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Transfer Function

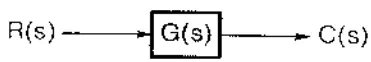
Transfer function

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input, under the assumption that all initial conditions are zero.

Note:

- A system is said to be linear if principle of superposition and principle of homogeneity applies.
- A differential equation is linear if the coefficients are constants or function only of independent variable.

Open Loop Transfer Function



- Transfer function (T.F.)

$$\text{T.F.} = \frac{C(s)}{R(s)} = G(s)$$

where, C(s) = Laplace transform of output

R(s) = Laplace transform of input

- **Impulse response of a system**

$$g(t) = c(t)$$

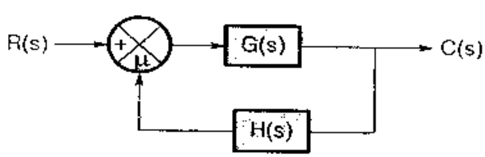
where, g(t) = Impulse response

c(t) = Output to the system

Note:

The transfer function of a system is the Laplace transform of its impulse response.

Closed Loop Transfer Function

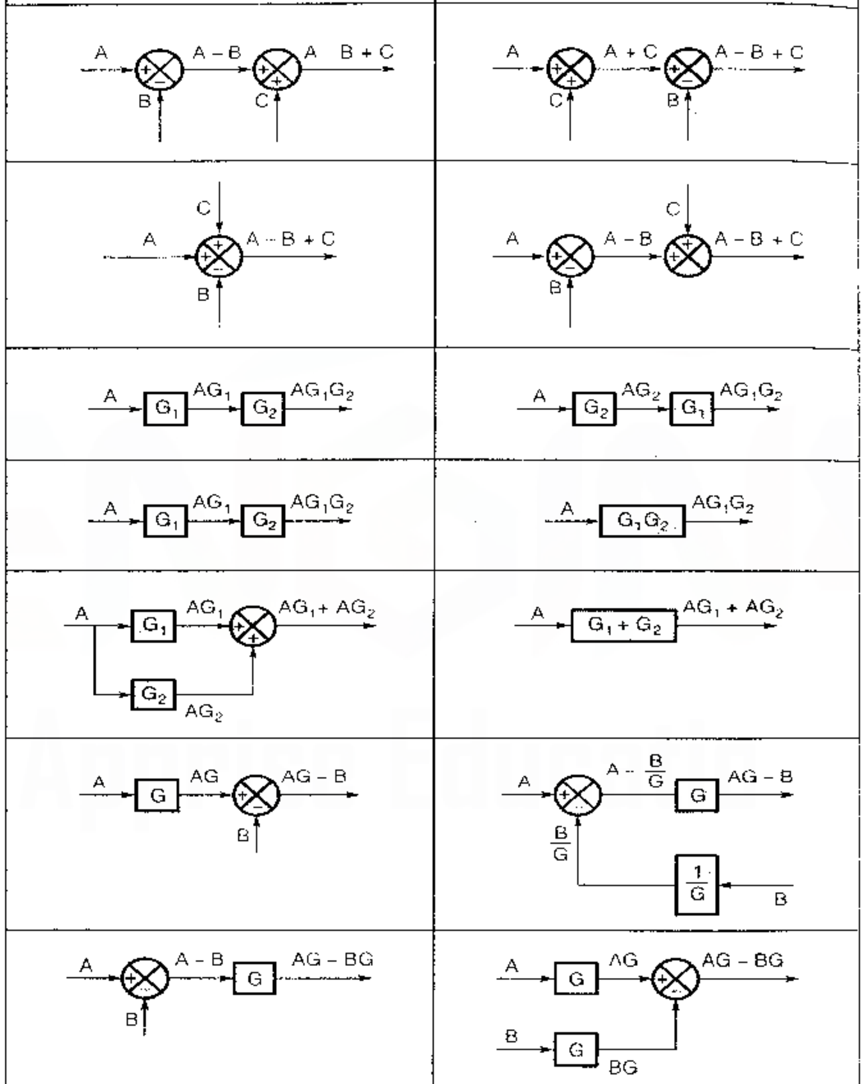


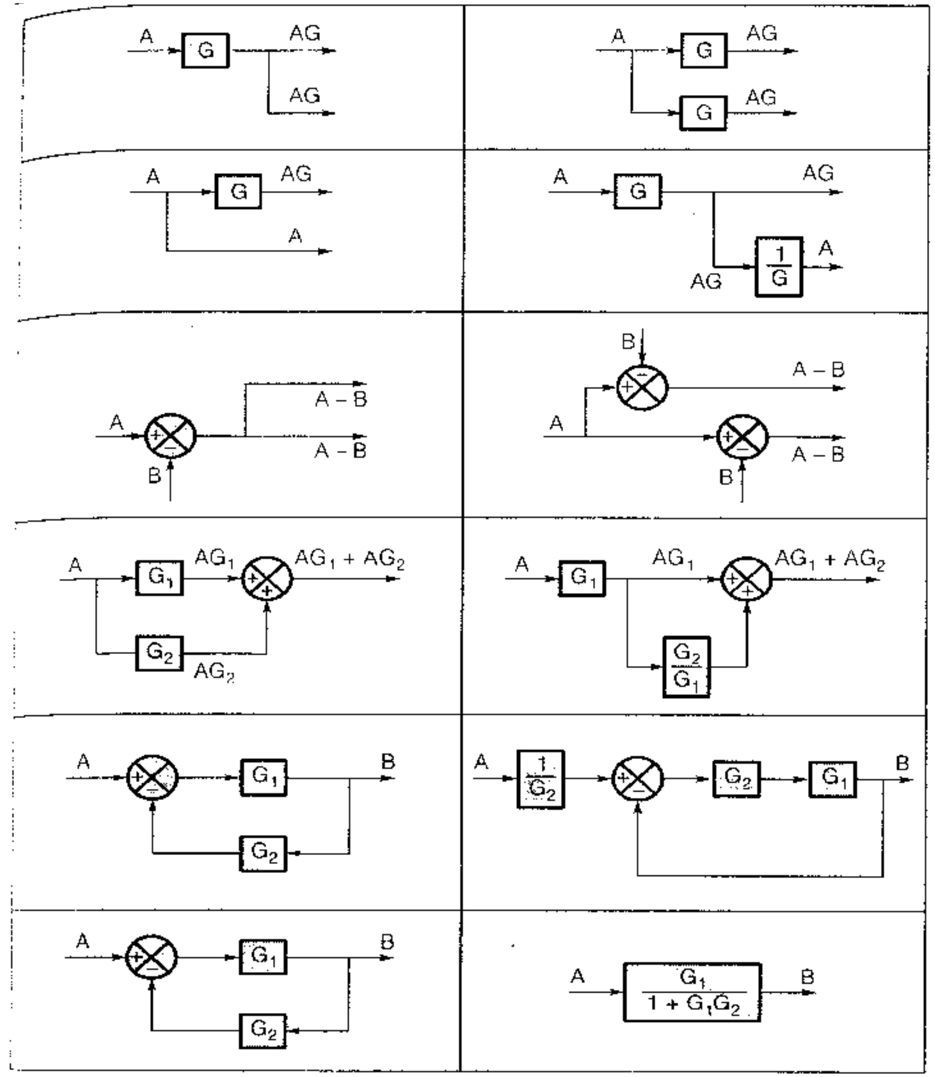
❑ Transfer function

$$T.F. = \frac{G(s)}{1 \pm G(s)H(s)}$$

Block Diagram Reduction

Equivalent Block Diagram	Original Block Diagram

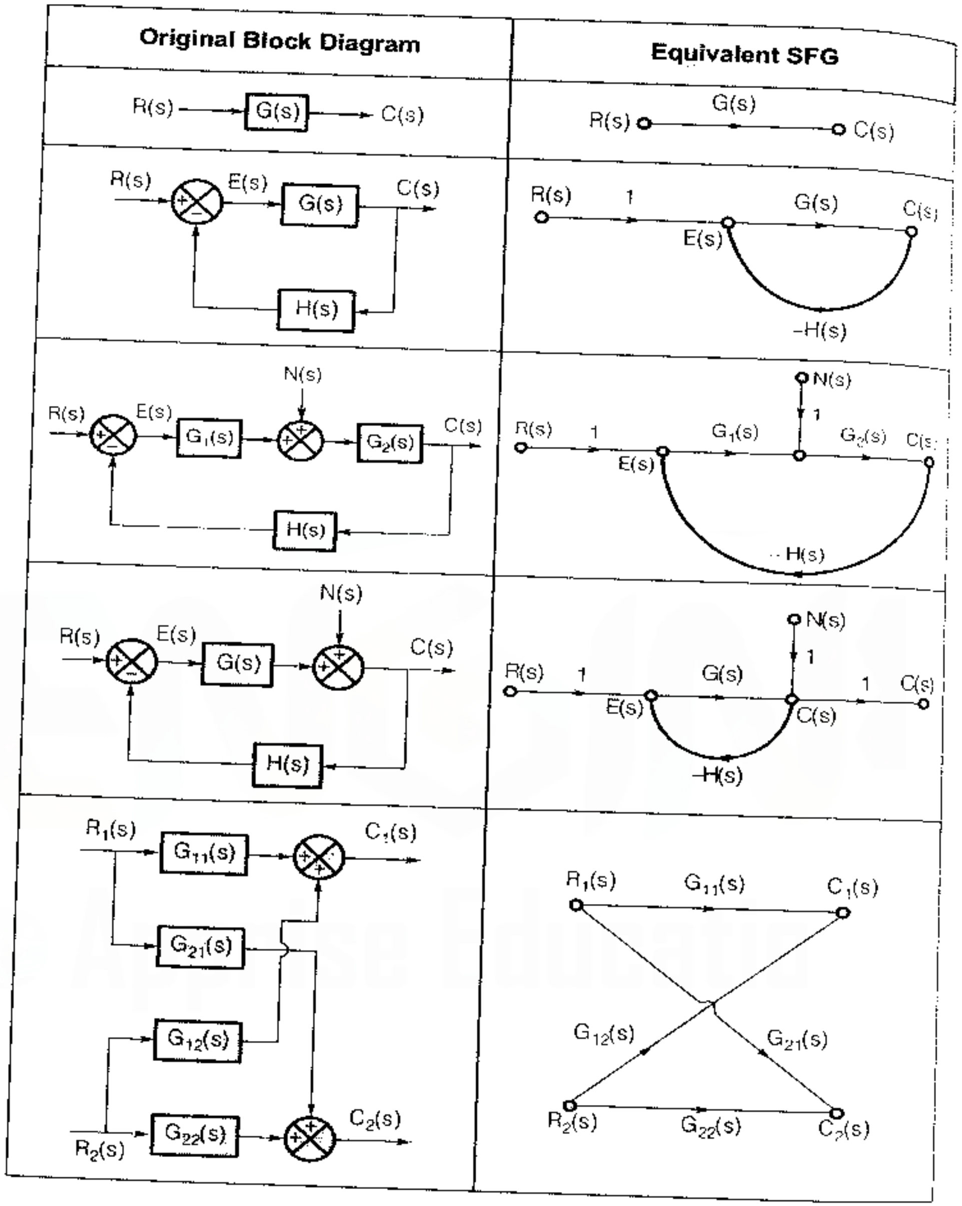




Rules for Drawing Signal Flow Graph From A Given Block Diagram

- While drawing SFG from a given block diagram, the adjacent summing points and take-off points (but not a take-off point preceding a summing point in the direction of SFG) are represented by a node and the block transfer function is represented by a line joining the respective nodes. The direction of signal flow is indicated by an arrow on the line.

2. If in the direction of signal flow, a take-off point precedes a summing point then such points are represented by two separate nodes with a transmittance of unity between them.



Mason's gain formula

$$P = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

where, P = Overall gain or transfer function

P_k = Path gain of k^{th} forward path

Δ = 1 – (sum of loop gains of all individual loops) + (sum of gain products of all possible combinations of two non-touching loops) – (sum of gain products of all possible combinations of three non-touching loops) + ...

Δ_k = Value of Δ obtained by removing all the loops touching k^{th} forward path

Note:

- Forward path is a path that can be traced through the graph from the input node to the output node without touching a node twice.
- Path gain is the product of all branch gain in a path.
- The individual loop is a closed path starting and ending at the same node.
- Loop gain is the product of all branch gains in a loop.
- Non touching loops are loops which do not have a common node.

Sensitivity

$$\text{Sensitivity} = \frac{\%\ \text{change in A}}{\%\ \text{change in K}}$$

$$S_K^A = \frac{\partial A / A}{\partial K / K} = \frac{\partial A}{\partial K} \times \frac{K}{A}$$

where, S_K^A = Sensitivity of variable A with respect to parameter K

Note:

It is preferable that the sensitivity function S_K^A should be minimum.

1. **Sensitivity of Overall Transfer Function M(s) With Respect to Forward Path Transfer Function G(s)**

- For open loop control system

$$S_G^M = \frac{G(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial G(s)} = 1$$

- For closed loop control system

$$S_G^M = \frac{1}{1 + G(s)H(s)}$$

2. **Sensitivity of Overall Transfer Function M(s) With Respect to Feedback Path Transfer Function H(s)**

$$S_H^M = -\frac{G(s)H(s)}{1 + G(s)H(s)}$$

Note:

Closed loop system is lesser sensitive to parameter variation. Hence, closed loop system is better.

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