10 Vectors

Fastrack Revision

- The sum of two vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ is $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$.
- $\overrightarrow{AB}$ = Position vector of point B − Position vector of point A.
- If position vector of points A and B are $\vec{a}$ and $\vec{b}$, then position vector of mid-point of $AB=\dfrac{\vec{a}+\vec{b}}{2}$.
- Position vector of point $P(a, b)=\overrightarrow{OP}=a\hat{i}+b\hat{j}$ and modulus of vector $\overrightarrow{OP}=OP=\sqrt{a^2+b^2}$
- Unit vector $=\dfrac{\text{Vector}}{\text{Modulus of vector}}$
- Suppose $\vec{r}$ is a position vector of point $P(x, y, z)$, then
  (i) $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$; $x\hat{i}, y\hat{j}$ and $z\hat{k}$ are component vectors and x, y and z are components of vector $\vec{r}$ along X, Y and Z-axes.
  And $|\vec{r}|=|x\hat{i}+y\hat{j}+z\hat{k}|=\sqrt{x^2+y^2+z^2}$
  (ii) If the vector $\vec{r}$ makes the angles α, β and γ with X, Y and Z-axes, then it makes **direction cosines** $\cos\alpha, \cos\beta$ and $\cos\gamma$ as $\cos\alpha=\dfrac{a}{\sqrt{a^2+b^2+c^2}}$, $\cos\beta=\dfrac{b}{\sqrt{a^2+b^2+c^2}}$,
  $$\cos\gamma=\frac{c}{\sqrt{a^2+b^2+c^2}}$$
  where a, b and c are direction ratios.
  (iii) If l, m and n are the direction cosines of a vector, then we always have $l^2+m^2+n^2=1$.
- If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are any two points in space, then direction ratios of $\overrightarrow{AB}$ are $(x_2-x_1), (y_2-y_1), (z_2-z_1)$.
- Three points A, B, C whose position vectors are given, will be collinear if $\overrightarrow{AC}=m\overrightarrow{AB}$ [where, m is any scalar]
- **Section Formulae:** Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and let P be a point dividing AB internally in the ratio $m:n$.
  Let $\overrightarrow{OP}=\vec{r}$, then
  $$\vec{r}=\frac{(m\vec{b}+n\vec{a})}{(m+n)} \quad \text{(internally)}$$
  $$\vec{r}=\frac{(m\vec{b}-n\vec{a})}{(m-n)} \quad \text{(externally)}$$
- Three vectors $\vec{a}, \vec{b}, \vec{c}$ will be coplanar, if
  $\vec{a}=\lambda\vec{b}+\mu\vec{c}$ [where, λ and μ are scalars]
- The scalar product of two vectors $\vec{a}$ and $\vec{b}$, having angle θ between them, is
  $$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta=ab\cos\theta$$
- Projection of $\vec{b}$ in the direction of $\vec{a}=\dfrac{\vec{a}\cdot\vec{b}}{a}=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$
- Projection of $\vec{a}$ in the direction of $\vec{b}=\dfrac{\vec{a}\cdot\vec{b}}{b}=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$
- Vector component of a vector $\vec{a}$ on $\vec{b}$
  $$=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\hat{b}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\cdot\frac{\vec{b}}{|\vec{b}|}=\frac{(\vec{a}\cdot\vec{b})}{|\vec{b}|^2}\vec{b}$$
- Vector component of a vector $\vec{b}$ on $\vec{a}=\dfrac{(\vec{a}\cdot\vec{b})}{|\vec{a}|^2}\vec{a}$
- Scalar product of unit vectors $\hat{i}, \hat{j}, \hat{k}$ is
  $$\hat{i}\cdot\hat{j}=1,\quad \hat{j}\cdot\hat{j}=1,\quad \hat{k}\cdot\hat{k}=1$$
  $$\hat{i}\cdot\hat{j}=0,\quad \hat{j}\cdot\hat{k}=0,\quad \hat{k}\cdot\hat{i}=0$$
- If the angle between two vectors $\vec{a}$ and $\vec{b}$ is θ, then
  $$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{ab}=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$
- Two vectors $\vec{a}$ and $\vec{b}$ will be perpendicular, if $\vec{a}\cdot\vec{b}=0$.
- The vector product of two vectors $\vec{a}$ and $\vec{b}$ having angle θ between them, is $\vec{a}\times\vec{b}=(|\vec{a}||\vec{b}|\sin\theta)\,\hat{n}=(ab\sin\theta)\,\hat{n}$
  where, $\hat{n}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$.
- If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$
  Then, $$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$
- The area of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}=|\vec{a}\times\vec{b}|$.
- Area of parallelogram $=\dfrac{1}{2}|\vec{d_1}\times\vec{d_2}|$
  where, $\vec{d_1}$ and $\vec{d_2}$ are the vectors of diagonals.
- The area of a quadrilateral $ABCD$ is $=\dfrac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}|$

- Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

  where, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are adjacent sides.
- Condition for collinearity of three points, whose position vectors are $\vec{a}, \vec{b}, \vec{c}$ is $(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = \vec{0}$.
- Unit vector perpendicular to vectors $\vec{a}$ and $\vec{b}$ is $\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.
- Vector product of unit vectors $\hat{i}, \hat{j}, \hat{k}$ is

  $\hat{i} \times \hat{i} = 0, \quad \hat{j} \times \hat{j} = 0, \quad \hat{k} \times \hat{k} = 0,$

  $\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$
- If the angle between two vectors $\vec{a}$ and $\vec{b}$ is θ, then

  $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{ab}$.

Practice Exercise

Multiple Choice Questions

Q 1. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following statement is not true?

(NCERT EXERCISE)

a. $\vec{b} = \lambda \vec{a}$ for some scalar

b. $\vec{a} = \pm \vec{b}$

c. the consecutive components of $\vec{a}$ and $\vec{b}$ are proportional

d. the direction of both vectors $\vec{a}$ and $\vec{b}$ are same but magnitudes are different

Q 2. *ABCD* is a rhombus whose diagonals intersect at *E*. Then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals:

(CBSE SQP 2023-24, CBSE 2020)

a. $\vec{0}$ b. $\overrightarrow{AD}$ c. $2\overrightarrow{BD}$ d. $2\overrightarrow{AD}$

Q 3. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is:

(CBSE 2023)

a. 1 b. 5

c. 7 d. 12

Q 4. A unit vector along the vector $4\hat{i} - 3\hat{k}$ is:

(CBSE 2023)

a. $\frac{1}{7}(4\hat{i} - 3\hat{k})$ b. $\frac{1}{5}(4\hat{i} - 3\hat{k})$

c. $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$ d. $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

Q 5. If $|\vec{a}| = 3$ and $-1 \le k \le 2$, then $|k\vec{a}|$ lies in the interval:

(NCERT EXEMPLAR)

a. [0, 6] b. [−3, 6]

c. [3, 6] d. [1, 2]

Q 6. Let *ABCD* be the parallelogram whose sides *AB* and *AD* are represented by the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ respectively. If $\vec{a}$ is a unit vector parallel to $\overrightarrow{AC}$, then $\vec{a}$ is equal to:

a. $\frac{1}{3}(3\hat{i} - 6\hat{j} + 2\hat{k})$ b. $\frac{1}{3}(3\hat{i} + 6\hat{j} + 2\hat{k})$

c. $\frac{1}{7}(3\hat{i} - 6\hat{j} - 3\hat{k})$ d. $\frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$

Q 7. The vectors $3\hat{i} + 5\hat{j} + 2\hat{k}$, $2\hat{i} - 3\hat{j} - 5\hat{k}$ and $5\hat{i} + 2\hat{j} - 3\hat{k}$ form the sides of:

a. isosceles triangle b. right triangle

c. scalene triangle d. equilateral triangle

Q 8. The vectors $\vec{a} = x\hat{i} - 2\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} + y\hat{j} - z\hat{k}$ are collinear, if:

a. $x = 1, y = -2, z = -5$ b. $x = 1/2, y = -4, z = -10$

c. $x = -1/2, y = 4, z = 10$ d. All of these

Q 9. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear, if: *(CBSE 2023)*

a. $a_1b_1 + a_2b_2 + a_3b_3 = 0$

b. $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

c. $a_1 = b_1, a_2 = b_2, a_3 = b_3$

d. $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

Q 10. The position vectors of the points *A*, *B*, *C* are $(2\hat{i} + \hat{j} - \hat{k})$, $(3\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 4\hat{j} - 3\hat{k})$ respectively. These points:

a. form an isosceles triangle

b. form a right angled triangle

c. are collinear

d. form a scalene triangle

Q 11. Consider the points, *A*, *B*, *C* and *D* with position vectors $7\hat{i} - 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. Then *ABCD* is a:

a. square b. rhombus

c. rectangle d. None of these

Q 12. The figure formed by the four points $\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} + 3\hat{j}$, $5\hat{j} - 2\hat{k}$ and $\hat{k} - \hat{j}$ is:

a. square b. rectangle

c. parallelogram d. None of these

Q 13. If *O* is origin and *C* is the mid-point of *A* (2, −1) and *B* (−4, 3), then the value of $\overrightarrow{OC}$ is:

a. $\hat{i} + \hat{j}$ b. $\hat{i} - \hat{j}$ c. $-\hat{i} + \hat{j}$ d. $-\hat{i} - \hat{j}$

Q 14. The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides *AB* and *AC*, respectively of a ΔABC. The length of the median through *A* is: *(NCERT EXEMPLAR)*

a. $\frac{\sqrt{34}}{2}$ b. $\frac{\sqrt{48}}{2}$
c. $\sqrt{18}$ d. None of these

Q 15. The angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+4\hat{k}$, is:

a. 90° b. 45° c. 30° d. 15°

Q 16. If $|\vec{a}|=3$, $|\vec{b}|=4$ then the value of λ for which $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{a}-\lambda\vec{b}$, is:

a. $\frac{9}{16}$ b. $\frac{3}{4}$ c. $\frac{3}{2}$ d. $\frac{4}{3}$

Q 17. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be vectors with magnitudes of 3, 4 and 5 respectively and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is: *(CBSE 2021 Term-1)*

a. 47 b. 25 c. 50 d. –25

Q 18. The value of λ for which two vectors $2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\lambda\hat{j}+\hat{k}$ are perpendicular, is: *(CBSE 2023)*

a. 2 b. 4 c. 6 d. 8

Q 19. The scalar projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is: *(CBSE SQP 2022-23)*

a. $\frac{7}{\sqrt{14}}$ b. $\frac{7}{14}$
c. $\frac{6}{13}$ d. $\frac{7}{2}$

Q 20. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}\cdot\vec{b}\geq 0$ only when: *(CBSE 2023)*

a. $0<\theta<\frac{\pi}{2}$ b. $0\leq\theta\leq\frac{\pi}{2}$
c. $0<\theta<\pi$ d. $0\leq\theta\leq\pi$

Q 21. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}\cdot\vec{b}=4$, then $|\vec{a}-2\vec{b}|$ is equal to: *(CBSE SQP 2022-23)*

a. $\sqrt{2}$ b. $2\sqrt{6}$ c. 24 d. $2\sqrt{2}$

Q 22. If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $|\vec{a}+\vec{b}|<1$, then:

a. $\theta=\frac{\pi}{2}$ b. $0<\frac{\pi}{3}$
c. $\pi\geq\theta\geq\frac{2\pi}{3}$ d. $\frac{\pi}{3}<\theta<\frac{2\pi}{3}$

Q 23. If $\vec{a}\cdot\hat{i}=\vec{a}\cdot(\hat{i}+\hat{j})=\vec{a}\cdot(\hat{i}+\hat{j}+\hat{k})=1$, then $\vec{a}$ is: *(CBSE 2023)*

a. $\hat{k}$ b. $\hat{i}$ c. $\hat{j}$ d. $\hat{i}+\hat{j}+\hat{k}$

Q 24. If $|\vec{a}|=|\vec{b}|=1$ and $|\vec{a}+\vec{b}|=\sqrt{3}$, then the value of $(3\vec{a}-4\vec{b})\cdot(2\vec{a}+5\vec{b})$ is:

a. –21 b. $-\frac{21}{2}$ c. 21 d. $\frac{21}{2}$

Q 25. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are perpendicular to $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ and $\vec{a}+\vec{b}$ respectively and if $|\vec{a}+\vec{b}|=6$, $|\vec{b}+\vec{c}|=8$ and $|\vec{c}+\vec{a}|=10$, then $|\vec{a}+\vec{b}+\vec{c}|$ is equal to:

a. $5\sqrt{2}$ b. 50
c. $10\sqrt{2}$ d. 10

Q 26. If $\hat{a}$ and $\hat{b}$ are two unit vectors inclined to *X*-axis at angles 30° and 120° respectively, then $|\hat{a}+\hat{b}|$ equals:

a. $\sqrt{\frac{2}{3}}$ b. $\sqrt{2}$ c. $\sqrt{3}$ d. 2

Q 27. The component of $\hat{i}$ in the direction of the vector $\hat{i}+\hat{j}+2\hat{k}$ is:

a. $\sqrt{6}$ b. 6 c. $6\sqrt{6}$ d. $\frac{\sqrt{6}}{6}$

Q 28. A unit vector $\vec{a}$ makes equal but acute angles on the coordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b}=5\hat{i}+7\hat{j}-\hat{k}$ is: *(CBSE 2023)*

a. $\frac{11}{15}$ b. $\frac{11}{5\sqrt{3}}$ c. $\frac{4}{5}$ d. $\frac{3}{5\sqrt{3}}$

Q 29. If $\vec{a}=4\hat{i}+6\hat{j}$ and $\vec{b}=3\hat{j}+4\hat{k}$, then the vector form of the component of $\vec{a}$ along $\vec{b}$ is: *(CBSE SQP 2023-24)*

a. $\frac{18}{5}(3\hat{i}+4\hat{k})$ b. $\frac{18}{25}(3\hat{j}+4\hat{k})$
c. $\frac{18}{5}(3\hat{i}+4\hat{k})$ d. $\frac{18}{25}(2\hat{i}+4\hat{j})$

Q 30. The sine of the angle between the vectors $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}+2\hat{k}$ is: *(CBSE 2023)*

a. $\sqrt{\frac{5}{21}}$ b. $\frac{5}{\sqrt{21}}$ c. $\sqrt{\frac{3}{21}}$ d. $\frac{4}{\sqrt{21}}$

Q 31. Let $\hat{a}$ and $\hat{b}$ be two units vectors and angle between them is θ, then $\hat{a}+\hat{b}$ is a unit vector if: *(NCERT EXERCISE)*

a. $\theta=\frac{\pi}{4}$ b. $\theta=\frac{\pi}{3}$ c. $\theta=\frac{\pi}{2}$ d. $\theta=\frac{2\pi}{3}$

Q 32. If $\hat{a}$ and $\hat{b}$ are unit vectors, then what is the angle between $\hat{a}$ and $\hat{b}$ for $\hat{a}-\sqrt{3}\hat{b}$ to be a unit vector? *(NCERT EXEMPLAR)*

a. 30° b. 45° c. 60° d. 90°

Q 33. The unit vector perpendicular to the vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{j}$ forming a right handed system is: *(NCERT EXEMPLAR)*

a. $\hat{k}$ b. $-\hat{k}$ c. $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$ d. $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Q 34. The area of a triangle with vertices A, B, C is given by: *(CBSE SQP 2022-23)*

a. $|\overrightarrow{AB} \times \overrightarrow{AC}|$ b. $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

c. $\frac{1}{4}|\overrightarrow{AC} \times \overrightarrow{AB}|$ d. $\frac{1}{8}|\overrightarrow{AC} \times \overrightarrow{AB}|$

Q 35. If $|\vec{a}|=8$, $|\vec{b}|=3$ and $|\vec{a} \times \vec{b}|=12$, then the value of $\vec{a} \cdot \vec{b}$ is: *(NCERT EXEMPLAR)*

a. $6\sqrt{3}$ b. $8\sqrt{3}$

c. $12\sqrt{3}$ d. None of these

Q 36. Let vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$ then $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is: *(NCERT EXERCISE)*

a. $\frac{\pi}{6}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$

Q 37. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$ and $|\vec{a} \cdot \vec{b}|=|\vec{a} \times \vec{b}|$, then θ is equal to: *(NCERT EXERCISE)*

a. 0 b. $\frac{\pi}{4}$ c. $\frac{\pi}{2}$ d. π

Assertion & Reason Type Questions

Directions (Q. Nos. 38-46): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 38. Assertion (A): The magnitude of the resultant of vectors $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ is $\sqrt{34}$.

Reason (R): The magnitude of a vector can never be negative.

Q 39. Assertion (A): The unit vector in the direction of sum of the vectors $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}-\hat{j}-\hat{k}$ and $2\hat{j}+6\hat{k}$ is $-\frac{1}{7}(3\hat{i}+2\hat{j}+6\hat{k})$.

Reason (R): Let $\vec{a}$ be a non-zero vector, then $\frac{\vec{a}}{|\vec{a}|}$ is a unit vector parallel to $\vec{a}$.

Q 40. Assertion (A): If the points $\vec{P}=(\vec{a}+\vec{b}-\vec{c})$, $\vec{Q}=(2\vec{a}+\vec{b})$ and $\vec{R}=(\vec{b}+t\vec{c})$ are collinear, where $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors, then the value of t is -2.

Reason (R): If P, Q, R are collinear, then $\overrightarrow{PQ} \parallel \overrightarrow{PR}$ or $\overrightarrow{PQ}=\lambda\overrightarrow{PR}$, $\lambda \in R$.

Q 41. Assertion (A): The adjacent sides of a parallelogram are along $\vec{a}=\hat{i}+2\hat{j}$ and $\vec{b}=2\hat{i}+\hat{j}$. The angle between the diagonals is 150°.

Reason (R): Two vectors are perpendicular to each other if their dot product is zero.

Q 42. Assertion (A): If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to -25.

Reason (R): If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the angle θ between $\vec{b}$ and $\vec{c}$ is given by $\cos\theta=\frac{\vec{a}^2-\vec{b}^2-\vec{c}^2}{2\vec{b}\vec{c}}$.

Q 43. Assertion (A): The length of projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is $\frac{7}{\sqrt{14}}$.

Reason (R): The projection of a vector $\vec{a}$ on another vector $\vec{b}$ is $\frac{(\vec{a}\cdot\vec{b})}{|\vec{b}|}$.

Q 44. Let $\vec{a}$ and $\vec{b}$ be proper vectors and θ be the angle between them.

Assertion (A): $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2 \neq (\vec{a})^2(\vec{b})^2$

Reason (R): $\sin^2\theta+\cos^2\theta=1$

Q 45. Assertion (A): If $\vec{a}=\hat{i}-\hat{j}+\hat{k}$, $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{c}$, where $\vec{c}=-2\hat{i}-\hat{j}+\hat{k}$, then $\vec{b}=(0,1,1)$.

Reason (R): If $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{b}=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$, then

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x_2 & y_2 & z_2\\ x_1 & x_2 & z_1\end{vmatrix}.$$

Q 46. Assertion (A): If $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=400$ and $|\vec{a}|=4$, then $|\vec{b}|=9$.

Reason (R): If $\vec{a}$ and $\vec{b}$ are any two vectors, then $(\vec{a}\times\vec{b})^2=(\vec{a})^2(\vec{b})^2-(\vec{a}\cdot\vec{b})^2$.

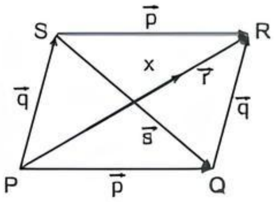
Answers

1. (d)	2. (a)	3. (c)	4. (b)	5. (a)	6. (d)	7. (d)	8. (d)	9. (b)	10. (a)
11. (d)	12. (d)	13. (c)	14. (a)	15. (a)	16. (b)	17. (d)	18. (d)	19. (a)	20. (b)
21. (b)	22. (c)	23. (b)	24. (b)	25. (d)	26. (b)	27. (d)	28. (a)	29. (b)	30. (a)
31. (d)	32. (a)	33. (a)	34. (b)	35. (c)	36. (b)	37. (b)	38. (b)	39. (d)	40. (a)
41. (d)	42. (b)	43. (a)	44. (d)	45. (c)	46. (d)				

Case Study Based Questions

Case Study 1

PQRS is a parallelogram whose adjacent sides are represented by the vectors $\vec{p}$ and $\vec{q}$. Three of its vertices are *P* (4, −2, 1), *Q* (3, −1 , 0) and *S* (1, −1 , −1).



Based on the above information, solve the following questions:

Q 1. The vector $\vec{p}+\vec{q}$ is:

a. $-4\hat{i}+2\hat{j}-3\hat{k}$ b. $4\hat{i}-2\hat{j}-3\hat{k}$

c. $-4\hat{i}+2\hat{j}+3\hat{k}$ d. $-\hat{i}+\hat{j}+\hat{k}$

Q 2. A unit vector along the vector $(\vec{p}+\vec{q})$ is:

a. $\frac{2\hat{i}-\hat{j}+\hat{k}}{\sqrt{6}}$ b. $\frac{-4\hat{i}+2\hat{j}-3\hat{k}}{\sqrt{29}}$

c. $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ d. $\frac{\hat{i}-2\hat{k}}{\sqrt{5}}$

Q 3. The diagonal $\vec{s}$ is:

a. $(2\hat{i}-\hat{j})$ b. $(\hat{j}+2\hat{k})$ c. $(2\hat{i}+\hat{k})$ d. $(\hat{i}-\hat{k})$

Q 4. Area of *PQRS*, whose adjacent sides are $\vec{p}$ and $\vec{q}$, is:

a. $\sqrt{2}$ sq. units b. $\sqrt{3}$ sq. units

c. $\sqrt{5}$ sq. units d. $\sqrt{6}$ sq. units

Q 5. The value of $\frac{1}{2}|\vec{r}\times\vec{s}|$ is:

a. $\sqrt{6}$ b. $2\sqrt{2}$ c. $\sqrt{3}$ d. $\sqrt{5}$

Solutions

1. Position vector of the points *P*, *Q* and *S* are

$$\overrightarrow{OP}=4\hat{i}-2\hat{j}+\hat{k},\ \overrightarrow{OQ}=3\hat{i}-\hat{j}$$

and $$\overrightarrow{OS}=\hat{i}-\hat{j}-\hat{k}$$

$\therefore$ $$\vec{p}=\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}$$
$$=(3\hat{i}-\hat{j})-(4\hat{i}-2\hat{j}+\hat{k})$$
$$=-\hat{i}+\hat{j}-\hat{k}$$

and $$\vec{q}=\overrightarrow{PS}=\overrightarrow{OS}-\overrightarrow{OP}$$
$$=(\hat{i}-\hat{j}-\hat{k})-(4\hat{i}-2\hat{j}+\hat{k})$$
$$=-3\hat{i}+\hat{j}-2\hat{k}$$

Now, $$\vec{p}+\vec{q}=(-\hat{i}+\hat{j}-\hat{k})+(-3\hat{i}+\hat{j}-2\hat{k})$$
$$=-4\hat{i}+2\hat{j}-3\hat{k}$$

So, option (a) is correct.

2. From part (1), $\vec{p}+\vec{q}=-4\hat{i}+2\hat{j}-3\hat{k}$

Now, $$|\vec{p}+\vec{q}|=|-4\hat{i}+2\hat{j}-3\hat{k}|$$
$$=\sqrt{(-4)^2+(2)^2+(-3)^2}$$
$$=\sqrt{16+4+9}=\sqrt{29}$$

$\therefore$ A unit vector along the vector $(\vec{p}+\vec{q})=\frac{(\vec{p}+\vec{q})}{|\vec{p}+\vec{q}|}$

$$=\frac{-4\hat{i}+2\hat{j}-3\hat{k}}{\sqrt{29}}$$

So, option (b) is correct.

3. Diagonal $\vec{s}=\vec{p}-\vec{q}=(-\hat{i}+\hat{j}-\hat{k})-(-3\hat{i}+\hat{j}-2\hat{k})$
$$=2\hat{i}+\hat{k}$$

So, option (c) is correct.

4. Now, $\vec{p}\times\vec{q}=(-\hat{i}+\hat{j}-\hat{k})\times(-3\hat{i}+\hat{j}-2\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & -1\\ -3 & 1 & -2\end{vmatrix}$$
$$=\hat{i}(-2+1)-\hat{j}(2-3)+\hat{k}(-1+3)$$
$$=-\hat{i}+\hat{j}+2\hat{k}$$

$\therefore$ Area of parallelogram *PQRS* $=|\vec{p}\times\vec{q}|$
$$=|-\hat{i}+\hat{j}+2\hat{k}|=\sqrt{(-1)^2+(1)^2+(2)^2}$$
$$=\sqrt{1+1+4}=\sqrt{6}\text{ sq. units}$$

So, option (d) is correct.

5. Diagonal $\vec{r}=\vec{p}+\vec{q}=-4\hat{i}+2\hat{j}-3\hat{k}$ (from part (1))

and diagonal $\vec{s}=2\hat{i}+\hat{k}$ (from part (3))

Now, $$\vec{r}\times\vec{s}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -4 & 2 & -3\\ 2 & 0 & 1\end{vmatrix}$$
$$=\hat{i}(2-0)-\hat{j}(-4+6)+\hat{k}(0-4)$$
$$=2\hat{i}-2\hat{j}-4\hat{k}$$

and $|\vec{r}\times\vec{s}| = |2\hat{i} - 2\hat{j} - 4\hat{k}|$

$$= \sqrt{(2)^2 + (-2)^2 + (-4)^2}$$
$$= \sqrt{4+4+16}$$
$$= \sqrt{24} = 2\sqrt{6}$$

$\therefore \quad \frac{1}{2}|\vec{r}\times\vec{s}| = \sqrt{6}$

So, option (a) is correct.

Case Study 2

Students of Class-XII appearing for a class test of Mathematics. The questions of test paper is based on vector algebra. All students were asked to attempt the following questions:

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors.

Based on the above information, solve the following questions:

Q 1. The position vector of the point which divides the join of points with position vectors $\vec{a}+2\vec{b}$ and $\vec{a}-2\vec{b}$ in the ratio 2 : 3 is:

a. $\frac{5\vec{a}+2\vec{b}}{5}$ b. $\frac{2\vec{a}+5\vec{b}}{5}$

c. $\frac{2\vec{a}+3\vec{b}}{5}$ d. $\frac{3\vec{a}+2\vec{b}}{5}$

Q 2. The projection of vector $\vec{a} = \hat{i} - 3\hat{j} + 2\hat{k}$ along $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ is:

a. $\frac{2}{3}$ b. $\frac{1}{\sqrt{14}}$

c. $\sqrt{7}$ d. $\frac{1}{\sqrt{7}}$

Q 3. The vector in the direction of the vector $3\hat{i} + 4\hat{k}$ that has magnitude 25, is:

a. $\frac{(3\hat{i}+4\hat{k})}{5}$ b. $(3\hat{i}+4\hat{k})$

c. $5(3\hat{i}+4\hat{k})$ d. $\frac{3\hat{i}+4\hat{k}}{25}$

Q 4. The value of λ such that the vectors $\vec{a} = \hat{i} - 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ are orthogonal, is:

a. 4 b. 3

c. 2 d. 1

Q 5. The vectors from origin to the points *A* and *B* are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ respectively, then the area of ΔOAB is:

a. 340 sq. units b. $\sqrt{255}$ sq. units

c. $\sqrt{229}$ sq. units d. $\frac{1}{2}\sqrt{229}$ sq. units

Solutions

1. Position vector of the required point

$$= \frac{3(\vec{a}+2\vec{b}) + 2(\vec{a}-2\vec{b})}{2+3}$$

TR!CK

The position vector of a point R dividing the line segment joining the points P and Q whose position vectors are $\vec{a}$ and $\vec{b}$ in the ratio m : n internally, is given by $\frac{n\vec{a}+m\vec{b}}{n+m}$.

$$= \frac{3\vec{a}+6\vec{b}+2\vec{a}-4\vec{b}}{5} = \frac{5\vec{a}+2\vec{b}}{5}$$

So, option (a) is correct.

2. Projection of a vector $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$= \frac{(\hat{i}-3\hat{j}+2\hat{k})\cdot(\hat{i}+2\hat{j}+3\hat{k})}{|\hat{i}+2\hat{j}+3\hat{k}|}$$
$$= \frac{(1)(1)+(-3)(2)+(2)(3)}{\sqrt{(1)^2+(2)^2+(3)^2}} = \frac{1-6+6}{\sqrt{1+4+9}} = \frac{1}{\sqrt{14}}$$

So, option (b) is correct.

3. Let $\vec{a} = 3\hat{i} + 4\hat{k} = 3\hat{i} + 0\hat{j} + 4\hat{k}$

and $|\vec{a}| = \sqrt{(3)^2+(0)^2+(4)^2}$
$$= \sqrt{9+0+16} = \sqrt{25} = 5$$

$\therefore$ The vector in the direction of $\vec{a}$ that has magnitude 25

$$= 25 \times \frac{\vec{a}}{|\vec{a}|} = 25 \times \frac{(3\hat{i}+4\hat{k})}{5} = 5(3\hat{i}+4\hat{k})$$

So, option (c) is correct.

4. Given, $\vec{a} = \hat{i} - 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$.

Since, $\vec{a}$ and $\vec{b}$ are orthogonal.

$\therefore \quad \vec{a}\cdot\vec{b} = 0$

$\Rightarrow \quad (\hat{i}-2\hat{j}+\lambda\hat{k})\cdot(3\hat{i}+\hat{j}-\hat{k}) = 0$

$\Rightarrow \quad (1)(3) + (-2)(1) + (\lambda)(-1) = 0$

$\Rightarrow \quad 3-2-\lambda = 0 \Rightarrow \lambda = 1$

So, option (d) is correct.

5. Given, $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$

Now, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$$= \hat{i}(-3-6) - \hat{j}(2-4) + \hat{k}(6+6)$$
$$= -9\hat{i} + 2\hat{j} + 12\hat{k}$$

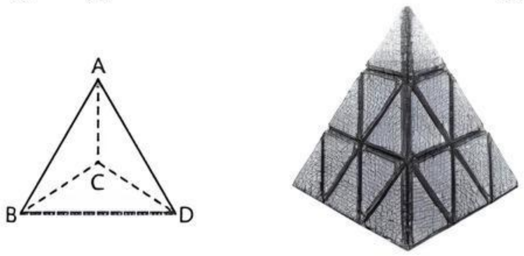
and $|\vec{a}\times\vec{b}| = \sqrt{(-9)^2+(2)^2+(12)^2}$
$$= \sqrt{81+4+144} = \sqrt{229}$$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}\sqrt{229}$

So, option (d) is correct.

Case Study 3

A building is to be constructed in the form of a triangular pyramid *ABCD* as shown in the figure.



Let its angular points be A (0, 1, 2), B (3, 0, 1), C (4, 3, 6) and D (2, 3, 2) and G be the point of intersection of the medians of ΔBCD.

Based on the above information, solve the following questions:

Q 1. The coordinates of point *G* are:

a. (2, 3, 3) b. (3, 3, 2) c. (3, 2, 3) d. (0, 2, 3)

Q 2. The length of vector $\overrightarrow{AG}$ is:

a. $\sqrt{17}$ units b. $\sqrt{11}$ units c. $\sqrt{13}$ units d. $\sqrt{19}$ units

Q 3. Area of ΔABC (in sq. units) is:

a. $\sqrt{10}$ b. $2\sqrt{10}$ c. $3\sqrt{10}$ d. $5\sqrt{10}$

Q 4. The sum of lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is:

a. 5 units b. 9.32 units c. 10 units d. 11 units

Q 5. The length of the perpendicular from the vertex *D* on the opposite face is:

a. $\frac{6}{\sqrt{10}}$ units b. $\frac{2}{\sqrt{10}}$ units

c. $\frac{3}{\sqrt{10}}$ units d. $8\sqrt{10}$ units

Solutions

1.

TR!CK

Intersection point of medians in a triangle is known as centroid.

Clearly, G is the centroid of ΔBCD, therefore coordinate of G are

$$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right) = (3, 2, 3)$$

So, option (c) is correct.

2. Since, $A = (0, 1, 2)$ and $G = (3, 2, 3)$

$$\therefore \quad \overrightarrow{AG} = (3-0)\hat{i} + (2-1)\hat{j} + (3-2)\hat{k}$$
$$= 3\hat{i} + \hat{j} + \hat{k}$$
$$\Rightarrow \quad |\overrightarrow{AG}|^2 = 3^2 + 1^2 + 1^2 = 9 + 1 + 1 = 11$$
$$\Rightarrow \quad |\overrightarrow{AG}| = \sqrt{11} \text{ units}$$

So, option (b) is correct.

3. Clearly, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Here, $$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-0 & 0-1 & 1-2 \\ 4-0 & 3-1 & 6-2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}(-4+2) - \hat{j}(12+4) + \hat{k}(6+4)$$
$$= -2\hat{i} - 16\hat{j} + 10\hat{k}$$
$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-2)^2 + (-16)^2 + 10^2}$$
$$= \sqrt{4 + 256 + 100}$$
$$= \sqrt{360} = 6\sqrt{10}$$

Hence, area of $\Delta ABC = \frac{1}{2} \times 6\sqrt{10} = 3\sqrt{10}$ sq. units

So, option (c) is correct.

4. Here, $\overrightarrow{AB} = 3\hat{i} - \hat{j} - \hat{k}$

$$\Rightarrow \quad |\overrightarrow{AB}| = \sqrt{9+1+1} = \sqrt{11}$$

Also, $\overrightarrow{AC} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

$$\Rightarrow \quad |\overrightarrow{AC}| = \sqrt{16+4+16} = \sqrt{36} = 6$$

Now, $|\overrightarrow{AB}| + |\overrightarrow{AC}| = \sqrt{11} + 6 = 3.32 + 6 = 9.32$ units

So, option (b) is correct.

5. The length of the perpendicular from the vertex D on the opposite face

$$= |\text{Projection of } \overrightarrow{AD} \text{ on } \overrightarrow{AB} \times \overrightarrow{AC}|$$

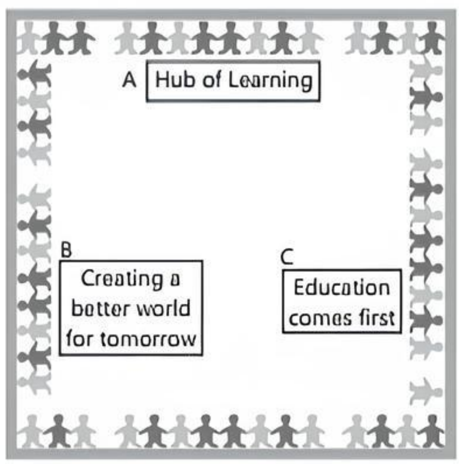
$$= \left|\frac{(2\hat{i} + 2\hat{j}) \cdot (-2\hat{i} - 16\hat{j} + 10\hat{k})}{\sqrt{(-2)^2 + (-16)^2 + 10^2}}\right|$$

$$= \left|\frac{-4-32}{\sqrt{360}}\right| = \frac{36}{6\sqrt{10}} = \frac{6}{\sqrt{10}} \text{ units}$$

So, option (a) is correct.

Case Study 4

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, −3, −2) and (−2, 2, 6) respectively.



Based on the given information, solve the following questions:

Q 1. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then $\vec{a} + \vec{b} + \vec{c}$ is equal to:

a. $2\hat{i} + 3\hat{j} + 6\hat{k}$ b. $2\hat{i} - 3\hat{j} - 6\hat{k}$

c. $2\hat{i} + 8\hat{j} + 3\hat{k}$ d. $2(7\hat{i} + 8\hat{j} + 3\hat{k})$

Q 2. Which of the following is not true?

a. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ b. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

c. $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$ d. $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

Q 3. Area of ΔABC is:

a. 19 sq. units b. $\sqrt{1937}$ sq. units

c. $\frac{1}{2}\sqrt{1937}$ sq. units d. $\sqrt{1837}$ sq. units

Q 4. Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ will be equal to:

a. −1 b. −2 c. 2 d. 0

Q 5. If $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, then unit vector in the direction of vector $\vec{a}$ is:

a. $\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$ b. $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

c. $\frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$ d. None of these

Solutions

1. $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$

and $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

So, option (a) is correct.

2. Using triangle law of addition in ΔABC, we get

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$, which can be rewritten as

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$ or $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

So, option (c) is correct.

3. We have, $A(1, 4, 2)$, $B(3, -3, -2)$ and $C(-2, 2, 6)$

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$$

$$= \hat{i}(-28-8) - \hat{j}(8-12) + \hat{k}(-4-21)$$

$$= -36\hat{i} + 4\hat{j} - 25\hat{k}$$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

$= \sqrt{1296 + 16 + 625} = \sqrt{1937}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{1937}$ sq. units

So, option (c) is correct.

4. If the given points lie on the straight line, then the points will be collinear and so area of $\Delta ABC = 0$.

$\Rightarrow |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$

[$\because$ If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three vertices A, B and C of ΔABC, then area of triangle $= \frac{1}{2}[|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|]$]

So, option (d) is correct.

5. Here, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36}$

$= \sqrt{49} = 7$

$\therefore$ Unit vector in the direction of vector $\vec{a}$ is

$$\hat{a} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

So, option (b) is correct.

Case Study 5

Rakesh purchased an air plant holder which is in the shape of a tetrahedron. Let P, Q, R and S be the coordinates of the air plant holder where $P \equiv (3, 3, 4)$, $Q \equiv (3, 1, 2)$, $R \equiv (2, 1, 3)$ and $S \equiv (1, 1, 1)$.

Based on the above information, solve the following questions.

Q 1. Find the position vector of $\overrightarrow{PS}$.

Q 2. Find the area of ΔPQR.

Q 3. Find the unit vector along $\overrightarrow{PS}$.

Or

Find the projection of $\overrightarrow{PQ}$ on $\overrightarrow{PR}$.

Solutions

1. Here, $\overrightarrow{OP} = 3\hat{i} + 3\hat{j} + 4\hat{k}$ and $\overrightarrow{OS} = \hat{i} + \hat{j} + \hat{k}$

$\therefore$ Position vector of $\overrightarrow{PS} = \overrightarrow{OS} - \overrightarrow{OP}$

$= (\hat{i} + \hat{j} + \hat{k}) - (3\hat{i} + 3\hat{j} + 4\hat{k}) = -2\hat{i} - 2\hat{j} - 3\hat{k}$

2. Here, $\overrightarrow{OQ} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{OR} = 2\hat{i} + \hat{j} + 3\hat{k}$

Now, position vector of $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (3\hat{i} + \hat{j} + 2\hat{k}) - (3\hat{i} + 3\hat{j} + 4\hat{k}) = -2\hat{j} - 2\hat{k}$

and position vector of $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$= (2\hat{i} + \hat{j} + 3\hat{k}) - (3\hat{i} + 3\hat{j} + 4\hat{k})$

$= -\hat{i} - 2\hat{j} - \hat{k}$

$$\text{Now, } \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & -2 \\ -1 & -2 & -1 \end{vmatrix}$$

$$= \hat{i}(2-4) - \hat{j}(0-2) + \hat{k}(0-2)$$

$$= -2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \quad |\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{(-2)^2 + (2)^2 + (-2)^2}$$

$$= \sqrt{4+4+4} = 2\sqrt{3}$$

$$\therefore \text{ Area of } \Delta PQR = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$$

$$= \frac{1}{2} \times 2\sqrt{3} = \sqrt{3} \text{ sq. units}$$

3. Unit vector along $\overrightarrow{PS} = \dfrac{\overrightarrow{PS}}{|\overrightarrow{PS}|}$

$$= \frac{-2\hat{i} - 2\hat{j} - 3\hat{k}}{\sqrt{(-2)^2 + (-2)^2 + (-3)^2}}$$

$$= \frac{-2\hat{i} - 2\hat{j} - 3\hat{k}}{\sqrt{4+4+9}} = -\frac{1}{\sqrt{17}}(2\hat{i} + 2\hat{j} + 3\hat{k})$$

Or

Here, $\overrightarrow{PR} = -\hat{i} - 2\hat{j} - \hat{k}$

and $\overrightarrow{PQ} = -2\hat{j} - 2\hat{k}$

$$\therefore \text{ Projection of } \overrightarrow{PQ} \text{ on } \overrightarrow{PR} = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PR}|}$$

$$= \frac{(-\hat{i} - 2\hat{j} - \hat{k}) \cdot (-2\hat{j} - 2\hat{k})}{|-\hat{i} - 2\hat{j} - \hat{k}|}$$

$$= \frac{(-2)(-2) + (-2)(-1)}{\sqrt{(-1)^2 + (-2)^2 + (-1)^2}}$$

$$= \frac{4+2}{\sqrt{1+4+1}} = \frac{6}{\sqrt{6}} = \sqrt{6}$$

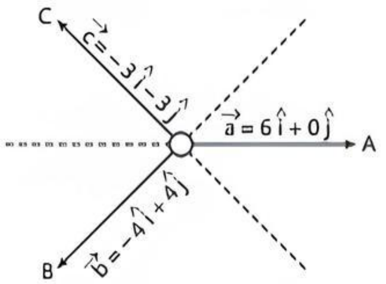
Case Study 6

Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area.

Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}$ kN,

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}$ kN,

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ kN



Based on the above information, solve the following questions: (CBSE SQP 2023-24)

Q 1. What is the magnitude of the force of Team A?

Q 2. Which team will win the game?

Q 3. Find the magnitude of the resultant force exerted by the teams.

Or

In what direction is the ring getting pulled?

Solutions

1. The magnitude of the force of team $A = |\vec{F_1}|$

$$= |6\hat{i} + 0\hat{j}| = \sqrt{6^2 + 0} = 6 \text{ kN}$$

2. Since, $|\vec{F_1}| = 6$ kN

Now, $|\vec{F_2}| = |-4\hat{i} + 4\hat{j}| = \sqrt{(-4)^2 + (4)^2}$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ kN}$$

and $|\vec{F_3}| = |-3\hat{i} - 3\hat{j}| = \sqrt{(-3)^2 + (-3)^2}$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ kN}$$

Since, 6 is larger, so team A wins.

3. The magnitude of the resultant force,

$|\vec{F}| = |\vec{F_1} + \vec{F_2} + \vec{F_3}|$

$$= |(6\hat{i} + 0\hat{j}) + (-4\hat{i} + 4\hat{j}) + (-3\hat{i} - 3\hat{j})|$$

$$= |-\hat{i} + \hat{j}| = \sqrt{(-1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$$

Or

We have, $\vec{F} = -\hat{i} + \hat{j}$

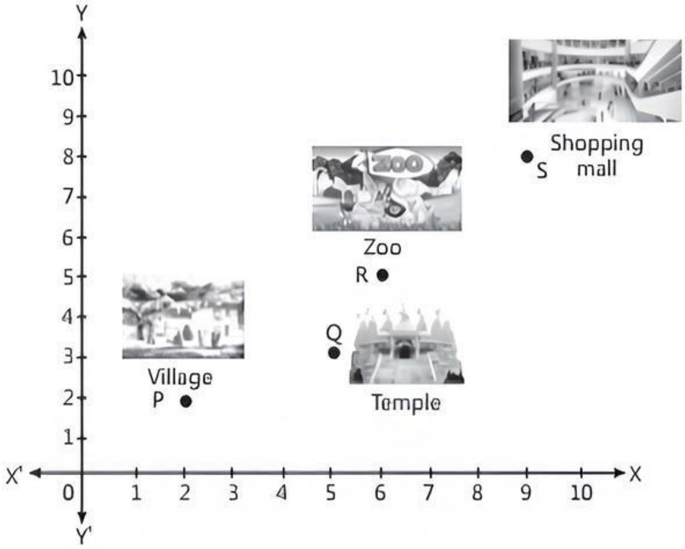
$$\therefore \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1}{-1}\right)$$

$$= \tan^{-1}(-1) = -\tan^{-1}(1)$$

$$= -\frac{\pi}{4}$$

Case Study 7

Tanya left from her village on weekend. First, she travelled up to temple. After this, she left for the zoo. After this she left for shopping in a mall. The positions of Tanya at different places is given in the following graph.



Based on the above information, solve the following questions:

Q 1. Find the vector $\overrightarrow{QR}$ in terms of $\hat{i}, \hat{j}$.

Q 2. Find the length of vector $\overrightarrow{PS}$.

Q 3. Find the unit vector of $\overrightarrow{PR}$.

Or

Find $\overrightarrow{PR} \times \overrightarrow{QS}$.

Solutions

1. Position vector of $Q = 5\hat{i} + 3\hat{j}$

and position vector of $R = 6\hat{i} + 5\hat{j}$

$\therefore \quad \overrightarrow{QR} = (6-5)\hat{i} + (5-3)\hat{j} = \hat{i} + 2\hat{j}$

2. Position vector of $P = 2\hat{i} + 2\hat{j}$

and position vector of $S = 9\hat{i} + 8\hat{j}$

$\therefore \quad \overrightarrow{PS} = (9-2)\hat{i} + (8-2)\hat{j} = 7\hat{i} + 6\hat{j}$

Now, $\quad |\overrightarrow{PS}|^2 = (7)^2 + (6)^2 = 49 + 36 = 85$

$\Rightarrow \quad |\overrightarrow{PS}| = \sqrt{85}$ units

3. $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = (6\hat{i} + 5\hat{j}) - (2\hat{i} + 2\hat{j})$

$= (4\hat{i} + 3\hat{j})$

$\therefore$ Unit vector of $\overrightarrow{PR}$ or $\hat{PR} = \dfrac{\overrightarrow{PR}}{|\overrightarrow{PR}|}$

$= \dfrac{4\hat{i} + 3\hat{j}}{|4\hat{i} + 3\hat{j}|} = \dfrac{4\hat{i} + 3\hat{j}}{\sqrt{16+9}}$

$= \dfrac{4\hat{i} + 3\hat{j}}{\sqrt{25}} = \dfrac{1}{5}(4\hat{i} + 3\hat{j})$

Or

$\overrightarrow{QS} = \overrightarrow{OS} - \overrightarrow{OQ} = (9\hat{i} + 8\hat{j}) - (5\hat{i} + 3\hat{j})$

$= 4\hat{i} + 5\hat{j}$

and $\quad \overrightarrow{PR} = 4\hat{i} + 3\hat{j}$

$\therefore \quad \overrightarrow{PR} \times \overrightarrow{QS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ 4 & 5 & 0 \end{vmatrix}$

$= (0-0)\hat{i} - (0-0)\hat{j} + (20-12)\hat{k} = 8\hat{k}$

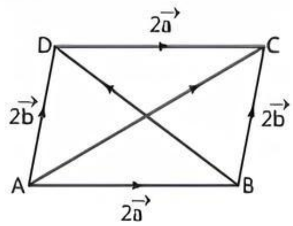
Case Study 8

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

Based on the above information, solve the following questions:

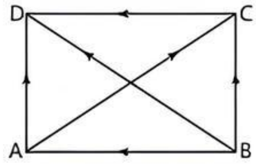
Q 1. If *ABCD* is a parallelogram and *AC* and *BD* are its diagonals, then find the value of $\overrightarrow{AC} + \overrightarrow{BD}$.

Q 2. If *ABCD* is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then find the value of $\overrightarrow{AC} - \overrightarrow{BD}$.

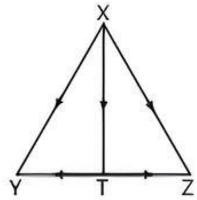


Or

If *ABCD* is a quadrilateral, whose diagonals are $\overrightarrow{AC}$ and $\overrightarrow{BD}$, then find the value of $\overrightarrow{BA} + \overrightarrow{CD}$.

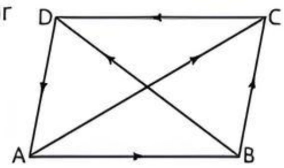


Q 3. If *T* is the mid-point of side *YZ* of ΔXYZ, then find the value of $\overrightarrow{XY} + \overrightarrow{XZ}$.



Solutions

1. From triangle law of vector addition,



$\overrightarrow{AC} + \overrightarrow{BD}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$

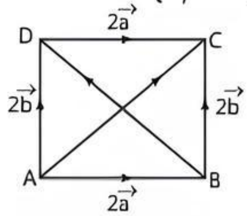
$= \overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

$= \overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB}$

$= 2\overrightarrow{BC}$ $\quad [\because \overrightarrow{AB} = -\overrightarrow{CD}]$

2. In ΔABC, $\overrightarrow{AC} = 2\vec{a} + 2\vec{b}$ ...(1)

[by triangle law of addition]



and in ΔABD, $2\vec{b} = 2\vec{a} + \overrightarrow{BD}$ ...(2)

[by triangle law of addition]

Adding eqs. (1) and (2), we have

$\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$

$\Rightarrow \quad \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$

Or

In ΔABC, $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$ (by triangle law) ...(1)

In ΔBCD, $\overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$ (by triangle law) ...(2)

From eqs. (1) and (2), $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BD} - \overrightarrow{CD}$

$\Rightarrow \quad \overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{AC} = \overrightarrow{BD} + \overrightarrow{CA}$

3. Since, *T* is the mid-point of *YZ*.

So, $\quad \overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\quad \overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$

(by triangle law)

$= 2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT}$ $[\because \overrightarrow{TY} = -\overrightarrow{YT}]$

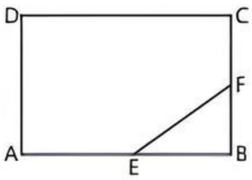
Very Short Answer Type Questions

Q 1. Write the associative law of vector addition.

Q 2. In ΔABC, prove $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$. *(NCERT EXERCISE)*

Q 3. Find the sum of vectors $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$, where *ABCD* is a quadrilateral.

Q 4. In the figure, *ABCD* is a parallelogram in which *E* and *F* are the mid-points of *AB* and *BC* respectively.



If $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{AD} = \vec{b}$, then find vector $\overrightarrow{EF}$.

Q 5. If the position vector of *P* is $\hat{i} + \hat{j}$ and position vector of *Q* is $4\hat{j} - 5\hat{k}$, then find $\overrightarrow{PQ}$. *(NCERT EXERCISE)*

Q 6. Three forces $2\hat{i} + 3\hat{j} + 4\hat{k}$, $-4\hat{j} + \hat{i}$ and $\hat{j} - 4\hat{k} - 3\hat{i}$ act on a particle. Prove that the particle is equilibrium.

Q 7. Find for what value of *a*, the vector $(2\hat{i} - 3\hat{j} + 4\hat{k})$ and $(a\hat{i} + 6\hat{j} - 8\hat{k})$ are collinear?

Q 8. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$. *(NCERT EXERCISE)*

Q 9. Find a unit vector along the vector $2\hat{i} - \hat{j} + 2\hat{k}$.

Q 10. Prove that $(\hat{i} \cdot \hat{j})\hat{k} + (\hat{j} \cdot \hat{k})\hat{i} + (\hat{k} \cdot \hat{i})\hat{j} = \vec{0}$.

Q 11. If vectors $a\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} + b\hat{j}$ are perpendicular, then prove that $3a + 2b = 0$. *(CBSE 2020, 19, 18)*

Q 12. If the vectors $2\hat{i} - \hat{j} + p\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ are perpendicular to each other, find the value of *p*.

Q 13. The magnitudes of two vectors $\vec{a}$ and $\vec{b}$ are 1 and 2 respectively and $\vec{a} \cdot \vec{b} = 1$, find the angle between these vectors. *(NCERT EXERCISE)*

Q 14. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$. *(CBSE 2018)*

Q 15. Find a vector perpendicular to vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$.

Q 16. If $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$, then find λ and μ.

Q 17. Find the area of that triangle whose two adjacent sides are represented by $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$.

Q 18. (i) Find $\hat{i} \cdot (\hat{j} \times \hat{k}) + (\hat{i} \times \hat{k}) \cdot \hat{j}$.

(ii) Find $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$.

(NCERT EXERCISE)

Short Answer Type-I Questions

Q 1. If *E* is the mid-point of side *AC* of ΔABC, then prove that $\overrightarrow{BE} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC})$.

Q 2. In a pentagon *ABCDE*, prove that $\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC} = 3\overrightarrow{AC}$.

Q 3. *ABCDEF* is a regular hexagon in which the forces $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$ and $\overrightarrow{AF}$ are acting at *A*, prove that their resultant is $3\overrightarrow{AD}$.

Q 4. Show that the line passing through the points (4, 7, 8) and (2, 3, 4) is parallel to the line passing through the points (–1, –2, 1) and (1, 2, 5).

Q 5. Find the coordinates of the points which divides the line joining two points (2, –5, 1) and (1, 4, – 6) in the ratio 2 : 3 internally.

Q 6. Find a unit vector, along the sum and difference of vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$. *(NCERT EXERCISE)*

Q 7. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ so that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal. *(CBSE SQP 2022-23)*

Q 8. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular. *(NCERT EXERCISE)*

Q 9. If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of *p*. *(CBSE 2023)*

Q 10. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. *(CBSE 2022 Term-2)*

Q 11. Find $|\vec{x}|$, if $(\vec{x} - \hat{a}) \cdot (\vec{x} + \hat{a}) = 12$, where $\hat{a}$ is a unit vector. *(CBSE SQP 2022-23)*

Q 12. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ such that $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, then find the value of λ. *(NCERT EXERCISE, CBSE 2022)*

Q 13. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then prove that the vector $\vec{a}$ and $\vec{b}$ are perpendicular. *(NCERT EXEMPLAR)*

Q 14. If the sum of unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$. *(CBSE 2019)*

Q 15. Find the angle between the vector $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and X-axis.

Q 16. If the angle between the vectors $a\hat{i}+\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$ is 60°, find the value of a.

Q 17. Find in which condition $(\vec{a}\cdot\vec{b})^2=a^2b^2$.

Q 18. If $\vec{a}$ and $\vec{b}$ are two vectors, then prove that $(\vec{a}\times\vec{b})^2=a^2b^2-(\vec{a}\cdot\vec{b})^2$. *(NCERT EXEMPLAR)*

Q 19. Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$. *(CBSE 2023)*

Q 20. Find a unit vector perpendicular to each of the vectors $2\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+4\hat{j}-\hat{k}$.

Q 21. If $\vec{a}=4\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+\hat{k}$, then find a unit vector along the vector $\vec{a}\times\vec{b}$. *(CBSE 2023)*

Q 22. If θ is the angle between two vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$, find $\sin\theta$. *(CBSE 2018)*

Q 23. If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=3$, $|\vec{b}|=\frac{2}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$. *(CBSE 2023)*

Q 24. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then find $\vec{b}$.

Q 25. If $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are four non-zero vectors such that $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=4\,\vec{b}\times\vec{d}$, then show that $(\vec{a}-2\vec{d})$ is parallel to $(2\vec{b}-\vec{c})$, where $\vec{a}\neq 2\vec{d}$, $\vec{c}\neq 2\vec{b}$. *(CBSE 2022 Term-2)*

Short Answer Type-II Questions

Q 1. D and E are the mid-points of sides AB and AC respectively of $\triangle ABC$. Prove that $\overrightarrow{BE}+\overrightarrow{DC}=\frac{3}{2}\overrightarrow{BC}$.

Q 2. $ABCD$ is a parallelogram and G is the point of intersection of its diagonals AC and BD. If O is any point, then prove that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=4\,\overrightarrow{OG}$.

Q 3. In a parallelogram $PQRS$, $\overrightarrow{PQ}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{PS}=-\hat{i}-2\hat{k}$. Find $|\overrightarrow{PR}|$ and $|\overrightarrow{QS}|$. *(CBSE 2022 Term-2)*

Q 4. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram. *(CBSE 2020)*

Q 5. The two adjacent sides of a parallelogram are represented by $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also. *(CBSE 2022 Term-2)*

Q 6. Prove that the points $A(-2\hat{i}+3\hat{j}+5\hat{k})$, $B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$ are collinear. *(NCERT EXERCISE)*

Q 7. The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$, is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$. *(CBSE 2019)*

Q 8. If $\vec{a},\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$, then prove that $3(\vec{a}\cdot\vec{b})+4(\vec{c}\cdot\vec{a})+5(\vec{b}\cdot\vec{c})+6=0$.

Q 9. If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that $|\hat{a}+\hat{b}|=2\cos\frac{\theta}{2}$, where θ is the angle between them. *(CBSE SQP 2022 Term-2)*

Q 10. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of point A,B,C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not. *(CBSE 2019)*

Q 11. If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$. *(CBSE 2017)*

Q 12. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$ and each one of them is perpendicular to the sum of other two vectors, then find $|\vec{a}+\vec{b}+\vec{c}|$. *(NCERT EXERCISE)*

Q 13. Three vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=2$. *(NCERT EXERCISE; CBSE 2023)*

Q 14. If $\vec{a}=2\hat{i}-\hat{j}-2\hat{k}$ and $\vec{b}=7\hat{i}+2\hat{j}-3\hat{k}$, then express $\vec{b}$ in the form of $\vec{b}=\vec{b}_1+\vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$. *(CBSE 2017)*

Q 15. For any two vectors $\vec{a}$ and $\vec{b}$, show that $|\vec{a}\cdot\vec{b}| \le |\vec{a}||\vec{b}|$. *(NCERT EXERCISE)*

Q 16. For any two vectors $\vec{a}$ and $\vec{b}$, show that $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$. *(NCERT EXERCISE)*

Q 17. If $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=4\hat{i}-7\hat{j}+\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=6$. *(CBSE 2017)*

Q 18. Let $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}\cdot\vec{a}=21$. *(CBSE 2018)*

Q 19. Find the area of a parallelogram *ABCD* whose side *AB* and the diagonal *DB* are given by the vectors $5\hat{i}+7\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$ respectively. *(CBSE 2017)*

Q 20. Find the area of parallelogram, whose diagonals are $\vec{d_1}=3\hat{i}+2\hat{j}-\hat{k}$ and $\vec{d_2}=\hat{i}-3\hat{j}+2\hat{k}$.

Q 21. If $\vec{a}=3\hat{i}-\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$, then find the area of that triangle whose two sides are represented by $\vec{a}$ and $\vec{b}$.

Q 22. Using vectors, find the area of ΔABC, with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$. *(CBSE 2020, 17)*

Q 23. Show that the points *A*, *B*, *C* with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively, are the vertices of a right angled triangle. Hence, find the area of the triangle. *(CBSE 2017)*

Q 24. Given that vectors $\vec{a}, \vec{b}, \vec{c}$ form a triangle such that $\vec{a}=\vec{b}+\vec{c}$. Find *p*, *q*, *r*, *s* such that area of triangle is $5\sqrt{6}$, where $\vec{a}=p\hat{i}+q\hat{j}+r\hat{k}$, $\vec{b}=s\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-2\hat{k}$. *(CBSE 2016)*

Q 25. Prove that the area of triangle, the position vector of whose vertices are $\vec{a}, \vec{b}, \vec{c}$, is: $\frac{1}{2}|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|$.

Q 26. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$, then find a unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$. *(CBSE 2023)*

Or

Find the unit normal vector to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$. *(NCERT EXERCISE)*

Q 27. If $\vec{a}\ne\vec{0}$, $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, then show that $\vec{b}=\vec{c}$. *(CBSE SQP 2022 Term-2)*

Long Answer Type Questions

Q 1. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram. *(CBSE 2022 Term-2)*

Q 2. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|\vec{c}|=\sqrt{3}$, then find the values of α and β.

Q 3. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vectors $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$, then find the value of $\vec{a}+2\vec{b}+6\vec{c}$.

Q 4. The scalar product of vector $\vec{r}$ from the vectors $3\hat{i}-5\hat{k}$, $2\hat{i}+7\hat{j}$ and $\hat{i}+\hat{j}+\hat{k}$ are respectively $-1, 6$ and 5. Find vector $\vec{r}$.

Q 5. Find the length of longer diagonal of parallelogram constructed on $5\vec{a}+2\vec{b}$ and $\vec{a}-3\vec{b}$, given that $|\vec{a}|=2\sqrt{2}$, $|\vec{b}|=3$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.

Q 6. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then show that $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$ does not exceed by 9.

Q 7. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}|=1$, $|\vec{v}|=2$, $|\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other, then find $|\vec{u}-\vec{v}+\vec{w}|$.

Q 8. Using vector method in ΔABC, prove that:

$$\cos A=\frac{b^2+c^2-a^2}{2bc}.$$ *(NCERT EXEMPLAR)*

Q 9. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$, then prove that the value of $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is $6(\vec{b}\times\vec{c})$ or $2(\vec{a}\times\vec{b})$ or $3(\vec{c}\times\vec{a})$.

Q 10. Using vector method in ΔABC, prove that:

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}.$$ *(NCERT EXEMPLAR)*

Solutions

Very Short Answer Type Questions

1. The associative law of vector addition is as follows:

$$\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$$

2. According to the triangle law of vector addition,

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}=-\overrightarrow{CA} \qquad [\because \overrightarrow{AC}=-\overrightarrow{CA}]$$

$$\Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{0}$$

Hence proved.

3. Required sum $=\overrightarrow{BA}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DA}$

$$=\overrightarrow{BA}+\overrightarrow{BD}+\overrightarrow{DA} \quad [\because \overrightarrow{BC}+\overrightarrow{CD}=\overrightarrow{BD}]$$

$$=\overrightarrow{BA}+\overrightarrow{BA} \quad [\because \overrightarrow{BD}+\overrightarrow{DA}=\overrightarrow{BA}]$$

$$=2\overrightarrow{BA}$$

4. In ΔEBF, $\overrightarrow{EF}=\overrightarrow{EB}+\overrightarrow{BF}$ [by triangle law]

$$=\frac{1}{2}\overrightarrow{AB}+\frac{1}{2}\overrightarrow{BC}=\frac{1}{2}(\overrightarrow{AB}+\overrightarrow{BC})$$

$$=\frac{1}{2}(\overrightarrow{AB}+\overrightarrow{AD})=\frac{1}{2}(\vec{a}+\vec{b}) \quad [\because \text{In} \parallel \text{gram}, \overrightarrow{BC}=\overrightarrow{AD}]$$

5. $\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$$=(4\hat{j}-5\hat{k})-(\hat{i}+\hat{j})=-\hat{i}+3\hat{j}-5\hat{k}$$

6. Resultant of these forces:

$$\vec{F}=(2\hat{i}+3\hat{j}+4\hat{k})+(-4\hat{j}+\hat{i})+(\hat{j}-4\hat{k}-3\hat{i})$$

$$=(2+1-3)\hat{i}+(3-4+1)\hat{j}+(4-4)\hat{k}=\vec{0}$$

Since, the resultant of these forces is zero, so the particle is in equilibrium. **Hence proved.**

7. Since, given vectors are collinear.

$$\therefore \quad \frac{2}{a}=\frac{-3}{6}=\frac{4}{-8}$$

$$\Rightarrow \quad a=-4$$

8. Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

$$\therefore \quad |\vec{a}|=\sqrt{(1)^2+(2)^2+(3)^2}=\sqrt{1+4+9}=\sqrt{14}$$

$\therefore$ Direction cosines $=\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$

9. Required unit vector $=\dfrac{2\hat{i}-\hat{j}+2\hat{k}}{|2\hat{i}-\hat{j}+2\hat{k}|}$

> **TiP**
> *Read the question carefully and practice more problems on unit vectors.*

$$=\frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{2^2+(-1)^2+2^2}}=\frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{4+1+4}}$$

$$=\frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{9}}=\frac{2\hat{i}-\hat{j}+2\hat{k}}{3}$$

10. $\because \quad \hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$

$$\therefore \quad \text{LHS}=(\hat{i}\cdot\hat{j})\hat{k}+(\hat{j}\cdot\hat{k})\hat{i}+(\hat{k}\cdot\hat{i})\hat{j}$$

$$=0\hat{k}+0\hat{i}+0\hat{j}=\vec{0}=\text{RHS}$$

Hence proved.

11. If vectors $(a\hat{i}+2\hat{j}+3\hat{k})$ and $(3\hat{i}+b\hat{j})$ are perpendicular, then

$$(a\hat{i}+2\hat{j}+3\hat{k})\cdot(3\hat{i}+b\hat{j})=0$$

$$\Rightarrow \quad (3)(a)+(b)(2)=0$$

$$\Rightarrow \quad 3a+2b=0$$

Hence proved.

12. Let $\vec{a}=2\hat{i}-\hat{j}+p\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$

Let $\vec{a}$ and $\vec{b}$ be perpendicular to each other then $\vec{a}\cdot\vec{b}=0$.

i.e., $\quad (2\hat{i}-\hat{j}+p\hat{k})\cdot(\hat{i}+\hat{j}-\hat{k})=0$

$$\Rightarrow \quad (2)(1)+(-1)+(p)(-1)=0$$

$$\Rightarrow \quad 2-1-p=0 \Rightarrow p=1$$

13. Given, $\vec{a}\cdot\vec{b}=1, |\vec{a}|=1$ and $|\vec{b}|=2$

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

$$\therefore \quad \cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{1}{1\cdot 2}=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$\therefore \quad \theta=\frac{\pi}{3}$$

14. Given that, the two vectors $\vec{a}$ and $\vec{b}$ having the same magnitude.

$$\therefore \quad |\vec{a}|=|\vec{b}|$$

> **TiP**
> *If θ is the angle between two non-zero vectors $\vec{a}$ and $\vec{b}$, then the scalar product is given by*
> $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$ *where,* $0\le\theta\le\pi$

Angle between $\vec{a}$ and $\vec{b}$ is 60° and their scalar product is $\frac{9}{2}$.

i.e., $\quad \vec{a}\cdot\vec{b}=\frac{9}{2}$

$$\therefore \quad |\vec{a}||\vec{b}|\cos\theta=\frac{9}{2}$$

[Here 'θ' is the angle between $\vec{a}$ and $\vec{b}$]

$$\Rightarrow \quad |\vec{a}||\vec{b}|\cos 60^\circ=\frac{9}{2} \quad [\because \theta=60^\circ]$$

$$\Rightarrow \quad |\vec{a}||\vec{a}|\times\frac{1}{2}=\frac{9}{2} \quad [\because |\vec{a}|=|\vec{b}|]$$

$$\Rightarrow \quad |\vec{a}|^2=9 \Rightarrow |\vec{a}|=3$$

Thus, $\quad |\vec{a}|=|\vec{b}|=3$

15. Given, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$

$$\therefore \quad \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & 3 & -1\end{vmatrix}$$

$$=\hat{i}(-1-3)-\hat{j}(-1-2)+\hat{k}(3-2)$$

$$=-4\hat{i}+3\hat{j}+\hat{k}$$

Therefore, a vector perpendicular to $\vec{a}$ and $\vec{b}$ is $-4\hat{i}+3\hat{j}+\hat{k}$.

16. $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$ [given]

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix}=\vec{0}$$

$$\Rightarrow \hat{i}(6\mu-27\lambda)-\hat{j}(2\mu-27)+\hat{k}(2\lambda-6)=\vec{0}$$

On comparing, we get

$$6\mu-27\lambda=0,\ 2\mu-27=0,\ 2\lambda-6=0$$

$$\Rightarrow \lambda=3 \text{ and } \mu=\frac{27}{2}$$

17. $$\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}=\hat{k}(21+20)=41\hat{k}$$

Hence, area of triangle $=\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{41}{2}$ sq. units.

18. (i) $\hat{i}\cdot(\hat{j}\times\hat{k})+(\hat{i}\times\hat{k})\cdot\hat{j}=\hat{i}\cdot(\hat{i})+(-\hat{j})\cdot\hat{j}=1-1=0$

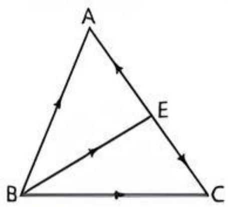
(ii) $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{k}\times\hat{i})+\hat{k}\cdot(\hat{i}\times\hat{j})$

$$=\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{j}+\hat{k}\cdot\hat{k}=1+1+1=3$$

Short Answer Type-I Questions

1. In ΔBAE, $\overrightarrow{BA}=\overrightarrow{BE}+\overrightarrow{EA}$ [by triangle law] ...(1)

In ΔBCE, $\overrightarrow{BC}=\overrightarrow{BE}+\overrightarrow{EC}$ [by triangle law]...(2)



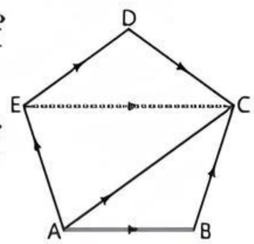
Adding eqs. (1) and (2), we get

$$\overrightarrow{BA}+\overrightarrow{BC}=2\overrightarrow{BE}+\overrightarrow{EA}+\overrightarrow{EC}=2\overrightarrow{BE}$$

[$\because$ $\overrightarrow{EA}$, $\overrightarrow{EC}$ are equal and opposite.]

$$\Rightarrow \overrightarrow{BE}=\frac{1}{2}(\overrightarrow{BA}+\overrightarrow{BC})$$ **Hence proved.**

2. $\overrightarrow{AB}+\overrightarrow{AE}+\overrightarrow{BC}+\overrightarrow{DC}+\overrightarrow{ED}+\overrightarrow{AC}$



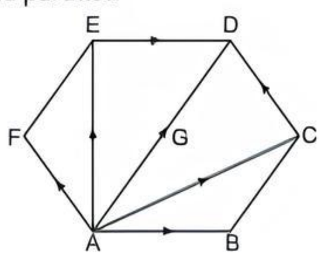
$=(\overrightarrow{AB}+\overrightarrow{BC})+(\overrightarrow{ED}+\overrightarrow{DC})+\overrightarrow{AE}+\overrightarrow{AC}$

$=\overrightarrow{AC}+\overrightarrow{EC}+\overrightarrow{AE}+\overrightarrow{AC}$

$=2\overrightarrow{AC}+(\overrightarrow{AE}+\overrightarrow{EC})$

$=2\overrightarrow{AC}+\overrightarrow{AC}=3\overrightarrow{AC}$ **Hence proved.**

3. We know that, the opposite sides of regular hexagon are equal and parallel.



$\therefore$ In regular hexagon $ABCDEF$,

$$\overrightarrow{AB}=\overrightarrow{ED}$$

and $$\overrightarrow{AF}=\overrightarrow{CD}$$

$\therefore$ $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$

$=\overrightarrow{ED}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{CD}$

$=(\overrightarrow{AE}+\overrightarrow{ED})+(\overrightarrow{AC}+\overrightarrow{CD})+\overrightarrow{AD}$

$=\overrightarrow{AD}+\overrightarrow{AD}+\overrightarrow{AD}=3\overrightarrow{AD}$ **Hence proved.**

4. Let the position vectors of the points A (4, 7, 8) and B (2, 3, 4) with respect to origin O be $\overrightarrow{OA}=4\hat{i}+7\hat{j}+8\hat{k}$ and $\overrightarrow{OB}=2\hat{i}+3\hat{j}+4\hat{k}$ respectively.

$\therefore$ $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

$=(2\hat{i}+3\hat{j}+4\hat{k})-(4\hat{i}+7\hat{j}+8\hat{k})$

$=2\hat{i}+3\hat{j}+4\hat{k}-4\hat{i}-7\hat{j}-8\hat{k}$

$=-2\hat{i}-4\hat{j}-4\hat{k}$

Again, let the position vectors of the point C (–1, –2, 1) and D (1, 2, 5) with respect to origin O are $\overrightarrow{OC}=-\hat{i}-2\hat{j}+\hat{k}$ and $\overrightarrow{OD}=\hat{i}+2\hat{j}+5\hat{k}$ respectively.

$\therefore$ $\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}$

$=(\hat{i}+2\hat{j}+5\hat{k})-(-\hat{i}-2\hat{j}+\hat{k})$

$=\hat{i}+2\hat{j}+5\hat{k}+\hat{i}+2\hat{j}-\hat{k}$

$=2\hat{i}+4\hat{j}+4\hat{k}$

$=-(-2\hat{i}-4\hat{j}-4\hat{k})=-\overrightarrow{AB}$

$\because$ $\overrightarrow{CD}=m\overrightarrow{AB}$ (where, $m=-1$)

Hence, the line passing through the points A and B is parallel to the line passing through the points C and D.

Hence proved.

5. Let the position vectors of the points P (2, –5, 1) and Q (1, 4, –6) be $\overrightarrow{OP}=2\hat{i}-5\hat{j}+\hat{k}$ and $\overrightarrow{OQ}=\hat{i}+4\hat{j}-6\hat{k}$ respectively.

Let the point R divide PQ internally in the ratio 2 : 3.

$\therefore$ Position vector of $R=\dfrac{2\overrightarrow{OQ}+3\overrightarrow{OP}}{2+3}$

$$=\frac{2(\hat{i}+4\hat{j}-6\hat{k})+3(2\hat{i}-5\hat{j}+\hat{k})}{5}$$

$$=\frac{(2+6)\hat{i}+(8-15)\hat{j}+(-12+3)\hat{k}}{5}$$

$$=\frac{8\hat{i}-7\hat{j}-9\hat{k}}{5}$$

6. Here, $\vec{a}+\vec{b}=(2\hat{i}+2\hat{j}-5\hat{k})+(2\hat{i}+\hat{j}+3\hat{k})$

$=4\hat{i}+3\hat{j}-2\hat{k}$

and $|\vec{a}+\vec{b}|=\sqrt{4^2+3^2+(-2)^2}=\sqrt{29}$

$\therefore$ Unit vector along the sum $= \dfrac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$

$$= \frac{1}{\sqrt{29}}(4\hat{i}+3\hat{j}-2\hat{k})$$

Again $\vec{a}-\vec{b} = (2\hat{i}+2\hat{j}-5\hat{k})-(2\hat{i}+\hat{j}+3\hat{k}) = \hat{j}-8\hat{k}$

and $|\vec{a}-\vec{b}| = \sqrt{1^2+(-8)^2} = \sqrt{65}$

$\therefore$ Unit vector along the difference $= \dfrac{\vec{a}-\vec{b}}{|\vec{a}-\vec{b}|}$

$$= \frac{1}{\sqrt{65}}(\hat{j}-8\hat{k})$$

COMMON ERROR

Some students find any vector instead of unit vector and others find the unit vector in the same direction.

7. Given, $\vec{a} = \hat{i}-\hat{j}+7\hat{k}$ and $\vec{b} = 5\hat{i}-\hat{j}+\lambda\hat{k}$

Now, $(\vec{a}+\vec{b}) = (\hat{i}-\hat{j}+7\hat{k})+(5\hat{i}-\hat{j}+\lambda\hat{k})$

$$= 6\hat{i}-2\hat{j}+(7+\lambda)\hat{k}$$

and $(\vec{a}-\vec{b}) = (\hat{i}-\hat{j}+7\hat{k})-(5\hat{i}-\hat{j}+\lambda\hat{k})$

$$= \hat{i}-\hat{j}+7\hat{k}-5\hat{i}+\hat{j}-\lambda\hat{k}$$

$$= -4\hat{i}+(7-\lambda)\hat{k}$$

$(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ will be orthogonal, if

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0$$

$\Rightarrow \{6\hat{i}-2\hat{j}+(7+\lambda)\hat{k}\}\cdot\{-4\hat{i}+(7-\lambda)\hat{k}\} = 0$

$\Rightarrow (6)(-4)+(7+\lambda)(7-\lambda) = 0$

$\Rightarrow -24+49-\lambda^2 = 0$

$\Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$

8. Given, $\vec{a} = 5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b} = \hat{i}+3\hat{j}-5\hat{k}$

Now, $\vec{a}+\vec{b} = (5\hat{i}-\hat{j}-3\hat{k})+(\hat{i}+3\hat{j}-5\hat{k})$

$$= 6\hat{i}+2\hat{j}-8\hat{k}$$

and $\vec{a}-\vec{b} = (5\hat{i}-\hat{j}-3\hat{k})-(\hat{i}+3\hat{j}-5\hat{k})$

$$= (4\hat{i}-4\hat{j}+2\hat{k})$$

$\therefore (\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = (6\hat{i}+2\hat{j}-8\hat{k})\cdot(4\hat{i}-4\hat{j}+2\hat{k})$

$$= (6)(4)+(2)(-4)+(-8)(2)$$

$$= 24-8-16 = 24-24 = 0$$

Therefore, vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular. **Hence proved.**

9.

TR!CK

The projection of $\vec{a}$ on $\vec{b}$ is given by $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = p\hat{i}+\hat{j}-2\hat{k}$

$\therefore$ The projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{(\hat{i}+\hat{j}+\hat{k})\cdot(p\hat{i}+\hat{j}-2\hat{k})}{\sqrt{(p)^2+(1)^2+(-2)^2}}$$

$\Rightarrow \dfrac{1}{3} = \dfrac{p+1-2}{\sqrt{p^2+5}} \Rightarrow \sqrt{p^2+5} = 3(p-1)$ ...(1)

Squaring both sides, we get

$p^2+5 = 9(p^2+1-2p) \Rightarrow 8p^2-18p+4 = 0$

$\Rightarrow 4p^2-9p+2 = 0 \Rightarrow (4p-1)(p-2) = 0$

$\Rightarrow p = 2, \dfrac{1}{4}$

But $p = \dfrac{1}{4}$ does not satisfy the eq. (1).

Hence, $p = 2$.

10. Given, $\vec{a} = 2\hat{i}-2\hat{j}+\hat{k}, \vec{b} = \hat{i}+2\hat{j}-2\hat{k}$

and $\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$

Now, $\vec{b}+\vec{c} = (\hat{i}+2\hat{j}-2\hat{k})+(2\hat{i}-\hat{j}+4\hat{k})$

$$= 3\hat{i}+\hat{j}+2\hat{k}$$

$\therefore$ Projection of the vector $(\vec{b}+\vec{c})$ on the vector $\vec{a}$

$$= \frac{(\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}|}$$

$$= \frac{(3\hat{i}+\hat{j}+2\hat{k})\cdot(2\hat{i}-2\hat{j}+\hat{k})}{\sqrt{(2)^2+(-2)^2+(1)^2}}$$

$$= \frac{(3)(2)+(1)(-2)+(2)(1)}{\sqrt{4+4+1}}$$

$= \dfrac{6-2+2}{\sqrt{9}} = \dfrac{6}{3} = 2.$ **Hence proved.**

11. Given, $\hat{a}$ is a unit vector.

$\therefore |\vec{a}| = 1$ $\left[\because \hat{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{\vec{a}}{1} = \vec{a}\right]$

We have, $(\vec{x}-\hat{a})\cdot(\vec{x}+\hat{a}) = 12$

$\Rightarrow (\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 12$

$\Rightarrow \vec{x}\cdot\vec{x}-\vec{a}\cdot\vec{x}+\vec{x}\cdot\vec{a}-\vec{a}\cdot\vec{a} = 12$

$\Rightarrow (\vec{x})^2-\vec{x}\cdot\vec{a}+\vec{x}\cdot\vec{a}-(\vec{a})^2 = 12$ $[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}]$

$\Rightarrow |\vec{x}|^2-|\vec{a}|^2 = 12$ $[\because \vec{a}^2 = |\vec{a}|^2]$

$\Rightarrow |\vec{x}|^2-(1)^2 = 12$

$\Rightarrow |\vec{x}|^2 = 12+1 = 13$

$\therefore |\vec{x}| = \sqrt{13}$

12. Given, $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$

$\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$

and $\vec{c} = 3\hat{i}+\hat{j}$

$$\vec{a}+\lambda\vec{b}=(2\hat{i}+2\hat{j}+3\hat{k})+\lambda(-\hat{i}+2\hat{j}+\hat{k})$$
$$=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$$

$\because$ $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$.

$\therefore$ $(\vec{a}+\lambda\vec{b})\cdot\vec{c}=0$

$\Rightarrow [(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}]\cdot[3\hat{i}+\hat{j}]=0$

$\Rightarrow$ $3(2-\lambda)+(1)(2+2\lambda)=0$

$\Rightarrow$ $6-3\lambda+2+2\lambda=0$

$-\lambda+8=0$

$\Rightarrow$ $\lambda=8$

TR!CK

$(a_1\hat{i}+b_1\hat{j}+c_1\hat{k})\cdot(a_2\hat{i}+b_2\hat{j}+c_2\hat{k})$
$=(a_1a_2+b_1b_2+c_2c_2)$

13. Given, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

Squaring both sides, we get

$$|\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$$

or $(\vec{a}+\vec{b})^2=(\vec{a}-\vec{b})^2$

[$\because$ square of vector = square of its modulus]

or $(\vec{a})^2+(\vec{b})^2+2(\vec{a}\cdot\vec{b})=(\vec{a})^2+(\vec{b})^2-2(\vec{a}\cdot\vec{b})$

or $4(\vec{a}\cdot\vec{b})=0$ or $\vec{a}\cdot\vec{b}=0$

Therefore, vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other. **Hence proved.**

14. Let $\vec{a}$ and $\vec{b}$ be unit vectors.

Then, according to question,

$$\vec{a}+\vec{b}=\vec{c} \quad ...(1)$$

Here, $\vec{c}$ is also a unit vector.

$\therefore$ $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ ...(2)

From eq. (1), $|\vec{a}+\vec{b}|=|\vec{c}|$

$\Rightarrow$ $|\vec{a}+\vec{b}|=1$

We know that, $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2(|\vec{a}|^2+|\vec{b}|^2)$

$\Rightarrow$ $1^2+|\vec{a}-\vec{b}|^2=2(1+1)$

$\Rightarrow$ $|\vec{a}-\vec{b}|^2=4-1=3$

$\Rightarrow$ $|\vec{a}-\vec{b}|=\sqrt{3}$

15. Given, $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$

Let the angle formed by vector $\vec{a}$ from X-axis = θ

$\because$ Unit vector along X-axis = $\hat{i}$

$\therefore$ Angle formed by vector $\vec{a}$ from X-axis

= Angle between the vectors $\vec{a}$ and $\hat{i}$ = θ

Then, $\cos\theta=\dfrac{\vec{a}\cdot\hat{i}}{|\vec{a}||\hat{i}|}=\dfrac{\vec{a}\cdot\hat{i}}{|\vec{a}|}$ $[\because |\hat{i}|=1]$

$$=\frac{(\hat{i}-2\hat{j}+\hat{k})\cdot\hat{i}}{\sqrt{(1)^2+(-2)^2+(1)^2}}=\frac{(1)(1)}{\sqrt{1+4+1}}=\frac{1}{\sqrt{6}}$$

$\Rightarrow$ $\theta=\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$, which is the required angle.

16. Let $\vec{p}=a\hat{i}+\hat{j}+3\hat{k}$ and $\vec{q}=3\hat{i}-2\hat{j}+\hat{k}$

$\because$ The angle between the vectors is 60°.

$\therefore$ $\dfrac{\vec{p}\cdot\vec{q}}{|\vec{p}||\vec{q}|}=\cos 60°=\dfrac{1}{2}$

$\Rightarrow$ $\dfrac{(a\hat{i}+\hat{j}+3\hat{k})\cdot(3\hat{i}-2\hat{j}+\hat{k})}{\sqrt{a^2+1^2+3^2}\sqrt{3^2+(-2)^2+1^2}}=\dfrac{1}{2}$

or $\dfrac{a\times3+1\times(-2)+3\times1}{\sqrt{a^2+10}\sqrt{14}}=\dfrac{1}{2}$

or $\dfrac{3a+1}{\sqrt{a^2+10}\sqrt{14}}=\dfrac{1}{2}$

or $[2(3a+1)]^2=(a^2+10)\cdot14$

[squaring on both sides]

$\Rightarrow$ $4(9a^2+6a+1)=14a^2+140$

$\Rightarrow$ $22a^2+24a-136=0$

or $11a^2+12a-68=0$

$\Rightarrow$ $11a^2+34a-22a-68=0$

or $a(11a+34)-2(11a+34)=0$

$\Rightarrow$ $(11a+34)(a-2)=0$

$\Rightarrow$ $a=2$ or $a=-\dfrac{34}{11}$

17. $(\vec{a}\cdot\vec{b})^2=a^2b^2$

$\Rightarrow$ $(ab\cos\theta)^2=a^2b^2$ where, $|\vec{a}|=a$ and $|\vec{b}|=b$

$\Rightarrow$ $a^2b^2\cos^2\theta=a^2b^2$

$\Rightarrow$ $\cos^2\theta=1$ $\Rightarrow$ $\cos\theta=\pm1$

$\Rightarrow$ $\theta=0°$ or $180°$

Therefore, the given relation will be true when the angle between two vectors is 0° or 180° *i.e.*, when two vectors are parallel.

18. $\because$ $\vec{a}\times\vec{b}=ab\sin\theta\,\hat{n}$

$\therefore$ LHS $=(\vec{a}\times\vec{b})^2=(ab\sin\theta\,\hat{n})^2$

$=a^2b^2\sin^2\theta(\hat{n}\cdot\hat{n})$

$=a^2b^2\sin^2\theta\cdot1$

$=a^2b^2(1-\cos^2\theta)$ $[\because \sin^2\theta+\cos^2\theta=1]$

$=a^2b^2-(ab\cos\theta)^2$

$=a^2b^2-(\vec{a}\cdot\vec{b})^2=$ RHS **Hence proved.**

19. Given, $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$

The area of parallelogram having adjacent sides $\vec{a}$ and $\vec{b}$ is given by

$$|\vec{a}\times\vec{b}|=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & -7 & 1\end{vmatrix}$$

$=|\hat{i}(-1+21)-\hat{j}(1-6)+\hat{k}(-7+2)|$

$=|20\hat{i}+5\hat{j}-5\hat{k}|=5|4\hat{i}+\hat{j}-\hat{k}|$

$=5\sqrt{(4)^2+(1)^2+(-1)^2}=5\sqrt{16+1+1}$

$=5\sqrt{18}=15\sqrt{2}$ sq. units

20. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$$

$$= \hat{i}(1-4) - \hat{j}(-2-3) + \hat{k}(8+3)$$

$$= -3\hat{i} + 5\hat{j} + 11\hat{k}$$

and $|\vec{a} \times \vec{b}| = |-3\hat{i} + 5\hat{j} + 11\hat{k}|$

$$= \sqrt{(-3)^2 + 5^2 + 11^2}$$

$$= \sqrt{9 + 25 + 121} = \sqrt{155}$$

Therefore, the unit vector perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k} = \dfrac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$

$$= \frac{1}{\sqrt{155}} \cdot (-3\hat{i} + 5\hat{j} + 11\hat{k})$$

21. Given $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}(-1+2) - \hat{j}(4-2) + \hat{k}(-8+2) = \hat{i} - 2\hat{j} - 6\hat{k}$$

and $|\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (-2)^2 + (-6)^2}$

$$= \sqrt{1 + 4 + 36} = \sqrt{41}$$

$\therefore$ The unit vector along the vector $\vec{a} \times \vec{b}$

$$= \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} = \frac{\hat{i} - 2\hat{j} - 6\hat{k}}{\sqrt{41}}$$

22. Given that, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

Now, $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= (-2+6)\hat{i} - (1-9)\hat{j} + (-2+6)\hat{k}$$

$$= 4\hat{i} + 8\hat{j} - 4\hat{k}$$

TR!CK

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,

then $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$$

and $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$$\Rightarrow |\vec{a} \times \vec{b}| = |4\hat{i} + 8\hat{j} - 4\hat{k}|$$

$$= \sqrt{(4)^2 + (8)^2 + (-4)^2}$$

$$= \sqrt{16 + 64 + 16} = \sqrt{96} = 4\sqrt{6}$$

Now, $|\vec{a}| = |\hat{i} - 2\hat{j} + 3\hat{k}| = \sqrt{(1)^2 + (-2)^2 + (3)^2}$

$$= \sqrt{1 + 4 + 9} = \sqrt{14}$$

and $|\vec{b}| = |3\hat{i} - 2\hat{j} + \hat{k}|$

$$= \sqrt{(3)^2 + (-2)^2 + (1)^2}$$

$$= \sqrt{9 + 4 + 1} = \sqrt{14}$$

Since, θ is the angle between $\vec{a}$ and $\vec{b}$.

TR!CK

Angle between two non-zero vectors is given by

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$\therefore \sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{4\sqrt{6}}{\sqrt{14}\sqrt{14}} = \frac{4\sqrt{16}}{14} = \frac{2}{7}\sqrt{16}$$

23. Given, $|\vec{a}| = 3, |\vec{b}| = \dfrac{2}{3}$ and $|\vec{a} \times \vec{b}| = 1$

$$\because \vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}||\sin\theta|$$

$$\Rightarrow 1 = 3 \times \frac{2}{3} \times |\sin\theta|$$

$$\Rightarrow |\sin\theta| = \frac{1}{2} \Rightarrow \theta = 30°$$

24. Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

Then, $\vec{a} \cdot \vec{b} = 1 \Rightarrow x + y + z = 1$ ...(1)

Also, $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k}$$

$$\Rightarrow \hat{i}(z-y) - \hat{j}(z-x) + \hat{k}(y-x) = \hat{j} - \hat{k}$$

On comparing, we get

$x - z = 1$ ...(2)

and $x - y = 1$ ...(3)

From eqs. (1), (2) and (3), we get

$x = 1,\ y = 0,\ z = 0$

$\therefore \vec{b} = \hat{i}$

25. Given, $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ ...(1)

and $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$ ...(2)

If $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$, then

$$(\vec{a} - 2\vec{d}) \times (2\vec{b} - \vec{c}) = \vec{0}$$

LHS $= (\vec{a} - 2\vec{d}) \times (2\vec{b} - \vec{c})$

$$= \vec{a} \times 2\vec{b} - \vec{a} \times \vec{c} - 2\vec{d} \times 2\vec{b} + 2\vec{d} \times \vec{c}$$

$$= 2\vec{a} \times \vec{b} - \vec{a} \times \vec{c} - 4\vec{d} \times \vec{b} + 2\vec{d} \times \vec{c}$$

$$= 2\vec{a} \times \vec{b} - \vec{a} \times \vec{c} + 4\vec{b} \times \vec{d} - 2\vec{c} \times \vec{d}$$

$= 2\vec{c}\times\vec{d} - 4\vec{b}\times\vec{d} + 4\vec{b}\times\vec{d} - 2\vec{c}\times\vec{d}$

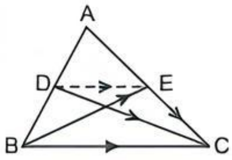
[from eqs. (1) and (2)]

$= \vec{0} = \text{RHS}$

Hence, $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$. **Hence proved.**

Short Answer Type-II Questions

1. $\because$ *D* and *E* are the mid-points of sides *AB* and *AC* respectively of ΔABC.



$\therefore$ *DE* and *BC* are parallel

and $DE = \frac{1}{2} BC$

$\Rightarrow \quad \overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$ ...(1)

In ΔBCE, $\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE}$

and in ΔDCE, $\overrightarrow{DC} = \overrightarrow{DE} + \overrightarrow{EC}$

$\therefore \overrightarrow{BE} + \overrightarrow{DC} = (\overrightarrow{BC} + \overrightarrow{CE}) + (\overrightarrow{DE} + \overrightarrow{EC})$

$= \overrightarrow{BC} + \overrightarrow{DE} + (\overrightarrow{CE} + \overrightarrow{EC})$

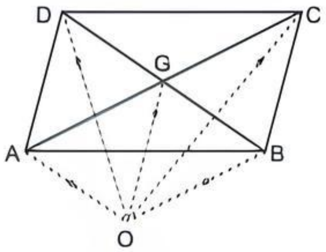
$= \overrightarrow{BC} + \overrightarrow{DE} + (-\overrightarrow{EC} + \overrightarrow{EC})$ $[\because \overrightarrow{CE} = -\overrightarrow{EC}]$

$= \overrightarrow{BC} + \overrightarrow{DE} + \vec{0} = \overrightarrow{BC} + \overrightarrow{DE} = \overrightarrow{BC} + \frac{1}{2}\overrightarrow{BC}$

[from eq. (1)]

$= \frac{3}{2}\overrightarrow{BC}$ **Hence proved.**

2. The diagonal of a parallelogram bisect each other.



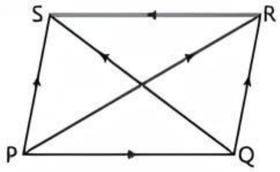
$\therefore$ Point of intersection *G* will be the mid-point of the diagonals *AC* and *BD* both.

Then, $\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OG}$ and $\overrightarrow{OB} + \overrightarrow{OD} = 2\overrightarrow{OG}$

Adding, we get $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OG}$

Hence proved.

3. Given, $\overrightarrow{PQ} = 3\hat{i} - 2\hat{j} + 2\hat{k}$



and $\overrightarrow{PS} = -\hat{i} - 2\hat{k}$

Now, diagonal $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$ [by triangle law]

$= \overrightarrow{PQ} + \overrightarrow{PS}$ $[\because \overrightarrow{QR} = \overrightarrow{PS}]$

$= (3\hat{i} - 2\hat{j} + 2\hat{k}) + (-\hat{i} - 2\hat{k})$

$= 2\hat{i} - 2\hat{j}$

$|\overrightarrow{PR}| = \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4+4} = \sqrt{8}$

$= 2\sqrt{2}$ units

and diagonal $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$

$= \overrightarrow{PS} - \overrightarrow{PQ}$

[$\because$ in ||gm, opposite sides are equal]

$= (-\hat{i} - 2\hat{k}) - (3\hat{i} - 2\hat{j} + 2\hat{k})$

$= -\hat{i} - 2\hat{k} - 3\hat{i} + 2\hat{j} - 2\hat{k}$

$= -4\hat{i} + 2\hat{j} - 4\hat{k}$

$|\overrightarrow{QS}| = \sqrt{(-4)^2 + (2)^2 + (-4)^2}$

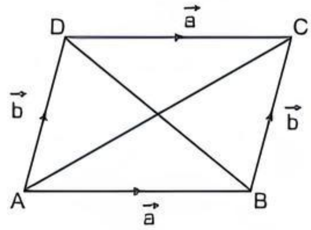
$= \sqrt{16+4+16} = \sqrt{36}$

$= 6$ units

4. Given two adjacent sides of parallelogram *ABCD* are $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$.

TiP

Practice problem based on parallel and perpendicular vectors.



Here, diagonal of parallelogram are $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

$\therefore \quad \overrightarrow{AC} = \vec{a} + \vec{b}$ [by triangle law]

and $\quad \overrightarrow{BD} = \vec{b} - \vec{a}$ [by triangle law]

Now, $\quad \overrightarrow{AC} = (\hat{i} + 2\hat{j} + 3\hat{k}) + (2\hat{i} + 4\hat{j} - 5\hat{k})$

$= 3\hat{i} + 6\hat{j} - 2\hat{k}$

and $\quad \overrightarrow{BD} = (2\hat{i} + 4\hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= \hat{i} + 2\hat{j} - 8\hat{k}$

$\therefore$ Unit vector parallel to the diagonal $\overrightarrow{AC}$

$= \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(3)^2 + (6)^2 + (-2)^2}}$

$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

and unit vector parallel to the diagonal $\overrightarrow{BD} = \frac{\overrightarrow{BD}}{|\overrightarrow{BD}|}$

$= \frac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{(1)^2 + (2)^2 + (-8)^2}} = \frac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{1+4+64}}$

$= \frac{1}{\sqrt{69}}(\hat{i} + 2\hat{j} - 8\hat{k})$

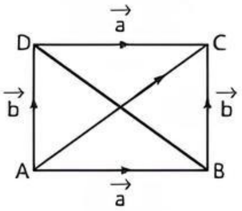
COMMON ERROR

Instead of finding the parallel vectors, some students take the cross product to find the perpendicular vector.

5. Given two adjacent sides of parallelogram *ABCD* are

$$\vec{a} = 2\hat{i} - 4\hat{j} - 5\hat{k}$$

and $$\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$



Here, diagonal of parallelogram are $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

$\therefore \quad \overrightarrow{AC} = \vec{a} + \vec{b}$ [by triangle law]

and $\quad \overrightarrow{BD} = \vec{b} - \vec{a}$ [by triangle law]

Now, $$\overrightarrow{AC} = (2\hat{i} - 4\hat{j} - 5\hat{k}) + (2\hat{i} + 2\hat{j} + 3\hat{k})$$
$$= 4\hat{i} - 2\hat{j} - 2\hat{k}$$

and $$\overrightarrow{BD} = (2\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} - 4\hat{j} - 5\hat{k})$$
$$= 6\hat{j} + 8\hat{k}$$

$\therefore$ Unit vector parallel to diagonal $\overrightarrow{AC} = \dfrac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$

$$= \frac{4\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{(4)^2 + (-2)^2 + (-2)^2}} = \frac{4\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{16+4+4}} = \frac{2\hat{i} - \hat{j} - \hat{k}}{\sqrt{6}}$$

and unit vector parallel to diagonal $\overrightarrow{BD} = \dfrac{\overrightarrow{BD}}{|\overrightarrow{BD}|}$

$$= \frac{6\hat{j} + 8\hat{k}}{\sqrt{(6)^2 + (8)^2}} = \frac{6\hat{j} + 8\hat{k}}{\sqrt{36+64}} = \frac{3\hat{j} + 4\hat{k}}{5}$$

Now, area of parallelogram $= \frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$

$$= \frac{1}{2}|2(2\hat{i} - \hat{j} - \hat{k}) \times 2(3\hat{j} + 4\hat{k})|$$

$$= \frac{4}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -1 \\ 0 & 3 & 4 \end{matrix}\right\| = 2|-\hat{i} - 8\hat{j} + 6\hat{k}|$$

$$= 2\sqrt{(-1)^2 + (-8)^2 + (6)^2}$$

$$= 2\sqrt{1+64+36} = 2\sqrt{101} \text{ sq. units}$$

COMMON ERROR

Some students get confused between the formula for areas of triangle and parallelogram.

6. Here, $\overrightarrow{OA} = -2\hat{i} + 3\hat{j} + 5\hat{k}$, $\overrightarrow{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$

and $\overrightarrow{OC} = 7\hat{i} - \hat{k}$

Now, $$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (-2\hat{i} + 3\hat{j} + 5\hat{k})$$
$$= 3\hat{i} - \hat{j} - 2\hat{k}$$
$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$
$$= (7\hat{i} - \hat{k}) - (-2\hat{i} + 3\hat{j} + 5\hat{k})$$
$$= 9\hat{i} - 3\hat{j} - 6\hat{k}$$

and $$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (7\hat{i} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$
$$= 6\hat{i} - 2\hat{j} - 4\hat{k}$$

$\Rightarrow$ $$AB = |\overrightarrow{AB}| = \sqrt{(3)^2 + (-1)^2 + (-2)^2}$$
$$= \sqrt{9+1+4} = \sqrt{14}$$

$\Rightarrow$ $$BC = |\overrightarrow{BC}| = \sqrt{(6)^2 + (-2)^2 + (-4)^2}$$
$$= \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$$

and $$AC = |\overrightarrow{AC}| = \sqrt{(9)^2 + (-3)^2 + (-6)^2}$$
$$= \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$$

Clearly, $AB + BC = AC$

Hence *A*, *B* and *C* are collinear. **Hence proved.**

7. Given that, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

Now, $$\vec{b} + \vec{c} = (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$$
$$= (\lambda + 2)\hat{i} + 6\hat{j} - 2\hat{k}$$

and $$|\vec{b} + \vec{c}| = \sqrt{(\lambda+2)^2 + (6)^2 + (-2)^2}$$
$$= \sqrt{\lambda^2 + 4 + 4\lambda + 36 + 4}$$
$$= \sqrt{\lambda^2 + 4\lambda + 44}$$

$\therefore$ Unit vector along $(\vec{b} + \vec{c}) = \dfrac{(\vec{b} + \vec{c})}{|\vec{b} + \vec{c}|}$

$$= \frac{(\lambda+2)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}$$

According to question,

$$\vec{a} \cdot \left(\frac{\vec{b} + \vec{c}}{|\vec{b} + \vec{c}|}\right) = 1$$

$\therefore$ $$(\hat{i} + \hat{j} + \hat{k}) \cdot \left(\frac{(\lambda+2)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}\right) = 1$$

$\Rightarrow (1)(\lambda+2) + (1)(6) + (1)(-2) = \sqrt{\lambda^2 + 4\lambda + 44}$

$\Rightarrow \quad (\lambda + 6)^2 = \lambda^2 + 4\lambda + 44$

[squaring on both sides]

$\Rightarrow \quad \lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44$

$\Rightarrow \quad 8\lambda = 8 \Rightarrow \lambda = 1$

Thus, unit vector along $(\vec{b} + \vec{c}) = \dfrac{(1+2)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + 4 \times 1 + 44}}$

$$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}} = \frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$$

8. Given that $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors.

So, $$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$\Rightarrow$ $$|\vec{a}|^2 = |\vec{b}|^2 = |\vec{c}|^2 = 1$$

$\Rightarrow$ $$\vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b} = \vec{c} \cdot \vec{c} = 1$$

and given, $\hat{a} + 2\hat{b} + 3\hat{c} = \vec{0}$

Taking the scalar product with $\hat{a}, \hat{b}$ and $\hat{c}$ respectively.

$$\vec{a}\cdot\vec{a}+2\vec{a}\cdot\vec{b}+3\vec{a}\cdot\vec{c}=0$$

or $$2\vec{a}\cdot\vec{b}+3\vec{c}\cdot\vec{a}+1=0 \quad ...(1)$$

and $$\vec{b}\cdot\vec{a}+2\vec{b}\cdot\vec{b}+3\vec{b}\cdot\vec{c}=0$$

$$\Rightarrow \vec{a}\cdot\vec{b}+3\vec{b}\cdot\vec{c}+2=0 \quad ...(2)$$

and $$\vec{c}\cdot\vec{a}+2\vec{c}\cdot\vec{b}+3\vec{c}\cdot\vec{c}=0$$

or $$\vec{c}\cdot\vec{a}+2\vec{b}\cdot\vec{c}+3=0 \quad ...(3)$$

Adding eqs. (1), (2) and (3), we get

$$3(\vec{a}\cdot\vec{b})+4(\vec{c}\cdot\vec{a})+5(\vec{b}\cdot\vec{a})+6=0$$ **Hence proved.**

9.

TiP

If $\vec{a}$ is a unit vector, then

$$|\vec{a}|=1$$

$$\because \quad \hat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{\vec{a}}{1} \quad \therefore \quad \hat{a}=\vec{a}$$

$$|\hat{a}+\hat{b}|^2=(\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})$$

$$=\hat{a}\cdot\hat{a}+\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{a}+\hat{b}\cdot\hat{b} \quad [\because \hat{a}\cdot\hat{a}=|\hat{a}|^2]$$

$$=|\hat{a}|^2+2\hat{a}\cdot\hat{b}+|\hat{b}|^2 \quad [\because \hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{a}]$$

$$=|\hat{a}|^2+2|\hat{a}||\hat{b}|\cos\theta+|\hat{b}|^2$$

$$=1^1+2(1)(1)\cos\theta+1^2 \quad [\because |\hat{a}|=|\hat{b}|=1]$$

$$=2+2\cos\theta=2(1+\cos\theta)$$

$$=2\left\{1+2\cos^2\frac{\theta}{2}-1\right\}=4\cos^2\frac{\theta}{2}$$

$$\Rightarrow |\hat{a}+\hat{b}|^2=\left\{2\cos\frac{\theta}{2}\right\}^2$$

$$\therefore \quad |\hat{a}+\hat{b}|=2\cos\frac{\theta}{2}$$ **Hence proved.**

10. Given that, position vector of the points *A, B, C* and *D* are $\vec{OA}=\hat{i}+\hat{j}+\hat{k}$, $\vec{OB}=2\hat{i}+5\hat{j}$, $\vec{OC}=3\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{OD}=\hat{i}-6\hat{j}-\hat{k}$.

Now, $\vec{AB}=\vec{OB}-\vec{OA}=(2\hat{i}+5\hat{j})-(\hat{i}+\hat{j}+\hat{k})$

$$=\hat{i}+4\hat{j}-\hat{k}$$

and $\vec{CD}=\vec{OD}-\vec{OC}=(\hat{i}-6\hat{j}-\hat{k})-(3\hat{i}+2\hat{j}-3\hat{k})$

$$=-2\hat{i}-8\hat{j}+2\hat{k}=-2(\hat{i}+4\hat{j}-\hat{k})$$

Let θ be the angle between the straight lines *AB* and *CD*.

TR!CK

Angle between two non-zero vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$.

$$\therefore \cos\theta=\frac{\vec{AB}\cdot\vec{CD}}{|\vec{AB}|\cdot|\vec{CD}|}=\frac{(\hat{i}+4\hat{j}-\hat{k})\cdot(-2\hat{i}-8\hat{j}+2\hat{k})}{|\hat{i}+4\hat{j}-\hat{k}|\cdot|-2\hat{i}-8\hat{j}+2\hat{k}|}$$

$$=\frac{(1)(-2)+(4)(-8)+(-1)(2)}{\sqrt{(1)^2+(4)^2+(-1)^2}\cdot\sqrt{(-2)^2+(-8)^2+(2)^2}}$$

$$=\frac{-2-32-2}{\sqrt{1+16+1}\cdot\sqrt{4+64+4}}=\frac{-36}{\sqrt{18}\cdot\sqrt{72}}$$

$$=\frac{-36}{3\sqrt{2}\cdot6\sqrt{2}}=\frac{-2}{2}=-1=\cos180°$$

$$\Rightarrow \quad \theta=180° \text{ or } \pi$$

So, angle between $\vec{AB}$ and $\vec{CD}$ is 180°.

$$\because \quad \vec{AB}=\hat{i}+4\hat{j}-\hat{k} \text{ and } \vec{CD}=-2(\hat{i}+4\hat{j}-\hat{k})$$

$$\therefore \quad \vec{AB}=-2\vec{CD} \quad \Rightarrow \quad \vec{AB}=\lambda\vec{CD}$$

Hence, $\vec{AB}$ and $\vec{CD}$ are collinear.

TR!CK

$\vec{a}$ and $\vec{b}$ are collinear (or parallel) iff

$\dfrac{a_1}{b_1}=\dfrac{a_2}{b_2}=\dfrac{a_3}{b_3}=\lambda$*, where λ is non-zero scalar.*

11. Given, $\vec{a},\vec{b},\vec{c}$ are vectors of equal magnitudes.

$$\therefore \quad |\vec{a}|=|\vec{b}|=|\vec{c}|=d \text{ (say)}$$

$\because$ $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors.

$$\therefore \quad \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$$

Let the vector $(\vec{a}+\vec{b}+\vec{c})$, makes the angles α,β,γ from the vectors $\vec{a},\vec{b},\vec{c}$ respectively.

TR!CK

Angle between two non-zero vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$.

Then, $$\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})=|\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\alpha$$

$$\Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=d|\vec{a}+\vec{b}+\vec{c}|\cos\alpha$$

$$\Rightarrow |\vec{a}|^2+0+0=d|\vec{a}+\vec{b}+\vec{c}|\cos\alpha$$

$$\Rightarrow d^2=d|\vec{a}+\vec{b}+\vec{c}|\cos\alpha$$

$$\Rightarrow d=|\vec{a}+\vec{b}+\vec{c}|\cos\alpha$$

$$\therefore \quad \cos\alpha=\frac{d}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\Rightarrow \quad \alpha=\cos^{-1}\frac{d}{|\vec{a}+\vec{b}+\vec{c}|}$$

Similarly, $$\beta=\cos^{-1}\frac{d}{|\vec{a}+\vec{b}+\vec{c}|}$$

and $$\gamma=\cos^{-1}\frac{d}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\therefore \quad \alpha=\beta=\gamma$$

Hence, the vector $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$. **Hence proved.**

12. Given, $\vec{a}\cdot(\vec{b}+\vec{c})=0, \vec{b}\cdot(\vec{c}+\vec{a})=0$

and $\vec{c}\cdot(\vec{a}+\vec{b})=0$

Now, $|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})$

$$=\vec{a}\cdot\vec{a}+\vec{a}\cdot(\vec{b}+\vec{c})+\vec{b}\cdot\vec{b}+\vec{b}\cdot(\vec{a}+\vec{c})+\vec{c}\cdot(\vec{a}+\vec{b})+\vec{c}\cdot\vec{c}$$

$$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2$$

$$=9+16+25=50$$

$\therefore \quad |\vec{a}+\vec{b}+\vec{c}|=\sqrt{50}=5\sqrt{2}$

13. $\because \quad \vec{a}+\vec{b}+\vec{c}=0$

$\therefore \quad \vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})=0 \Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$

$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=-|\vec{a}|^2=-9 \qquad ...(1)$

Again, $\vec{b}\cdot(\vec{a}+\vec{b}+\vec{c})=0 \Rightarrow \vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{b}\cdot\vec{c}=0$

$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}=-|\vec{b}|^2=-16 \qquad ...(2)$

and $\vec{c}\cdot(\vec{a}+\vec{b}+\vec{c})=0 \Rightarrow \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}\cdot\vec{c}=0$

$\Rightarrow \quad \vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-|\vec{c}|^2=-4 \qquad ...(3)$

Adding eqs. (1), (2) and (3), we get

$$2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-29$$

$\Rightarrow \quad 2\mu=-29 \Rightarrow \mu=\frac{-29}{2}$

14. Given, $\vec{a}=2\hat{i}-\hat{j}-2\hat{k}, \vec{b}=7\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=\vec{b}_1+\vec{b}_2$

where, $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

As $\vec{b}_1$ is parallel to $\vec{a}$.

$\therefore \quad \vec{b}_1=\lambda\vec{a}=\lambda(2\hat{i}-\hat{j}-2\hat{k})$

Also, $\quad \vec{b}=\vec{b}_1+\vec{b}_2 \Rightarrow \vec{b}_2=\vec{b}-\vec{b}_1$

$\Rightarrow \quad \vec{b}_2=(7\hat{i}+2\hat{j}-3\hat{k})-\lambda(2\hat{i}-\hat{j}-2\hat{k})$

$\Rightarrow \quad \vec{b}_2=(7-2\lambda)\hat{i}+(2+\lambda)\hat{j}+(-3+2\lambda)\hat{k}$

Now, $\vec{b}_2$ is perpendicular to $\vec{a}$.

$\therefore \quad \vec{b}_2\cdot\vec{a}=0$

TR!CK

If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then scalar product $\vec{a}\cdot\vec{b}=a_1a_2+b_1b_2+c_1c_2$,

$[\because \hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1, \hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0]$

$\Rightarrow \{(7-2\lambda)\hat{i}+(2+\lambda)\hat{j}+(-3+2\lambda)\hat{k}\}\cdot(2\hat{i}-\hat{j}-2\hat{k})=0$

$\Rightarrow \quad 2(7-2\lambda)-(2+\lambda)-2(-3+2\lambda)=0$

$\Rightarrow \quad 14-4\lambda-2-\lambda+6-4\lambda=0 \Rightarrow -9\lambda+18=0$

$\Rightarrow \quad \lambda=\frac{18}{9}=2$

$\therefore \quad \vec{b}_1=2(2\hat{i}-\hat{j}-2\hat{k})=4\hat{i}-2\hat{j}-4\hat{k}$

and $\quad \vec{b}_2=(7-2\times2)\hat{i}+(2+2)\hat{j}+(-3+2\times2)\hat{k}$

$$=(7-4)\hat{i}+4\hat{j}+(-3+4)\hat{k}=3\hat{i}+4\hat{j}+\hat{k}$$

Hence, $\vec{b}=(4\hat{i}-2\hat{j}-4\hat{k})+(3\hat{i}+4\hat{j}+\hat{k})$

15. Here, $|\vec{a}\cdot\vec{b}|=|\vec{a}|\,|\vec{b}|\,|\cos\theta|$

$\because \quad -1\le\cos\theta\le1$

$\Rightarrow \quad 0\le|\cos\theta|\le1$

$\Rightarrow \quad |\vec{a}|\,|\vec{b}|\,|\cos\theta|\le|\vec{a}|\,|\vec{b}|$

$\Rightarrow \quad |\vec{a}\cdot\vec{b}|\le|\vec{a}|+|\vec{b}|$ **Hence proved.**

16. The inequality holds trivially in case either $\vec{a}=0$

or $\quad \vec{b}=0$.

Let $|\vec{a}|\ne\vec{0}\ne|\vec{b}|$, then

$$|\vec{a}+\vec{b}|^2=(\vec{a}+\vec{b})^2=(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})$$

$$=\vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}$$

$$=|\vec{a}|^2+2\,\vec{a}\cdot\vec{b}+|\vec{b}|^2$$

[scalar product is commutative]

$\le|\vec{a}|^2+2|\vec{a}\cdot\vec{b}|+|\vec{b}|^2 \quad [\because x\le|x|\,\forall x\in R]$

$\le|\vec{a}|^2+2|\vec{a}||\vec{b}|+|\vec{b}|^2 \quad [\because |\vec{a}\cdot\vec{b}|\le|\vec{a}||\vec{b}|]$

$=(|\vec{a}|+|\vec{b}|)^2$

Therefore, $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$. **Hence proved.**

17. Given that, $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=4\hat{i}-7\hat{j}+\hat{k}$

Let $\quad \vec{c}=x\hat{i}+y\hat{j}+z\hat{k}$

TR!CK

If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then scalar product of $\vec{a}$ and $\vec{b}$ is $\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$, where $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$ and $\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$.

$\because \quad \vec{a}\cdot\vec{c}=6$

$\therefore \quad (2\hat{i}+\hat{j}-\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k})=6$

$\Rightarrow \quad 2x+y-z=6 \qquad ...(1)$

Now, $\quad \vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ x & y & z\end{vmatrix}$

$$=(z+y)\hat{i}-(2z+x)\hat{j}+(2y-x)\hat{k}$$

TR!CK

If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then vector product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

$$=\hat{i}(a_2b_3-b_2a_3)-\hat{j}(a_1b_3-b_1a_3)+\hat{k}(a_1b_2-b_1a_2)$$

$\because \quad \vec{a}\times\vec{c}=\vec{b}$

$\therefore \quad (z+y)\hat{i}-(2z+x)\hat{j}+(2y-x)\hat{k}=4\hat{i}-7\hat{j}+\hat{k}$

On comparing coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$$z + y = 4 \quad \Rightarrow \quad z = 4 - y \qquad ...(2)$$
$$-(2z + x) = -7 \quad \Rightarrow \quad x = 7 - 2z \qquad ...(3)$$
and $$2y - x = 1 \quad \Rightarrow \quad x = 2y - 1 \qquad ...(4)$$

From eqs. (3) and (4), we get
$$7 - 2z = 2y - 1$$
$$\Rightarrow \quad 2y + 2z = 8$$
$$\Rightarrow \quad y + z = 4 \qquad ...(5)$$

From eqs. (2) and (5), we get
$$y - (4 - y) = -3$$
$$\Rightarrow \quad y - 4 + y = -3$$
$$\Rightarrow \quad 2y = 1 \quad \Rightarrow \quad y = \frac{1}{2}$$

From eq. (2), $z = 4 - \frac{1}{2} = \frac{7}{2}$

From eq. (4), $x = 2 \times \frac{1}{2} - 1 = 1 - 1 = 0$

Thus, $\vec{c} = x\hat{i} + y\hat{i} + z\hat{k} = 0\hat{i} + \frac{1}{2}\hat{j} + \frac{7}{2}\hat{k}$
$$= \frac{1}{2}(\hat{j} + 7\hat{k})$$

18. We have, $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$

and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$

> **TR!CK**
>
> *$\lambda(\vec{a} \times \vec{b})$ is perpendicular to the plane, which contains $\vec{a}$ and $\vec{b}$.*

Since, $\vec{d}$ is perpendicular to both $\vec{c}$ and $\vec{b}$.

$$\vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix}$$
$$= \lambda\{(5-4)\hat{i} - (15+1)\hat{j} + (-12-1)\hat{k}\}$$
$$= (\hat{i} - 16\hat{j} - 13\hat{k})\lambda \qquad ...(1)$$

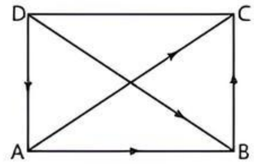
Also, it is given that $\vec{d} \cdot \vec{a} = 21$
$$\lambda(\hat{i} - 16\hat{j} - 13\hat{k}) \cdot (4\hat{i} + 5\hat{j} - \hat{k}) = 21$$
$$\Rightarrow \quad \lambda\{(1)(4) + (-16)(5) + (-13)(-1)\} = 21$$

> **TR!CK**
>
> *If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then scalar product of $\vec{a}$ and $\vec{b}$ is*
>
> $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
>
> *where, $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$ and $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$*

$$\Rightarrow \lambda(4 - 80 + 13) = 21$$
$$\Rightarrow \quad \lambda = -\frac{21}{63} = -\frac{1}{3}$$

From eq. (1), we get $\vec{d} = -\frac{1}{3}(\hat{i} - 16\hat{j} - 13\hat{k})$

19. Given, *ABCD* is a parallelogram in which
$$\overrightarrow{AB} = 5\hat{i} + 7\hat{k}$$
and $$\overrightarrow{DB} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$



By triangle law of vector addition,
$$\overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{DB}$$
$$\overrightarrow{DA} = \overrightarrow{DB} - \overrightarrow{AB}$$
$$= (2\hat{i} + 2\hat{j} + 3\hat{k}) - (5\hat{i} + 7\hat{k}) = -3\hat{i} + 2\hat{j} - 4\hat{k}$$
$$\Rightarrow \quad \overrightarrow{DA} \times \overrightarrow{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & -4 \\ 5 & 0 & 7 \end{vmatrix}$$
$$= (14 + 0)\hat{i} - (-21 + 20)\hat{j} + (0 - 10)\hat{k}$$
$$= 14\hat{i} + \hat{j} - 10\hat{k}$$

> **TR!CK**
>
> *If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,*
>
> *then* $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
>
> $= (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$

So, the area of a parallelogram $ABCD = |\overrightarrow{DA} \times \overrightarrow{AB}|$
$$= |14\hat{i} + \hat{j} - 10\hat{k}| = \sqrt{(14)^2 + (1)^2 + (-10)^2}$$

> **TR!CKS**
>
> - *If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram, then its area is given by $|\vec{a} \times \vec{b}|$.*
> - *If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then modulus $\vec{r}$ i.e., $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$*

$$= \sqrt{196 + 1 + 100} = \sqrt{297} \text{ sq. units}$$
$$= 3\sqrt{33} \text{ sq. units}$$

> **COMMON ERR(!)R**
>
> *Mostly students get confused in deciding the formula to be used as a side and a diagonal are given.*

20. Given, $\vec{d_1} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{d_2} = \hat{i} - 3\hat{j} + 2\hat{k}$

> **TR!CK**
>
> *The area of parallelogram when diagonals*
>
> $\vec{d_1} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
>
> *and* $\vec{d_2} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ *is given by*
>
> $\vec{d_1} \times \vec{d_2} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$$\therefore \quad \vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= (4-3)\hat{i} - (6+1)\hat{j} + (-9-2)\hat{k} = \hat{i} - 7\hat{j} - 11\hat{k}$$

$$|\vec{d_1} \times \vec{d_2}| = \sqrt{(1)^2 + (-7)^2 + (-11)^2}$$

$$= \sqrt{1 + 49 + 121} = \sqrt{171}$$

Hence, area of the parallelogram $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

$$= \frac{1}{2}\sqrt{171} \text{ sq. units}$$

COMMON ERROR

Some students use the formula to find the area of parallelogram when sides are given.

21. Given, $\vec{a} = 3\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

TiP

Practice more problems related to area of triangle and parallelogram.

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 5 \\ 1 & 2 & -1 \end{vmatrix}$

$$= (1-10)\hat{i} - (-3-5)\hat{j} + (6+1)\hat{k}$$

$$= -9\hat{i} + 8\hat{j} + 7\hat{k}$$

So, area of the triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$$= \frac{1}{2}|-9\hat{i} + 8\hat{j} + 7\hat{k}|$$

$$= \frac{1}{2}\sqrt{(-9)^2 + (8)^2 + (7)^2}$$

$$= \frac{1}{2}(81 + 64 + 49) = \frac{1}{2} \cdot \sqrt{194} \text{ sq. units}$$

22. Let the position vectors of the vertices A (1, 2, 3), $B(2, -1, 4)$ and C (4, 5, −1) of the ΔABC with respect to origin be:

$$\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k},$$

$$\vec{OB} = 2\hat{i} - \hat{j} + 4\hat{k},$$

$$\vec{OC} = 4\hat{i} + 5\hat{j} - \hat{k}$$

$$\therefore \vec{AB} = \vec{OB} - \vec{OA} = (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \hat{i} - 3\hat{j} + \hat{k}$$

and $\vec{AC} = \vec{OC} - \vec{OA}$

$$= (4\hat{i} + 5\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= 3\hat{i} + 3\hat{j} - 4\hat{k}$$

TR!CK

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then vector product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - b_1a_3)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Now, $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$

$$= (12-3)\hat{i} - (-4-3)\hat{j} + (3+9)\hat{k}$$

$$= 9\hat{i} + 7\hat{j} + 12\hat{k}$$

So, area of $\Delta ABC = \frac{1}{2}|\vec{AB} \times \vec{AC}|$

TR!CK

If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then its area is given by $\frac{1}{2}|\vec{a} \times \vec{b}|$.

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then modulus of $\vec{r}$ i.e.,

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$= \frac{1}{2}|9\hat{i} + 7\hat{j} + 12\hat{k}|$$

$$= \frac{1}{2}\sqrt{(9)^2 + (7)^2 + (12)^2} = \frac{1}{2}\sqrt{81 + 49 + 144}$$

$$= \frac{1}{2}\sqrt{274} \text{ sq. units}$$

23. Let the given vectors be $\vec{A}, \vec{B}$ and $\vec{C}$ respectively.

TiP

Practice more problems based on area of the triangle.

$$\therefore \quad \vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{OB} = \hat{i} - 3\hat{j} - 5\hat{k} \text{ and}$$

$$\vec{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

Now, $\vec{AB} = \vec{OB} - \vec{OA}$

$$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$$

$$= -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$= (3\hat{j} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$$

$$= 2\hat{j} - \hat{j} + \hat{k} \text{ and}$$

$$\vec{CA} = \vec{OA} - \vec{OC}$$

$$=(2\hat{j}-\hat{i}+\hat{k})-(3\hat{i}-4\hat{j}-4\hat{k})$$
$$=-\hat{i}+3\hat{j}+5\hat{k}$$
$$|\overrightarrow{AB}|=\sqrt{(-1)^2+(-2)^2+(-6)^2}$$
$$=\sqrt{1+4+36}=\sqrt{41}$$
$$|\overrightarrow{BC}|=\sqrt{(2)^2+(-1)^2+(1)^2}$$
$$=\sqrt{4+1+1}=\sqrt{6}$$
$$|\overrightarrow{CA}|=\sqrt{(-1)^2+(3)^2+(5)^2}$$
$$=\sqrt{1+9+25}=\sqrt{35}$$

We see that, $|\overrightarrow{BC}|^2+|\overrightarrow{CA}|^2=|\overrightarrow{AB}|^2$

Therefore, given vectors are the vertices of a right angled triangle. **Hence proved.**

$\therefore$ Angle between $\overrightarrow{BC}$ and $\overrightarrow{CA}$ is 90°.

$\therefore$ Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{BC}\times\overrightarrow{CA}|$

TR!CK

If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ *and* $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$,

then $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}$

$=(a_2b_3-b_2a_3)\hat{i}-(a_1b_3-b_1a_3)\hat{j}+(a_1b_2-a_2b_1)\hat{k}$

$$=\frac{1}{2}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&1\\-1&3&5\end{vmatrix}$$
$$=\frac{1}{2}|(-5-3)\hat{i}-(10+1)\hat{j}+(6-1)\hat{k}|$$

TR!CK

If $\vec{r}=a\hat{i}+b\hat{j}+c\hat{k}$ *then* $|\vec{r}|=\sqrt{a^2+b^2+c^2}$

$$=\frac{1}{2}|-8\hat{i}-11\hat{j}+5\hat{k}|=\frac{1}{2}\sqrt{(-8)^2+(-11)^2+(5)^2}$$
$$=\frac{1}{2}\sqrt{64+121+25}=\frac{1}{2}\sqrt{210}\text{ sq. units}$$

24. Given that, $\vec{a}=p\hat{i}+q\hat{j}+r\hat{k}$, $\vec{b}=s\hat{i}+3\hat{j}+4\hat{k}$

and $\vec{c}=3\hat{i}+\hat{j}-2\hat{k}$

Also, $\vec{a}=\vec{b}+\vec{c}$

$\Rightarrow(p\hat{i}+q\hat{j}+r\hat{k})=(s\hat{i}+3\hat{j}+4\hat{k})+(3\hat{i}+\hat{j}-2\hat{k})$

$\Rightarrow(p\hat{i}+q\hat{j}+r\hat{k})=(s+3)\hat{i}+4\hat{j}+2\hat{k}$

On comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$p=s+3, q=4$ and $r=2$

Given, area of triangle $=5\sqrt{6}$

But area of triangle $=\frac{1}{2}|\vec{a}\times\vec{b}|$

$$\therefore\quad 5\sqrt{6}=\frac{1}{2}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\p&4&2\\p-3&3&4\end{vmatrix}$$

$$=\frac{1}{2}|(16-6)\hat{i}+(-6-2p)\hat{j}+(-p+12)\hat{k}|$$

$\Rightarrow\quad 10\sqrt{6}=|(10)\hat{i}+(2p+6)\hat{j}+(12-p)\hat{k}|$

$\Rightarrow\quad 10\sqrt{6}=\sqrt{(10)^2+(2p+6)^2+(12-p)^2}$

Squaring on both sides, we get

$600=100+4p^2+36+24p+144+p^2-24p$

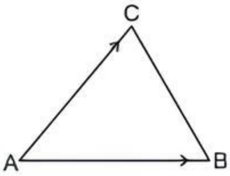
$\Rightarrow\quad 5p^2=600-280=320$

$\Rightarrow\quad p^2=64\Rightarrow p=\pm 8$

If $p=8$, then $S=8-3=5$

If $p=-8$, then $S=-8-3=-11$

25. Let O be origin.



Then, $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$ and $\overrightarrow{OC}=\vec{c}$

$\therefore\quad \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\vec{b}-\vec{a}$

and $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\vec{c}-\vec{a}$

Now, $\overrightarrow{AB}\times\overrightarrow{AC}=(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})$

$=\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{a}\times\vec{c}+\vec{a}\times\vec{a}$

$=\vec{b}\times\vec{c}+\vec{a}\times\vec{b}+\vec{c}\times\vec{a}\qquad[\because\vec{a}\times\vec{a}=\vec{0}]$

Therefore, area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$=\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$ sq. units

Hence proved.

26. Given, $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

$\therefore\quad \vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{a}-\vec{b}=-\hat{j}-2\hat{k}$

A vector, which is perpendicular to both $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ is

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&4\\0&-1&-2\end{vmatrix}$$
$$=(-6+4)\hat{i}-(-4-0)\hat{j}+(-2-0)\hat{k}$$
$$=-2\hat{i}+4\hat{j}-2\hat{k}$$

Now, $|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|=\sqrt{(-2)^2+(4)^2+(-2)^2}$

$=\sqrt{4+16+4}=\sqrt{24}=2\sqrt{6}$

So, required unit vector $=\dfrac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|}$

$$=\frac{-2\hat{i}+4\hat{j}-2\hat{k}}{2\sqrt{6}}$$
$$=\frac{-1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$$

27. We have, $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$

$\Rightarrow\quad \vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}=0$

$\Rightarrow\quad \vec{a}\cdot(\vec{b}-\vec{c})=0$

$\Rightarrow\quad \vec{b}-\vec{c}=0$ or $\vec{a}\perp(\vec{b}-\vec{c})\qquad[\because\vec{a}\neq\vec{0}]$

$\Rightarrow\quad \vec{b}=\vec{c}$ or $\vec{a}\perp(\vec{b}-\vec{c})$

Also, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$

$\Rightarrow$ $\vec{a}\times\vec{b}-\vec{a}\times\vec{c}=0$

$\Rightarrow$ $\vec{a}\times(\vec{b}-\vec{c})=0$

$\Rightarrow$ $\vec{b}-\vec{c}=0$ or $\vec{a}\,||\,(\vec{b}-\vec{c})$ $\quad[\because \vec{a}\neq\vec{0}]$

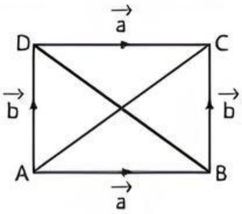
$\Rightarrow$ $\vec{b}=\vec{c}$ or $\vec{a}\,||\,(\vec{b}-\vec{c})$

Here, $\vec{a}$ cannot be both perpendicular to $(\vec{b}-\vec{c})$ and parallel to $(\vec{b}-\vec{c})$.

Hence, $\vec{b}=\vec{c}$ **Hence proved.**

Long Answer Type Questions

1. Given two adjacent sides of parallelogram *ABCD* are $\vec{a}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}-3\hat{k}$.



Here, diagonal of parallelogram are $\vec{AC}$ and $\vec{BD}$.

$\therefore$ $\vec{AC}=\vec{a}+\vec{b}$ [by triangle law]

Now, $\vec{AC}=(2\hat{i}-4\hat{j}+5\hat{k})+(\hat{i}-2\hat{j}-3\hat{k})$

$=3\hat{i}-6\hat{j}+2\hat{k}$

$\therefore$ Unit vector parallel to diagonal $\vec{AC}=\dfrac{\vec{AC}}{|\vec{AC}|}$

$$=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{(3)^2+(-6)^2+(2)^2}}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}}$$

$$=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$

$=(12+10)\vec{i}-(-6-5)\vec{j}+(-4+4)\vec{k}$

$=22\vec{i}+11\vec{j}$

Since, $\vec{a}$ and $\vec{b}$ are two adjacent sides of parallelogram ABCD.

So, area of parallelogram $=|\vec{a}\times\vec{b}|$

$=|22\vec{i}+11\vec{j}|$

$=\sqrt{(22)^2+(11)^2}$

$=11\sqrt{4+1}=11\sqrt{5}$ sq. units.

Hence, unit vector parallel to one of its diagonals is $\dfrac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$ and area of parallelogram is $11\sqrt{5}$ sq. units.

2. If $\vec{a},\vec{b},\vec{c}$ are linearly dependent vectors, then $\vec{c}$ should be a linear combination of $\vec{a}$ and $\vec{b}$.

Let $\vec{c}=p\vec{a}+q\vec{b}$

i.e., $\hat{i}+\alpha\hat{j}+\beta\hat{k}=p(\hat{i}+\hat{j}+\hat{k})+q(4\hat{i}+3\hat{j}+4\hat{k})$

Equating coefficients of $\hat{i},\hat{j},\hat{k}$ on both sides, we get

$1=p+4q,\ \alpha=p+3q,\ \beta=p+4q$

From first and third, $\beta=1$

Now, $|\vec{c}|=\sqrt{3}$ [given]

$\Rightarrow$ $1+\alpha^2+\beta^2=3 \Rightarrow \alpha^2=1$

$\Rightarrow$ $1+\alpha^2+1=3 \Rightarrow \alpha=\pm1$

Hence, $\alpha=\pm1,\beta=1$

3. If $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$, then

$\vec{a}+2\vec{b}=t\vec{c}$ ...(1)

Also, if $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$, then

$\vec{b}+3\vec{c}=\lambda\vec{a} \Rightarrow \vec{b}=\lambda\vec{a}-3\vec{c}$

Putting the value of $\vec{b}$ in eq. (1), we get

$\vec{a}+2(\lambda\vec{a}-3\vec{c})=t\vec{c}$

$\Rightarrow$ $\vec{a}+2\lambda\vec{a}-6\vec{c}=t\vec{c}$

$\Rightarrow$ $(\vec{a}-6\vec{c})=t\vec{c}-2\lambda\vec{a}$

On comparing, we get

$1=-2\lambda \Rightarrow \lambda=-1/2$ and $-6=t \Rightarrow t=-6$

From eq. (1), we get

$\vec{a}+2\vec{b}=-6\vec{c} \Rightarrow \vec{a}+2\vec{b}+6\vec{c}=\vec{0}$

4. Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

Given, $\vec{a}=3\hat{i}-5\hat{k}=3\hat{i}+0\hat{j}-5\hat{k}$,

$\vec{b}=2\hat{i}+7\hat{j}+0\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+\hat{k}$

According to the question,

$\vec{r}\cdot\vec{a}=-1$ or $3\times x+0\times y+(-5)z=-1$

or $3x-5z=-1$ ...(1)

and $\vec{r}\cdot\vec{b}=6 \Rightarrow 2x+7y=6$ ...(2)

and $\vec{r}\cdot\vec{c}=5 \Rightarrow x+y+z=5$ ...(3)

Multiplying eq. (3) by 5 and adding in eq. (1), we get

$8x+5y=24$ ...(4)

Multiply eq. (2) by 4, we get

$8x+28y=24$ ...(5)

Subtracting eq. (4) from eq. (5), we get

$23y=0 \Rightarrow y=0$

From eq. (2), $2x=6 \Rightarrow x=3$

From eq. (3), $3+0+z=5 \Rightarrow z=2$

$\therefore$ Required vector $\vec{r}=3\hat{i}+0\hat{j}+2\hat{k}=3\hat{i}+2\hat{k}$

5. Given that $|\vec{a}|=2\sqrt{2}$ and $|\vec{b}|=3$

Since, the parallelogram is constructed on $5\vec{a}+2\vec{b}$ and $\vec{a}-3\vec{b}$.

∴ Its one diagonal

$$= (5\vec{a} + 2\vec{b}) + (\vec{a} - 3\vec{b})$$

$$= 6\vec{a} - \vec{b}$$

And its other diagonal

$$= (5\vec{a} + 2\vec{b}) - (\vec{a} - 3\vec{b})$$

$$= 5\vec{a} + 2\vec{b} - \vec{a} + 3\vec{b} = 4\vec{a} + 5\vec{b}$$

Now, length of one diagonal

$$= |6\vec{a} - \vec{b}| = (6\vec{a} - \vec{b})^{2 \times \frac{1}{2}}$$

$$= \{(6\vec{a} - \vec{b}) \cdot (6\vec{a} - \vec{b})\}^{\frac{1}{2}}$$

$$= (36\,\vec{a} \cdot \vec{a} - 6\,\vec{b} \cdot \vec{a} - 6\,\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b})^{\frac{1}{2}}$$

$$= (36\,|\vec{a}|^2 - 6\,\vec{a} \cdot \vec{b} - 6\,\vec{a} \cdot \vec{b} + |\vec{b}|^2)^{\frac{1}{2}}$$

$$= (36 \times 8 - 12\,\vec{a} \cdot \vec{b} + 9)^{\frac{1}{2}}$$

$$= \left(288 + 9 - 12\,|\vec{a}|\,|\vec{b}| \cos\frac{\pi}{4}\right)^{\frac{1}{2}}$$

[∵ angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$]

$$= \left(297 - 12 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}$$

$$= (297 - 72)^{\frac{1}{2}} = (225)^{\frac{1}{2}} = 15$$

Length of other diagonal $= |4\vec{a} + 5\vec{b}|$

$$= (4\vec{a} + 5\vec{b})^{2 \times \frac{1}{2}} = \{(4\vec{a} + 5\vec{b}) \cdot (4\vec{a} + 5\vec{b})\}^{\frac{1}{2}}$$

$$= \{16\,\vec{a} \cdot \vec{a} + 20\,\vec{b} \cdot \vec{a} + 20\,\vec{a} \cdot \vec{b} + 25\,\vec{b} \cdot \vec{b}\}^{\frac{1}{2}}$$

$$= \{16\,|\vec{a}|^2 + 20\,\vec{a} \cdot \vec{b} + 20\,\vec{a} \cdot \vec{b} + 25\,|\vec{b}|^2\}^{\frac{1}{2}}$$

$$= \{16 \times 8 + 40\,\vec{a} \cdot \vec{b} + 25 \times 9\}^{\frac{1}{2}}$$

$$= \left(128 + 225 + 40\,|\vec{a}|\,|\vec{b}| \cos\frac{\pi}{4}\right)^{\frac{1}{2}}$$

$$= \left(353 + 40 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}$$

$$= (353 + 240)^{\frac{1}{2}} = \sqrt{593} = 24.35$$

Thus, length of the longest diagonal is $\sqrt{593}$ or 24.35.

6. Since, $\vec{a}, \vec{b}, \vec{c}$ are unit vectors.

$$\therefore \quad |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$$

$$= 2\left[|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right] - 2\Sigma(\vec{a} \cdot \vec{b})$$

$$= 2(1 + 1 + 1) - 2\Sigma(\vec{a} \cdot \vec{b}) = 6 - 2\Sigma(\vec{a} \cdot \vec{b}) \quad \text{...(1)}$$

But $(\vec{a} + \vec{b} + \vec{c})^2 \geq 0$

$$\Rightarrow \quad (1 + 1 + 1) + 2\Sigma(\vec{a} \cdot \vec{b}) \geq 0$$

$$\therefore \quad 3 \geq -2\Sigma(\vec{a} \cdot \vec{b}) \quad \text{...(2)}$$

From eqs. (1) and (2), we get

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 6 + 3 = 9$$

Hence proved.

7. Given, $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$

The projection of $\vec{v}$ along $\vec{u} = \dfrac{\vec{v} \cdot \vec{u}}{|\vec{u}|}$

and the projection of $\vec{w}$ along $\vec{u} = \dfrac{\vec{w} \cdot \vec{u}}{|\vec{u}|}$

According to the given question,

$$\frac{\vec{v} \cdot \vec{u}}{|\vec{u}|} = \frac{\vec{w} \cdot \vec{u}}{|\vec{u}|} \Rightarrow \vec{v} \cdot \vec{u} = \vec{w} \cdot \vec{u} \quad \text{...(1)}$$

and $\vec{v}, \vec{w}$ are perpendicular to each other.

$$\therefore \quad \vec{v} \cdot \vec{w} = 0 \quad \text{...(2)}$$

Now, $|\vec{u} - \vec{v} + \vec{w}|^2$

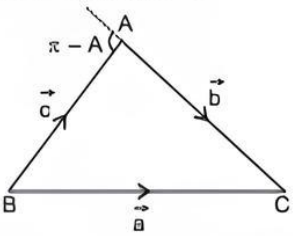
$$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2\,\vec{u} \cdot \vec{v} + 2\,\vec{u} \cdot \vec{w} - 2\,\vec{v} \cdot \vec{w}$$

$$= 1 + 4 + 9 - 2\,\vec{u} \cdot \vec{v} + 2\,\vec{u} \cdot \vec{w} - 0$$

$$\Rightarrow \quad |\vec{u} - \vec{v} + \vec{w}|^2 = 14 \quad \text{[from eqs. (1) and (2)]}$$

$$\Rightarrow \quad |\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$$

8. Let $\vec{BC} = \vec{a}, \vec{BA} = \vec{c}$ and $\vec{AC} = \vec{b}$



In $\Delta ABC, \vec{BC} = \vec{BA} + \vec{AC}$ or $\vec{a} = \vec{c} + \vec{b}$ (by triangle law)

or $$\vec{a} \cdot \vec{a} = (\vec{c} + \vec{b}) \cdot (\vec{c} + \vec{b})$$

or $$\vec{a} \cdot \vec{a} = \vec{c} \cdot \vec{c} + 2\,\vec{b} \cdot \vec{c} + \vec{b} \cdot \vec{b}$$

Since, angle between vectors $\vec{b}$ and $\vec{c}$ = the angle between CA produced and AB. [∵ $\theta = \pi - A$]

or $$a^2 = c^2 + 2bc\cos(\pi - A) + b^2$$

or $$a^2 = c^2 - 2bc\cos A + b^2$$

TR!CK

$\cos(\pi - \theta) = -\cos\theta$

or $$2bc\cos A = b^2 + c^2 - a^2$$

or $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$ **Hence proved.**

9. Given, $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ ...(1)

$\Rightarrow \quad \vec{a}=-2\vec{b}-3\vec{c}$

Taking the vector product of both sides by vector $\vec{b}$,

$\vec{a}\times\vec{b}=(-2\vec{b}-3\vec{c})\times\vec{b}=-2\vec{b}\times\vec{b}-3\vec{c}\times\vec{b}$

$=-2\cdot 0+3\vec{b}\times\vec{c}$

$[\because \vec{b}\times\vec{b}=0 \text{ and } \vec{c}\times\vec{b}=-\vec{b}\times\vec{c}]$

$=3\vec{b}\times\vec{c}$ ...(2)

Again, from eq. (1), $2\vec{b}=-\vec{a}-3\vec{c}$

Taking the vector product of both sides by vector $\vec{c}$,

$2\vec{b}\times\vec{c}=(-\vec{a}-3\vec{c})\times\vec{c}=-\vec{a}\times\vec{c}-3\vec{c}\times\vec{c}$

$=\vec{c}\times\vec{a}-3\cdot 0$ $[\because \vec{c}\times\vec{c}=0 \text{ and } \vec{a}\times\vec{c}=-\vec{c}\times\vec{a}]$

$=\vec{c}\times\vec{a}$

$\Rightarrow \vec{b}\times\vec{c}=\frac{1}{2}\vec{c}\times\vec{a}$ ...(3)

Again from eq. (1), $3\vec{c}=-\vec{a}-2\vec{b}$

Taking the vector product of both sides by vector $\vec{a}$

$3\vec{c}\times\vec{a}=(-\vec{a}-2\vec{b})\times\vec{a}=-\vec{a}\times\vec{a}-2\vec{b}\times\vec{a}$

$=-0+2\vec{a}\times\vec{b}$ $[\because \vec{a}\times\vec{a}=0 \text{ and } \vec{b}\times\vec{a}=-\vec{a}\times\vec{b}]$

$=2\vec{a}\times\vec{b}$

$\Rightarrow \vec{c}\times\vec{a}=\frac{2}{3}\vec{a}\times\vec{b}$ ...(4)

Adding eqs. (2), (3) and (4),

$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=3\vec{b}\times\vec{c}+\frac{1}{2}\vec{c}\times\vec{a}+\frac{2}{3}\vec{a}\times\vec{b}$

$=3\vec{b}\times\vec{c}+\vec{b}\times\vec{c}+2\vec{b}\times\vec{c}$ ...(5)

[from eqs. (3) and (4)]

$=6(\vec{b}\times\vec{c})$

$=2(3\vec{b}\times\vec{c})=2(\vec{a}\times\vec{b})$ ...(6)

[from eq. (2)]

$=3\cdot\frac{2}{3}(\vec{a}\times\vec{b})=3(\vec{c}\times\vec{a})$ ...(7)

[from eq. (4)]

Therefore, from eqs. (5), (6) and (7),

$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=6(\vec{b}\times\vec{c})$

or $2(\vec{a}\times\vec{b})$ or $3(\vec{c}\times\vec{a})$ **Hence proved.**

10. Let $\vec{BC}=\vec{a}$, $\vec{CA}=\vec{b}$

and $\vec{AB}=\vec{c}$

$\because$ By triangle law, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

or $\vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{0}=\vec{0}$

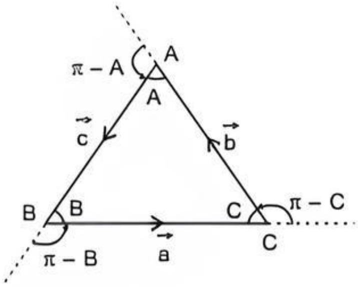
or $\vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{0}$

or $\vec{0}+\vec{a}\times\vec{b}-\vec{c}\times\vec{a}=\vec{0}$

or $\vec{a}\times\vec{b}=\vec{c}\times\vec{a}$

$\Rightarrow \quad |\vec{a}\times\vec{b}|=|\vec{c}\times\vec{a}|$ ...(1)

Similarly, $|\vec{b}\times\vec{c}|=|\vec{c}\times\vec{a}|$ ...(2)



$\therefore$ From eqs. (1) and (2),

$|\vec{a}\times\vec{b}|=|\vec{b}\times\vec{c}|=|\vec{c}\times\vec{a}|$

or $ab\sin(\pi-C)=bc\sin(\pi-A)=ca\sin(\pi-B)$

TR!CK

$\sin(\pi-\theta)=-\sin\theta$

or $ab\sin C=bc\sin A=ca\sin B$

or $\frac{\sin C}{c}=\frac{\sin A}{a}=\frac{\sin B}{b}$ **Hence proved.**

Chapter Test

Multiple Choice Questions

Q 1. $(\vec{a}\cdot\hat{i})^2+(\vec{a}\cdot\hat{j})^2+(\vec{a}\cdot\hat{k})^2$ **is equal to:**

a. 1

b. $\vec{a}$

c. $-\vec{a}$

d. $|\vec{a}|^2$

Q 2. If $|\vec{a}|=5, |\vec{b}|=13$ **and** $|\vec{a}\times\vec{b}|=25$**, find** $\vec{a}\cdot\vec{b}$**.**

a. 10 b. 40

c. 60 d. 25

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): The vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$, if $\vec{a}$ and $\vec{b}$ are equal vectors.

Reason (R): The values of k for which $|k\vec{a}|<|\vec{a}|$ and $k\vec{a}+\frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true, are $k \in (-1, 1)$.

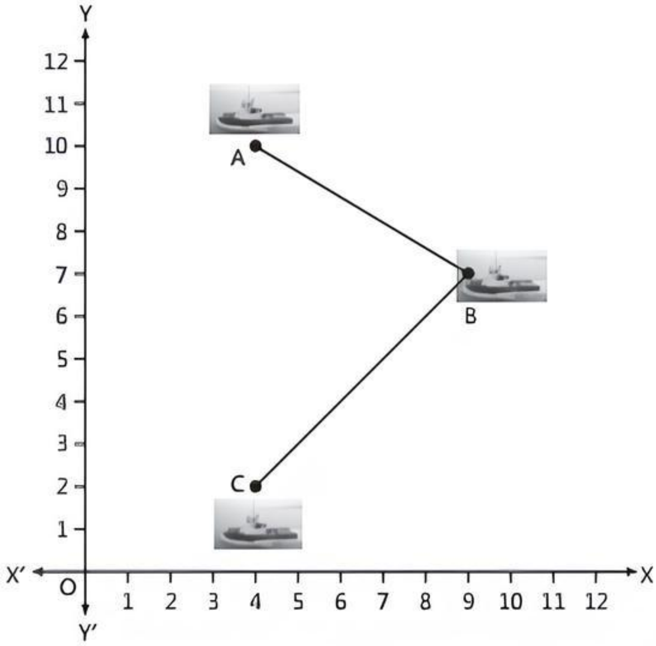
Q 4. Assertion (A): The number of vectors of unit length perpendicular to the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$ is two.

Reason (R) If $\vec{a}$ and $\vec{b}$ are adjacent sides of a rhombus, then $\vec{a}\cdot\vec{b}=0$.

Case Study Based Questions

Q 5. Case Study 1

A barge is pulled into harbour by two tug boats as shown in the figure.



Based on the above figure, solve the following questions:

(i) Find the position vector of A.

(ii) Find the vector $\overrightarrow{AC}$ in terms of $\hat{i}$, $\hat{j}$.

(iii) If $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$, then its unit vector.

Or

Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CB}$.

Q 6. Case Study 2

Solar panels have to be installed carefully so that the tilt of the roof and the direction to the sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof, where solar panels are to be mounted.

In the picture, suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters $P_1(6,8,4)$, $P_2(21,8,4)$, $P_3(21,16,10)$ and $P_4(6,16,10)$.

Based on the above information, solve the following questions:

(i) What are the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 − PV of P_1 and $\vec{B}$ = PV of P_4 − PV of P_1? (where PV stands for position vector).

(ii) What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$?

(iii) What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ on the surface of the roof?

Or

The Sun is located along the unit vector $\vec{S}=\frac{1}{2}\hat{i}-\frac{6}{7}\hat{j}+\frac{1}{7}\hat{k}$. If the flow of solar energy is given by the vector $\vec{F}=910$ in units Watts/m^2, what is the dot product of vectors $\vec{F}$ with $\vec{N}$ and the units for this quantity?

Very Short Answer Type Questions

Q 7. Prove that $\vec{a}=\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{b}=4\hat{i}+2\hat{j}-4\hat{k}$ are mutually perpendicular.

Q 8. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}+\hat{k}$, then find $\vec{a}\times\vec{b}$.

Short Answer Type-I Questions

Q 9. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=8$, then find $|\vec{x}|$.

Q 10. Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$.

Short Answer Type-II Questions

Q 11. Show that the points $2\hat{i}, -\hat{i}-4\hat{j}, -\hat{i}+4\hat{j}$ form an isosceles triangle.

Q 12. If the angle between two unit vectors $\hat{a}$ and $\hat{b}$ is θ, then prove that $\sin\frac{\theta}{2}=\frac{1}{2}|\hat{a}-\hat{b}|$.

Long Answer Type Questions

Q 13. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

Q 14. Prove that $|\hat{i}\times\vec{a}|^2+|\hat{j}\times\vec{a}|^2+|\hat{k}\times\vec{a}|^2=2|\vec{a}|^2$.