CBSE Sample Paper-04
SUMMATIVE ASSESSMENT –I
Class – IX MATHEMATICS

Time allowed: 3 hours | Maximum Marks: 90

General Instructions:

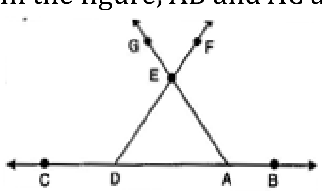
a) All questions are compulsory.
b) The question paper consists of 31 questions divided into four sections – A, B, C and D.
c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
d) Use of calculator is not permitted.

Section A

1. Is the number $\left(3-\sqrt{7}\right)\left(3+\sqrt{7}\right)$ rational or irrational?
2. Find the value of 513^2-512^2.
3. If ΔABC is an isosceles triangle and $\angle$ B $=65^0$, find x.
4. If the perimeter of a rhombus is 20cm and one of the diagonals is 8cm. Find the area of the rhombus.

Section B

5. Multiply $\left(3-\sqrt{5}\right)$ by $\left(6+\sqrt{2}\right)$
6. Using factor theorem, show that (x – y) is a factor of $x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)$
7. Find the remainder when $4x^3-3x^2+2x-4$ is divided by $\left(x-4\right)$.
8. In the figure, AB and AC are opposite rays and $\angle$DAE = $\angle$ADE. Prove that $\angle$BAE = $\angle$CDE.



9. The angles of a triangle are in the ratio 3 : 5 : 10. Find the measure of each angle.
10. Find out the quadrant in which the following points lie:
(i) Point A = $\left(3,-4\right)$ (ii) Point B = $\left(-3,4\right)$
(iii) Point C = $\left(-3,-4\right)$ (iv) Point D = (3, 4)

Section C

11. Simplify: $\dfrac{\sqrt{25}}{\sqrt[3]{64}}+\left(\dfrac{256}{625}\right)^{-\frac{1}{4}}+\dfrac{1}{\left(\dfrac{64}{125}\right)^{\frac{2}{3}}}$

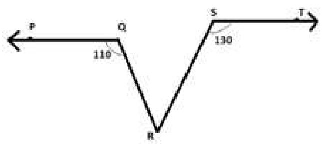
12. Express $0.0\overline{15}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

13. Factories: $\frac{r^3}{8} - \frac{s^3}{343} - \frac{t^3}{216} - \frac{1}{28}rst.$

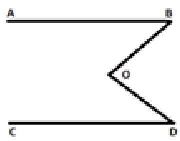
14. Without actually calculating the cubes, find the value of: $\left(\frac{-3}{4}\right)^3 + \left(\frac{-5}{8}\right)^3 + \left(\frac{11}{8}\right)^3$.

15. State any 2 axioms given by Euclid. Give an example for each.

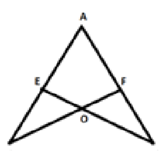
16. In the given figure, if $PQ \parallel ST$, $\angle PQR = 110°$ and $\angle RST = 130°$, find $\angle QRS$.



17. In the given figure, $AB \parallel CD$. Find the measure of reflex $\angle BOD$.



18. In the following figure, it is given that $\angle B = \angle C$ and $BA = BC$. Prove that $\Delta BAF \cong \Delta CAE$.



19. The following table gives measures (in degrees) of two acute angles of a right triangle

X	10	20	30	40	50	60	70	80
Y	80	70	60	50	40	30	20	10

Plot the point and join them.

20. Manish has a vegetable garden in the shape of a rhombus. The length of each side of the garden is 35m and its diagonal is 42m long. After growing the vegetables in it, he wants to divide it in four equal parts. Find the area of each part.

Section D

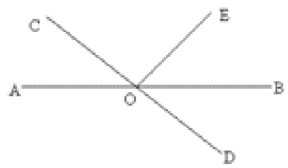
21. If $a = \frac{1}{7 - 4\sqrt{3}}$ *and* $b = \frac{1}{7 + 4\sqrt{3}}$, then find the value of:

(i) $a^2 + b^2$
(ii) $a^3 + b^3$

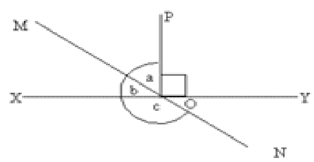
22. Rationalize the denominators of the following: (i) $\frac{1}{\sqrt{5} + \sqrt{2}}$ (ii) $\frac{1}{\sqrt{7} - 2}$

23. If the polynomials $(3x^3 + ax^2 + 3x + 5)$ and $(4x^3 + x^2 - 2x + a)$ leave the same remainder when divided by $(x - 2)$, then find the value of a. Also find the remainder in each case.

24. If the polynomial az3 + 3z – 4 and z3 – 4z + a leave the same remainder when divided by z – 3, find the value of a.
25. The volume of a cuboid is given by the expression $3x^2 - 12x$. Find the possible expressions for its dimensions.
26. Using remainder theorem factorise $x^3 - 3x^2 - x + 3$.
27. In the figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.



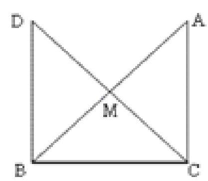
28. In the following figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, find c.



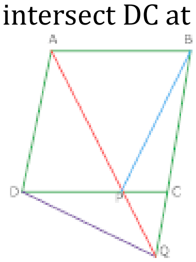
29. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that:

(i) $\Delta AMC \cong \Delta BMD$ (ii) $\angle DBC$ is a right angle

(iii) $\Delta DBC \cong \Delta ACB$ (iv) $CM = \frac{1}{2} AB$



30. Prove that difference of any two sides of a triangle is less than the third side.
31. Show that ar (BPC) = ar (DPQ) if BC is produced to a point Q such that AD = CQ and AQ intersect DC at P



CBSE Sample Paper-04
SUMMATIVE ASSESSMENT –II
Class – IX MATHEMATICS

(Solutions)

SECTION-A

1. $\left(3-\sqrt{7}\right)\left(3+\sqrt{7}\right)=(3)^2-\left(\sqrt{7}\right)^2$
   =9-7=2 which is rational.
2. $513^2-512^2=(513+512)(513-512)=1025\times1=1025$
3. 50^0
4. 24 sq cm
5. $\left(3-\sqrt{5}\right)\left(6+\sqrt{2}\right)$
   $=3\left(6-\sqrt{2}\right)-\sqrt{5}\left(6+\sqrt{2}\right)$
   $=18+3\sqrt{2}-6\sqrt{5}-\sqrt{5}\times\sqrt{2}$
   $=18+3\sqrt{2}-6\sqrt{5}-\sqrt{10}$
6. Let $p(x)=x(y^2-z^2)+y(z^2-x^2)+z(x^2-y^2)$
   putting x = y in given polynomial p(x), we get
   $p(y)=y(y^2-z^2)+y(z^2-y^2)+z(y^2-y^2)$
   $=y(y^2-z^2)-y(y^2-z^2)=0$
   $\therefore$ (x – y) is a factor of given polynomial p(x).
7. By remainder theorem,
   $f(4)=4(4)^3-3(4)^2+2\times4-4 \quad \Rightarrow \quad f(4)=4\times64-3\times16+2\times4-4$
   $\Rightarrow \quad f(4)=256-48+8-4=212$
8. $\angle$BAE + $\angle$EAC = 180° [Linear pair] ……….(i)
   And $\angle$EDA + $\angle$EDC = 180° [Linear pair] ……….(ii)
   From eq. (i) and (ii), we have
   $\angle$BAE + $\angle$EAC = $\angle$EDA + $\angle$EDC
   $\Rightarrow$ $\angle$BAE + $\angle$EAC = $\angle$DAE + $\angle$EDC [Given $\angle$DAE = $\angle$ADE]
   $\Rightarrow$ $\angle$BAE = $\angle$CDE
9. Let a triangle ABC and $\angle$A : $\angle$B : $\angle$C = 3 : 5 : 10
   Let the angles be $\angle$A = $3x$, $\angle$B = $5x$ and $\angle$C = $10x$
   $\because$ $\angle$A + $\angle$B + $\angle$C = 180°
   $\Rightarrow \quad 3x+5x+10x=180^\circ \quad \Rightarrow \quad 18x=180^\circ \quad \Rightarrow \quad x=10$
   $\therefore$ Angles are $30^\circ, 50^\circ$ and 100°.
10. (i) Point A lies in the fourth quadrant, since its abscissa is positive and ordinate is negative.
    (ii) Point A lies in the second quadrant, since its abscissa is negative and ordinate is positive.

(iii) Point A lies in the third quadrant, since both abscissa and ordinate arenegative.
(iv) Point A lies in the first quadrant, since both abscissa and ordinate are positive

11. $\frac{\sqrt{25}}{\sqrt[3]{64}}+\left(\frac{256}{625}\right)^{-\frac{1}{4}}+\frac{1}{\left(\frac{64}{125}\right)^{\frac{2}{3}}}$

$=\frac{\sqrt{5\times5}}{\sqrt[3]{4\times4\times4}}+\left(\frac{625}{256}\right)^{\frac{1}{4}}+\left(\frac{125}{64}\right)^{\frac{2}{3}}$

$=\frac{5}{4}+\left(\frac{5^4}{4^4}\right)^{\frac{1}{4}}+\left(\frac{5^3}{4^3}\right)^{\frac{2}{3}}$

$=\frac{5}{4}+\left(\frac{5}{4}\right)^{4\times\frac{1}{4}}+\left(\frac{5}{4}\right)^{3\times\frac{2}{3}}$

$=\frac{5}{4}+\frac{5}{4}+\left(\frac{5}{4}\right)^2=\frac{5}{4}+\frac{5}{4}+\frac{25}{16}$

$=\frac{20+20+25}{16}=\frac{65}{16}$

12. Let x= $0.0\overline{15}$
Then, x=2.0151515….
$\Rightarrow\ 10x=20.151515\ldots$
$10x=20+0.51515\ldots$
Let y=0.151515…
$\Rightarrow\ 100y=15.151515\ldots$
Subtracting (ii) from (iii), we get
100y-y=15.1515….-0.151515…..
99y=15
$y=\frac{15}{99}\Rightarrow y=\frac{5}{33}$
Now $10x=20+\frac{5}{33}\Rightarrow 10x=\frac{660+5}{33}$
$\Rightarrow 10x=\frac{665}{33}\Rightarrow x=\frac{665}{330}\Rightarrow x=\frac{133}{66}$

13. $\frac{r^3}{8}-\frac{s^3}{343}-\frac{t^3}{216}-\frac{1}{28}rst.$

$=\left(\frac{r}{2}\right)^3+\left(\frac{-s}{7}\right)^3+\left(\frac{-t}{16}\right)^3-3\left(\frac{r}{2}\right)\left(\frac{-s}{7}\right)\left(\frac{-t}{6}\right)$

$=\left[\frac{r}{2}+\left(\frac{-s}{7}\right)+\left(\frac{-t}{6}\right)\right]\left[\left(\frac{r}{2}\right)^2+\left(\frac{-s}{7}\right)^2+\left(\frac{-t}{6}\right)^2-\frac{r}{2}\left(\frac{-s}{7}\right)-\left(\frac{-s}{7}\right)\left(\frac{-t}{6}\right)-\left(\frac{-t}{6}\right)\left(\frac{r}{2}\right)\right]$

$$=\left(\frac{r}{2}-\frac{s}{7}-\frac{t}{6}\right)\left(\frac{r^2}{4}+\frac{s^2}{49}+\frac{t^2}{36}+\frac{rs}{14}-\frac{st}{42}+\frac{tr}{12}\right)$$

14. Let $a=\frac{-3}{4}, b=\frac{-5}{8}, c=\frac{11}{8}$

$\therefore \quad a+b+c=\frac{-3}{4}-\frac{5}{8}+\frac{11}{8}$

$=\frac{-6-5+11}{8}=0$

If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

$\therefore \quad \left(\frac{-3}{4}\right)^3+\left(\frac{-5}{8}\right)^3+\left(\frac{11}{8}\right)^3=3\left(\frac{-3}{4}\right)\left(\frac{-5}{8}\right)\left(\frac{11}{8}\right)=\left(\frac{495}{256}\right)$

15. AXIOMS

1. Things which are equal to the same thing are equal to one another.
2. If equals are added to equals , the wholes are equal.

Give examples yourself.

16. Through point R , draw a line ARB parallel to PQ and parallel to ST .

Let $\angle ARQ=x$ and $\angle BRS=y$

Since, $PQ \| AR$ and QR is transversal

$110°+x=180°$ (co-interior angles are supplementary)

$x=70°$

Similarly, $y=50°$

Since, ARB is straight line, $\therefore \angle ARB=180°$

$\Rightarrow x+\angle QRS+y=180°$

$\Rightarrow \angle QRS=60°$

17. Draw $EO \| AB$

Then, $\angle 1+\angle 2=x$

Now, $EO \| AB$ and BO is the transversal,

$\therefore \angle 1+\angle ABO=180°$ (co-int. angles)

$\angle 1=140°$

Similarly, $\angle 2=145°$

$\therefore \angle 1+\angle 2=285°$

Hence, reflex $\angle BOD=285°$

18. In ΔBOE and ΔCOF we have

$\angle B = \angle C$

And $\angle BOE = \angle COF$ (vertically opposite angles)

$\therefore \angle B + \angle BOE = \angle C + \angle COF$

$\Rightarrow 180° - \angle BEO = 180° - \angle CFO$ (by angle sum property)

$\Rightarrow \angle BEO = \angle CFO$ (1)

Now, $\angle BEO + \angle OEA = 180°, \angle CFO + \angle OFA = 180°$ (angles of a linear pair)

$\therefore \angle BEO + \angle OEA = \angle CFO + \angle OFA$

$\angle OEA = \angle OFA, \angle CEA = \angle BFA$ (using (1), $\angle OEA = \angle CEA$ and $\angle OFA = \angle BFA$ ....(2))

Now, in ΔBAF and ΔCAE, we have

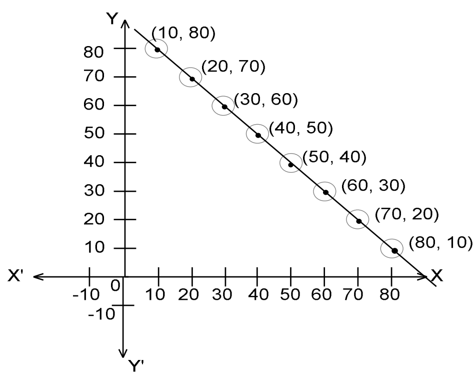
$\angle B = \angle C$ (given)

$\angle CEA = \angle BFA$ (from (2))

$BA = AC$ (given)

$\therefore \Delta BAF \cong \Delta CAE$ (by AAS)

19.



20. Let $ABCD$ be the garden

$DC = 35m, DB = 42m$

Draw $CE \perp DB$

$DE = \frac{1}{2} DB = \frac{1}{2} \times 42 = 21m$

$CE = 28m$ (by pythagoras theorem)

Area of $\Delta DBC = \frac{1}{2} \times DB \times CE = 588 sq.m$

$\therefore$ area of garden $ABCD = 2 \times 588 = 1176 sq.m$

Area of each part $= \frac{1176}{4} = 294 sq.m$

21. $a=\dfrac{1}{7-4\sqrt{3}}=\dfrac{1}{7-4\sqrt{3}}\times\dfrac{7+4\sqrt{3}}{7+4\sqrt{3}}=\dfrac{7+4\sqrt{3}}{7^2-\left(4\sqrt{3}\right)^2}$

$=\dfrac{7-4\sqrt{3}}{49-16\times 3}=\dfrac{7-4\sqrt{3}}{49-48}$

$\therefore\ a=\dfrac{1}{7-4\sqrt{3}}=7+4\sqrt{3}$

$b=\dfrac{1}{7+4\sqrt{3}}=\dfrac{1}{7+4\sqrt{3}}\times\dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}=\dfrac{7-4\sqrt{3}}{7^2-\left(4\sqrt{3}\right)^2}$

$\therefore\ \dfrac{7-4\sqrt{3}}{49-16\times 3}=\dfrac{7-4\sqrt{3}}{49-48}$

$\therefore\ a+b=7+4\sqrt{3}+7-4\sqrt{3}=14$

And $ab=\left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)$

$=7^2-\left(4\sqrt{3}\right)^2=49-16\times 3$=49-48

$\Rightarrow$ab=1

Now, $a^2+b^2=\left(a+b\right)^2-2ab=\left(14\right)^2-2\times 1=196-2$

$\therefore\ a^2+b^2=194$

Now, $a^3+b^3=\left(a+b\right)^3-3ab\left(a+b\right)=\left(14\right)^2-3\times 1\left(14\right)$

$\therefore\ a^3+b^3=2744-42=2702$

22. (i) $\dfrac{1}{\sqrt{5}+\sqrt{2}}$

We need to multiply the numerator and denominator of $\dfrac{1}{\sqrt{5}+\sqrt{2}}$ by $\sqrt{5}-\sqrt{2}$, to get

$\dfrac{1}{\sqrt{5}+\sqrt{2}}\times\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\dfrac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$.

We need to apply the formula $\left(a-b\right)\left(a+b\right)=a^2-b^2$ in the denominator to get

$\dfrac{1}{\sqrt{7}-2}=\dfrac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-\left(2\right)^2}$

$=\dfrac{\sqrt{7}+2}{7-4}$

$$=\frac{\sqrt{7}+2}{3}$$

$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$, we get $\frac{\sqrt{5}-\sqrt{2}}{3}$.

(ii) $\frac{1}{\sqrt{7}-2}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-2}$ by $\sqrt{7}+2$, to get

$$\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-2}$, we get $\frac{\sqrt{7}+2}{3}$.

23. Let $f(x)=3x^3+ax^2+3x+5$ and $p(x)=4x^3+x^2-2x+a$

Divisor = $(x-2)$ then remainder = $f(2)$ and $p(2)$.

According to the question,

$f(2)=p(2)$

$\Rightarrow 3(2)^3+a(2)^2+3(2)+5=4(2)^3+(2)^2-2(2)+a$

$\Rightarrow (3\times8)+4a+6+5=4\times8+4-4+a$

$\Rightarrow 24+4a+11=32+a$

$\Rightarrow 35+4a=32+a$

$\Rightarrow 3a=-3$

$\Rightarrow a=-1$

24. Let $p(z)=az^3+4z^2+3z-4 \quad and \quad q(z)=z^3-4z+a$

When p(z) is divided by z – 3 the remainder is given by,

$$p(3) = a\times 3^3 + 4\times 3^2 + 3\times 3 - 4 = 27a + 36 + 9 - 4$$
$$p(3) = 27a + 41 \qquad ...(i)$$

When q(z) is divided by z – 3 the remainder is given by,

$$q(3) = 3^3 - 4\times 3 + a = 27 - 12 + a$$
$$q(3) = 15 + a \qquad ...(ii)$$

According to questions, p(3) =q(3)

$$\Rightarrow 27a + 41 = 15 + a \quad \Rightarrow \quad 27a - a = -41 + 15$$
$$26a = -26$$
$$\Rightarrow a = \frac{-26}{26} \quad \Rightarrow \quad a = -1$$

25. The volume of cuboid is given by

$3x^3 - 12x = 3x(x^2 - 4)$ = 3x (x+2) (x-2)

Dimensions of the cuboid are given by 3x, (x+2) and (x-2)

P(1) = $1^3 - m\times 1^2 - 13\times 1 + n = 0$

= 1-m-13+n = 0

= -m+n =12 (1)

x+3 is factor of P(x)

$\therefore$ P(-3) = 0

P(-3) = $(3)^3 - m(-3)^2 - 13\times(-3) + n = 0$

= -27-9m+39+n = 0

= -9m+n 12 =0 (2)

= -9m+n = -12

Subtracting eq. (2) from (1)

8m = 24, m = 3

Put m = 3 in eq (1)

m = 3 and n = 15

26. $x^3 - 3x^2 - x + 3$

Coefficient of x^3 is 1

Constant =3

3×1 = 3

$\therefore$ We can Put x= $\pm$ 3 and ($\bar{X}$) and check

Put= x=1

$1^3 - 3\times 1^2 - 1 + 3$

1 – 3 – 1 + 3 = 0

Remainder =0

$\because x-1$ *is factor of* x^3-3x^2-x+3

$$
\begin{array}{r|l}
 & x^2-2x-3 \\ \hline
x-1 & x^3-3x^2+3 \\
 & \underline{x^3-x^2} \\
 & -2x^2-x+3 \\
 & \underline{-2x^2+2x} \\
 & -3x+3 \\
 & \underline{-3x+3} \\
 & 0
\end{array}
$$

$\therefore\ x^3-3x^2x+3=(x-1)\ (x^2-2x-3)$

$= (x-1)\ (x^2-3x+x-3)$

$= (x-1)\ [x(x-3)\ +1\ (x-3)]$

$= (x-1)\ (x-3)\ (x+1)$

27. Given:- $\angle AOC=\angle BOE=70°$ …………………… equation (i)

And $\angle BOD=40°$

Now, putting the value of equation (ii) in equation (i),

$\angle AOC=\angle BOE=70°$

$\Rightarrow 40°+\angle BOE=70°$

$\Rightarrow \angle BOE=70°-40°$

$\Rightarrow \angle BOE=30°$

Now, $\angle AOC+\angle BOE+\angle COE=180°$

(Angles at a common point on a line)

$\Rightarrow 70°+\angle COE=180°$

$\Rightarrow \angle COE=180°-70°$

$\Rightarrow \angle COE=110°$

Reflex $\angle COE=360°-110°=250°$

Hence, $\angle BOE=360°$

And reflex $\angle COE=250°$

28. Given:- $\angle POY=90°$

And a : b = 2 : 3

Therefore, $\frac{a}{b}=\frac{2}{3}$

$\Rightarrow a = \frac{2}{3}b$ …………………….. equation (i)

Now, $\angle POX + \angle POY = 180°$

$\angle POX + 90° = 180°$

$\angle POX = 180° - 90°$

$\angle POX = 90°$

$a + b = 90°$ (therefore, $\angle POX = a + b$)

$\frac{2}{3}b + b = 90°$

$\frac{2b + 3b}{3} = 90°$

= 2b + 3b = $90° \times 3$

= 5b = $270°$

$= b = \frac{270°}{5}$

$\Rightarrow b = 54°$

Putting the value of b in equation (i)

$a = \frac{2}{3}b$

Or, $a = \frac{2}{3} \times 54° = 36°$

Now, $b + c = 180°$ {Angles at a common point on a line}

$\Rightarrow c = 126°$

29. In ΔAMC and ΔBMD

BM = AM (M is midpoint)

DM = CM (given)

$\angle DMB = \angle AMC$ (opposite angles)

So, $\Delta AMC \cong \Delta BMD$

Hence, DB = AC

$\angle DBA = \angle BAC$

So, $DB \parallel AC$ (alternate angles are equal)

So, $\angle BDC = \angle ACB$ = right angle

(internal angels are complementary in

Case of tranversal of parallel lines)

ΔDBC and ΔACB

DB = AC (proved earlier)

BC = BC (Commo side)

$\angle BDC = \angle ACB$ (proved earlier)

So, $\Delta DBC \cong \Delta ACB$

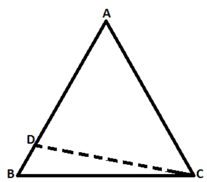
So, AB = DC

SO, AM = BM = CM = DM

So, $CM = \frac{1}{2} AB$

30. **To prove**: $AB - AC < BC$

Construction: From AB cut $AD = AC$. Join D and C



Proof: $AD = AC$

$\Rightarrow \angle ADC = \angle ACD$ (angles opposite to equal sides are equal)

In ΔADC, ext. $\angle BDC > \angle ACD$ (ext. angle of a triangle is greater than its int. opp. Angle)

$\Rightarrow \angle BDC > \angle ADC$

Similarly, in ΔBDC

Ext. $\angle ADC > \angle BCD$

$\Rightarrow \angle BDC > \angle ADC > \angle BCD$

$\Rightarrow \angle BDC > \angle BCD$

$\therefore$ In $\Delta BDC, \angle BDC > \angle BCD$

$\Rightarrow BC > BD$

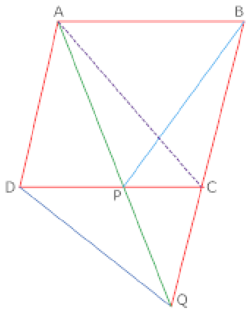
$BD < BC$

$AB - AD < BC$

$AB - AC < BC (\because AD = AC)$

31. Join AC

$$ar(\Delta BCP) = ar(\Delta APC)\ldots\ldots\ldots(i)$$
$$AD = CQ$$
$$AD \parallel BC$$
$$AD \parallel CQ$$



Hence, a pair of opposite side AD and CQ of the quadrilateral ADQC is equal and parallel.

In ΔAPC and ΔQPD,

$$AP = QP$$
$$CP = DP$$
$$\angle APC = \angle QPD$$
$$\Delta APC \cong \Delta QPD$$
$$ar(\Delta APC) = ar(\Delta QPD)\ldots\ldots\ldots(ii)$$

From (i) and (ii)

$$ar(\Delta BCP) = ar(\Delta QPD)$$
$$ar(BPC) = ar(DPQ)$$