

# MATHEMATICS (860)

## Aims:

1. To enable candidates to acquire knowledge and to develop an understanding of the terms, concepts, symbols, definitions, principles, processes and formulae of Mathematics at the Senior Secondary stage.
2. To develop the ability to apply the knowledge and understanding of Mathematics to unfamiliar situations or to new problems.
3. To develop an interest in Mathematics.
4. To enhance ability of analytical and rational thinking in young minds.
5. To develop skills of -
  - (a) Computation.
  - (b) Logical thinking.
  - (c) Handling abstractions.
  - (d) Generalizing patterns.
  - (e) Solving problems using multiple methods.
  - (f) Reading tables, charts, graphs, etc.
6. To develop an appreciation of the role of Mathematics in day-to-day life.
7. To develop a scientific attitude through the study of Mathematics.

A knowledge of Arithmetic, Basic Algebra (Formulae, Factorization etc.), Basic Trigonometry and Pure Geometry is assumed.

As regards to the standard of algebraic manipulation, students should be taught:

- (i) To check every step before proceeding to the next particularly where minus signs are involved.
- (ii) To attack simplification piecemeal rather than en block.
- (iii) To observe and act on any special features of algebraic form that may be obviously present.

## CLASS XI

The syllabus is divided into **three** sections A, B and C.

Section A is compulsory for all candidates. Candidates will have a choice of attempting questions from **EITHER** Section B **OR** Section C.

There will be one paper of **three** hours duration of 100 marks.

**Section A (80 Marks):** Candidates will be required to attempt **all** questions. Internal choice will be provided in three questions of four marks each and two questions of six marks each.

**Section B/ Section C (20 Marks):** Candidates will be required to attempt **all** questions **EITHER** from Section B or Section C. Internal choice will be provided in two questions of four marks each.

S.No.	UNIT	TOTAL WEIGHTAGE
SECTION A: 80 Marks		
1.	Sets and Functions	22 Marks
2.	Algebra	34 Marks
3.	Coordinate Geometry	8 Marks
4.	Calculus	8 Marks
5.	Statistics & Probability	8 Marks
SECTION B: 20 marks		
6.	Conic Section	12 Marks
7.	Introduction to Three Dimensional Geometry	4 Marks
8.	Mathematical Reasoning	4 Marks
OR		
SECTION C: 20 Marks		
9.	Statistics	6 Marks
10.	Correlation Analysis	6 Marks
11.	Index Numbers & Moving Averages	8 Marks
	TOTAL	100 Marks

## SECTION A

### 1. Sets and Functions

#### (i) Sets

Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of a set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Practical problems on union and intersection of two and three sets. Difference of sets. Complement of a set. Properties of Complement of Sets.

#### (ii) Relations & Functions

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto  $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$ ). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Function as a type of mapping, types of functions (one to one, many to one, onto, into) domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotient of functions.

- **Sets:** Self-explanatory.
- **Basic concepts of Relations and Functions**
  - Ordered pairs, sets of ordered pairs.
  - Cartesian Product (Cross) of two sets, cardinal number of a cross product.
  - Relations as:
    - an association between two sets.
    - a subset of a Cross Product.
  - Domain, Range and Co-domain of a Relation.
  - Functions:
    - As special relations, concept of writing “y is a function of x” as  $y = f(x)$ .
  - Introduction of Types: one to one, many to one, into, onto.

- Domain and range of a function.
- Sketches of graphs of exponential function, logarithmic function, modulus function, step function and rational function.

#### (iii) Trigonometry

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity  $\sin^2 x + \cos^2 x = 1$ , for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing  $\sin(x \pm y)$  and  $\cos(x \pm y)$  in terms of  $\sin x$ ,  $\sin y$ ,  $\cos x$  &  $\cos y$  and their simple applications. Deducing the identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y},$$

$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

Identities related to  $\sin 2x$ ,  $\cos 2x$ ,  $\tan 2x$ ,  $\sin 3x$ ,  $\cos 3x$  and  $\tan 3x$ . General solution of trigonometric equations of the type  $\sin y = \sin a$ ,  $\cos y = \cos a$  and  $\tan y = \tan a$ . Properties of triangles (proof and simple applications of sine rule cosine rule and area of triangle).

#### • Angles and Arc lengths

- Angles: Convention of sign of angles.
- Magnitude of an angle: Measures of Angles; Circular measure.
- The relation  $S = r\theta$  where  $\theta$  is in radians. Relation between radians and degree.
- Definition of trigonometric functions with the help of unit circle.

- Truth of the identity  $\sin^2 x + \cos^2 x = 1$

**NOTE:** Questions on the area of a sector of a circle are required to be covered.

- **Trigonometric Functions**

- Relationship between trigonometric functions.
- Proving simple identities.
- Signs of trigonometric functions.
- Domain and range of the trigonometric functions.
- Trigonometric functions of all angles.
- Periods of trigonometric functions.
- Graphs of simple trigonometric functions (only sketches).

**NOTE:** Graphs of  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $\sec x$ ,  $\operatorname{cosec} x$  and  $\cot x$  are to be included.

- **Compound and multiple angles**

- Addition and subtraction formula:  $\sin(A \pm B)$ ;  $\cos(A \pm B)$ ;  $\tan(A \pm B)$ ;  $\tan(A + B + C)$  etc., Double angle, triple angle, half angle and one third angle formula as special cases.
- Sum and differences as products  

$$\sin C + \sin D = 2 \sin \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right), \text{ etc.}$$
- Product to sum or difference i.e.  $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$  etc.

**Trigonometric Equations**

- Solution of trigonometric equations (General solution and solution in the specified range).
- Equations expressible in terms of  $\sin \theta = 0$  etc.
- Equations expressible in terms i.e.  $\sin \theta = \sin \alpha$  etc.
- Equations expressible multiple and sub-multiple angles i.e.  $\sin^2 \theta = \sin^2 \alpha$  etc.

- Linear equations of the form  $a \cos \theta + b \sin \theta = c$ , where  $|c| \leq \sqrt{a^2 + b^2}$  and  $a, b \neq 0$

- Properties of  $\Delta$

$$\text{Sine formula: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C};$$

Cosine formula:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \text{ etc}$$

$$\text{Area of triangle: } \Delta = \frac{1}{2} bc \sin A, \text{ etc}$$

Simple applications of the above.

## 2. Algebra

### (i) Principle of Mathematical Induction

Process of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.

Using induction to prove various summations, divisibility and inequalities of algebraic expressions only.

### (ii) Complex Numbers

Introduction of complex numbers and their representation, Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Square root of a complex number. Cube root of unity.

- Conjugate, modulus and argument of complex numbers and their properties.
- Sum, difference, product and quotient of two complex numbers additive and multiplicative inverse of a complex number.
- Locus questions on complex numbers.
- Triangle inequality.
- Square root of a complex number.
- Cube roots of unity and their properties.

### (iii) Quadratic Equations

Statement of Fundamental Theorem of Algebra, solution of quadratic equations (with real coefficients).

- Use of the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In solving quadratic equations.

- Equations reducible to quadratic form.
- Nature of roots
  - Product and sum of roots.
  - Roots are rational, irrational, equal, reciprocal, one square of the other.
  - Complex roots.
  - Framing quadratic equations with given roots.

**NOTE:** Questions on equations having common roots are to be covered.

- **Quadratic Functions.**

Given  $\alpha, \beta$  as roots then find the equation whose roots are of the form  $\alpha^3, \beta^3$ , etc.

Case I:  $a > 0$   $\left\{ \begin{array}{l} \rightarrow \text{Real roots} \\ \rightarrow \text{Complex roots} \\ \rightarrow \text{Equal roots} \end{array} \right.$

Case II:  $a < 0$   $\left\{ \begin{array}{l} \rightarrow \text{Real roots} \\ \rightarrow \text{Complex roots,} \\ \rightarrow \text{Equal roots} \end{array} \right.$

Where 'a' is the coefficient of  $x^2$  in the equations of the form  $ax^2 + bx + c = 0$ .

Understanding the fact that a quadratic expression (when plotted on a graph) is a parabola.

- **Sign of quadratic**

Sign when the roots are real and when they are complex.

- **Inequalities**

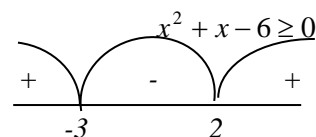
- Linear Inequalities

Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical representation of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables.

*Self-explanatory.*

- Quadratic Inequalities

Using method of intervals for solving problems of the type:



A perfect square e.g.  $x^2 - 6x + 9 \geq 0$ .

- Inequalities involving rational expression of type

$$\frac{f(x)}{g(x)} \leq a \text{ etc. to be covered.}$$

### (iv) Permutations and Combinations

Fundamental principle of counting. Factorial n. (n!) Permutations and combinations, derivation of formulae for  ${}^n P_r$  and  ${}^n C_r$  and their connections, simple application.

- Factorial notation  $n!$ ,  $n! = n(n-1)!$
- Fundamental principle of counting.
- Permutations
  - ${}^n P_r$ .
  - Restricted permutation.
  - Certain things always occur together.
  - Certain things never occur.
  - Formation of numbers with digits.
  - Word building - repeated letters - No letters repeated.
  - Permutation of alike things.
  - Permutation of Repeated things.

- Circular permutation – clockwise counterclockwise – Distinguishable / not distinguishable.
- **Combinations**
  - ${}^nC_r, {}^nC_n = 1, {}^nC_0 = 1, {}^nC_r = {}^nC_{n-r}, {}^nC_x = {}^nC_y$ , then  $x + y = n$  or  $x = y$ ,  ${}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$ .
  - When all things are different.
  - When all things are not different.
  - Mixed problems on permutation and combinations.

#### (v) Binomial Theorem

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications.

- *Significance of Pascal's triangle.*
- *Binomial theorem (proof using induction) for positive integral powers,*

$$\text{i.e. } (x + y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + \dots + {}^nC_n y^n.$$

*Questions based on the above.*

#### (vi) Sequence and Series

Sequence and Series. Arithmetic Progression (A. P.). Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of first  $n$  terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M. Formulae for the following special sums  $\sum n, \sum n^2, \sum n^3$ .

- **Arithmetic Progression (A.P.)**
  - $T_n = a + (n - 1)d$
  - $S_n = \frac{n}{2} \{2a + (n - 1)d\}$
  - Arithmetic mean:  $2b = a + c$
  - Inserting two or more arithmetic means between any two numbers.
  - Three terms in A.P. :  $a - d, a, a + d$
  - Four terms in A.P.:  $a - 3d, a - d, a + d, a + 3d$

- **Geometric Progression (G.P.)**

$$T_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r - 1},$$

$$S_\infty = \frac{a}{1 - r}; |r| < 1 \quad \text{Geometric}$$

$$\text{Mean, } b = \sqrt{ac}$$

- Inserting two or more Geometric Means between any two numbers.
- Three terms are in G.P.  $ar, a, ar^{-1}$
- Four terms are in GP  $ar^3, ar, ar^{-1}, ar^{-3}$
- **Arithmetico Geometric Series**  
Identifying series as A.G.P. (when we substitute  $d = 0$  in the series, we get a G.P. and when we substitute  $r = 1$  the A.P).
- **Special sums**  $\sum n, \sum n^2, \sum n^3$   
Using these summations to sum up other related expression.

### 3. Coordinate Geometry

#### (i) Straight Lines

Brief recall of two dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point-slope form, slope- intercept form, two-point form, intercept form and normal form. General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.

- *Basic concepts of Points and their coordinates.*
- *The straight line*
  - Slope or gradient of a line.
  - Angle between two lines.
  - Condition of perpendicularity and parallelism.
  - Various forms of equation of lines.
  - Slope intercept form.
  - Two-point slope form.
  - Intercept form.
  - Perpendicular /normal form.
  - General equation of a line.

- Distance of a point from a line.
- Distance between parallel lines.
- Equation of lines bisecting the angle between two lines.
- Equation of family of lines
- Definition of a locus.
- Equation of a locus.

#### (ii) Circles

- Equations of a circle in:
  - Standard form.
  - Diameter form.
  - General form.
  - Parametric form.
- Given the equation of a circle, to find the centre and the radius.
- Finding the equation of a circle.
  - Given three non collinear points.
  - Given other sufficient data for example centre is  $(h, k)$  and it lies on a line and two points on the circle are given, etc.
- Tangents:
  - Condition for tangency
  - Equation of a tangent to a circle

### 4. Calculus

#### (i) Limits and Derivatives

Derivative introduced as rate of change both as that of distance function and geometrically.

Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Definition of derivative relate it to slope of tangent of the curve, Derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

- Limits
  - Notion and meaning of limits.
  - Fundamental theorems on limits (statement only).
  - Limits of algebraic and trigonometric functions.

- Limits involving exponential and logarithmic functions.

**NOTE:** Indeterminate forms are to be introduced while calculating limits.

#### • Differentiation

- Meaning and geometrical interpretation of derivative.
- Derivatives of simple algebraic and trigonometric functions and their formulae.
- Differentiation using first principles.
- Derivatives of sum/difference.
- Derivatives of product of functions.
- Derivatives of quotients of functions.

### 5. Statistics and Probability

#### (i) Statistics

Measures of dispersion: range, mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances.

- Mean deviation about mean and median.
- Standard deviation - by direct method, short cut method and step deviation method.

**NOTE:** Mean, Median and Mode of grouped and ungrouped data are required to be covered.

#### (ii) Probability

Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, 'not', 'and' and 'or' events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories studied in earlier classes. Probability of an event, probability of 'not', 'and' and 'or' events.

- Random experiments and their outcomes.
- Events: sure events, impossible events, mutually exclusive and exhaustive events.
  - Definition of probability of an event
  - Laws of probability addition theorem.

## SECTION B

### 6. Conic Section

Sections of a cone, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola.

- *Conics as a section of a cone.*

- *Definition of Foci, Directrix, Latus Rectum.*
- *$PS = ePL$  where  $P$  is a point on the conics,  $S$  is the focus,  $PL$  is the perpendicular distance of the point from the directrix.*

- (i) *Parabola*

$$e = 1, y^2 = \pm 4ax, x^2 = 4ay, y^2 = -4ax,$$

$$x^2 = -4ay, (y - \beta)^2 = \pm 4a(x - \alpha),$$

$$(x - \alpha)^2 = \pm 4a(y - \beta).$$

- *Rough sketch of the above.*
- *The latus rectum; quadrants they lie in; coordinates of focus and vertex; and equations of directrix and the axis.*
- *Finding equation of Parabola when Foci and directrix are given, etc.*
- *Application questions based on the above.*

- (ii) *Ellipse*

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, e < 1, b^2 = a^2(1 - e^2)$$

$$\frac{(x - \alpha)^2}{a^2} + \frac{(y - \beta)^2}{b^2} = 1$$

- *Cases when  $a > b$  and  $a < b$ .*
- *Rough sketch of the above.*
- *Major axis, minor axis; latus rectum; coordinates of vertices, focus and centre; and equations of directrices and the axes.*
- *Finding equation of ellipse when focus and directrix are given.*
- *Simple and direct questions based on the above.*
- *Focal property i.e.  $SP + SP' = 2a$ .*

- (iii) *Hyperbola*

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, e > 1, b^2 = a^2(e^2 - 1)$$

$$\frac{(x - \alpha)^2}{a^2} - \frac{(y - \beta)^2}{b^2} = 1$$

- *Cases when coefficient  $y^2$  is negative and coefficient of  $x^2$  is negative.*
- *Rough sketch of the above.*
- *Focal property i.e.  $SP - S'P = 2a$ .*
- *Transverse and Conjugate axes; Latus rectum; coordinates of vertices, foci and centre; and equations of the directrices and the axes.*

- *General second degree equation*  

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

- *Case 1: pair of straight line if  $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ,*
- *Case 2:  $abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$ , then represents a parabola if  $h^2 = ab$ , ellipse if  $h^2 < ab$ , and hyperbola if  $h^2 > ab$ .*
- *Condition that  $y = mx + c$  is a tangent to the conics, general equation of tangents, point of contact and locus problems.*

### 7. Introduction to three-dimensional Geometry

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

- *As an extension of 2-D*
- *Distance formula.*
- *Section and midpoint form*

### 8. Mathematical Reasoning

Mathematically acceptable statements. Connecting words/ phrases - consolidating the understanding of "if and only if (necessary and sufficient) condition", "implies", "and/or", "implied by", "and", "or", "there exists" and their use through variety of examples related to the Mathematics and real life. Validating the statements involving the connecting words, Difference between contradiction, converse and contrapositive.

*Self-explanatory.*

## SECTION C

### 9. Statistics

- Combined mean and standard deviation.
- The Median, Quartiles, Deciles, Percentiles and Mode of grouped and ungrouped data.

### 10. Correlation Analysis

- Definition and meaning of covariance.
- Coefficient of Correlation by Karl Pearson.  
If  $x - \bar{x}$ ,  $y - \bar{y}$  are small non-fractional numbers, we use

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2} \sqrt{\sum (y - \bar{y})^2}}$$

If  $x$  and  $y$  are small numbers, we use

$$r = \frac{\sum xy - \frac{1}{N} \sum x \sum y}{\sqrt{\sum x^2 - \frac{1}{N} (\sum x)^2} \sqrt{\sum y^2 - \frac{1}{N} (\sum y)^2}}$$

Otherwise, we use assumed means

$A$  and  $B$ , where  $u = x - A$ ,  $v = y - B$

$$r = \frac{\sum uv - \frac{1}{N} (\sum u)(\sum v)}{\sqrt{\sum u^2 - \frac{1}{N} (\sum u)^2} \sqrt{\sum v^2 - \frac{1}{N} (\sum v)^2}}$$

- Rank correlation by Spearman's (Correction included).

### 11. Index Numbers and Moving Averages

#### (i) Index Numbers

- Price index or price relative.
- Simple aggregate method.
- Weighted aggregate method.
- Simple average of price relatives.
- Weighted average of price relatives (cost of living index, consumer price index).

#### (ii) Moving Averages

- Meaning and purpose of the moving averages.
- Calculation of moving averages with the given periodicity and plotting them on a graph.
- If the period is even, then the centered moving average is to be found out and plotted.