Square root of Complex Number

Q.1. Find the square root of 5 – 12i.

Solution : 1

Let $\sqrt{(5 - 12i)} = x + iy$

Squaring both sides, we get

$5 - 12i = x^2 + 2ixy + (iy)^2 = x^2 - y^2 + 2xyi.$

Equating real and imaginary parts , we get

$5 = x^2 - y^2$ ----------- (1) & $xy = -6$ ------------ (2)

Squaring (1), we get

$25 = (x^2 - y^2)^2 = (x^2 + y^2)^2 - 4x^2y^2$

Or, $25 = (x^2 + y^2)^2 - 4(-6)^2$

Or, $(x^2 + y^2)^2 = 169$

Or, $x^2 + y^2 = 13$ ---------- (3)

Adding (1) and (3) we get

$2x^2 = 18 \Rightarrow x = \pm 3.$

Subtracting (1) from (3) we get

$2y^2 = 8 \Rightarrow y = \pm 2.$

Therefore, $\sqrt{(5 - 12i)} = 3 + 2i$ or, $3 - 2i$ or, $-3 + 2i$ or, $-3 - 2i$.

As imaginary part of 5 – 12i is negative, the square root is $\pm(3 - 2i)$.

Q.2. Find the square root of : $-15 - 8i$.

Solution : 2

Let $\sqrt{(-15 - 8i)} = x + iy$.

Squaring both sides, we get

$-15 - 8i = (x + iy)^2 = x^2 - y^2 + 2ixy$

Equating real and imaginary parts, we get

$-15 = x^2 - y^2$ -------------------------------- (1)

And $-4 = xy$ -------------------------------- (2)

Squaring (1), we get

$225 = (x^2 - y^2)^2 = (x^2 + y^2)^2 - 4x^2y^2$

Or, $225 = (x^2 + y^2)^2 - 4 (-4)^2$

Or, (x2 + y2)2 = 289

Or, $x^2 + y^2 = 17$ ------------------------------- (3)

Adding (1) and (3) , we get

$2x^2 = 2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$.

Subtracting (1) from (3), we get

$2y^2 = 32 \Rightarrow y^2 = 16 \Rightarrow y \pm 4$.

Therefore, $\sqrt{(-15 - 8i)} = 1 + 4i$ or, $1 - 4i$, $-1 + 4i$, $-1 - 4i$.

As imaginary part of $-15 - 8i$ is negative, the square root is $\pm (1 - 4i)$.

Q.3. Find the square root of i.

Solution : 3

Let $\sqrt{i} = x + iy$.

Squaring, we get

$i = (x + iy)^2 = x^2 - y^2 + 2xyi$

Equating real and imaginary parts, we get

$x^2 - y^2 = 0$ ---------------------------- (1)

$2xy = 1$ ---------------------------- (2)

Squaring (1), we get

$(x^2 - y^2)^2 = 0$

Or, $(x^2 + y^2)^2 - 4x^2y^2 = 0$

Or, $(x^2 + y^2)^2 = 4x^2y^2 = (2xy)^2 = (1)^2 = 1$

Or, $x^2 + y^2 = 1$ --------------------------------- (3)

Solving (1) and (3) we get $x = \pm 1/\sqrt{2}$ and $y = \pm 1/\sqrt{2}$.

As imaginary part of i is positive, the square root is $\pm (1/\sqrt{2} + 1/\sqrt{2})$

$= \pm 1/\sqrt{2} (1 + i)$.