DAY FOUR

Laws of Motion

Learning & Revision for the Day

- Concept of Forces
- Inertia
- Newton's Laws of Motion
- Principle of Conservation of Linear Momentum
- Free Body Diagram
- Connected Motion
- Equilibrium of concurrent Forces
- Friction

Concept of Forces

A push or a pull exerted on any object, is defined to be a force. It is a vector quantity. Force can be grouped into two types:

- **Contact forces** are the forces that act between two bodies in contact, e.g. tension, normal reaction, friction etc.
- **Non-contact forces** are the forces that act between two bodies separated by a distance without any actual contact. e.g. gravational force between two bodies and electrostatic form between two charges etc.

Inertia

The inability of a body to change by itself its state of rest or state of uniform motion along a straight line is called inertia of the body.

As inertia of a body is measured by the mass of the body. Heavier the body, greater the force required to change its state and hence greater is its inertia. There are three type of inertia (i) inertia of rest (ii) inertia of motion (iii) inertia of direction.

Newton's Laws of Motion

First Law of Motion (Law of Inertia)

It states that a body continues to be in a state of rest or of uniform motion along a straight line, unless it is acted upon by some external force the change the state. This is also called law of inertia.

If $F = 0, \Rightarrow v = \text{constant} \Rightarrow a = 0$

- This law defines force.
- The body opposes any external change in its state of rest or of uniform motion.
- It is also known as the **law of inertia** given by Galileo.

Linear Momentum

It is defined as the total amount of motion of a body and is measured as the product of the mass of the body and its velocity. The momentum of a body of mass m moving with a velocity $\mathbf{v}$ is given by $\mathbf{p} = m\mathbf{v}$.

Its unit is kg-ms^{-1} and dimensional formula is $[\text{ML T}^{-1}]$

Second Law of Motion

The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

According to second law, $F \propto \frac{dp}{dt}$ or $F = k\frac{dp}{dt}$

where, k is constant.

as, $\frac{dp}{dt} = \frac{d}{dt}(mv) = ma$ or $\frac{m\,dv}{dt} = ma$

i.e., second law can be written as

$$F = \frac{dp}{dt} = ma$$

The SI unit of force is newton (N) and in CGS system is dyne.

$$1\text{ N} = 10^5 \text{ dyne}$$

Impulse

Impulse received during an impact is defined as the product of the average force and the time for which the force acts.

Impulse, $\mathbf{I} = \mathbf{F}_{\text{av}}\, t$

Impulse is also equal to the total change in momentum of the body during the impact.

Impulse, $\mathbf{I} = \mathbf{p}_2 - \mathbf{p}_1$

Impulse = Change in momentum

Third Law of Motion

To every action, there is an equal and opposite reaction.

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

- Action and reaction are mutually opposite and act on two different bodies.
- The force acting on a body is known as action.
- When a forc acts on a body, then the reaction acts normally to the surface of the body.

Principle of Conservation of Linear Momentum

It states that if no external force is acting on a system, the momentum of the system remains constant.

According to second law of motion, $\mathbf{F} = \frac{d\mathbf{p}}{dt}$

If no force is acting, then $\mathbf{F} = 0$

$\therefore$ $$\frac{d\mathbf{p}}{dt} = 0 \Rightarrow \mathbf{p} = \text{constant}$$

or $$m_1\mathbf{v}_1 = m_2\mathbf{v}_2 = \text{constant}$$

Applications of Conservation of Linear Momentum

The propulsion of rockets and jet planes is based on the principle of conservation of linear momentum.

- Upward thrust on the rocket, $F = -\frac{u\,dm}{dt} - mg$ and if effect of gravity is neglected, then $F = -\frac{u\,dm}{dt}$.
- Instantaneous upward velocity of the rocket

  $$v = u \ln\left(\frac{m_0}{m}\right) - gt$$

  and neglecting the effect of gravity

  $$v = u \ln\left(\frac{m_0}{m}\right) = 2.303\, u \log_{10}\left(\frac{m_0}{m}\right)$$

  where, m_0 = initial mass of the rocket including that of the fuel,

  u = initial velocity of the rocket at any time t,

  m = mass of the rocket left,

  v = velocity acquired by the rocket,

  $\frac{dm}{dt}$ = rate of combination of fuel.
- **Burnt out speed** of the rocket is the speed attained by the rocket when the whole of fuel of the rocket has been burnt.

  Burnt out speed of the rocket

  $$v_b = u \log_e\left(\frac{m_0}{m_r}\right) = 3.303\, u \log_{10}\left(\frac{m_0}{m_r}\right)$$

Apparent Weight of a Boy in a Lift

Actual weight of the body is mg. Here we consider the apparent weight of a man standing in a moving lift.

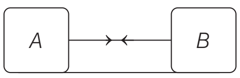
- If lift is accelerating upward with acceleration a, then apparent weight of the body is $R = m(g + a)$.
- If lift is accelerating downward at the rate of acceleration a, then apparent weight of the body is $R = m(g - a)$
- If lift is moving upward or downward with constant velocity, then apparent weight of the body is equal to actual weight.
- If the lift is falling freely under the effect of gravity, then it is called weightlessness condition.

Free Body Diagram

A free body diagram (FBD) consists of a diagramatize representation of a single body or sub-system of bodies isolated from its surroundings showing all forces acting on it.

While sketching a free body diagram the following points should be kept in mind.

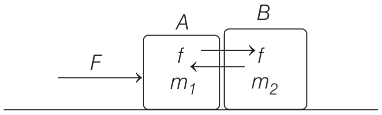
- Normal reaction (N) always acts normal to the surface on which the body is kept.



- When two objects A and B are connected by a string, the tension for object A is towards B and for object B, it is towards A.

Connected Motion

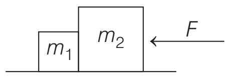
- If two blocks of masses m_1 and m_2 are placed on a perfectly smooth surface and are in contact, then



Acceleration of the blocks, $a = \dfrac{F}{m_1 + m_2}$

and the contact force (acting normally) between the two blocks is $f = m_2 a = \dfrac{F\,m_2}{(m_1 + m_2)}$.

- A block system is shown in the figure

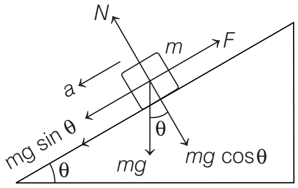


Acceleration of the blocks $a = \dfrac{F}{m_1 + m_2}$

Contact force between two blocks

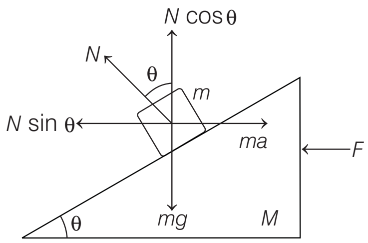
$$f = m_1 a = \frac{m_1 F}{(m_1 + m_2)}$$

- For a block of mass m placed on a fixed, perfectly smooth inclined plane of angle θ, the forces acting on the block are as shown in the figure. Obviously, here $a = g\sin\theta$



- If a block of mass m is placed on a smooth movable wedge of mass M, which in turn is placed on smooth surface, then a force **F** is applied on the wedge, horizontally.

The acceleration of the wedge and the block is

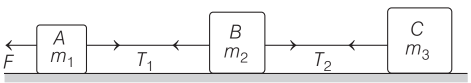


$$a = \frac{F}{(M + m)}$$

Force on the block, $F = (M + m)a = (M + m)g\tan\theta$

- For a block system shown in the figure, acceleration of the system

$$a = \frac{F}{m_1 + m_2 + m_3}$$



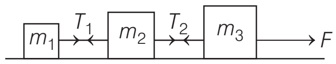
Tension in the string,

$$T_1 = (m_2 + m_3)a = \frac{F(m_3 + m_2)}{(m_1 + m_2 + m_3)}$$

and tension $T_2 = m_3 a = \dfrac{F\,m_3}{m_1 + m_2 + m_3}$

- For a block system shown in the figure, acceleration of the system

$$a = \frac{F}{m_1 + m_2 + m_3}$$

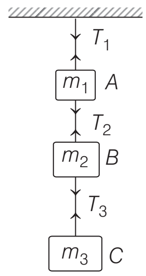


Tension in the string,

$$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$$

and $T_2 = \dfrac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

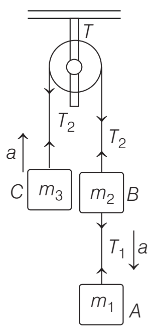
- For a block system suspended freely from a rigid support as shown in the figure, the acceleration of the system $a = 0$.
String tension,



$$T_1 = (m_1 + m_2 + m_3)g$$
$$T_2 = (m_2 + m_3)g$$
and $T_3 = m_3 g$

- For a block system and a pulley as shown in the figure, value of the acceleration of the system

$$a = \frac{(m_1 + m_2 - m_3)g}{(m_1 + m_2 + m_3)}$$



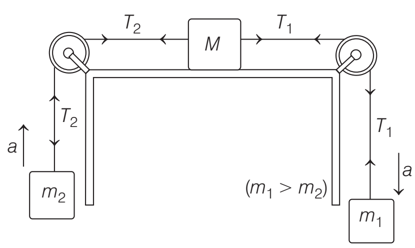
Tension, $T_1 = \dfrac{2m_1 m_3 g}{(m_1 + m_2 + m_3)}$

Tension, $T_2 = \dfrac{2m_3(m_1 + m_2)g}{(m_1 + m_2 + m_3)}$

and tension T

$$T = 2T_2 = \frac{4m_3(m_1 + m_2)g}{(m_1 + m_2 + m_3)}$$

- For the pulley and block arrangement as shown in the figure, we have



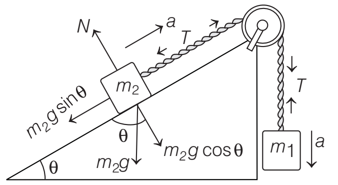
Net acceleration,

$$a = \frac{\text{Net accelerating force}}{\text{Total mass}} = \frac{(m_1 - m_2)g}{(m_1 + m_2 + M)}$$

Tension, $T_1 = m_1(g - a) = \dfrac{(M + 2m_2)m_1 g}{(M + m_1 + m_2)}$

and Tension, $T_2 = m_2(g + a) = \dfrac{(M + 2m_1)m_2 g}{(M + m_1 + m_2)}$

- For the system of block and pulley, with a smooth inclined plane as shown in the figure, we have



Net acceleration,

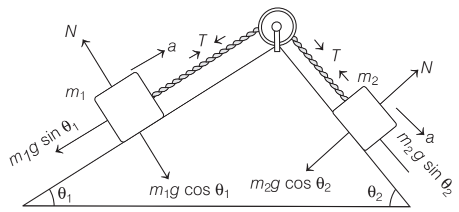
$$a = \frac{(m_1 - m_2 \sin\theta)g}{(m_1 + m_2)}, \text{ if } m_1 g > m_2 g \sin\theta$$

and $a = \dfrac{(m_2 \sin\theta - m_1)g}{m_1 + m_2}$, if $m_1 g < m_2 g \sin\theta$

and tension in the string

$$T = m_1(g - a) = \frac{m_1 m_2 (1 + \sin\theta)}{(m_1 + m_2)}$$

- For a pulley and block system on a smooth double inclined plane as shown in the figure, we have



Net acceleration, $a = \dfrac{(m_2 \sin\theta_2 - m_1 \sin\theta_1)g}{(m_1 + m_2)}$,

for $\theta_2 > \theta_1, m_2 > m_1$

and tension in the string,

$$T = \frac{m_1 m_2 (\sin\theta_1 + \sin\theta_2)g}{(m_1 + m_2)}$$

Equilibrium of Concurrent Forces

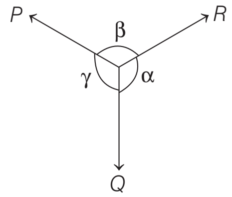
If a number of forces act at the same point, they are called concurrent forces.

The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.

Mathematically for equilibrium,

$$\Sigma \mathbf{F}_{\text{net}} = 0 \text{ or } \Sigma F_x = 0, \Sigma F_y = 0 \text{ and } \Sigma F_z = 0$$

Lami Theorem For three concurrent forces in equilibrium position.



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Friction

Whenever an object actually slides or rolls over the surface of another body or tends to do so, a force opposing the relative motion starts acting between these two surfaces in contact. It is known as **friction** or the force due to friction. Force of friction acts in a tangential direction to the surfaces in contact.

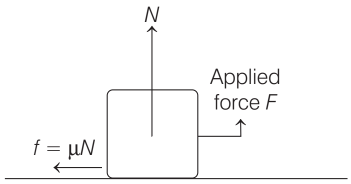
Types of Friction

The four types of friction are given below

1. **Static Friction** It is the opposing force that comes into play when one body is at rest and a force acts to move it over the surface of another body.

   It is a self adjusting force and is always equal and opposite to the applied force.

2. **Limiting Friction** It is the limiting (maximum) value of static friction when a body is just on the verge of starting its motion over the surface of another body.



   The force of limiting friction f_l between the surfaces of two bodies is directly proportional to the normal reaction at the point of contact. Mathematically,

$$f_l \propto N \text{ or } f_l = \mu_l N$$

$$\Rightarrow \qquad \mu_l = \frac{f_l}{N}$$

   where, μ_l is the **coefficient of limiting friction** for the given surfaces in contact.

3. **Kinetic Friction** It is the opposing force that comes into play when one body is actually slides over the surface of another body. Force of kinetic friction f_k is directly proportional to the normal reaction N and the ratio $\dfrac{f_k}{N}$ is called **coefficient of kinetic friction** μ_k, value of μ_k is slightly less than $\mu_e (\mu_k < \mu_l)$.

   Whenever limiting friction is converted into kinetic friction, body started motion with a lurch.

4. **Rolling Friction** It is the opposing force that comes into play when a body of symmetric shape (wheel or cylinder or disc, etc.) rolls over the surface of another body. Force of rolling friction f_r is directly proportional to the normal reaction N and inversely proportional to the radius (r) of the wheel.

Thus, $$f_r \propto \frac{N}{r} \quad \text{or} \quad f_r = \mu_r \frac{N}{r}$$

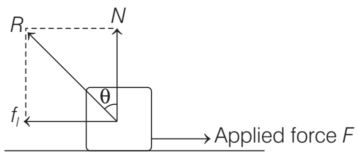
The constant μ_r is known as the **coefficient of rolling friction** μ_r has the unit and dimensions of length. Magnitudewise $\mu_r << \mu_k$ or μ_l.

- The value of rolling friction is much smaller than the value of sliding friction.
- Ball bearings are used to reduce the wear and tear and energy loss against friction.

Angle of Friction

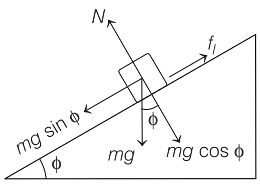
Angle of friction is defined as the angle θ which the resultant R of the force of limiting friction f_l and normal reaction N, subtends with the normal reaction.

The tangent of the angle of friction is equal to the coefficient of friction. i.e. $\mu = \tan\theta$



Angle of Repose

Angle of repose is the least angle of the inclined plane (of given surface) with the horizontal such that the given body placed over the plane, just begins to slide down, without getting accelerated.



The tangent of the angle of repose is equal to the coefficient of friction.

Hence, we conclude that angle of friction (θ) is equal to the angle of repose (ϕ).

In limiting condition, $f_1 = mg \sin\phi$

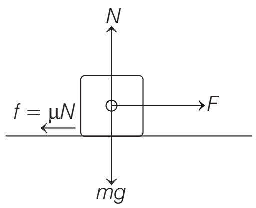
$$\Rightarrow \quad N = mg\cos\phi$$

$$\frac{f_1}{N} = \tan\phi$$

$$\therefore \quad \frac{f_1}{N} = \mu_s = \tan\phi$$

Acceleration of a Block on Applying a Force on a Rough Surface

- Acceleration of a block on a horizontal surface is as shown in the figure.



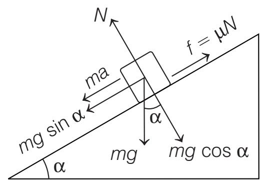
$$a = \frac{F - f}{m} = \frac{F - \mu mg}{m}$$

or $$a = \frac{F}{m} - \mu g$$

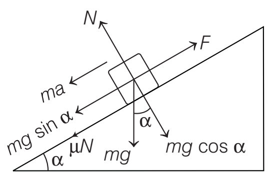
where, μ = coefficient of kinetic friction between the two surfaces in contact.

- Acceleration of block sliding down a rough inclined plane as shown in the figure is given by

$$a = g(\sin\alpha - \mu\cos\alpha)$$



- Retardation of a block sliding up a rough inclined plane as shown in the figure is $a = g(\sin\alpha + \mu\cos\alpha)$



DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 Five forces inclined at an angle of 72° w.r.t. each other act on a particle of mass m placed at the origin. Four forces are of magnitude F_1 and one has a magnitude F_2. Find the resultant acceleration of the particle.

(a) $\frac{F_2 - F_1}{m}$ (b) Zero
(c) $\frac{F_2 + F_1}{m}$ (d) $\frac{F_2 - 4F_1}{m}$

2 A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves by 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. (take, $g = 10\ \text{ms}^{-2}$)

(a) 4 N (b) 16 N (c) 20 N (d) 22 N

3 A player catches a cricket ball of mass 150 g, moving at a rate of 20 ms^{-1}. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to

(a) 150 N (b) 3 N (c) 30 N (d) 300 N

4 A ball of mass m is thrown vertically upwards with a velocity v. If air exerts an average resisting force F, the velocity with which the ball returns to the thrower is

(a) $v\sqrt{\frac{mg}{mg+F}}$ (b) $v\sqrt{\frac{F}{mg+F}}$
(c) $v\sqrt{\frac{mg-F}{mg+F}}$ (d) $v\sqrt{\frac{mg+F}{mg}}$

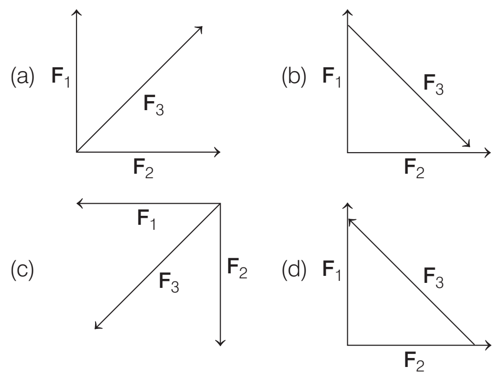
5 A block of mass 200 g is moving with a velocity of 5 ms^{-1} along the positive x-direction. At time $t = 0$, when the body is at $x = 0$, a constant force 0.4 N is directed along the negative x-direction , is applied on the body for 10 s. What is the position x of the body at $t = 2.5$ s?

(a) $x = 6.75$ m (b) $x = 6.25$ m
(c) $x = 6$ m (d) $x = 6.50$ m

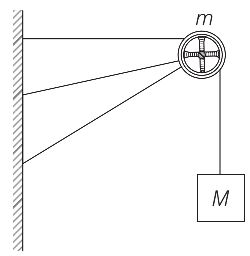
6 Two trains A and B are running in the same direction on parallel tracks such that A is faster than B. If packets of equal weight are exchanged between the two, then

(a) A will be retarded but B will be accelerated
(b) A will be accelerated but B will be retarded
(c) there will not be any change in the velocity of A but B will be accelerated
(d) there will not be any change in the velocity of B, but A will be accelerated

7 Which of the four arrangements in the figure correctly shows the vector addition of two forces F_1 and F_2 to yield the third force F_3?



8 A string of negligible mass, going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by

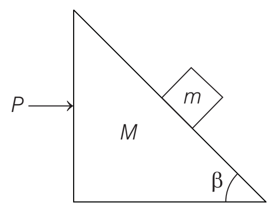


(a) $\sqrt{2}Mg$ (b) $\sqrt{[(M+m)^2 + m^2]}g$
(c) $2Mg$ (d) $\sqrt{[(M+m)+m]^2}g$

9 A bullet is fired from a gun. The force on the bullet is given by $F = (600 - 2 \times 10^5 t)$, where F is in newton and t is in second. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet?

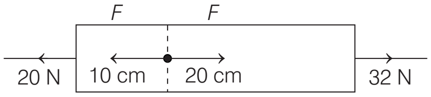
(a) 9 N-s (b) Zero (c) 0.9 N-s (d) 1.8 N-s

10 Two wooden blocks are moving on a smooth horizontal surface, such that the mass m remains stationary with respect to the block of mass M as shown in the figure. The magnitude of force P is



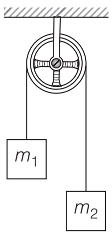
(a) $g \tan\beta$ (b) $mg\cos\beta$
(c) $(M+m)\ \text{cosec}\ \beta$ (d) $(M+m)\, g \tan\beta$

11 The figure below shows a uniform rod of length 30 cm having a mass of 3.0 kg. The strings as shown in the figure are pulled by constant forces of 20 N and 32 N. Find the force exerted by the 20 cm part of the rod on the 10 cm part. All the surfaces are smooth and the strings are light



(a) 36 N (b) 12 N (c) 64 N (d) 24 N

12 Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg, tied to a string, are hanging over a light frictionless pulley. What is the acceleration of the masses produced when system is free to move? (take, $g = 9.8\ \text{ms}^{-2}$)



(a) $0.2\ \text{ms}^{-2}$ (b) $9.8\ \text{ms}^{-2}$
(c) $5\ \text{ms}^{-2}$ (d) $4.8\ \text{ms}^{-2}$

13 A light string passing over a smooth light pulley, connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $(g/8)$, then the ratio of masses is

(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3

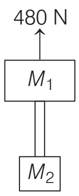
14 Two bodies of equal masses are connected by a light inextensible string passing over a smooth frictionless pulley. The amount of mass that should be transferred from one to another, so that both the masses move 50 m in 5 s is

(a) 30% (b) 40% (c) 70% (d) 50%

15 A man slides down a light rope, whose breaking strength is η times his weight. What should be his maximum acceleration, so that the rope does not break?

(a) $g(1-\eta)$ (b) ηg (c) $\dfrac{g}{1+\eta}$ (d) $\dfrac{g}{1-\eta}$

16 Two blocks of mass $M_1 = 20$ kg and $M_2 = 12$ kg are connected by a metal rod of mass 8 kg. The system is pulled vertically up by applying a force of 480 N as shown in figure. The tension at the mid-point of the rod is



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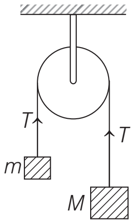
(a) 144 N (b) 96 N (c) 240 N (d) 190 N

17 Two blocks of masses m and M are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in the figure. The system is then released. If $M = 2m$, then the stress produced in the wire is **→ JEE Main (Online) 2013**



(a) $\dfrac{2\,mg}{3A}$ (b) $\dfrac{4\,gm}{3A}$ (c) $\dfrac{gm}{A}$ (d) $\dfrac{3\,mg}{4A}$

18 A satellite in a force-free space sweeps out stationary interplanetary dust at a rate $\dfrac{dM}{dt} = \alpha v$, where M is the mass and v is the velocity of the satellites and α is a constant. The deceleration of the satellite is

(a) $\dfrac{2\alpha v^2}{M}$ (b) $-\dfrac{\alpha v^2}{M}$ (c) $-\dfrac{\alpha v^2}{2M}$ (d) $-\alpha v^2$

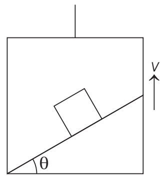
19 A lift is moving down with an acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively

(a) g, g (b) a, a (c) $(g-a), g$ (d) a, g

20 A spring balance is attached to the ceiling of a lift. A man hangs his bag on the string and the balance reads 49 N, when the lift is stationary. If the lift moves downwards with an acceleration of $5\ \text{ms}^{-2}$, the reading of the spring balance would be

(a) 24 N (b) 74 N (c) 15 N (d) 49 N

21 A block A is able to slide on the frictionless incline of angle θ and length l, kept inside an elevator going up with uniform velocity v. Time taken by the block to slide down the length of the incline, if released from rest is



(a) $\sqrt{\dfrac{2l}{(g+a)\sin\theta}}$ (b) $\sqrt{\dfrac{2l}{g}}$

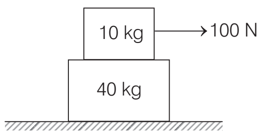
(c) $\sqrt{\dfrac{2l}{g\sin\theta}}$ (d) $\sqrt{\dfrac{2l}{\sin\theta}}$

22 A plane is inclined at an angle θ with the horizontal. A body of mass m rests on it. If the coefficient of friction is μ, then the minimum force that has to be applied parallel to the inclined plane, so as to make the body to just move up the inclined plane, is

(a) $mg\sin\theta$ (b) $\mu mg\cos\theta$
(c) $\mu mg\cos\theta - mg\sin\theta$ (d) $\mu mg\cos\theta + mg\sin\theta$

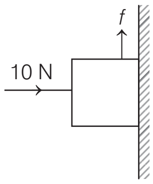
23 A 40 kg slab rests on a frictionless floor. A 10 kg block rests on the top of the slab. The static coefficients of friction between the block and the slab is 0.60, while the kinetic coefficient is 0.40.

The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8\ \text{ms}^{-2}$, the resultant acceleration of the slab will be



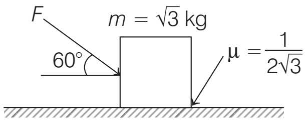
(a) $0.98\ \text{ms}^{-2}$ (b) $1.47\ \text{ms}^{-2}$
(c) $1.52\ \text{ms}^{-2}$ (d) $6.1\ \text{ms}^{-2}$

24 A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is



(a) 20 N (b) 50 N
(c) 100 N (d) 2 N

25 What is the maximum value of the force F, such that the block as shown in the arrangement, does not move?



(a) 20 N (b) 10 N (c) 12 N (d) 15 N

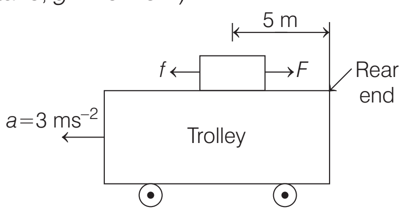
26 A block of mass M is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction between the block and the wall is μ and the acceleration due to gravity is g, then minimum force required to be applied by the finger to hold the block against the wall?

(a) $\frac{Mg}{2\mu}$ (b) $\frac{Mg}{\mu}$ (c) $\frac{2M}{\mu g}$ (d) $\frac{2Mg}{\mu}$

27 A wooden block of mass M resting on a rough horizontal surface, is pulled with a force F at an angle with the horizontal. If μ is the coefficient of kinetic friction between block and the surface, then acceleration of the block is

(a) $\frac{F}{M}(\cos\phi + \mu\sin\phi) - \mu g$ (b) $\frac{F}{M}\sin\phi$
(c) $\mu F\cos\phi$ (d) $\mu F\sin\phi$

28 A block of mass 10 kg is placed at a distance of 5 m from the rear end of a long trolley as shown in the figure. The coefficient of friction between the block and the surface below is 0.2. Starting from rest, the trolley is given an uniform acceleration of 3 ms^{-2}. At what distance from the starting point will the block fall off the trolley? (take, $g = 10\ ms^{-2}$)

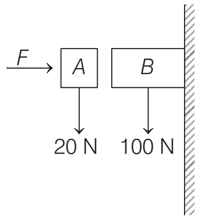


(a) 15 m (b) 25 m (c) 20 m (d) 10 m

29 A body starts from rest on a long inclined plane of slope 45°. The coefficient of friction between the body and the plane varies as $\mu = 0.3\,x$, where x is distance travelled down the plane. The body will have maximum speed (for $g = 10\ ms^{-2}$) when x is equal to → **JEE Main (Online) 2013**

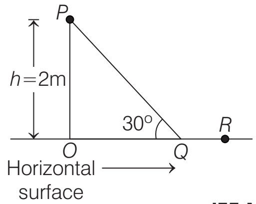
(a) 9.8 m (b) 27 m (c) 12 m (d) 3.33 m

30 Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown in figure. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall in block B is → **JEE Main 2015**



(a) 100 N (b) 80 N (c) 120 N (d) 150 N

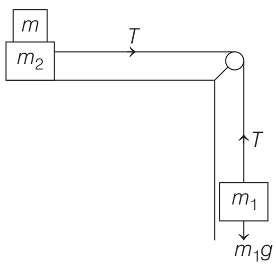
31 A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x(= QR)$, are respectively close to



→ **JEE Main 2016 (Offline)**

(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m
(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

32 Two masses $m_1 = 5$ kg and $m_2 = 10$ kg connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is → **JEE Main 2018**

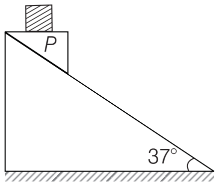


(a) 18.3 kg (b) 27.3 kg (c) 43.3 kg (d) 10.3 kg

33 The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$, then the ratio $\frac{F_1}{F_2}$ is

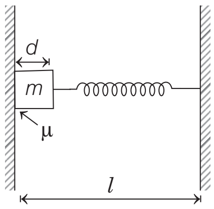
(a) 4 (b) 1 (c) 2 (d) 3

34 A box of mass 80 kg kept on a horizontal weighing machine of negligible mass, attached to a massless platform P that slides down at 37° incline. The weighing machine read 72 kg. Box is always at rest w.r.t. weighing machine. Then, coefficient of friction between the platform and incline is



(a) $\frac{12}{13}$ (b) $\frac{13}{24}$ (c) $\frac{3}{5}$ (d) $\frac{11}{24}$

35 A block of mass m is placed against a vertical surface by a spring of unstretched length l. If the coefficient of friction between the block and the surface is μ, then choose the correct statement.



(a) If spring constant $k = \frac{2mg}{\mu d}$, block will not be in equilibrium.
(b) Minimum spring constant k_{min} to keep the block of mass m in equilibrium is $\frac{mg}{\mu d}$.
(c) If spring constant is $k = \frac{2mg}{\mu d}$, the normal reaction is $\frac{mg}{\mu}$.
(d) In the part (c), force of friction is $2mg$.

Direction (Q. Nos. 36-40) *Each of these questions contains two statements : Statement* I (*Assertion*) *and Statement* II (*Reason*). *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below*

(a) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true, Statement II is not the correct explanation for Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

36 **Statement I** When the car accelerates horizontally along a straight road, the accelerating force is given by the push of the rear axle on the wheels.

Statement II When the car accelerates, the rear axle rotates with a greater frequency.

37 **Statement I** It is easier to pull a heavy object than to push it on a level ground.

Statement II The magnitude of frictional force depends on the nature of the two surfaces in contact.

38 **Statement I** A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.

Statement II For every action there is an equal and opposite reaction.

39 **Statement I** A bullet is fired from a rifle. If the rifle recoils freely, the kinetic energy of the rifle is less than that of the bullet.

Statement II In the case of a rifle-bullet system, the law of conservation of momentum is violated.

40 **Statement I** Newton's second law is applicable on a body with respect to an inertial frame of reference.

Statement I In order to apply Newton's second law on a body observed from a non-inertial frame of reference. We apply line pseudo force an imaginary force.

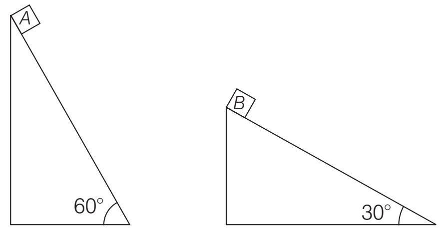
DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A flexible uniform chain of mass m and length l suspended vertically, so that its lower end just touches the surface of a table. When the upper end of the chain is released, it falls with each link coming to rest the instant it strikes the table. The force exerted by the chain on the table at the moment when y part of the chain has already rested on the table is

(a) $\frac{3myg}{l}$ (b) $\frac{3mg}{l}$ (c) $\frac{2}{3}\frac{mg}{l}$ (d) $\frac{1}{3}\frac{mg}{l}$

2 Two fixed frictionless inclined plane making the angles 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction

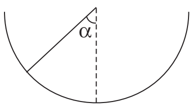
3 If M is the mass of a rocket, r is the rate of ejection of gases with respect to the rocket, then acceleration of the rocket, $\frac{dv}{dt}$ is equal to

(a) $\frac{ru}{(M-rt)}$ (b) $\frac{(M-rt)}{ru}$ (c) $\frac{ru}{(M+rt)}$ (d) $\frac{ru}{M}$

4 A person 40 kg is managing to be at rest between two vertical walls by pressing one wall A by his hands and feet and B with his back. The coefficient of friction is 0.8 between his body and the wall. The force with which the person pushes the wall is

(a) 100 N (b) 50 N (c) 150 N (d) 200 N

5 An insect crawls up a hemispherical surface very slowly. The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



(a) $\cot\alpha = 3$ (b) $\sec\alpha = 3$
(c) $\operatorname{cosec}\alpha = 3$ (d) None of these

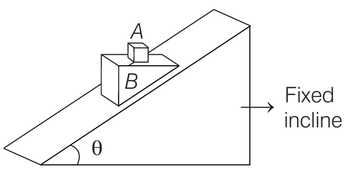
6 A given object takes n times more time to slide down a 45° rough inclined plane in comparison to slides down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

(a) $\frac{1}{1-n^2}$ (b) $1-\frac{1}{n^2}$ (c) $\sqrt{1-\frac{1}{n^2}}$ (d) $\sqrt{\frac{1}{1-n^2}}$

7 When a body slides down from rest along a smooth inclined plane making an angle of 45° with the horizontal,it takes time T. When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance,it is seen to take time pT, where p is some number greater than 1. The coefficient of friction between the body and the rough plane will be

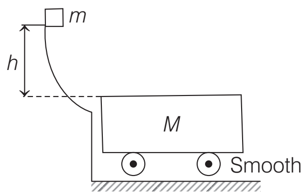
(a) $1-p^2$ (b) $1-\frac{1}{p^2}$ (c) p^2-1 (d) p^2

8 Block A of mass m is placed over a wedge of the same mass m. Both the block and wedge are placed on a fixed inclined plane. Assuming all surfaces to be smooth. Then, displacement of the block A in ground frame in 1 s is



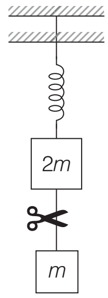
(a) $\left[\frac{g\sin^2\theta}{1+\sin^2\theta}\right]$ (b) $\left[\frac{g\cos^2\theta}{1+\cos^2\theta}\right]$
(c) $\left[\frac{g\tan^2\theta}{1+\tan^2\theta}\right]$ (d) $\left[\frac{g\sin^2\theta}{1+\sin\theta}\right]$

9 A carriage of mass M and length l is joined to the end of a slope as shown below. A block of mass m is released from the slope from height h. It slides till end of the carriage. The coefficient of friction between block and carriage is μ (the friction between other surfaces are negligible). Then, minimum height h is



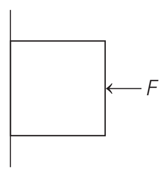
(a) $\mu\left(1+\frac{M}{m}\right)l$ (b) $\mu\left(2+\frac{m}{M}\right)l$
(c) $\mu\left(1+\frac{m}{M}\right)l$ (d) $2\mu\left(1+\frac{m}{M}\right)l$

10 System as shown in figure is in equilibrium and at rest. The spring and string are massless, now the string is cut. The acceleration of the masses $2m$ and m just after the string is cut, will be



(a) $\frac{g}{2}$ upwards, g downwards
(b) g upwards, $\frac{g}{2}$ downwards
(c) g upwards, $2g$ downwards
(d) $2g$ upwards, g downwards

11 A block of mass m is at rest under the action of a force F, acting against a wall, as shown in the figure. Which of the following statement is incorrect?



(a) $f = mg$ (where, f is the frictional force)
(b) $F = N$ (where, N is the normal force)
(c) F will not produce torque
(d) N will not produce torque

12 The upper-half of an inclined plane with an inclination ϕ, is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is given by

(a) $2\sin\phi$ (b) $2\cos\phi$ (c) $2\tan\phi$ (d) $\tan\phi$

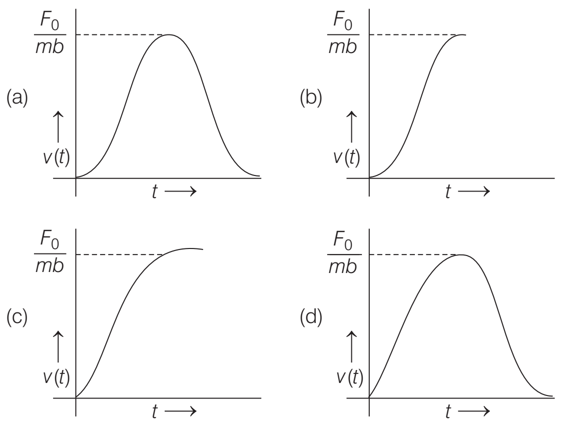
13 A block of mass m is placed on a surface with a vertical cross-section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

→ **JEE Main 2014**

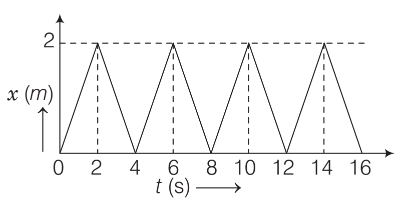
(a) 1/6 m (b) 2/3 m (c) 1/3 m (d) 1/2 m

14 A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x-direction. Its speed $v(t)$ is depicted by which of the following curves?

→ JEE Main 2013

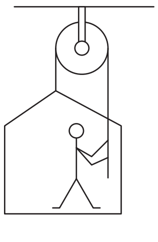


15 The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



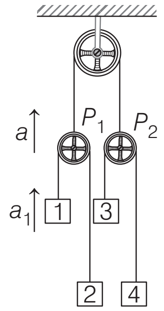
(a) 0.4N - s (b) 0.8 N - s
(c) 1.6N - s (d) 0.2 N - s

16 A worker is raising himself and the crate on which he stands with an acceleration of 5 ms^{-2} by a massless rope and pulley arrangement. Mass of the worker is 100 kg and that of the crate is 50 kg. If T is the tension in the rope and F be the force of contact between the worker and the floor and if $g = 10\,\text{ms}^{-2}$, then



(a) $T = 2250$N, $F = 1125$N
(b) $T = 1125$N, $F = 2250$N
(c) $T = 1125$N, $F = 375$N
(d) $T = 1125$N, $F = 750$N

17 In the system shown in figure, masses of the blocks are such that when system is released, acceleration of pulley P_1 is a upwards and acceleration of block 1 is a_1, upwards. It is found that acceleration of block 3 is same as that of 1 both in magnitude and direction.



Given that, $a_1 > a > \frac{a_1}{2}$.

Match the following.

	Column I		Column II
A.	Acceleration of 2	1.	$2a + a_1$
B.	Acceleration of 4	2.	$2a - a_1$
C.	Acceleration of 2 w.r.t.3	3.	Upwards
D.	Acceleration of 2 w.r.t. 4	4.	Downwards

Codes

	A	B	C	D
(a)	2,3	1,4	4	3
(b)	2,4	1	4	1,3
(c)	4	1,3	2	1,2

(d) No above matching is correct

ANSWERS

SESSION 1

1 (a)	**2** (d)	**3** (c)	**4** (c)	**5** (b)	**6** (a)	**7** (a,c)	**8** (b)	**9** (c)	**10** (d)
11 (d)	**12** (a)	**13** (b)	**14** (b)	**15** (a)	**16** (d)	**17** (b)	**18** (b)	**19** (c)	**20** (a)
21 (c)	**22** (d)	**23** (a)	**24** (d)	**25** (a)	**26** (b)	**27** (a)	**28** (a)	**29** (d)	**30** (c)
31 (c)	**32** (b)	**33** (d)	**34** (b)	**35** (b)	**36** (a)	**37** (b)	**38** (b)	**39** (c)	**40** (a)

SESSION 2

1 (a)	**2** (d)	**3** (a)	**4** (d)	**5** (a)	**6** (b)	**7** (b)	**8** (a)	**9** (c)	**10** (a)
11 (d)	**12** (c)	**13** (a)	**14** (c)	**15** (b)	**16** (c)	**17** (a)			

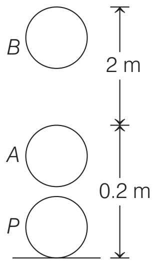
Hints and Explanations

SESSION 1

1 According to polygon law, resultant of four forces, each of magnitude F_1 acting at an angle of 72°, is along the fifth side of the polygon taken an in opposite order. As F_2 is acting along this side of polygon, therefore the net force on the particle $= F_2 - F_1$

Acceleration $(a) = \dfrac{F_2 - F_1}{m}$

2 The situation is as shown in the figure. At an initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B, let acceleration of ball during the motion from P to A is $a\,\text{ms}^{-2}$ [assumed to be constant] in an upward direction and velocity of ball at A is $v\ \text{ms}^{-1}$.



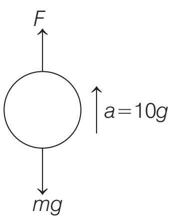
Then for PA, $v^2 = 0^2 + 2a \times 0.2$

For AB, $0 = v^2 - 2 \times g \times 2$

$\Rightarrow \quad v^2 = 2g \times 2$

From above equations,

$$a = 10g = 100\ \text{ms}^{-2}$$



Then for PA, from FBD of ball is

$$F - mg = ma$$

[F is the force exerted by hand on the ball]

$\Rightarrow \quad F = m(g + a) = 0.2(11g) = 22\,\text{N}$

3 This is the question based on impulse-momentum theorem.

$$|F\Delta t| = |\text{Change in momentum}|$$

$\Rightarrow \quad F \times 0.1 = |p_f - p_i|$

As the ball will stop after catching,

$p_i = mv_i = 0.15 \times 20 = 3$ and $p_f = 0$

$\Rightarrow \quad F \times 0.1 = 3$

$\Rightarrow \quad F = 30\,\text{N}$

4 For an upward motion

Retarding force $= mg + F$

Retardation $(a) = \dfrac{mg + F}{m}$

Distance, $s = \dfrac{v^2}{2a} = \dfrac{v^2 m}{2(mg + F)}$ ... (i)

For the downward motion, net force

$$= mg - F$$

$\therefore \quad$ Acceleration $(a') = \dfrac{mg - F}{m}$

Distance $(s') = \dfrac{v'^2}{2a'} = \dfrac{v'^2 m}{2(mg - F)}$ As, $s = s'$

$\therefore \quad v' = v\sqrt{\dfrac{mg - F}{mg + F}}$

5 Given, $u = 5\,\text{ms}^{-1}$, along positive x-direction

$$F = -0.4\,\text{N},$$

along negative x-direction

$$M = 200\text{g} = 0.2\ \text{kg}$$

Thus, the acceleration

$$a = \frac{F}{M} = -\frac{0.4}{0.2} = -2\ \text{ms}^{-2}$$

The negative sign showing the retardation.

The position of the object at time t is given by

$$x = x_0 + ut + \frac{1}{2}at^2$$

At $t = 0$, the body is at $x = 0$, therefore $x_0 = 0$.

Hence, $\quad x = ut + \dfrac{1}{2}at^2$

Since, the force acts during the time interval from $t = 0$ to $t = 10$ s, the motion is accelerated only within this time interval. The position of the body at $t = 2.5$ s is given by

$$x = 5 \times 2.5 + \frac{1}{2} \times (-2) \times (2.5)^2$$

$$= 6.25\,\text{m}$$

6 Initially, the momentum of the packet in train A is more than in train B. When packets are changed, the packet reaching train A being of lower momentum will retard the train A but packet reaching train B, being of higher momentum will accelerate B.

7 Parallelogram law of vector addition

$$\mathbf{F}_3 = \mathbf{F}_1 + \mathbf{F}_2$$

So, option (a) and (c) both can be satisfied.

8 Force on the pulley, by the clamp

= Resultant of forces $(M + m)g$ acting along horizontally and mg acting vertically downwards

$= \sqrt{(Mg + mg)^2 + (mg)^2}$

$= \sqrt{[(M + m)^2 + m^2]}\,g$

9 As, $F = 600 - 2 \times 10^5 t$

At, $\quad t = 0, F = 600\,\text{N}$

According to question,

$F = 0$, on leaving the barrel,

$\Rightarrow \quad 0 = 600 - 2 \times 10^5 t$

$\therefore \quad t = \dfrac{600}{2 \times 10^5} = 3 \times 10^{-3}\,\text{s}$

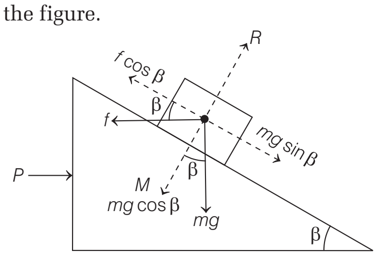
This is the time spent by the bullet in the barrel

Average force $= \dfrac{600 + 0}{2} = 300\,\text{N}$

Average impulse imparted $= F \times t$

$$= 300 \times 3 \times 10^{-3} = 0.9\ \text{N-s}$$

10 Different forces involved are as shown in the figure.



Observing the figure, we have acceleration of the system,

$$a = \frac{P}{M + m}$$

Force on block of mass $(m) = \dfrac{Pm}{M + m}$

If f is pseudo force on m in the direction opposite to force P, then

$$f = \frac{Pm}{M + m}$$

As it is clear from the figure,

$$f \cos\beta = mg \sin\beta$$

$$\frac{Pm}{(M+m)}\cos\beta = mg\sin\beta$$

$$P = g(M+m)\frac{\sin\beta}{\cos\beta}$$

or, $$P = (M+m)\,g\tan\beta$$

11 Net force on the rod,

$$f = 32 - 20 = 12\,\text{N}$$

Acceleration of the rod

$$= \frac{f}{m} = \frac{12}{3} = 4\,\text{ms}^{-2}$$

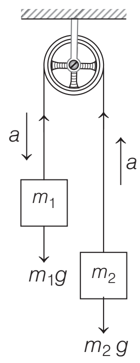
Equation of motion of the 10 cm part is

$$F - 20 = m \times a = 1 \times 4,$$
$$F = 4 + 20 = 24\ \text{N}$$

Similarly, equation of motion of 20 cm part is

$$32 - F = m \times a = 2 \times 4,$$
$$F = 32 - 8 = 24\ \text{N}$$

12 On releasing, the motion of the system will be according to the figure



The equations of motion of blocks are,

$$m_1 g - T = m_1 a \quad \ldots\text{(i)}$$

and $$T - m_2 g = m_2 a \quad \ldots\text{(ii)}$$

On solving,

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \quad \ldots\text{(iii)}$$

Here, $m_1 = 5$ kg,

$$m_2 = 4.8\ \text{kg}$$
$$g = 9.8\ \text{ms}^{-2}$$

$$\therefore \quad a = \left(\frac{5 - 4.8}{5 + 4.8}\right)\times 9.8$$

$$= \frac{0.2}{9.8}\times 9.8$$

$$= 0.2\,\text{ms}^{-2}$$

13 As, $a = \dfrac{(m_1 - m_2)\,g}{m_1 + m_2} = \dfrac{g}{8}$,

$$\frac{m_1 - m_2}{m_1 + m_2} = \frac{1}{8}$$

or $8m_1 - 8m_2 = m_1 + m_2$

or $7m_1 = 9m_2$

$$\frac{m_1}{m_2} = \frac{9}{7}$$

14 As, $s = ut + \dfrac{1}{2}at^2$

$$\Rightarrow 50 = 0 \times 5 + \frac{1}{2} \times a \times (5)^2$$

$$\therefore \quad a = \frac{100}{25} = 4\ \text{ms}^{-2}$$

Let, mass of one become m_1 and that of other m_2, where $m_1 > m_2$. As m_1 moves downwards with acceleration $a = 4\ \text{ms}^{-2}$

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

So, $$4 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)10$$

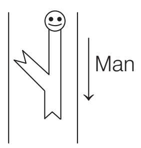
$$\left(\frac{m_1 - m_2}{m_1 + m_2}\right) = \frac{a}{g} = \frac{4}{10} = \frac{2}{5}$$

$\therefore$ Percentage of mass transferred

$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\times 100$$

$$= \frac{2}{5}\times 100 = 40\%$$

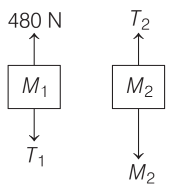
15 As, $mg - R = ma$



$$mg - \eta mg = ma$$
$$mg(1-\eta) = ma \Rightarrow a = g(1-\eta)$$

16 For block of mass M_1,

$$\frac{480 - T_1 - 20\,g}{20} = a$$



Also, for block of mass M_2,

$$\frac{T_2 - 12\,g}{12} = a$$

Since, a is common for all the individuals of the system

$$\Rightarrow \quad \frac{480 - T_1 - 20g}{20} = \frac{T_2 - 12g}{12}$$

After taking $g = 10\,\text{ms}^{-2}$ this gives

$$5T_2 + 3T_1 = 1440 \quad \ldots\text{(i)}$$

Now, for the metal rod, tension at both of its end are dissimilar and

$$T_1 - T_2 = 80 \quad \ldots\text{(ii)}$$
$$(\because g = 10\ \text{ms}^{-2})$$

Now, from Eqs. (i) and (ii), we get

$$T_1 = 230\,\text{N} \text{ and } T_2 = 150\,\text{N}$$

$\therefore$ Tension at mid-point

$$= T_1 - 4g = 190\ \text{N}$$

17 Tension, $T = \left(\dfrac{2m_1 m_2}{m_1 + m_2}\right)g$

$$= \left(\frac{2m \times 2m}{m + 2m}\right)g$$

(where, $m_1 = m$ and $m_2 = 2m$)

$$= \frac{4}{3}mg$$

$$\therefore \quad \text{Stress} = \frac{\text{Force (Tension)}}{\text{Area}}$$

$$= \frac{\frac{4}{3}mg}{A} = \frac{4\,mg}{3A}$$

18 It is known that the thrust

$$= -v\left(\frac{dM}{dt}\right) = -v(\alpha v)$$

Hence, the retardation produced

$$= \frac{\text{thrust}}{\text{mass}} = -\frac{\alpha v^2}{M}$$

19 When ball dropped, acceleration of the ball is g as will be observed by a man standing stationary on the ground. The man inside the lift is having its own downward acceleration, a. Therefore, relative acceleration of the ball as observed by the man in the lift will must be $= (g - a)$.

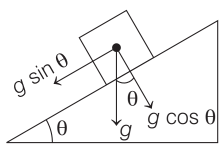
20 When the lift is stationary, then

$$R = mg$$
$$49 = m \times 9.8$$
$$m = \frac{49}{9.8}\,\text{kg} = 5\,\text{kg}$$

If a is the downward acceleration of the lift then, $R = m(g - a)$

$$= 5(9.8 - 5) = 24\ \text{N}$$

21 The situation is given in figure below



From equation of motion,

$$\frac{1}{2}\,g\sin\theta\, t^2 = l$$

$$\Rightarrow \quad t = \sqrt{\frac{2l}{g\sin\theta}}$$

22 To move the body up the inclined plane, the force required,

$$= mg\sin\theta + \mu R$$
$$= mg\sin\theta + \mu mg\cos\theta$$

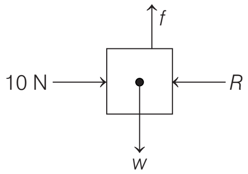
23 Limiting force of friction of block on slab $\mu\, m_1 g = 0.6 \times 10 \times 9.8 = 58.8\,\text{N}$

Since, the applied force = 100 N on block, which is greater than the force of limiting friction, the block will accelerate on the slab, due to which, the force acting on the slab will be that due to the kinetic friction $(\mu_k m_1 g)$.

Hence, acceleration of the slab,

$$a = \frac{\mu_k m_1 g}{m_2} = \frac{0.4 \times 10 \times 9.8}{40} = 0.98\ \text{ms}^{-2}$$

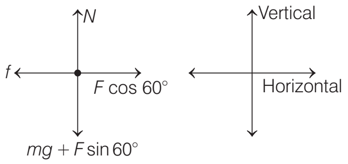
24 Let, R be the normal contact force by wall on the block.



$R = 10\text{N},\ f_L = w$ and $f = \mu R$

$\therefore\quad \mu R = w$ or $w = 0.2 \times 10 = 2\,\text{N}$

25 Free body diagram (FBD) of the block (shown by a dot) is as shown in the figure



For vertical equilibrium of the block

$$N = mg + F\sin 60^\circ$$
$$= \sqrt{3}\,g + \sqrt{3}\,\frac{F}{2} \qquad \dots \text{(i)}$$

For no motion, force of friction

$$f \geq F\cos 60^\circ$$

or $\quad \mu N \geq F\cos 60^\circ$

or $\quad \dfrac{1}{2\sqrt{3}}\left(\sqrt{3}\,g + \dfrac{\sqrt{3}\,F}{2}\right) \geq \dfrac{F}{2}$

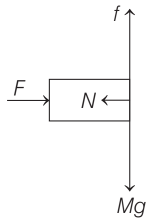
or $\quad g \geq \dfrac{F}{2}$ or $F \leq 2g$ or $20\,\text{N}$

Therefore, maximum value of F is 20 N.

26 Given, mass of the block = M

Coefficient of friction between the block and the wall = μ

Let, a force F be applied on the block to hold the block against the wall. The normal reaction of mass be N and force of friction acting upward be f. In equilibrium, vertical and horizontal forces should be balanced separately.



$$f = Mg \qquad \dots \text{(i)}$$

and $\qquad F = N \qquad \dots \text{(ii)}$

But force of friction $(f) = \mu N$

$$= \mu F \qquad \dots \text{(iii)}$$

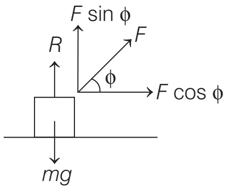
[using Eq. (ii)]

From Eqs. (i) and (iii), we get

$$\mu F = Mg \quad \text{or} \quad F = \frac{Mg}{\mu}$$

27 Here, $\ R = Mg - F\sin\phi$

$\therefore\qquad f = \mu R = \mu(Mg - F\sin\phi)$



Net force, $\ F\cos\phi - f = Ma$

$\therefore\quad a = \dfrac{1}{M}[F\cos\phi - f]$

$\Rightarrow\quad a = \dfrac{1}{M}[F\cos\phi - \mu\,(Mg - F\sin\phi)]$

$$= \frac{F}{M}\cos\phi - \mu g + \frac{\mu F}{M}\sin\phi$$
$$= \frac{F}{M}(\cos\phi + \mu\sin\phi) - \mu g$$

28 Given, acceleration of the trolley $(a) = 3\ \text{ms}^{-2}$.

Therefore, the force acting on the block is $\quad F = ma = 10 \times 3 = 30\,\text{N}$.

The weight mg of the block is balanced by the normal reaction R. The force of limiting friction is given by

$$\mu = \frac{f}{R} = \frac{f}{mg}$$

$$f = \mu mg = 0.2 \times 10 \times 10 = 20\,\text{N}$$

The net force on the block is towards right and is given by

$$F' = F - f = 30 - 20 = 10\,\text{N}$$

So, $\quad a' = \dfrac{F'}{m} = \dfrac{10}{10} = 1\ \text{ms}^{-2}$.

Let, t be the time taken for the block to fall off from the rear end for the trolley. Then, the block has to travel a distance $s' = 5\,\text{m}$ to fall off. Now, since the trolley starts from rest. So, $u = 0$ and using $s = ut + \dfrac{1}{2}at^2$, we can determine t as $\sqrt{10}$ s.

The distance covered by the trolley in this time,

$$s' = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 3 \times 10 = 15\text{m}$$

29 From Newton's second law,

$$\frac{mg\sin\theta - \mu\ mg\cos\theta}{m} = a$$

Now, distance covered by the particle,

$$v^2 = u^2 + 2as$$

$$\Rightarrow v = \sqrt{2\left(\frac{mg\sin\theta - \mu\ mg\cos\theta}{m}\right)x}$$

$$= \sqrt{2\,gx\sin\theta - 0.6\,x^2\,g\cos\theta}$$

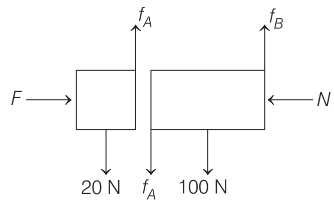
v should be maximum when $\dfrac{dv}{dx} = 0$

$$\Rightarrow \quad \frac{d\sqrt{2gx\sin\theta - 0.6\,x^2\,g\cos\theta)}}{dx} = 0$$

By differentiating, we get $x = 3.33$ m

30 In vertical direction, weights are balanced by frictional forces.

Consider FBD of block A and B as shown in diagram below.



As the blocks are in equilibrium, balance forces are in horizontal and vertical direction.

The system of blocks $(A + B)$

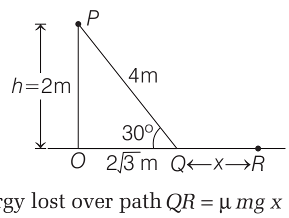
$$F = N$$

For block A, $f_A = 20\,\text{N}$

and for block B,

$$f_B = f_A + 100 = 20 + 100 = 120\ \text{N}$$

31 Energy lost over path $PQ = \mu\ mg\cos\theta \times 4$



Energy lost over path $QR = \mu\,mg\,x$

i.e. $\ \mu\,mg\,\cos 30^\circ \times 4 = \mu\,mg\,x$

$(\because \theta = 30^\circ)$

$$x = 2\sqrt{3} = 3.45\,\text{m}$$

From Q to R energy loss is half of the total energy loss.

i.e. $\ \mu\,mgx = \dfrac{1}{2} \times mgh \ \Rightarrow\ \mu = 0.29$

The values of the coefficient of friction μ and the distance $x(= QR)$ are 0.29 and 3.5.

32 Motion stops when pull due to $m_1 \le$ force of friction between m and m_2 and surface.

$\Rightarrow \quad m_1 g \le \mu(m_2 + m)g$

$\Rightarrow \quad 5 \times 10 \le 0.15(10 + m) \times 10$

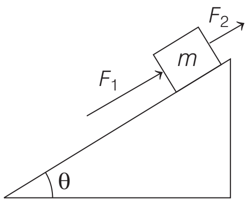
$\Rightarrow \quad m \ge 23.33$ kg

Here, nearest value is 27.3 kg

So, $m_{\min} = 27.3$ kg

33 $F_1 = mg\,(\sin\theta + \mu\cos\theta)$ [as body just in position to move up, friction force downward]

$F_2 = mg(\sin\theta - \mu\cos\theta)$ [as body just in position to slide down, friction upward]



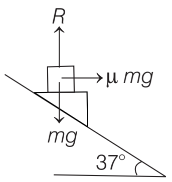
$$\therefore \quad \frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}$$

$$= \frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = 3$$

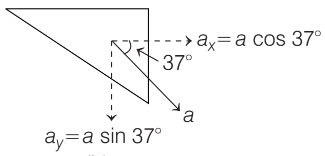
34 Here, $(80 - 72)g = ma_y$

or $\quad a_y = 1\ \text{m/s}^2$

$\therefore a\sin 37° = 1\ \text{m/s}^2$ or $a = \frac{5}{3}\ \text{m/s}^2$



(a)



(b)

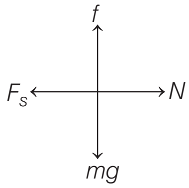
Now, we apply Newton's second law of motion on the box in the direction of acceleration,

$$mg\sin 37° - \mu\, mg\cos 37° = m \times \frac{5}{3}$$

or $\mu = \frac{13}{24}$

Hence, (b) is the correct option.

35 Free body diagram of the block is



Here, $N = F_s = kd$

and $mg = f \le \mu N = \mu kd$

or $k \ge \frac{mg}{\mu d}$

Hence, (b) is the correct option.

36 When a car accelerates, the engine rotates the rear axle which exerts a push on the wheels to move.

37 Both Statements are correct. But Statement II does not explain correctly, Statement I.

Correct explanation is there is increase in normal reaction when the object is pushed and there is decrease in normal reaction when the object is pulled (but strictly, not horizontally).

38 The cloth can be pulled out without dislodging the dishes from the table due to law of inertia, which is Newton's first law. While, Statement II is true, but it is Newton's third law.

39 If the bullet is fired from the rifle, the momentum of bullet-rifle system is conserved.

It means, $\quad M_b v_b = M_r v_r \quad$ ... (i)

and
$$\frac{E_{k(b)}}{E_{k(r)}} = \frac{\frac{1}{2}M_b v_b^2}{\frac{1}{2}M_r v_r^2} = \frac{M_r}{M_b}$$

As, $M_r > M_b$ (mass of rifle is greater than the mass of bullet).

Hence, $E_{k(b)} > E_{k(r)}$. So, the kinetic energy of bullet is greater than the kinetic energy of rifle.

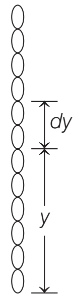
40 In order to apply Newton's second law on a body observed from a non-inertial frame of reference pseudo force is considered in a direction opposite to real acceleration.

SESSION 2

1 Suppose, F = force on the table due to the weight of the chain on the table + momentum of the chain transmitted on the table

$\Rightarrow \quad F = F_1 + F_2$ (Let)

Now, $\quad F_1 = \frac{m}{l}\,yg,$



$$dp = dmv = \frac{m}{l}\,dy\,\sqrt{2gy}$$

$$\frac{dp}{dt} = F_2 = \frac{m}{l}\frac{dy}{dt}\sqrt{2gy}$$

$$= \frac{m}{l}(2gy)\left[\frac{dy}{dt} = \sqrt{2gy}\right]$$

$$\therefore \quad F = \frac{m}{l}\,yg + \frac{2myg}{l} = \frac{3myg}{l}$$

2 Force applying on the block

$F = mg\sin\theta$

or $\quad mg\sin\theta = ma$

$\therefore \quad a = g\sin\theta$

where, a is along the inclined plane.

$\therefore$ Vertical component of acceleration is $g\sin^2\theta$.

$\therefore$ Relative vertical acceleration of A with respect to B is

$g\,(\sin^2 60° - \sin^2 30°]$

$= \frac{g}{2} = 4.9\ \text{ms}^{-2}$

[in vertical direction]

3 Here, initial mass of the rocket $= M$

$\frac{dm}{dt} = r$

Relative velocity of gases w.r.t. rocket $= v$

then, acceleration of the rocket

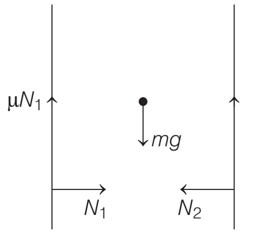
$$a = \frac{F}{m} = \frac{u(dm/dt)}{\left(M - \frac{dm}{dt} \times t\right)} = \frac{ur}{(M - rt)}$$

4 Balanced horizontal force, $N_1 = N_2$

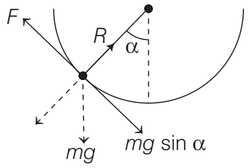
Balanced vertical force, $2\mu N_1 = mg$

$$\mu N_1 = \frac{mg}{2} = \frac{40 \times 10}{2} = 200\ \text{N}$$

$\therefore$ Man pushes the wall with 200 N



5 As, it is clear from the figure



$$F = mg\sin\alpha$$

and $\quad R = mg\cos\alpha$

$\Rightarrow \quad \dfrac{F}{R} = \tan\alpha$

i.e. $\quad \mu = \tan\alpha = \dfrac{1}{3}$

$\Rightarrow \quad \cot\alpha = 3$

6 On smooth inclined plane, the acceleration of body,

$$a = g\sin\theta$$

If s be the distance travelled, then

$$s = \frac{1}{2}g\sin\theta t_1^2 \quad \ldots\text{(i)}$$

On rough inclined plane, the acceleration is given by

$$a = g\sin\theta - \mu g\cos\theta$$

$$\therefore \quad s = \frac{1}{2}(g\sin\theta - \mu g\cos\theta)t_2^2 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{t_2^2}{t_1^2} = \frac{\sin\theta}{g\sin\theta - g\cos\theta}$$

But $\quad t_1 = nt_1 \quad$ or $\quad \dfrac{t_2^2}{t_1^2} = n^2$

$$\therefore \quad n^2 = \frac{\sin\theta}{\sin\theta - \mu\cos\theta}$$

Solving, we get

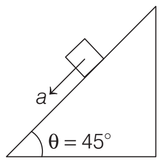
$$\mu = \frac{n^2-1}{n^2} \times \frac{\sin\theta}{\cos\theta}$$

$$= \frac{n^2-1}{n^2} \times \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{n^2-1}{n^2} \times \left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)$$

$$= \frac{n^2-1}{n^2} = 1 - \frac{1}{n^2}$$

$$\mu = 1 - \frac{1}{n^2}$$

7 **On smooth inclined plane** Acceleration of a body sliding down a smooth inclined plane, $a = g\sin\theta$



Here, $\theta = 45^\circ$

$$\therefore \quad a = g\sin 45^\circ = \frac{g}{\sqrt{2}}$$

Let the travelled distance be s.

Using equation of motion,

$$s = ut + \frac{1}{2}at^2, \text{ we get}$$

$$s = 0.t + \frac{1}{2}\frac{g}{\sqrt{2}}T^2 \quad \text{or} \quad s = \frac{gT^2}{2\sqrt{2}} \quad \ldots\text{(i)}$$

On rough inclined plane Acceleration of the body

$$a = g(\sin\theta - \mu\cos\theta)$$

$$= g(\sin 45^\circ - \mu\cos 45^\circ) = \frac{g(1-\mu)}{\sqrt{2}}$$

$$\left(\text{As}, \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}\right)$$

Again using equation of motion

$$s = ut + \frac{1}{2}at^2, \text{ we get}$$

$$s = 0(pT) + \frac{1}{2}\frac{g(1-\mu)}{\sqrt{2}}(pT)^2$$

or $\quad s = \dfrac{g(1-\mu)p^2T^2}{2\sqrt{2}} \quad \ldots\text{(ii)}$

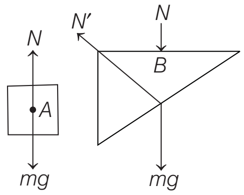
From Eqs. (i) and (ii), we get

$$\frac{gT^2}{2\sqrt{2}} = \frac{g(1-\mu)p^2T^2}{2\sqrt{2}}$$

or $\quad (1-\mu)p^2 = 1$

or $\quad 1-\mu = \dfrac{1}{p^2}$ or $\mu = \left(1 - \dfrac{1}{p^2}\right)$

8 Free body diagram for A and B is



For *A*

$mg - N = m(a\sin\theta) \quad \ldots\text{(i)}$

[as block has only vertically downward acceleration]

For *B*

$(N + mg)\sin\theta = ma \quad \ldots\text{(ii)}$

On solving Eqs. (i) and (ii), we get

$$a = \left(\frac{2g\sin\theta}{1+\sin^2\theta}\right)$$

The acceleration of block A is

$$a_A = a\sin\theta = \frac{2g\sin^2\theta}{1+\sin^2\theta}$$

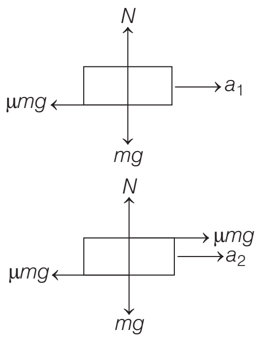
$\therefore$ Displacement of block A in 1 s is

$$s = 0.1 + \frac{1}{2}\left(\frac{2g\sin^2\theta}{1+\sin^2\theta}\right) \times (1)^2$$

or $\quad s = \left[\dfrac{g\sin^2\theta}{1+\sin^2\theta}\right]$

Hence, (a) is the correct option.

9



Velocity of block, just before reaching the carriage is

$$v = \sqrt{2gh}$$

Now, acceleration of block,

$$a_1 = \frac{-\mu mg}{m} = -\mu g$$

Acceleration of carriage,

$$a_2 = \frac{\mu mg}{M}$$

At $t = 0$, motion of block as seen from the carriage is

$$u_{\text{rel}} = v = \sqrt{2gh}$$

and $\quad a_{\text{rel}} = a_1 - a_2 = -\mu g\left(1 + \dfrac{m}{M}\right)$

Now, relative velocity of block when block moves through distance x with respect to carriage,

$$v_{\text{rel}}^2 = u_{\text{rel}}^2 + 2a_{\text{rel}}\, x$$

$$\Rightarrow \quad 0^2 = 2gh - 2\mu g\left(1 + \frac{m}{M}\right)\cdot l$$

(when $x = l$, then $v_{\text{rel}} = 0$)

$$\Rightarrow \quad 2gh = 2\mu g\left(1 + \frac{m}{M}\right)\cdot l$$

$$\Rightarrow \quad h = \mu\left(1 + \frac{m}{M}\right)\cdot l$$

Hence, (c) is the correct option.

10 Initially under the equilibrium of mass m, $T = mg$

Now, the string is cut. Therefore, $T = mg$ force is decreased on the mass m upwards and downwards on the mass $2m$.

$\therefore \quad a_m = \dfrac{mg}{m} = g \quad$ (downwards)

and $\quad a_{2m} = \dfrac{mg}{2m} = \dfrac{g}{2} \quad$ (upwards)

11 This is the equilibrium of coplanar forces.

Hence, $\Sigma F_x = 0$

$\therefore \quad F = N$

$\therefore \quad \Sigma F_y = 0, \; f = mg, \; \Sigma \tau_c = 0$

$\therefore \quad \tau_N + \tau_f = 0$

Since, $\quad \tau_f \neq 0$

$\therefore \quad \tau_N \neq 0$

Thus, N will produce torque.

12 According to the work-energy theorem,

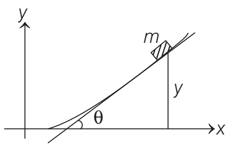
$$\Sigma W = \Delta K = 0$$

$\Rightarrow$ Work done by friction + Work done by gravity = 0

$$\Rightarrow -(\mu \, mg \cos\phi)\frac{l}{2} + mgl \sin\phi = 0$$

or $\frac{\mu}{2}\cos\phi = \sin\phi$ or $\mu = 2\tan\phi$

13 A block of mass m is placed on a surface with a vertical cross-section, then



$$\tan\theta = \frac{dy}{dx} = \frac{d\left(\frac{x^3}{6}\right)}{dx} = \frac{x^2}{2}$$

At limiting equilibrium, we get

$$\mu = \tan\theta$$

$$0.5 = \frac{x^2}{2} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Now, putting the value of x in

$y = \frac{x^3}{6}$, we get

When $x = 1$ $\qquad$ When $x = -1$

$y = \frac{(1)^3}{6} = \frac{1}{6}$ $\qquad$ $y = \frac{(-1)^3}{6} = \frac{-1}{6}$

So, the maximum height above the ground at which the block can be placed without slipping is 1/6 m.

14 As the force is exponentially decreasing, so it's acceleration, i.e. rate of increase of velocity will decrease with time. Thus, the graph of velocity will be an increasing curve with decreasing slope with time.

$$a = \frac{F}{m} = \frac{F_0}{m}e^{-bt} = \frac{dv}{dt}$$

$$\Rightarrow \int_0^v dv = \int_0^t \frac{F_0}{m}e^{-bt}$$

$$\Rightarrow \quad v = \frac{F_0}{m}\left(\frac{1}{-b}\right)e^{-bt}\Bigg|_0^t = \frac{F_0}{mb}e^{-bt}\Bigg|_t^0$$

$$= \frac{F_0}{mb}(e^0 - e^{-bt})$$

$$= \frac{F_0}{mb}(1 - e^{-bt})$$

with $v_{\max} = \frac{F_0}{mb}\,[a + t = \infty]$

15 From the graph, it is a straight line so, motion is uniform because of impulse direction of velocity changes as can be seen from the slope of the graph.

Initial velocity, $v_1 = \frac{2}{2} = 1 \text{ ms}^{-1}$

Final velocity, $v_2 = -2/2 = -1 \text{ ms}^{-1}$

$p_i = mv_1 = 0.4$ N - s

and $\quad p_f = mv_2 = -0.4$ N - s

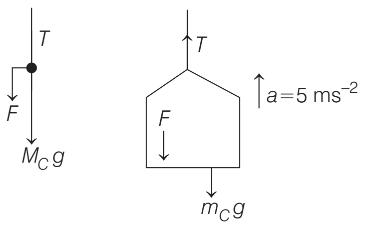
Now, impulse,

$$\mathbf{J} = p_f - p_i = -0.4 - 0.4$$

$$= -0.8 \text{ N - s}$$

$\Rightarrow \quad |\mathbf{J}| = 0.8$ N - s

16 Free body diagram for crate



So, for vertical equilibrium of the crate.

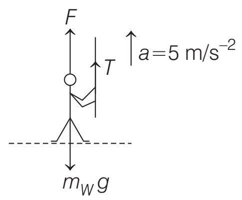
$$T - F - M_c g = M_c a$$

[$\because M_c$ = mass of crate = 50 kg]

$\Rightarrow \quad T - F - 500 = 250$

$\Rightarrow \quad T - F = 750 \qquad \ldots$(i)

Free body diagram for worker



$$T + F - m_w g = m_w a$$

[$\because m_w$ = mass of worker]

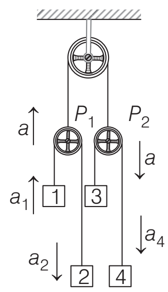
$\Rightarrow \quad T + F - 1000 = 500$

$\Rightarrow \quad T + F = 1500 \qquad \ldots$(ii)

Solving Eqs. (i) and (ii), we get

$T = 1125$ N and $F = 375$ N

17 Let the accelerations of various blocks are as shown in figure. Pulley P_2 will have downward acceleration a.



Analysing the diagram,

$$a = \frac{a_1 + a_2}{2} \Rightarrow a_2 = 2a - a_1 > 0$$

So, acceleration of block 2 is upward

Hence, (A)$\rightarrow$ (2,3)

and $\quad a = \frac{a_1 + a_4}{2}$

$\Rightarrow \quad a_4 = 2a + a_1 > 0$

So, acceleration of block 4 is downward.

Hence, (B)$\rightarrow$ (1,4).

This is downwards.

Hence, (c)$\rightarrow$ (4)

Acceleration of block 2 with respect to block 4.

$a_2/4 = a_2 - (-a_4) = 4\,a > 0$

This is upward.

Hence (D)$\rightarrow$(3) (C)$\rightarrow$ (4)

Hence, A $\rightarrow$ (2,3), B $\rightarrow$ (1,4), C $\rightarrow$ (4), D $\rightarrow$ (3)