 1 \end{cases}$$

(a) $f(x)$ is continuous at $x=0$

✓(b) $f(x)$ is discontinuous at $x=\frac{1}{2}$

(c) $f(x)$ is continuous at $x=1$

(d) all are true

Sol^n: (a) $x=0$

$x>0$ → $\frac{1}{2}-x$ → $\frac{1}{2}$

$x=0$ → 0

$x<0$ → 0

(b) $x=\frac{1}{2}$

$x>\frac{1}{2}$

$x=\frac{1}{2}$ → $\frac{1}{2}$

$x<\frac{1}{2}$ → $\frac{1}{2}-x$ → $\frac{1}{2}-\frac{1}{2}$ → 0

Q:- What should be the value of λ the $f(x)$ is continuous at $x = \pi/2$ when the function is

$$f(x) \begin{cases} \dfrac{\lambda \cos x}{\pi/2 - x} & \text{if } x \neq \pi/2 \\ 1 & \text{if } x = \pi/2 \end{cases}$$

Sol:-

$$\cancel{\text{L.H.L.} = \lim_{x \to \pi/2^-} f(x) = \lim_{h \to \pi/2} f(a-h)}$$

$$\lim_{x \to \pi/2} \frac{\lambda \cos x}{\pi/2 - x} \quad \left(\frac{0}{0}\right)$$

$$\lim_{x \to \pi/2} \frac{-\lambda \sin x}{-1} = 1$$

$$\boxed{\lambda = 1}$$

Q:- $\displaystyle\lim_{\substack{x \to 0 \\ y \to 0}} \frac{xy}{x^2+y^2}$ (2016)

$$\lim_{\substack{x \to 0 \\ y \to 0}} \frac{xy}{x^2\left[1+\frac{y^2}{x^2}\right]}$$

$$\lim_{\substack{x \to 0 \\ y \to 0}} \frac{y}{x\left(1+\frac{y^2}{x^2}\right)}$$

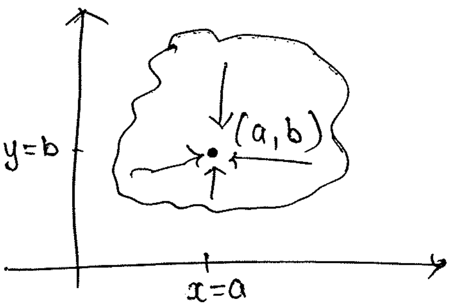
$$\lim_{\substack{x \to 0 \\ y \to 0}} \frac{1}{1\left(1+\frac{2y}{2x}\right)}$$

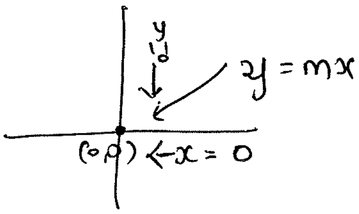
We can approch it from any side i.e. by $x=0$ OR $y=0$ OR $y = mx$ OR $y = mx^2$

∴ Not defined

Limit of two variables

-A number l is said to be limit of a function of multi variable if I take a limit from every path comes to be equal otherwise limit does not exist





$$\lim_{x\to 0} \frac{x \cdot mx}{x^2+m^2x^2}$$

$$\lim_{x\to 0} \frac{m\cancel{x^2}}{\cancel{x^2}(1+m^2)}$$

$$\frac{m}{1+m^2}$$

Different approach will getting different answer. So limit doesn't exists.