ICSE SEMESTER 2 EXAMINATION

SAMPLE PAPER - 4

MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hours

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 10 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

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*Attempt **all** questions from **Section A** and **any three** questions from **Section B.***

SECTION A

*(Attempt **all** questions from this Section.)*

Section-A (Attempt all questions)

Question 1.

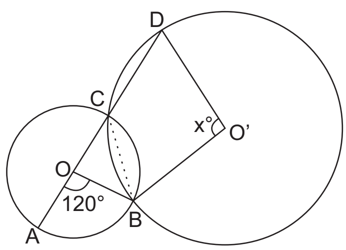
Choose the correct answers to the questions from the given options. (Do not copy the question, write the correct answer only.)

(i) The ratio in which the line segment joining of (– 1, 3) and (4, 2) divided by the y-axis is:

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

(ii) In the given Fig. $\angle AOB = 120°$ $\angle BO'D =$

(a) 100° (b) 80° (c) 120° (d) 90°



(iii) If the radius of a right circular cone is halved and the height is doubled the percentage of increase/decrease in its volume is:

(a) Increased by 50% (c) Decreased by 25%

(b) Increased by 100% (d) Decreased by 50%

(iv) The reflection of the point (2 , 3) in the line $y = 1$ is:

(a) (1,3) (b) (3,3) (c) (2,4) (d) (2 , – 1)

(v) $\dfrac{\cos A}{1-\sin A} - \tan A =$

(a) cos A (b) sec A (c) sin A (d) cosec A

(vi) In a frequency distribution mid value of a class is 10 and the class width is 6, then the lower limit of the class is:

(a) 4 (b) 7 (c) 5 (d) 3

(vii) If the line $2y = 3x + 2$ and $y = ax + 5$ are perpendicular to each other, find the value of a:

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $-\frac{3}{2}$

(viii) The height and the radius of a right circular cylinder is in the ratio 3:2 and its volume is $96\pi cm^3$. Its curved surface area is:

(a) $52\pi\ cm^2$ (b) $48\pi\ cm^2$ (c) $96\pi\ cm^2$ (d) $24\pi\ cm^2$

(ix) The median class of the following frequency distribution table is:

Class	Frequency
0-10	9
10-20	3
20-30	12
30-40	4
40-50	12

(a) 10-20 (b) 20-30 (c) 30-40 (d) 40-50

(x) If a pair of dice is tossed, the probability of getting the sum of the numbers on the top more than 12 is:

(a) 1 (b) 1/36 (c) 1/2 (d) 0

Section-B (Attempt any three questions from this Section.)

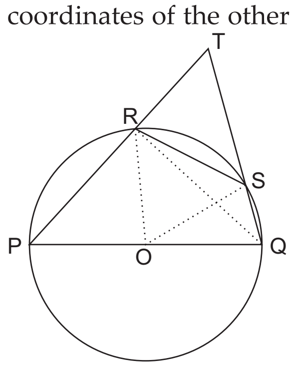
Question 2.

(i) One end of the diameter of a circle is (– 2, 3) and its centre is (4, 5), find the coordinates of the other end.

(ii) One card is drawn from a pack of 52 playing cards, find the probability that the drawn card is:

(a) Black face card (b) Numbered card of hearts

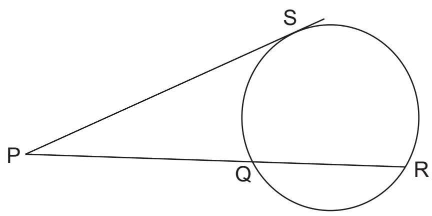
(iii) PQ is the diameter of the circle, RS is a chord equal to the radius of the circle. PR and QS when extended intersect at point T. Find ∠PTQ.



(iv) The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 20 m vertically above the first is 30°. Find the height of the tower.

Question 3.

(i) In the given figure, PS = 18 cm and PQ = 12 cm, then find the length of QR.



(ii) A well with inner radius 4 m is dug 7 m deep. Earth taken out of it has been spread evenly all around a width of 3 m to form an embankment. Find the height of the embankment.

(iii) Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

(iv) Draw Less than ogive for the following distribution:

Class	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	10	8	12	24	6	25	15

Question 4.

(i) Find the equation of a straight line which cuts the positive axis with sum of its intercept is 4 and passing through (1, 1).

(ii) Given below are weekly wages of 200 workers in a small factory:

Weekly wages in ₹	80-100	100-120	120-140	140-160	160-180
No. of workers	20	30	20	40	90

Calculate the mean weekly wages of the workers.

(iii) A roadroller, in the shape of cylinder, has a diameter 1.4 m and its width is 2.5 m. Find the least number of revolutions that the roller must make in order to level a playground of size 55 m by 35 m.

(iv) The points (0,3), (3,5) and (3,0) are the vertices of a triangle:

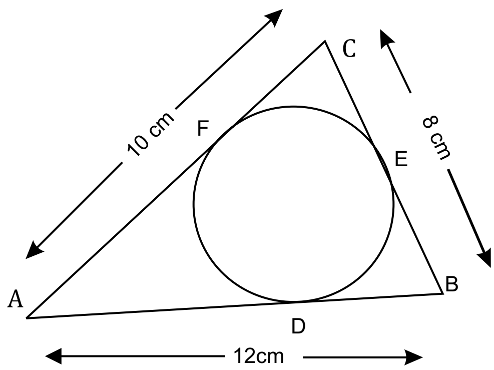
(a) Plot the points on a graph paper

(b) Draw the triangle by reflecting the points on Y axis

(c) Write whether the triangles are congruent or not

Question 5.

(i) A circle is inscribed in a triangle ABC having sides 8 cm, 10 cm and 12 cm as shown in the figure. Find AD, BE and CF.



(ii) Prove that : $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

(iii) A straight line passing through the points (1, 3) and (5, 2) and cuts the x-axis and y-axis at A and B respectively and M is the mid point of AB:

(a) Find the equation of the line.

(b) Find the co-ordinates of A and B.

(c) Find the co-ordinates of M.

(iv) Draw the histogram of the following data with class size 4.

Class Mark	7	11	15	19	23	27
Frequency	8	5	9	3	4	5

Question 6.

(i) Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is:

(a) 8 (b) 13 (c) less than or equal to 12

(ii) Find the point that divides (2, 5) and (– 3, 12) in the ratio 2 : 3.

(iii) An airplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the airplane in km/hr.

(iv) The mean of the following frequency table is 50. But the frequencies f_1 and f_2 in class 20-40 and 60-80 are missing . Find the missing frequencies.

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

Answers

Section-A

Answer 1.

(i) (c) 1 : 4

Explanation :

Let the ratio be k : 1.

Hence by section formula,

$$\frac{k \times 4 + 1 \times -1}{k+1} = 0 \quad \text{(as } x\text{-coordinate on the } y\text{-axis is zero)}$$

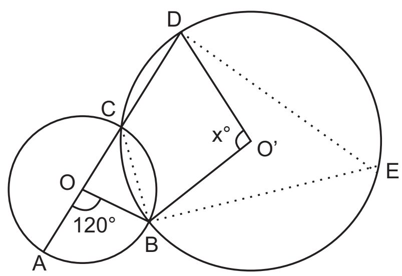
i.e., $4k - 1 = 0$

$\Rightarrow$ $k = \frac{1}{4}$

$\therefore$ Ratio is 1 : 4.

(ii) (c) 120°

Explanation :



$\angle$BOC = 180° – 120° (Linear Pair)

= 60°

$\angle$OCB = $\angle$OBC = 60° (OC = OB - Radius of the same triangle and hence ΔOBC is equilateral)

$\angle$BCD = 180° – 60° = 120° (Linear Pair)

$\angle$DEB = 180° – 120° = 60° (Opposite angles of a cyclic quadrilateral are supplementary)

$\angle$BO'D = 2 × 60° = 120°.

(iii) (d) Decreased by 50%

Explanation :

Let the radius of the cone be 'r' and its height be 'h'.

$$\text{Original volume (V)} = \frac{1}{3} \times \pi \times r^2 \times h$$

If the radius is halved and height is doubled,

$$\text{New volume} = \frac{1}{3} \times \pi \times \left(\frac{r}{2}\right)^2 \times 2h$$

$$= \frac{1}{2} \times \frac{1}{3} \times \pi \times r^2 \times h = \left(\frac{1}{2}\right) \text{V}$$

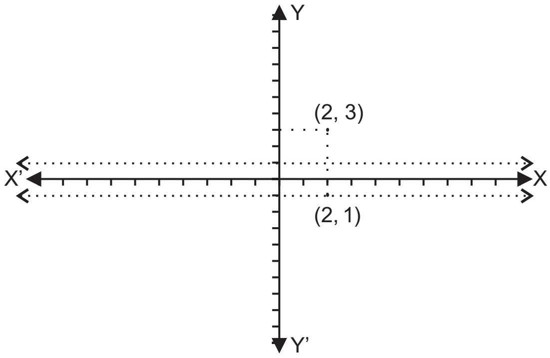
$\therefore$ Decreased volume = $\text{V} - \left(\frac{1}{2}\right)\text{V} = \left(\frac{1}{2}\right)\text{V}$

and Original Volume = V

Hence, Reduced volume = $\frac{\left(\frac{1}{2}\right)\text{V}}{\text{V}} \times 100$ = 50% Decrease.

(iv) (d) (2, – 1)

Explanation :



(v) (b) sec A

Explanation :

$$\frac{\cos A}{1-\sin A}-\tan A = \frac{\cos A(1+\sin A)}{(1-\sin A)(1+\sin A)}-\tan A$$

$$= \frac{\cos A(1+\sin A)}{(1-\sin^2 A)}-\tan A$$

$$= \frac{\cos A(1+\sin A)}{\cos^2 A}-\tan A$$

$$= \frac{(1+\sin A)}{\cos A}-\tan A$$

$$= \frac{1}{\cos A}+\frac{\sin A}{\cos A}-\tan A$$

= sec A + tan A – tan A

= sec A.

(vi) (b) 7

Explanation :

Let x_1 and x_2 be the lower and upper limit of the class.

Hence, $x_2 - x_1 = 6$

and $\frac{x_1 + x_2}{2} = 10$

i.e., $x_2 + x_1 = 20$ ...(1)

and $x_2 - x_1 = 6$

$-\quad+\quad-$ ...(2)

Subtracting, $2x_1 = 14$

$\Rightarrow$ $x_1 = 7$

(vii) (c) $-\frac{2}{3}$

Explanation :

Given, $2y = 3x + 2$

$\Rightarrow$ $y = \left(\frac{3}{2}\right)x + 1$

$\therefore$ slope $m_1 = \frac{3}{2}$

and $y = ax + 5$

$\therefore$ slope $m_2 = a$

But as the lines are perpendicular,

$\therefore$ $m_1 m_2 = -1$

$$\left(\frac{3}{2}\right)a = -1$$

$\Rightarrow$ $a = -\frac{2}{3}$.

(viii) (b) 48π cm^2

Explanation :

Let $r = 2a$ and $h = 3a$

Given Volume = $\pi r^2 h = 96\pi$ cm^3

$\Rightarrow$ $\pi \times (2a)^2 \times 3a = 96\pi$

$\Rightarrow$ $12a^3 = 96$

$\Rightarrow$ $a^3 = 8$

$\Rightarrow$ $a = 2$

$\therefore$ $r = 2 \times 2 = 4$ cm and $h = 2 \times 3 = 6$ cm

Hence, Curved surface area $= 2\pi rh$

$= 2 \times \pi \times 4 \times 6$

$= 48\pi$ cm^2.

(ix) (b) 20-30

Explanation :

Class	Frequency	Cumulative frequency (less than)
0-10	9	9
10-20	3	12
20-30	12	24
30-40	4	28
40-50	12	40
Total (N)	40	

N = 40

$\therefore$ $\frac{N}{2} = \frac{40}{2} = 20$

$\because$ 20 belongs to the class 20-30.

Hence, the median class is 20-30.

(x) (d) 0

Explanation :

The possible outcomes are {(1, 1), (1, 2), (1, 3), … (6, 6)}

Total No. of possible outcomes = 36

$\because$ The maximum number as sum is 6 + 6 = 12

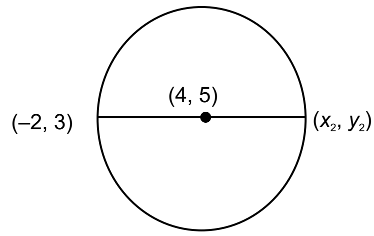
Hence, there is no favourable outcome.

$$\text{P(getting sum more than 12)} = \frac{0}{36} = 0.$$

Section-B

Answer 2.

(i) $$\frac{-2+x_2}{2} = 4, \frac{3+y_2}{2} = 5$$



$\Rightarrow$ $$-2+x_2 = 8, 3+y_2 = 10$$

$\Rightarrow$ $$x_2 = 10, y_2 = 7$$

Hence, the coordinate of the other end is (10, 7).

(ii) (a) Total number of cards = 52

No. of black face cards = 6

$$\text{P(Black face card)} = \frac{\text{No. of favourable outcome}}{\text{Total number of cards}} = \frac{6}{52} = \frac{3}{26}$$

(b) Total number of cards = 52

No. of numbered cards of hearts = 9

$$\text{P(Numbered card of hearts)} = \frac{\text{No. of favourable outcomes}}{\text{Total number of cards}} = \frac{9}{52}$$

(iii) $\angle PRQ = 90°$ ($\because$ angle substended on a semi circle is 90°)

$\because$ RS = OR = OS ($\because$ ORS is an equilateral triangle)

$\therefore$ $\angle ROS = 60°$

$\angle RQS = 30°$ (angle subtended on chord RS on point Q of the circle is half of the angle subtended at the center *i.e.*, $\angle ROS$)

Now, $\angle QRT + \angle RQT + \angle RTQ = 180°$ (angle sum property)

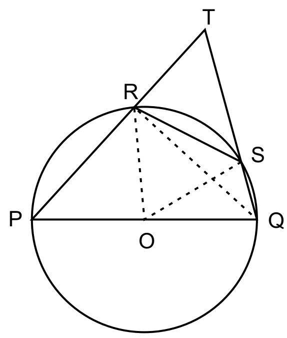
$\Rightarrow$ $90° + 30° + \angle RTQ = 180°$

$\Rightarrow$ $120° + \angle RTQ = 180°$

$\Rightarrow$ $\angle RTQ = 180° - 120°$

$\Rightarrow$ $\angle RTQ = 60°$

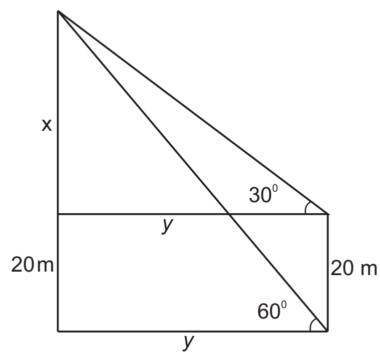
$\Rightarrow$ $\angle PTQ = 60°$.



(iv) $$\tan 30° = \frac{x}{y}$$

$\Rightarrow$ $$\frac{1}{\sqrt{3}} = \frac{x}{y}$$

$\Rightarrow$ $$y = x\sqrt{3} \quad \ldots\text{(i)}$$



and $$\tan 60° = \frac{x+20}{y}$$

$\Rightarrow$ $$\sqrt{3} = \frac{x+20}{y}$$

$\Rightarrow$ $$\frac{x+20}{x\sqrt{3}} = \sqrt{3}$$ [from (i)]

$\Rightarrow \quad x + 20 = 3x$

$\Rightarrow \quad 2x = 20$

$\Rightarrow \quad x = 10$

$\therefore$ Height of the tower $= 20 + x = 20 + 10 = 30$ m.

Answer 3.

(i) By tangent secant theorem, $PS^2 = PR \times PQ$

$\Rightarrow \quad 18^2 = 12 \times PR$

$\Rightarrow \quad 324 = 12 \times PR$

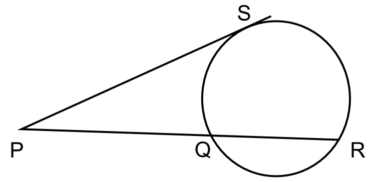
$\Rightarrow \quad PR = 27$

$\therefore \quad PQ + QR = 27$

$\Rightarrow \quad 12 + QR = 27$

$\Rightarrow \quad QR = 27 - 12$

$\Rightarrow \quad QR = 15$ cm



(ii) Given, Radius of well, $r = 4$ m,

Height of well, $h = 7$ m,

Outer radius of embankment $R = 4 + 3$

$= 7$ m

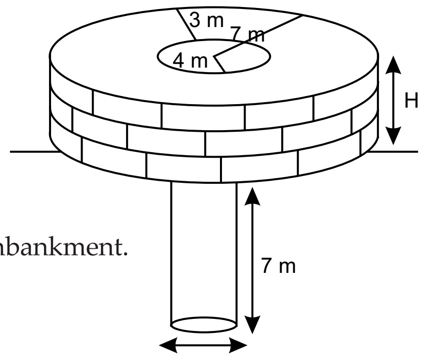
Height of embankment H = ?

Volume of the earth taken out will be equal to the volume of the embankment.

i.e., $\pi r^2 h = \pi(R^2 - r^2)H$

$\Rightarrow \quad 4 \times 4 \times 7 = (7^2 - 4^2) \cdot H$

$\Rightarrow \quad H = \dfrac{112}{33} = 3{\cdot}39$ m.



(iii) L.H.S. $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A \sec^2 A + 2 \cos A \sec A$

$= \sin^2 A + \cos^2 A + (1 + \tan^2 A) + (1 + \cot^2 A) + 2 \sin A \operatorname{cosec} A + 2 \cos A \sec A$

(As $\sec^2 A = 1 + \tan^2 A$ and $\operatorname{cosec}^2 A = 1 + \cot^2 A$)

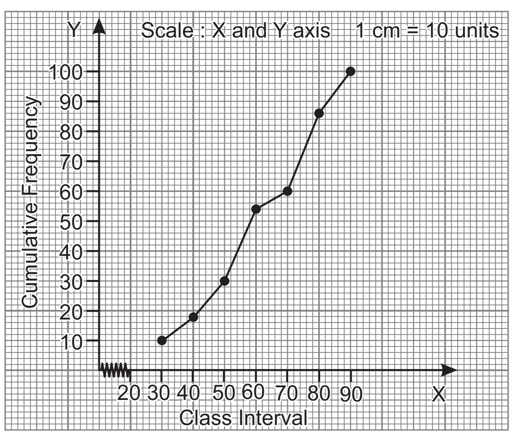
$= 1 + 1 + \tan^2 A + 1 + \cot^2 A + 2 + 2$

(As $\sin A \operatorname{cosec} A = 1$, $\cos A \sec A = 1$ and $\sin^2 A + \cos^2 A = 1$)

$= 7 + \tan^2 A + \cot^2 A =$ R.H.S.

(iv)

Class	Frequency	Less than *c.f.*
20-30	10	10
30-40	8	18
40-50	12	30
50-60	24	54
60-70	6	60
70-80	25	85
80-90	15	100
	100	



Answer 4.

(i) Let the coordinates of the intercepts be $(a, 0)$ and $(0, b)$.

Given, $$a + b = 4$$

Let the equation of the line be

$$\frac{x}{a} + \frac{y}{b} = 1$$

as it is passing through (1, 1).

$$\frac{1}{a} + \frac{1}{b} = 1$$

$\Rightarrow$ $b + a = ab$

$\Rightarrow$ $(4 - a) + a = a(4 - a)$

$\Rightarrow$ $4 = 4a - a^2$

$\Rightarrow$ $a^2 - 4a + 4 = 0$

$\therefore$ $a = 2$

and $b = 4 - a = 4 - 2 = 2$

Hence, equation is $\frac{x}{2} + \frac{y}{2} = 1$

or $x + y = 2$.

(ii)

Weekly wages in ₹	No. of workers (f)	x	$d = \frac{x - A}{h}$	$f.d$
80 – 100	20	90	– 2	– 40
100 – 120	30	110	– 1	– 30
120 – 140	20	130 = A	0	0
140 – 160	40	150	1	40
160 – 180	90	170	2	180
	$\Sigma f = 200$			$\Sigma(fd) = 150$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma fd}{\Sigma f} \times h$$

$$\bar{x} = 130 + \frac{150}{200} \times 20 \qquad (\because h = 20)$$

$$\bar{x} = 130 + 15 = 145$$

$\therefore$ Mean = ₹ 145.

(iii) Given, Diameter of cylindrical roller = 1.4 m

$\therefore$ Radius of roller (r) $= \frac{1.4}{2}$

$= 0.7$ m

and, width of roller (h) = 2.5 m

$\therefore$ CSA of roller $= \pi r^2 h$

$= \frac{22}{7} \times (0.7)^2 \times 2.5$

$= 3.85 \text{ m}^2$

Also, Area of playground $= 55 \times 35$

$= 1925 \text{ m}^2$

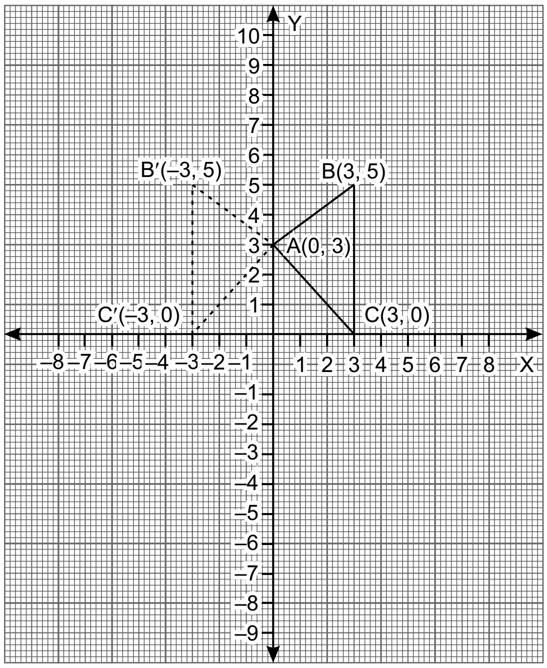
$\therefore$ Number of revolutions $= \dfrac{\text{Area of playground}}{\text{CSA of roadroller}}$

$= \dfrac{1925}{3.85}$

$= 500$

Hence, the least number of revolutions is 500.

(iv)



(a) Points A(0, 3), B(3, 5) and C(3, 0) are plotted on the graph.

(b) Points B′(– 3, 5) and C'(– 3, 0) are reflection points of B(3, 5) and C(3, 0) on Y-axis.

(c) As the corresponding sides of the given triangles are equal to the corresponding sides of the image, the triangles are congruent by SSS rule.

Answer 5.

(i) Let BD $= x$

$\Rightarrow$ AD $= 12 - x$ …(1)

Hence, BE $= x$ (tangents from same point to the circle)

CE $= 8 - x$

FC $=$ CE $= 8 - x$ (tangents from same point to the circle)

$\therefore$ AF $= 10 - (8 - x)$

$\Rightarrow$ AF $= 2 + x$ …(2)

But AD = AF

$\Rightarrow$ $12 - x = 2 + x$ (from 1 and 2)

$\Rightarrow$ $2x = 10$

$\Rightarrow$ $x = 5$ cm

$\therefore$ AD $= 12 - x = 12 - 5 = 7$ cm

BE $= x = 5$ cm

and CF $= 8 - x = 8 - 5 = 3$ cm.

(ii) L.H.S. $= \dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \dfrac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$

$$= \frac{\sin A(1-2(1-\cos^2 A))}{\cos A(2\cos^2 A-1)}$$

$$= \frac{\sin A(1-2+2\cos^2 A)}{\cos A(2\cos^2 A-1)}$$

$$= \frac{\sin A(2\cos^2 A-1)}{\cos A(2\cos^2 A-1)} = \frac{\sin A}{\cos A} = \tan A = \text{R.H.S.}$$

(iii) (a) Equation of line is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow \quad y - 3 = \frac{2-3}{5-1}(x-1)$

$\Rightarrow \quad 4(y - 3) = (-1)(x - 1)$

$\Rightarrow \quad 4y - 12 = -x + 1$

$\Rightarrow \quad x + 4y = 13$

(b) For $x = 0$, $y = \frac{13}{4}$

For $y = 0$, $x = 13$

Hence, coordinates of A is (13, 0)

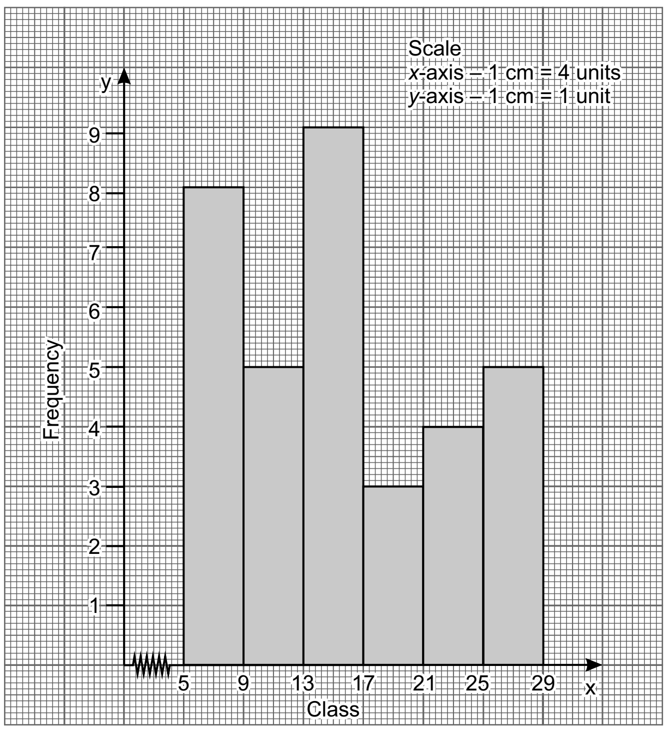
and coordinates of B is $\left(0, \frac{13}{4}\right)$

(c) Coordinates of M is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{13+0}{2}, \frac{0+13/4}{2}\right) = \left(\frac{13}{2}, \frac{13}{8}\right)$$

(iv)

Class Marks	Class	Frequency
7	5-9	8
11	9-13	5
15	13-17	9
19	17-21	3
23	21-25	4
27	25-29	5



Answer 6.

(i) (a) Total no. of outcomes = 36

Let E be the event of getting the sum 8.

Favourable outcomes are {(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)}

No. of favourable outcomes to E = 5

$$P(E) = \frac{\text{No. favourable outcomes}}{\text{Total no. of outcomes}}$$

$$= \frac{5}{36}$$

(b) Total no. of outcomes = 36

The maximum sum is 12, hence the no. of favourable outcome = 0

P(getting sum 13) = 0.

(c) Total no. of outcome = 36

All the outcomes have sum less than or equal to 12.

Hence, No. of favourable outcomes = 36

P(getting sum less than or equal to 12) = $\frac{36}{36} = 1$.

(ii) Section Formula, $x = \frac{mx_2 + nx_1}{m+n}$, $y = \frac{my_2 + ny_1}{m+n}$

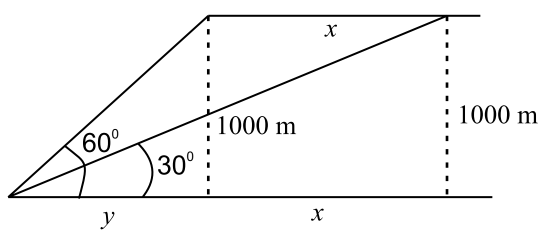
$\Rightarrow$ $x = \frac{2\times(-3)+3\times(2)}{2+3}$, $y = \frac{2\times(12)+3\times(5)}{2+3}$

$\Rightarrow$ $x = \frac{-6+6}{2+3}$, $y = \frac{24+15}{2+3}$

$\Rightarrow$ $x = 0$, $y = \frac{39}{5}$

Hence, the coordinates of the point are $\left(0, \frac{39}{5}\right)$.

(iii) From the figure, $\tan 60° = \frac{1000}{y}$



$\Rightarrow$ $\sqrt{3} = \frac{1000}{y}$

$\Rightarrow$ $y = \frac{1000}{\sqrt{3}}$

and $\tan 30° = \frac{1000}{x+y}$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{1000}{x + \frac{1000}{\sqrt{3}}}$

$\Rightarrow$ $\sqrt{3}x + 1000 = 3000$

$\Rightarrow$ $\sqrt{3}x = 2000$

$x = \frac{2000}{\sqrt{3}}$

$\therefore$ Distance travelled in 10 seconds = $\frac{2000}{\sqrt{3}}$ m

Speed of the airplane = $\frac{2000}{\sqrt{3} \times 10}$ m/s

$= \frac{200}{\sqrt{3}} \times \frac{18}{5}$ km/hr

$= 40 \times \sqrt{3} \times 6$

$= 240\sqrt{3}$ = 240 × 1.732

= 415.68 km/hr.

(iv)

Class Interval	**Class Mark x_i**	**Frequency(f_i)**	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
0-20	10	17	– 2	– 34
20-40	30	f_1	– 1	$-f_1$
40-60	50	32	0	0
60-80	70	f_2	1	f_2
80-100	90	19	2	38
Total		$120 = 68 + f_1 + f_2$		$4 - f_1 + f_2$

$$120 = 68 + f_1 + f_2$$

$\Rightarrow$ $f_1 + f_2 = 120 - 68$

$\Rightarrow$ $f_1 + f_2 = 52$ …(i)

and Mean = 50 (given)

$\therefore$ Mean = A + $\frac{\Sigma f u_i}{\Sigma f_i} \times i$

$\Rightarrow$ $50 = 50 + 20 \times \frac{4 - f_1 + f_2}{120}$

$\Rightarrow$ $f_1 - f_2 = 4$ …(ii)

Adding equation (i) and (ii), we get

$$2f_1 = 56$$

$\Rightarrow$ $f_1 = 56/2$

$\Rightarrow$ $f_1 = 28$

$\therefore$ $f_2 = 52 - 28$ [from (i)]

$\Rightarrow$ $f_2 = 24$

Hence, the missing frequencies are

$f_1 = 28$ and $f_2 = 24$.

❑❑