Short Answer Type Questions – I

[2 MARKS]

Que 1. Diagonals of a quadrilateral PQRS bisect each other. If $\angle P = 40^0$, Determine $\angle Q$.

Sol. Since the diagonals of quadrilateral PQRS bisect each other, therefore it must be a parallelogram.

$\therefore \angle P + \angle Q = 180^0$ (Angles on the same side of the transversal)

$\Rightarrow \quad 40^0 + \angle Q = 180^0$

$\Rightarrow \quad \angle Q = 180^0 - 40^0 \Rightarrow \angle Q = 140^0$

Que 2. In Fig. 8.16, ABCD is a parallelogram. If $\angle DAB = 60^0$ and $\angle DBC = 80^0$, Find $\angle CDB$.

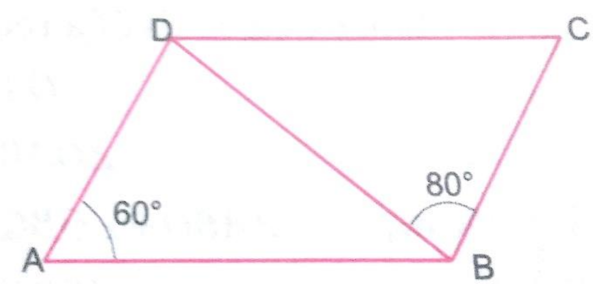

Fig. 8.16

Sol. We have, $\angle C = \angle A$ (Opposite angles of parallelogram)

$\Rightarrow \quad \angle C = 60^0$

Now, in ΔBDC

$\angle C + \angle CDB + \angle DBC = 180^0$

$\Rightarrow \quad 60^0 + \angle CDB + 80^0 = 180^0$

$\Rightarrow \quad \angle CDB = 180^0 - 140^0$

$\Rightarrow \quad \angle CDB = 40^0$

Que 3. In Fig. 8.17, ABCD is a rhombus. Find the value of x.

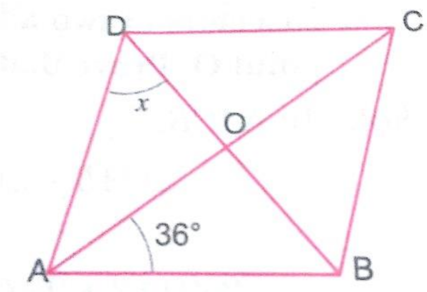

Fig. 8.17

Sol. $\angle AOB = 90^0$

(Diagonals of rhombus bisect each other at 90^0)

In ΔAOB, we have

$\angle OAB + \angle ABO + 90^0 = 180^0$

$36^0 + \angle ABO + 90^0 = 180^0 \quad \Rightarrow \quad \angle ABO = 180^0 - 126^0$

$\Rightarrow \quad \angle ABO = 54^0$

$\Rightarrow \quad \angle ADB = \angle ABD$ (Angles opposite to equal sides)

$\Rightarrow \quad \angle ADB = 54^0 \Rightarrow x = 54^0$

Que 4. In fig. 8.18, ABCD is a square. Determine ∠DAC.

Sol. As ABCD is a square,

$\therefore \quad AD = DC \quad$ and $\quad \angle ADC = 90^0$

$i.e \quad \angle DAC = \angle DCA$ and $\angle ADC = 90^0$

(Angles opposite to equal sides)

Now, in ΔADC, we have

$\angle 1 + \angle 2 + \angle ADC = 180^0$

$\Rightarrow \angle 1 + \angle 2 + 90^0 = 180^0 \quad \Rightarrow \quad 2\angle 1 = 90^0$ ($\because \angle 1 = \angle 2$)

$\Rightarrow \quad \angle 1 = 45^0 \quad$ or $\quad \angle DAC = 45^0$

Que 5. In ΔABC, median AM is produced to D such that AM = MD [Fig. 8.19].

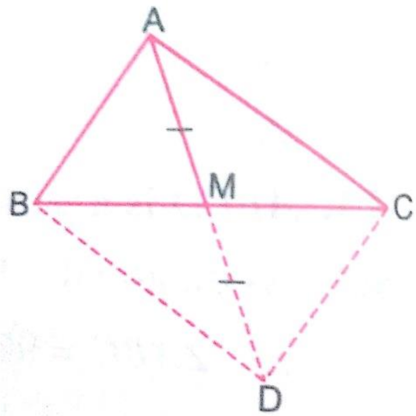

Fig. 8.19

Prove that ABCD is a Parallelogram.

Sol. In quadrilateral ABDC, we have

AM = MD (given)

BM = MC (AM is the Median)

As, diagonals AD and BC bisect each other. Therefore,

ABDC is a parallelogram.

Que 6. ABCD is a trapezium [Fig. 8.20] in which AB||CD and ∠A = ∠B = 45^0. Find ∠D of the trapezium.

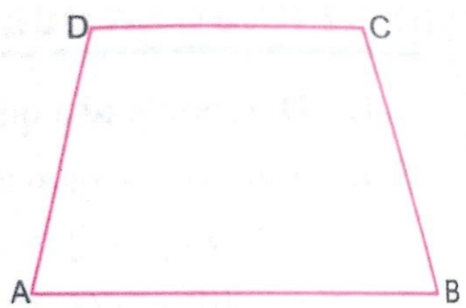

Fig. 8.20

Sol. Since, AB||CD and AD is the transversal.

∴ $\angle A + \angle D = 180^o$

$45^o + \angle D = 180^o$

⇒ $\angle D = 180^o - 45^o$ ⇒ $\angle D = 135^o$

Que 7. In rectangle ABCD, ∠BAC = 32º, Find the measure of ∠DBC.

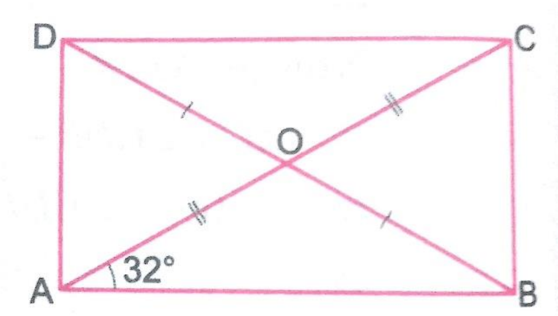

Fig. 8.21

Sol. Let AC and BD intersect at O [Fig 8.21].

Since diagonals of a rectangle bisect each other and are equal

∴ OA = OB

⇒ ∠OAB = ∠OBA = 32º

Now, ∠ABO + ∠OBC = 90º

⇒ ∠OBC = 90º – 32º = 58º

⇒ ∠DBC = ∠OBC = 58º

Que 8. Bisectors of two adjacent angles A and B of quad. ABCD intersect at a point O. Prove that 2∠AOB = ∠C + ∠D.

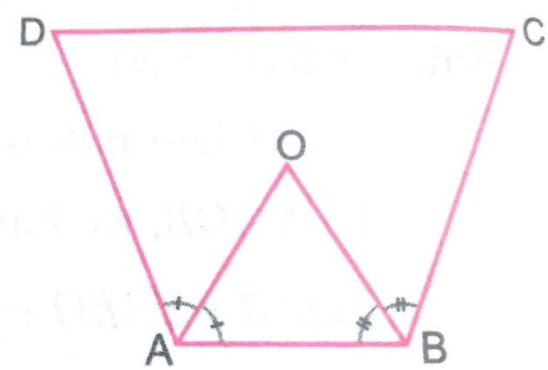

Fig. 8.22

Sol. In ΔAOB

∠OAB + ∠OBA + ∠AOB = 180º

(Angle sum property of Δ)

2∠OAB + 2∠OBA + 2∠AOB = 360º

$\angle A + \angle B + 2\angle AOB = 360^o$

But $\angle A + \angle B + \angle C + \angle D = 360^o$ (Angle sum property of quad.) ..(ii)

From (i) and (ii)

$\angle A + \angle B + 2\angle AOB = \angle A + \angle B + \angle C + \angle D$

$2\angle AOB = \angle C + \angle D$

Que 9. The sides BA and DC of quad. ABCD are produced as shown in Fig. 8.23. Prove that x + y = a + b.

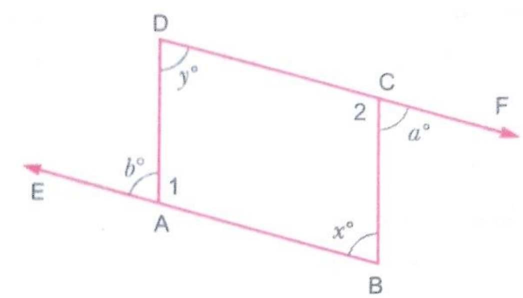

Fig. 8.23

Sol. Proof: $b + \angle 1 = 180^o$ (Linear pair)

$\angle 1 = 180^o - b$ ..(i)

Again $a + \angle 2 = 180^o$ (Linear pair)

$\angle 2 = 180^o - a$

But $\angle 1 + x + y + \angle 2 = 360^o$ (Angle sum property of quad.)

$180^o - b + x + y + 180^o - a = 360^o$ [From (i) and (ii)]

$x + y = a + b$

Que 10. In fig. 8.24, ABCD is a trapezium in which $\angle A = x + 25^o$, $\angle B = y^o$, $\angle C = 95^o$ and CD = $2x + 5^o$, then Find the value of x and y.

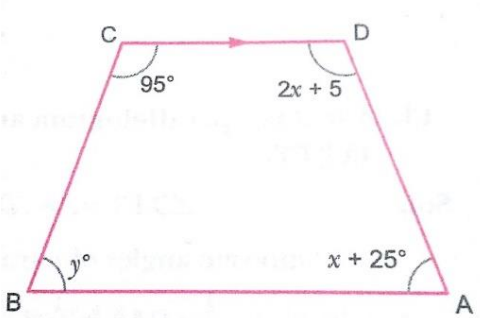

Fig. 8.24

Sol. As CD||BA

$\angle C + \angle B = 180^o$ (Co-interior angles)

∴ $95^o + y = 180^o$ ⇒ $y = 85^o$

Again, $\angle D + \angle A = 180^o$

$(2x + 5) + (x + 25) = 180^o$

$3x + 30^o = 180^o$

⇒ $3x = 150^o$ ⇒ $x = 50^o$

∴ $x = 50^o$, $y = 85^o$

Que 11. Two adjacent angles of a||^{gm} are in the ratio 2:3. Find all the four angles of the parallelogram.

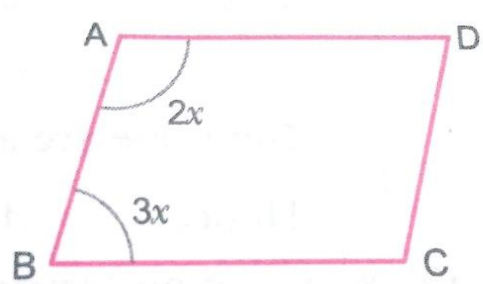

Fig. 8.25

Sol. Let the angles be 2x and 3x.

Also, $\angle A + \angle B = 180^o$ (Co-interior angles)

∴ $2x + 3x = 180^o$ ⇒ $5x = 180^o$

$X = \frac{180^o}{5} = 36^o$

∴ $\angle A = 2 \times 36^o = 72^o$

$\angle B = 3 \times 36^o = 108^o$

Again $\angle A = \angle C = 72^o$ (Opposite angles of parallelogram)

$\angle B = \angle D = 108^o$ (Opposite angles of parallelogram)

Que 12. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.

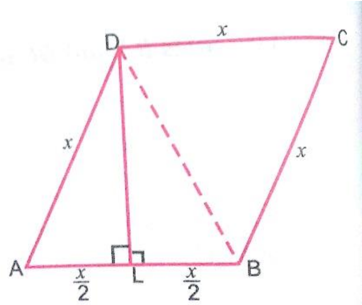

Fig. 8.26

Sol. Let sides of a rhombus be AB = BC = CD = DA = x

Now, join DB.

In ΔALD and ΔBLD, ∠DLA = ∠DLB = 90º

[Since, DL is a perpendicular bisector of AB]

AL = BL = $\frac{x}{2}$

And DL = DL [common side]

∴ ΔALD ≅ ΔBLD [by SAS congruence rule]

AD = BD [by CPCT]

Now, in ΔADB, AD = AB = DB = x

Then, ΔADB is an equilateral triangle,

∴ ∠A = ∠ADB = ∠ABD = 60º

Similarly, ΔDBC is an equilateral triangle,

∴ ∠C = ∠BDC = ∠DBC = 60º

Also, ∠A = ∠C

∴ ∠D = ∠B = 180º – 60º = 120º

[Since, sum of interior angles is 180º]

Que 13. ABCD is a parallelogram and line segments AX, CY bisect ∠A and ∠C respectively. Show that AX||CY.

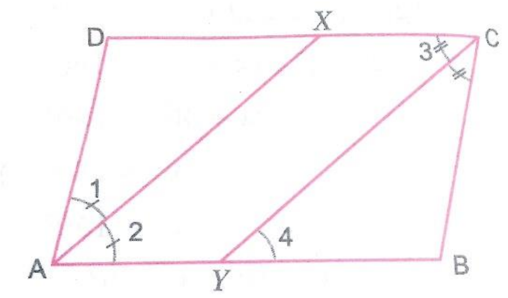

Fig. 8.27

Sol. ∠DAB = ∠BCD

(Opposite angles of parallelogram)

$\frac{1}{2}$ ∠DAB = $\frac{1}{2}$ ∠BCD

Or ∠2 = ∠3

But ∠3 = ∠4 (Alt. angles)

∴ ∠2 = ∠4

But these are alt. angles.

Hence AX||CY.

Que 14. In Fig. 8.28, ABCD is a parallelogram. Find the value of x, y and z.

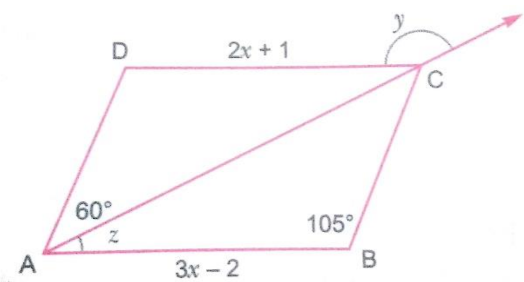

Fig. 8.28

Sol. 3x – 2 = 2x + 1 (Opposite sides of a parallelogram)

X = 3

∠B = ∠D = 105^o (Opposite angles of a parallelogram)

In ΔADC. ∠y = 60^o + ∠D (ext. angle property of Δ)

∠y = 60^o + 105^o = 165^o

Again, ∠D + ∠A

$60^o + z + 105^o = 180^o$

$Z + 165^o = 180^o \quad \Rightarrow \quad z = 15^o$

Que 15. In Fig. 8.29, ABCD is a parallelogram with perimeter 40cm. find x and y.

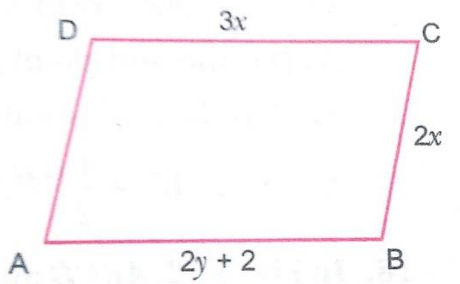

Fig. 8.29

Sol. Perimeter of parallelogram = 2(l+b)

∴ $40 = 2(l+b) = 20 = l + b$

$20 = (2y + 2) + 2x$

$20 = 2y + 2x + 2$

$18 = 2y + 2x$

$9 = y + x$

Also, opposite sides of a parallogram are equal.

∴ $3x = 2y + 2 \quad \Rightarrow \quad 3x - 2y = 2$

On putting (i) in (ii), we get

$3(9 - y) - 2y = 2$

$27 - 3y - 2y = 2 \quad \Rightarrow \quad 25 = 5y \quad \Rightarrow \quad 5 = y$

Putting the value of y in (i), we get

$x = 9 - 5 = 4$

Hence x = 4 cm and y = 5 cm.

Que 16. In fig. 8.30, D, E and F are the mid-points of the sides BC, CA and AB respectively of ΔABC. If AB = 6.2 cm, BC = 5.6 cm and CA = 4.6 cm, find the perimeter of:

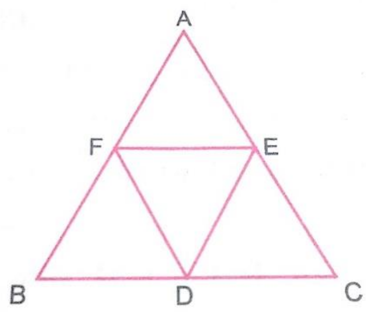

Fig. 8.30

(i) Trapezium FBCE and (ii) ΔDEF

Sol. (i) Perimeter of FBCE

$= FB + BC + CE + EF$

$= \frac{1}{2} AB + 5.6 + \frac{1}{2} AC + 2.8$

$= \frac{1}{2} (6.2) + 5.6 + \frac{1}{2} \times (4.6) + 2.8 = 13.8$ cm

(ii) F and E are the mid-points of AB and AC respectively.

∴ $FE = \frac{1}{2} BC$ (Midpoint theorem)

$FE = \frac{1}{2} \times 5.6 = 2.8$

Again, $DE = \frac{1}{2} AB = 3.1$ cm and $DF = \frac{1}{2} AC = 2.3$ cm

∴ Perimeter of ΔDEF = DE + EF + FD

$= 3.1 + 2.8 + 2.3 = 8.2$ cm

Que 17. In Fig. 8.31, D is the mid-point of AB and PC = $\frac{1}{2}$ AP = 3 cm. If AD = DB = 4 cm and DE||BP. Find AE.

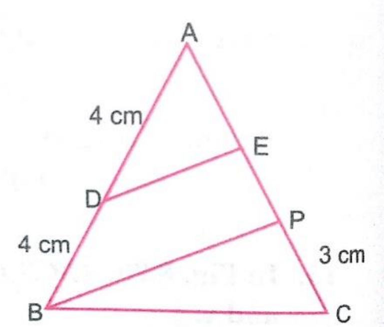

Fig. 8.31

Sol. $\frac{1}{2}$ AP = 3cm (Given)

⇒ AP = 6cm

As D is the mid-point and DE||BP

⇒ E is the mid-point of AP (By converse of midpoint theorem)

∴ AE = $\frac{1}{2}$ AP = $\frac{1}{2} \times 6$ = 3 cm

Que 18. In Fig. 8.32. ABCD and PQRB are rectangle where Q is the mid-point of BD. If QR = 5 cm, Find the measure of AB.

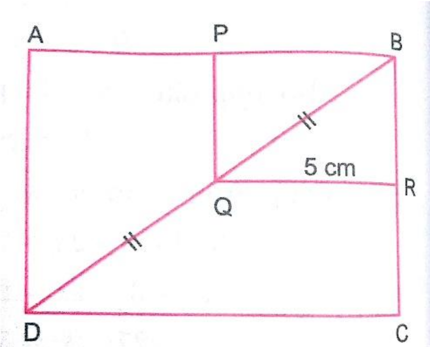

Fig. 8.32

Sol. In ΔBDC, Q is the mid-point of BD.

Again, QR||DC (As ABCD is rectangle and PQRB is a rectangle)

⇒ R is the mid-point of BC (by converse of mid-point theorem)

Again, in ΔBDC, Q and R are the mid-point of BD and BC.

⇒ QR = $\frac{1}{2}$ DC

5 = $\frac{1}{2}$ DC

So, DC = 10 cm

Also, DC = AB

(Opposite sides of rectangle)

∴ DC = AB = 10 cm

Que 19. Prove that each of a rectangle is a right angle.

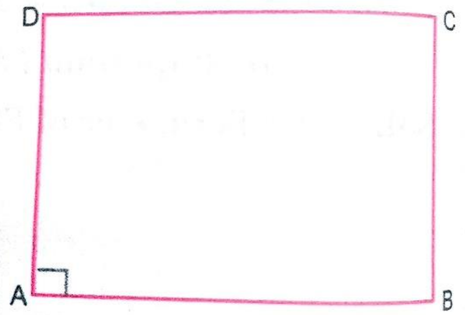

Fig. 8.33

Sol. Let ABCD be a rectangle and ∠A = 90º

⇒ ABCD is parallelogram also.

∴ AD||BC

⇒ ∠A + ∠B = 180º

Again ∠A = ∠C = 90º (Opposite angles of parallelogram)

∠B = ∠D = 90º (Opposite angles of parallelogram)

Que 20. In Fig. 8.34, side AC of ΔABC produced to E so that CE = $\frac{1}{2}$ AC. D is the mid-point of BC and ED produced meets AB at F. RC, DP are drawn parallel to BA. Prove that FD = $\frac{1}{3}$ FE.

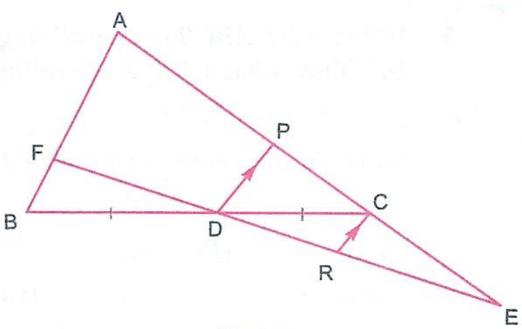

Fig. 8.34

Sol. In ΔABC, D is the mid-point of BC and DP||BA. (Given)

∴ P is the mid-point of AC ..(i) (By the converse of mid-point theorem)

Now, FA||DP||RC and AC is the transversal such that AP=PC and FDR is the other transversal on them.

∴ FD = DR ..(ii)

Also $\quad CE = \frac{1}{2} AC$

Or $\quad CE = PC \quad$ [Using (i)]

Now in ΔEDP, C is the mid-point of DE.

[By the converse of mid-point theorem]

$DR = RE$

$\therefore \quad FD = DR = RE \quad$ [From (ii) and (iii)]

$FD = \frac{1}{3} FE \quad$ [$\because$ FE = FD + DR + RE]