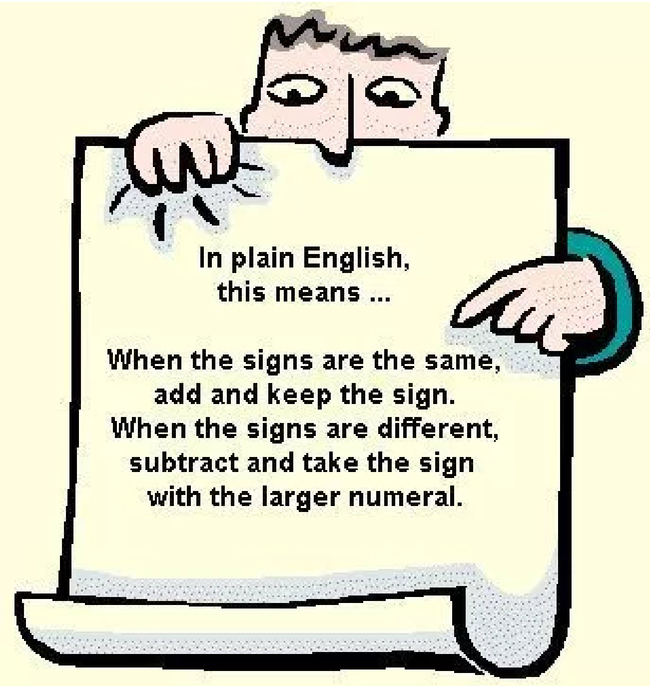
3. Simplifying

Adding Signed Numbers

Rules for Adding:

1. When you add two numbers with the **same signs,**
    1. add the absolute values, and
    2. write the sum (the answer) with the sign of the numbers.
2. When you add two numbers with **different signs,**
    1. subtract the absolute values, and
    2. write the difference (the answer) with the sign of the number having the larger absolute value.



Examples: Add :

1. $-9 + (-7) = \mathbf{-16}$

2. $-20 + 15 = \mathbf{-5}$

3. $(-23) + (-7) = \mathbf{-30}$

4. $(+3) + (+5) = \mathbf{8}$

5. $6 + (-2) = \mathbf{4}$

6. $(-21) + 21 = \mathbf{0}$

7. $-3 + 8 = \mathbf{5}$

8. $-9 + 6 = \mathbf{-3}$

9. Add -9 and -5. Answer: **-14**

10. Add (-7), (+3), and (-12). Answer: **-16**

Multiplying and Dividing Signed Numbers

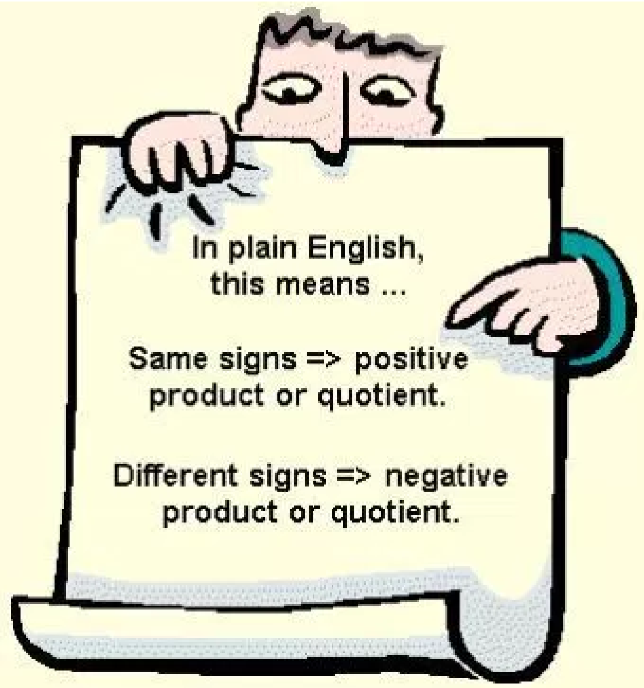
Rules for Multiplying/Dividing:

Multiplying

1. The product of two numbers with the same signs is positive.
2. The product of two numbers with different signs is negative.

Dividing

1. The quotient of two numbers with the same sign is positive.
2. The quotient of two numbers with different signs is negative.



Examples:

Perform the indicated operations:

1. $(+3) \bullet (-5) = -15$

2. $(-8) \bullet (7) = -56$

3. $(-6) \bullet (-5) = 30$

4. $(+4)(+3) = 12$

5. $-12 \div (-3) = 4$

6. $-27 \div 3 = -9$

7. $-6 \div (-8) = 0.75$

8. $-18 / 2 = -9$

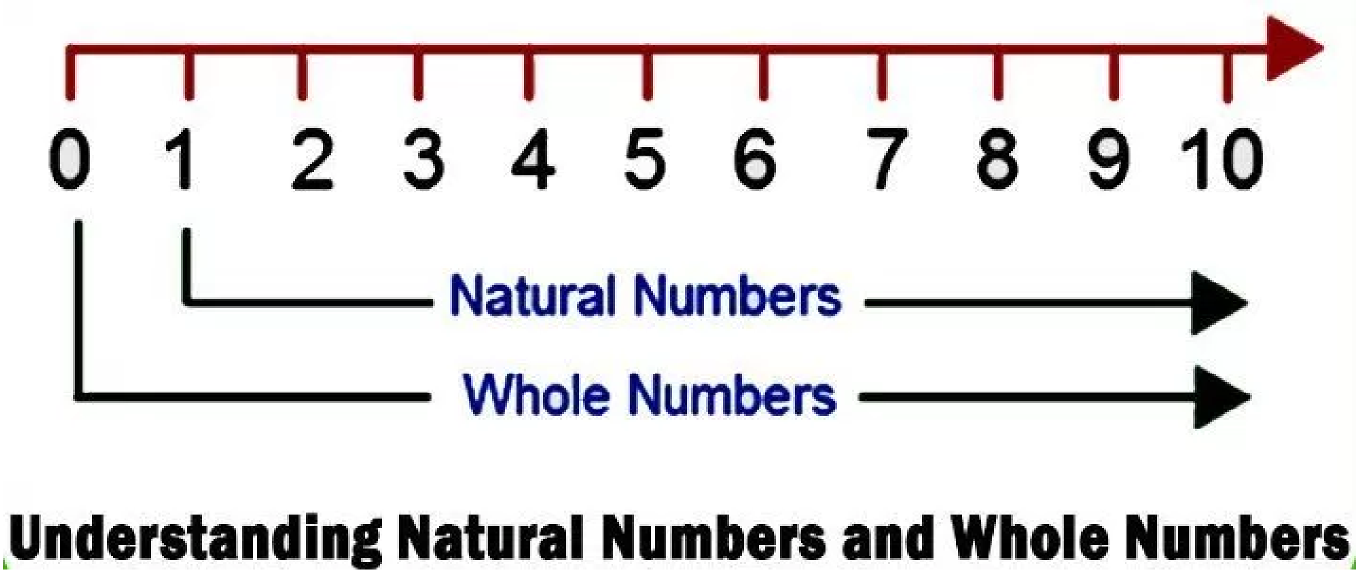
9. Multiply (-5) by (-3).
$(-5) \times (-3) = +15$

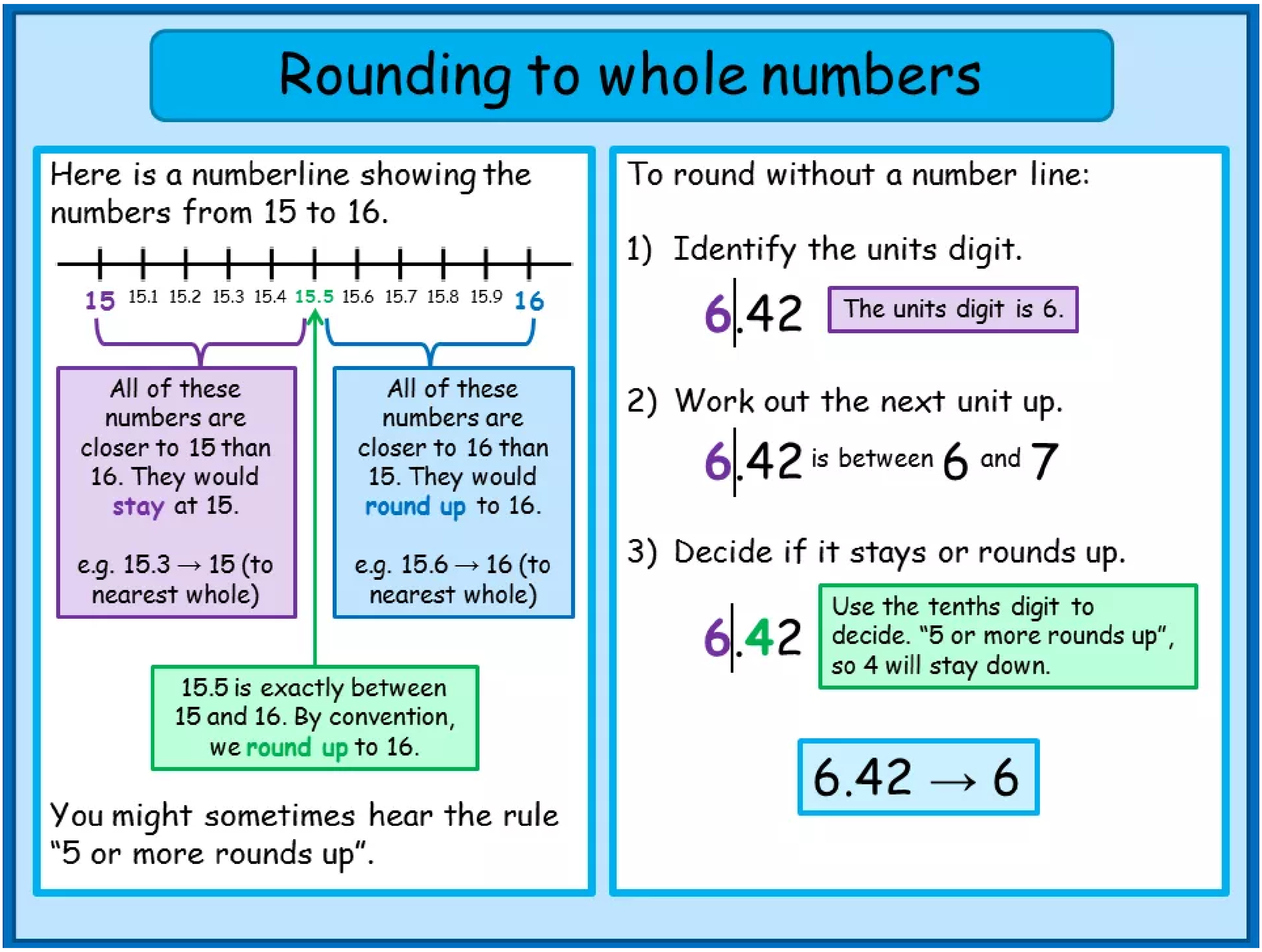
10. Divide (-24) by (+6).
$(-24) / (+6) = -4$

Whole Numbers And Its Properties

WHOLE NUMBERS

Now if we add zero (0) in the set of natural numbers, we get a new set of numbers called the **whole numbers**. Hence the set of whole numbers consists of zero and the set of natural numbers. It is denoted by W. i.e., W = {0, 1, 2, 3, . . .}. Smallest whole number is zero.





Properties of whole numbers

All the properties of numbers satisfied by natural numbers are also satisfied by whole numbers. Now we shall learn some fundamental properties of numbers satisfied by whole numbers.

Properties of Addition

(a) Closure Property: The sum of two whole numbers is always a whole number. Let a and b be two whole numbers, then a + b = c is also a whole number.
This property is called the closure property of addition

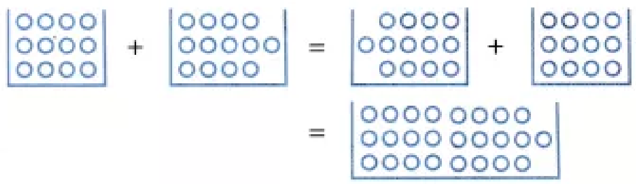
Example: 1 + 5 = 6 is a whole number.

$3 + 7 = 10$ is a whole number.

(b) Commutative Property: The sum of two whole numbers remains the same if the order of numbers is changed. Let a and b be two whole numbers, then a + b = b + a
This property is called the commutative property of addition.

Examples: $12 + 13 = 13 + 12$

$$(25) = (25)$$



$$0 + 8 = 8 + 0$$

$$(8) = (8)$$

the sum remains same

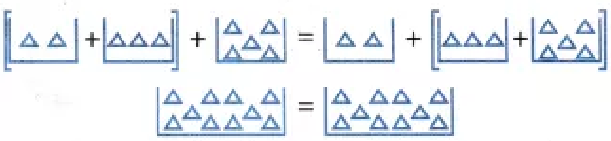
(c) Associative Property: The sum of three whole numbers remains the same even if the grouping is changed. Let a, b, and c be three whole numbers, then (a + b) + c = a + (b + c)

This property is called the associative property of addition.

Examples: $(2+ 3) + 5 = 2 + (3 +5)$

$$5 + 5 = 2 + 8$$

$$10 = 10$$



$$12 + (13 + 7) = (12 + 13) + 7$$

$$12 + 20 = 25 + 7$$

$$32 = 32$$

(d) Identity Element: If zero is added to any whole number, the sum remains the number itself. As we can see that 0+a=a=a+0 where a is a whole number.

Examples: $0 + 3 = 3 = 3 + 0$

$$0 + 312 = 312 = 312 + 0$$

$$0 + 27 = 27 = 27 + 0$$

Therefore, the number zero is called the additive identity, as it does not change the value of the number when addition is performed on the number.

Properties of Subtraction

(a) Closure Property: The difference of two whole numbers will not always be a whole number. Let a and b be two whole numbers, then a – b will be a whole number if a > b or a = b. If a < b, then the result will not be a whole number. Hence, closure property does not hold good for subtraction of whole numbers.

Examples

17 – 5 = 12 is a whole number.
5 – 17 = – 12 is not a whole number.

(b) Commutative Property: If a and b are two whole numbers, then a – b ≠ b – a. It shows that subtraction of two whole numbers is not commutative. Hence, commutative property does not hold good for subtraction of whole numbers, i.e.,
a – b ≠ b – a.

Example: 3 – 4 = – 1 and 4 – 3 = 1
∴ 3 – 4 ≠ 4 – 3

(c) Associative Property: If a, b, and c are whole numbers, then (a – b) – c ≠ a – (b – c). It shows that subtraction of whole numbers is not associative. Hence, associative property does not hold good for subtraction of whole numbers.

Example: (40 – 25) – 10 = 15 – 10 = 5
40 – (25 – 10) =40- 15 = 25
∴ (40 – 25) – 10 ≠ 40 – (25 – 10)

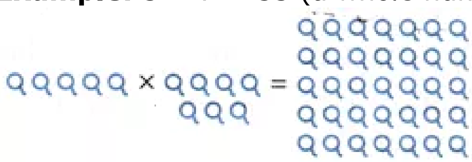
(d) Property of Zero: If we subtract zero from any whole number, the result remains the number itself.

Example: 7 – 0 = 7
5 – 0 = 5

Properties of Multiplication

(a) Closure Property: If a and b are two whole numbers, then a × b = c will always be a whole number. Hence, closure property holds good for multiplication of whole numbers.

Example: 5 × 7 = 35 (a whole number)

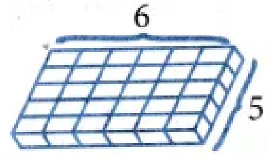


6 × 1 = 6 (a whole number)

(b) Commutative Property: If a and b are two whole numbers, then the product of two whole numbers remains unchanged if the order of the numbers is interchanged, i.e.,
a × b = b × a.

Example: 6 × 5 = 5 × 6
30 = 30

i. e., 6 rows of 5 or 5 rows of 6 give the same results.



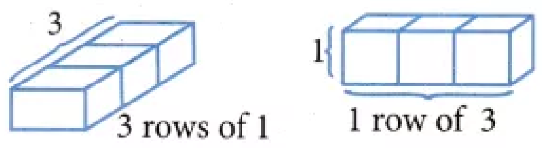
so, 6 × 5 = 30 = 5 × 6

(c) Associative Property: If a, b, and c are whole numbers, then the product of three whole numbers remains unchanged even if they are multiplied in any order. Hence, associative property does hold good for multiplication of whole numbers, i.e.,
(a × b) × c = a × (b × c)

Example:

(4 × 5) × 8 = 4 × (5 × 8)
20 × 8 = 4 × 40
160 = 160

(d) Multiplicative Identity: If any whole number is multiplied by 1, the product remains the number itself. Let a whole number be a, then
a × 1 = a = 1 × a.



3 × 1 = 3 = 1 × 3

Examples

75 × 1 = 75 = 1 × 75
3 × 1 = 3 = 1 × 3
Hence, 1 is called the multiplicative identity.

(e) Multiplicative Property of Zero: Any whole number multiplied by zero gives the product as zero.If a is any whole number, then 0 × a = a × 0 = 0.

Example: 3 × 0 = 0 × 3 = 0

Properties of Division

(a) Closure Property: If a and b are whole numbers, then a ÷ b is not always a whole number. Hence, closure property does not hold good for division of whole numbers.

Example: 7 ÷ 5 = $\frac{7}{5}$ is not a whole number.
7 ÷ 7 = 1 is a whole number.

(b) Commutative Property: If a and b are whole numbers, then a ÷ b ≠ b ÷ a. Hence, commutative property does not hold good for division of whole numbers.

Example: 18 ÷ 3 = 6 is a whole number.
3 ÷ 18 = $\frac{3}{18}$ = $\frac{1}{6}$ is not a whole number.
∴ 3 ÷ 18 ≠ 18 ÷ 3

(c) Associative Property: If a, b, and c are whole numbers then (a ÷ b) ÷ c ≠ a ÷ (b ÷ c). Hence, associative property does not hold good for division of whole numbers.

Example: (15 ÷ 3) ÷ 5 = 5 ÷ 5 = 1
15 ÷ (3 ÷ 5) = 15 ÷ 3/5 = 15 × 5/3
= 25
∴ (15 ÷ 3) ÷ 5 ≠ 15 ÷ (3 ÷ 5)

(d) Property of Zero: If a is a whole number then 0 ÷ a = 0 but a ÷ 0 is undefined.

Example: $6 \div 0$ is undefined.

Note:

- Product of zero and a whole number gives zero.
  $a \times 0 = 0$
- Zero divided by any whole number gives zero.
  $0 \div a = 0$
  $a \div 0 =$ undefined
- Any number divided by 1 is the number itself.
  $a \div 1 = a$

DISTRIBUTIVE PROPERTY

You are distributing something as you separate or break it into parts.

Example: Raj distributes 4 boxes of sweets. Each box comprises 6 chocolates and 10 candies. How many sweets are there in these 4 boxes?

∴ Chocolates in 1 box = 6
Choclolates in 4 boxes = $4 \times 6 = 24$
Candies in 1 box =10
Candies in 4 boxes = $4 \times 10 = 40$
Total number of sweets in 4 boxes
$= 4 \times 6 + 4 \times 10 = 4 \times (6 + 10)$
$= 4 \times 16 = 64$

Hence, we conclude the following:

(a) Multiplication distributes over addition, i.e., $a(b + c) = ab + ac$, where a, b, c are whole numbers.

Example: $10 \times (6 + 5) = 10 \times 6 + 10 \times 5$
$10 \times 11 = 60 + 50$
$110 = 110$
This property is called the distributive property of multiplication over addition.

(b) Similarly, multiplication distributes over subtraction, i.e., $a \times (b - c) = ab - ac$ where a, b, c are whole numbers and $b > c$.

Example: $10 \times (6 - 5) = 10 \times 6 - 10 \times 5$
$10 \times 1 = 60 - 50$
$10 = 10$
This property is called the distributive property of multiplication over subtraction.

Example 1: Determine the following by suitable arrangement.
$2 \times 17 \times 5$

Solution: $2 \times 17 \times 5 = (2 \times 5) \times 17$
$= 10 \times 17 = 170$

Example 2: Solve the following using distributive property.
97×101

Solution: $97 \times 101 = 97 \times (100 + 1)$
$= 9700 + 97 = 9797$

Example 3: Tina gets 78 marks in Mathematics in the half-yearly Examination and 92 marks in the final Examination. Reenagets 92 marks in the half- yearly Examination and 78 marks in the final Examination in Mathematics. Who has got the higher total marks?

Solution: Tina gets the following marks = 78 + 92 = 170 Total marks
Reena gets the following marks = 92 + 78 = 170 Total marks
So, both of them got equal marks.

Example 4: A fruit seller placed 12 bananas, 10 oranges, and 6 apples in a fruit basket. Tarun buys 3 fruit baskets for a function. What is the total number of fruits in these 3 baskets?

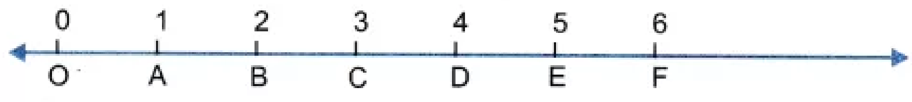
Solution: Number of bananas in 3 baskets = 12 × 3 = 36 bananas
Number of oranges in 3 baskets = 10 × 3 = 30 oranges
Number of apples in 3 baskets = 6×3 = 18 apples
Total number of fruits = 36 + 30+ 18 = 84

Alternative Method

Total number of fruits in 3 baskets
= 3 × [12 +(10 + 6)]
= 3 × [12 + 16]
= 3 × 28 = 84

Representation Of Whole Numbers On A Number Line

We can represent whole numbers-on a straight line. To represent a set of whole numbers on a number line, let's first draw a straight line and mark a point O on it. After that, mark points A, B, C, D, E, F on the line at equal distance, on the right side of point O.



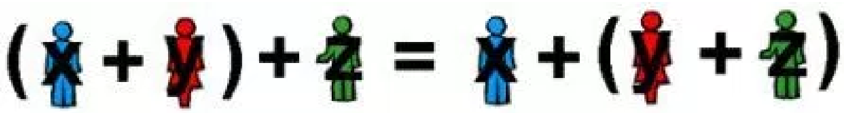
Now, OA = AB = BC = CD and so on
Let OA = 1 unit
OB = OA + AB = 1 + 1 = 2 units
OC = OB + BC = 2 + 1 = 3 units
OD = OC + CD = 3 + 1 = 4 units and so on.
Let the point O correspond to the whole number 0, then points A, B, C, D, E, ….. correspond to the whole numbers 1, 2, 3, 4, 5,…. In this way every whole number can be represented on the number line.

Hints for Remembering the Properties of Real Numbers

Commutative Property – interchange or switch the elements

Example shows commutative property for addition:
X + Y = Y + X
Think of the elements as "commuting" from one location to another. "They get in their cars and drive to their new locations." This explanation will help you to remember that the elements are "moving" (physically changing places).

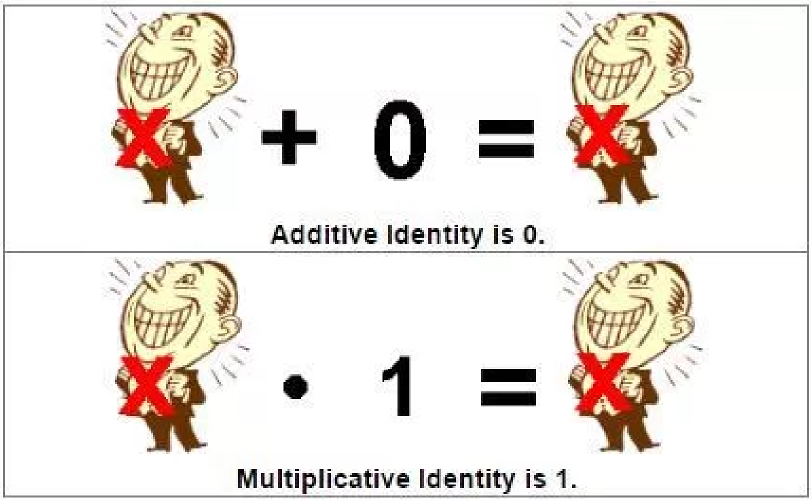
Associative Property – regroup the elements
Example shows associative property for addition:
(X + Y) + Z = X + (Y + Z)
The associative property can be thought of as illustrating "friendships" (associations). The parentheses show the grouping of two friends. In the example below, the red girl (y) decides to change from the blue boyfriend (x) to the green boyfriend (z). "I don't want to associate with you any longer!" Notice that the elements do not physically move, they simply change the person with whom they are "holding hands" (illustrated by the parentheses.)



Identity Property – What returns the input unchanged?

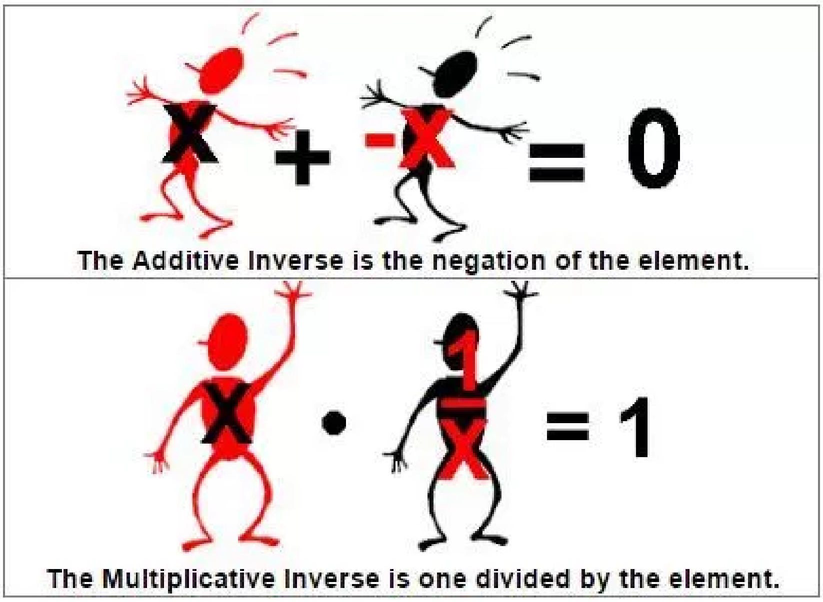
X + 0 = X   Additive Identity
X • 1 = X   Multiplicative Identity
Try to remember the "I" in the word identity. Variables can often times have an "attitude". "I am the

most important thing in the world and I do not want to change!" The identity element allows the variable to maintain this attitude.

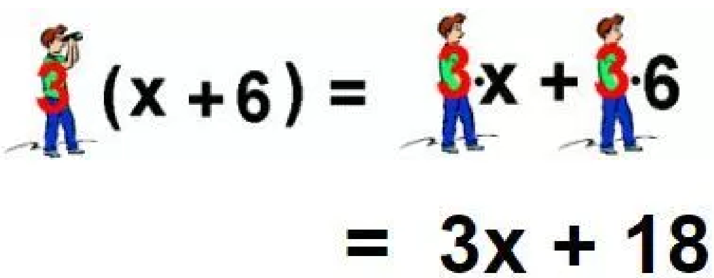


Inverse Property – What brings you back to the identity element using that operation?

X + -X = 0 Additive Inverse
X • 1/X = 1 Multiplicative Inverse
Think of the inverse as "inventing" an identity element. What would you need to add (multiply) to this element to turn it into an identity element?



Distributive Property – multiply across the parentheses. Each element inside the parentheses is multiplied by the element outside the parentheses.
a(b + c) = a•b + a•c
Let's consider the problem 3(x + 6). The number in front of the parentheses is "looking" to distribute (multiply) its value with all of the terms inside the parentheses.



Proportions

1. A **proportion** is a comparison of ratios.

2. A proportion is an equation that states that two ratios are equal, such as
$$\frac{4}{8}=\frac{1}{2}$$
3. Proportions always have an **EQUAL** sign!
4. A proportion can be written in two ways:
$$\frac{4}{8}=\frac{1}{2} \quad \text{or} \quad 4:8=1:2$$
Both are read "4 is to 8 as 1 is to 2".
5. In each proportion the first and last terms (4 and 2) are called the **extremes.** The second and third terms (8 and 1) are called the **means.**

You can tell if a simple proportion is true by just examining the fractions. If the fractions both reduce to the same value, the proportion is true.	$\frac{5}{15}=\frac{2}{6}$ This is a true proportion, since both fractions reduce to 1/3.
You can often use this same approach when solving for a missing part of a simple proportion. Remember that both fractions must represent the same value. Notice how we solve this problem by getting a common denominator for the two fractions.	$\frac{1}{3}=\frac{x}{15}$ $\frac{5}{15}=\frac{x}{15}$ To change the denominator of 3 to 15 requires multiplying by 5. The SAME must be done to the top to keep the fractions equal. Answer: x = 5

This simple approach may not be sufficient when working with more complex proportions. You need a rule:
Some people call this rule **Cross Multiply!!**
A more precise statement of the rule is:

RULE: In a true proportion, the product of the means equals the product of the extremes.
Proportions can also be solved by multiplying each side of the proportion by the common denominator for both fractions.

Example 1: Solve for x algebraically in this proportion:
$$\frac{25}{x}=\frac{5}{2}$$
Solution:

Method 1:
Appling the rule that the "in a true proportion, the product of the means equals the product of the extremes".
$$\frac{25}{x} \times \frac{5}{2}$$
$$5x=(25)(2)$$
$$5x=50$$
$$x=10 \text{ Answer}$$

Method 2:
Multiplying by the common denominator, 2x.
$$2x\cdot\frac{25}{x}=\frac{5}{2}\cdot 2x$$
$$2\cdot 25=5\cdot x$$
$$50=5x$$
$$10=x \text{ Answer}$$

Example 2: The length of a stadium is 100 yards and its width is 75 yards. If 1 inch represents 25 yards, what would be the dimensions of the stadium drawn on a sheet of paper?

Solution: This problem can be solved by an intuitive approach, such as:
100 yards by 75 yards
100 yards = 4 inches (HINT: 100 / 25)

75 yards = 3 inches (HINT: 75 / 25)
Therefore, the dimensions would be 4 inches by 3 inches.

Solution by proportion: (Notice that the inches are all on the top and the yards are all on the bottom for this solution. Other combinations are possible.)

Length:

$$\frac{1}{25} = \frac{x}{100}$$

$$25x = 100$$

$$x = 4 \text{ inches}$$

Width:

$$\frac{1}{25} = \frac{y}{75}$$

$$25y = 75$$

$$y = 3 \text{ inches}$$

What is a Ratio and Proportion

Ratio and Proportion

- A ratio is a comparison of two values expressed as a quotient
  - Example: A class has 12 girls and 18 boys. The ratio of girls to boys is $\frac{12}{18}$
  - This ratio can also be expressed as an equivalent fraction $\frac{2}{3}$
- A proportion is an equation stating that two ratios are equal.
  - Example: $\frac{12}{18} = \frac{2}{3}$

1. **Ratio:**

   The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

   In the ratio $a : b$, we call a as the first term or **antecedent** and b, the second term or **consequent**.

   Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

   Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

   Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. **Proportion:**

   The equality of two ratios is called proportion.

   If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

   Here a and d are called **extremes**, while b and c are called **mean terms**.

   Product of means = Product of extremes.

   Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. **Fourth Proportional:**

   If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

   Third Proportional:

   $a : b = c : d$, then c is called the third proportion to a and b.

   Mean Proportional:

   Mean proportional between a and b is $\sqrt{ab}$.

4. **Comparison of Ratios:**

   We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

   Compounded Ratio:

   The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

5. **Duplicate Ratios:**

   Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

   Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

   Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

   Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

6. **Variations:**

   We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

   We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

RATIO

In our day-to-day life, we compare one quantity with another quantity of the same kind by using the 'method of subtraction' and 'method of division'.

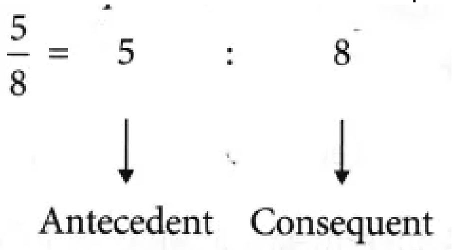
Example: The height of Seema is 1 m 67 cm and that of Reema is 1 m 62 cm. The difference in their heights is:
167 cm – 162 cm = 5 cm
Thus, we say Seema is 5 cm taller than Reema.
Similarly, suppose the weight of Seema is 60 kg and the weight of Reema is 50 kg. We can compare their weights by division, i.e.,

$$\frac{\text{Weight of Seema}}{\text{Weight of Reema}} = \frac{50\text{ kg}}{60\text{ kg}}$$

$$= \frac{6}{5}$$

So, the weight of Seema is $\frac{6}{5}$ times the weight of Reema.
When we compare two similar quantities by division, the comparison is called the **'ratio'**. It is denoted by **':'** and read as **'is to'**.

Example: $\frac{5}{8}$ = 5 : 8 (read as 5 is to 8).
As shown in the above example a ratio is like a fraction or comparison of two numbers, where a numerator and a denominator is separated by a colon (:). The first term or the quantity (5), called antecedent means 'that precedes' and the second term, called consequent means 'that follows'.



Properties of Ratio

When we compare two quantities, the following points must be taken care of:

1. A ratio is usually expressed in its simplest form.
   Example:
   $\frac{12}{36} = \frac{1}{3} = 1 : 3$
2. Both the quantities should be in the same unit. So, ratio is a number with no unit involved in it.
   Example: 200 g : 2 kg
   = 200 g : 2000 g
   $\frac{200}{2000} = \frac{1}{10} = 1 : 10$
3. The order of the quantities of a ratio is very important.
   Example: 5 : 6 is different from 6 : 5.
   They are not equal.
   5 : 6 ≠ 6 : 5

Equivalent Ratios

A ratio is similar to a fraction. So, if we divide or multiply the numerator (antecedent) and denominator (consequent) by the same number, we get an equivalent fraction (ratio).

Example: 5 : 6 = $\frac{5}{6}$

If we multiply $\frac{5}{6}$ by the same number, say 3, we get the ratio

$$\frac{5 \times 3}{6 \times 3} = \frac{15}{18} \quad \left(\begin{array}{l}\text{Multiplying by the} \\ \text{same number 3}\end{array}\right)$$

$$= 15 : 18 \text{ which is equivalent to } 5 : 6.$$

Similarly,

$$\frac{12}{36} = \frac{12 \div 12}{36 \div 12} = \frac{1}{3} \quad \left(\begin{array}{l}\text{Dividing by the} \\ \text{same number 12}\end{array}\right)$$

So, 12 : 36 is equivalent to 1 : 3.

Comparison of Ratios

To compare two ratios, we have to follow these steps:
Step 1: Convert each ratio into a fraction in its simplest form.
Step 2: Find the LCM of denominators of the fractions obtained in step 1.
Step 3: Convert the denominators equal to LCM obtained in step 2 in each fraction.
Step 4: Now, compare the numerators of the fractions; the fraction with a greater numerator will be greater than the other.

Example 1: Compare the ratio 5: 6 and 7: 8.

Solution: Here, 5 : 6 = 5/6 and 7 : 8 = 7/8.

Now, compare the two fractions $\frac{5}{6}$ and $\frac{7}{8}$ making their denominators equal.
LCM of 6 and 8 = 24

$$\therefore \quad \frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

$$\text{and} \quad \frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

$$\because \quad 21 > 20,$$

$$\text{So,} \quad \frac{21}{24} > \frac{20}{24} \quad \text{or,} \quad \frac{7}{8} > \frac{5}{6}$$

Hence, $7 : 8 > 5 : 6$

Example 2: Convert the ratio 125 : 275 in its simplest form.

Solution:

$$\text{Here,} \quad 125 : 275 = \frac{125}{275}$$

HCF of 125 and 275 is 25.

$$\text{So,} \quad \frac{125 \div 25}{275 \div 25} = \frac{5}{11}$$

So, 5 : 11 is the simplest form of 125 : 275.

Example 3: Write the following ratios in descending order:
4 : 3, 4 : 7, 7 : 10

Solution: We have,

$$4 : 3 = \frac{4}{3}, 4 : 7 = \frac{4}{7}, 7 : 10 = \frac{7}{10}$$

Now, compare these fractions making their denominators equal.

LCM of 3 , 7, and 10 = 210

$$\therefore \quad \frac{4}{3} = \frac{4 \times 70}{3 \times 70} = \frac{280}{210},$$

$$\frac{4}{7} = \frac{4 \times 30}{7 \times 30} = \frac{120}{210},$$

$$\frac{7}{10} = \frac{7 \times 21}{10 \times 21} = \frac{147}{210}$$

$$\because \quad 280 > 147 > 120$$

$$\text{So,} \quad \frac{280}{210} > \frac{147}{210} > \frac{120}{210}$$

$$\text{or,} \quad \frac{4}{3} > \frac{7}{10} > \frac{4}{7}$$

$$\text{Hence,} \quad 4 : 3 > 7 : 10 > 4 : 7$$

Example 4: Mr Lai divides a sum of Rs. 1500 between his two sons in the ratio 2 : 3. How much money does each son get?

Solution: Let the first son get 2x and the second son get 3x.

$$2x + 3x = ₹\, 1500$$

$$5x = ₹\, 1500$$

$$x = ₹\, 300$$

$$\text{First son's share} = 2 \times ₹\, 300 = ₹\, 600$$

$$\text{Second son's share} = 3 \times ₹\, 300 = ₹\, 900$$

Example 5: Two numbers are in the ratio 3 : 5 and their sum is 96. Find the numbers.

Solution: Let the first number be 3x and the second number be 5x.
Then, their sum = 3x + 5x = 96
8x = 96
x = 12
The first number = 3x = 3 × 12 = 36
The second number 5x = 5 × 12 = 60

Example 6: In a pencil box there are 100 pencils. Out of which 60 are red pencils and the restare blue pencils. Find the ratio of:

(a) blue pencils to the total number of pencils.
(b) red pencils to the total number of pencils.
(c) red pencils to blue pencils.

Solution: Total number of pencils in the pencil box = 100
Number of red pencils = 60
∴ Number of blue pencils = 100 – 60 = 40

(*a*) The ratio of blue pencils to the total number of pencils

$$= 40 : 100 = \frac{40}{100} = \frac{2}{5} = 2 : 5$$

(*b*) The ratio of red pencils to the total number of pencils

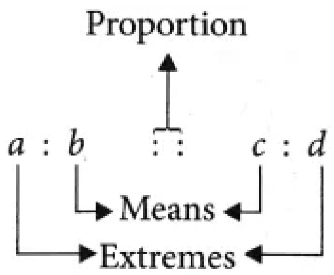
$$= 60 : 100 = \frac{60}{100} = \frac{3}{5} = 3 : 5$$

(*c*) The ratio of red pencils to blue pencils

$$= 60 : 40 = \frac{60}{40} = \frac{3}{2} = 3 : 2$$

PROPORTION
A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal. When two ratios are equal then such type of equality of ratios is called proportion and their terms are said to be in proportion.

Example: If the cost of 3 pens is Rs. 21, and that of 6 pens is Rs. 42, then the ratio of pens is 3 : 6, and the ratio of their costs is 21 : 42. Thus, 3 : 6 = 21:42. Therefore, the terms 3,6,21, and 42 are in proportion.
Generally, the four terms, a, b, c, and d are in proportion if a : b = c : d.
Thus, a : b : : c : d means a/b = c/d or ad = ad = bc
Conversely, if ad = be, then a/b = c/d or a : b : : c : d
Here, a is the first term, b is the second term, c is the third term, and d is the fourth term. The first and the fourth terms are called extreme terms or extremes and the second and third terms are called middle terms or means.



Product of extremes	=	Product of means
ad	=	bc

Continued proportion

In a proportion, if the second and third terms are equal then the proportion is called continued proportion.
Example: If 2 : 4 : : 4 : 8, then we say that 2, 4, 8 are in continued proportion.

Mean proportion

If the terms a, b, and c are in continued proportion, then 'b' is called the mean proportion of a and c.
Example: If a, b, c are in continued proportion, then

$$a : b :: b : c$$

$$\downarrow$$

$$\frac{a}{b} = \frac{b}{c}$$

Mean proportion = b^2 = ac

Third proportion

If the terms a, b, c are in continued proportion, then c is called the third proportion.

Properties of proportions:
Convertendo: If a : b :: c : d ,then a : (a - b) :: c : (c – d) .
Invertendo: If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$.
Alternendo: If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$.
Componendo: If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$.
Dividendo: $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$
Componendo and Dividendo: If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$.

Example 1: Find x, where x : 3 : : 4 : 12.

Solution: Here, x, 3, 4, and 12 are in proportion.

$$\therefore \quad \frac{x}{3} = \frac{4}{12}$$

or, $12 \times x = 3 \times 4$

or, $x = \frac{3 \times 4}{12}$

or, $x = \frac{12}{12}$

or, $x = 1$

Example 2: Find the third proportion of 10 and 20.

Solution: If a, b, c are in proportion, then $b^2 = ac$.

$$\therefore \quad (20)^2 = 10 \times c$$

or, $10c = 400$

or, $c = \frac{400}{10} = 40$

So, the third proportion = 40

Example 3: Find the value of x, if 14, 42, x are in continued proportion.

Solution: Here 14, 42, and x are in proportion.

$$\therefore \quad 14 : 42 :: 42 : x$$

or, $\frac{14}{42} = \frac{42}{x}$

or, $14 \times x = 42 \times 42$

or, $x = \frac{42 \times 42}{14}$

or, $x = 126$

Example 4: The cost of 1 dozen bananas is Rs. 24. How much do 50 bananas cost?

Solution: Let the cost of 50 bananas be x.

Bananas	*Cost* (in ₹)
12	24
50	x

$$12 : 24 :: 50 : x$$

or, $\frac{12}{24} = \frac{50}{x}$

or, $12 \times x = 24 \times 50$

or, $x = \frac{24 \times 50}{12}$

or, $x = ₹\ 100$

Example 5: Rajesh drives his car at a constant speed of 12 km per 10 minutes. How long will he take to cover 48 km?

Solution: Let Rajesh take x mins, to cover 48 km.

Speed (in km)	*Time (in minutes)*
12	10
48	x

$$12 : 10 :: 48 : x$$

$$12 \times x = 10 \times 48$$

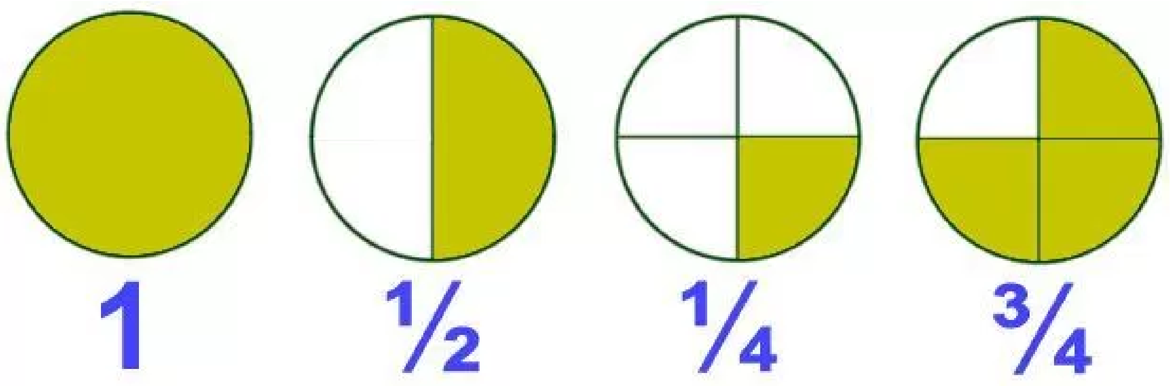
$$x = \frac{10 \times 48}{12}$$

$$= 40 \text{ minutes}$$

What is Fraction and How many Types of Fractions are there

Fraction

A number that compares part of an object or a set with the whole, especially the quotient of two whole numbers is written in the form of xly is called a **fraction**. The fraction 1/3, which means 1 divided by 3, can be represented as 1 pencil out of a box of 3 pencils.
A fraction is a (i) part of a whole. (ii) part of a collection.

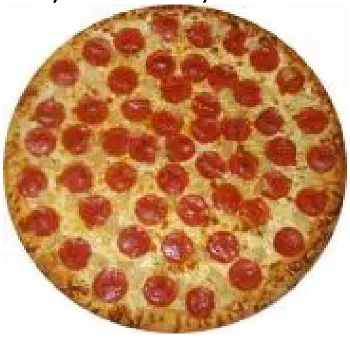


A fraction comprises two numbers separated by a horizontal line. The number above-the horizontal line is called the numerator and the number below the horizontal line is called the denominator of the fraction.

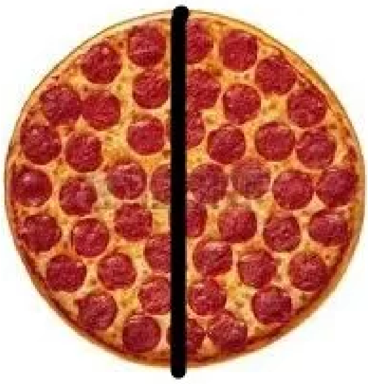
$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{\leftarrow 4}{\leftarrow 7}$$

Fraction as a part of a whole
A fraction is a part of a whole. Imagine a pizza cut into slices. All of the slices make 1 whole pizza. Each slice is a fraction of a pizza.

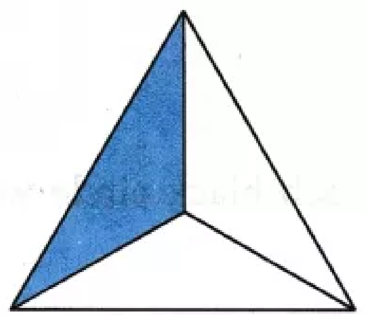
Tanya and Sanya want to share a pizza equally



They decide to cut the pizza from the middle and divide it into two equal parts. Each part is called the half of the whole and written as $\frac{1}{2}$. Both the sisters get equal share. The $\frac{1}{2}$ part of the whole is a fraction.



Similarly, we can take many examples from our daily life to show fraction as a part of a whole.



In this figure we have divided a triangle into 3 equal parts. The shaded part shows one part out of three, i.e., $\frac{1}{3}$. Here, $\frac{1}{3}$ is a fraction, which is a part of the whole triangle.

Fraction is a part of a collection

A fraction represents parts of a collection, the numerator being the number of parts we have and the denominator being the total number of parts in the collection.

Let us take a collection of 12 stars and we want to shade $\frac{3}{4}$ of the collection.

In order to find $\frac{3}{4}$ out of the 12 stars, we divide the 12 stars into four equal parts.

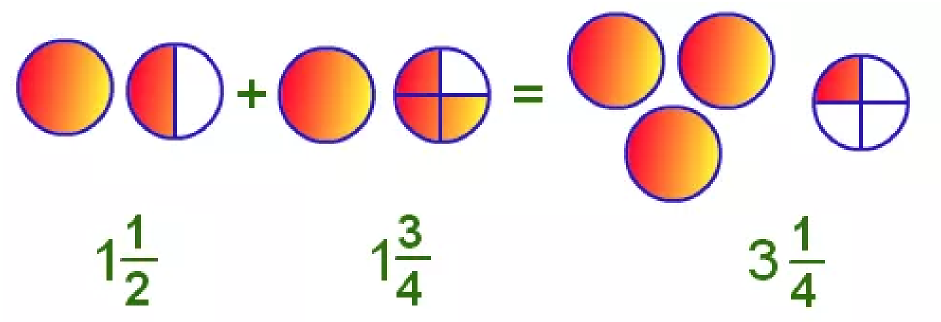
Each part contains 3 stars. Now, we can shade 3 parts out of 4 parts.

On counting, we find that the total number of shaded stars is 9.

In other words, $\frac{3}{4}$ of 12 stars = 9 stars.

Types of fractions

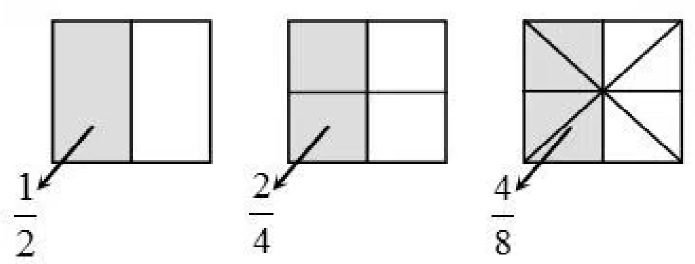
1. **Like fractions:** Fractions having the same denominators are called like fractions.
   Examples: $\frac{1}{7}$, $\frac{3}{7}$, $\frac{2}{7}$, $\frac{6}{7}$ etc. are like fractions.
2. **Unlike fractions:** Fractions having different denominators are called unlike fractions.
   Examples: $\frac{2}{3}$, $\frac{5}{7}$, $\frac{6}{8}$, $\frac{1}{3}$ etc. are unlike fractions.
3. **Unit fraction:** A fraction having a numerator as 1 is called a unit fraction.
   Examples: $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{8}$, $\frac{1}{5}$ etc. are all unit fractions
4. **Proper fraction:** A fraction, whose numerator is smaller than its denominator is called a proper fraction.
   Examples: $\frac{2}{3}$, $\frac{5}{7}$, $\frac{1}{6}$, $\frac{3}{9}$ etc. are all proper fractions.
5. **Improper fraction:** A fraction, whose numerator is greater than or equal to its denominator is called an improper fraction.
   Examples: $\frac{4}{3}$, $\frac{7}{5}$, $\frac{9}{9}$ etc. are all improper fractions.
6. **Mixed fraction:** A fraction, which is a combination of a whole number and a proper fraction is called a mixed fraction. All improper fractions can be written in the form of mixed fractions.
   Example: 2 $\frac{1}{4}$ is a mixed fraction, since 2 is a 4 whole number and $\frac{1}{4}$ is a proper fraction.



7. **Equivalent Fraction:** If $\frac{c}{d} = \frac{m \times a}{m \times b}$, then the fractions $\frac{a}{b}$ and $\frac{c}{d}$ are called equivalent fractions because they represent the same portion of the whole.

   For example, $\frac{4}{6} = \frac{2 \times 2}{3 \times 2}$; $\frac{15}{48} = \frac{5 \times 3}{16 \times 3}$

   For example, the shaded parts of each of the following figures are same but they are represented by different fractional numbers.



   They are called equivalent fractions.

   So we write $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$, etc.
8. **Decimal fractions:** A fraction whose denominator is any of the number 10,100,1000 etc. is called a decimal fraction.

   For example : $\frac{8}{10}, \frac{11}{100}, \frac{17}{1000}$ etc. are decimal fractions.
9. **Vulgar fractions:** A fraction whose denominator is a whole number, other than 10,100,1000 etc. is called a vulgar fractions.

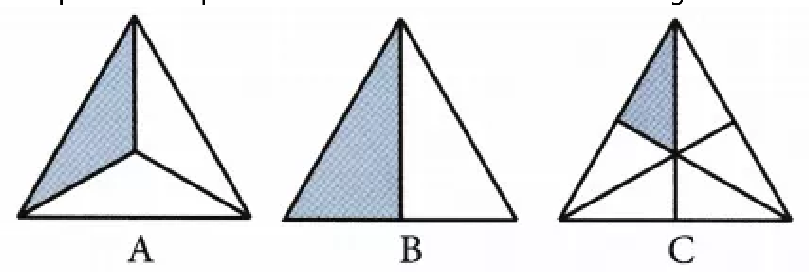
   For example $\frac{2}{7}, \frac{3}{8}, \frac{11}{17}$ etc. are vulgar fractions.

What is Comparing and Ordering of Fractions

Comparison of fractions are divided into three categories.

1. Fraction with the same numerator

Let us consider the following fractions with the same numerator: $\frac{1}{3}, \frac{1}{2}, \frac{1}{6}$
The pictorial representation of these fractions are given below:



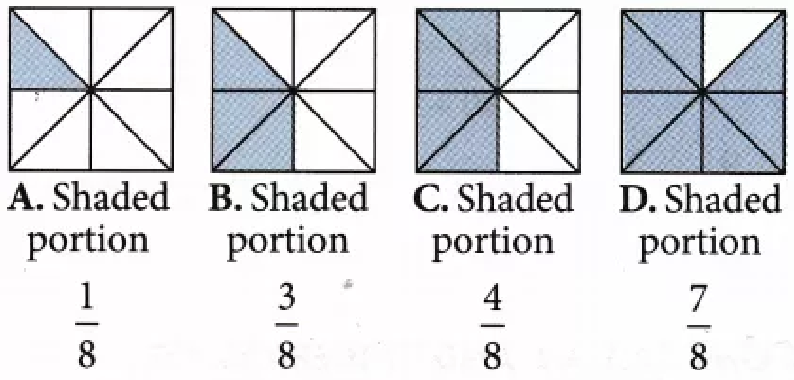
Shaded portion is $\frac{1}{3}$ Shaded portion is $\frac{1}{2}$ Shaded portion is $\frac{1}{6}$

By looking at the shaded portion in these pictures, we can conclude that
shaded area of B > shaded area of A > shaded area of C
or $\frac{1}{2} > \frac{1}{3} > \frac{1}{6}$
Thus, we conclude that if two or more fractions have the same numerator, then the fraction with a smaller denominator is greater.

2. Fraction with the same denominator

Let us consider the following fractions with the same denominator $\frac{1}{8}, \frac{3}{8}, \frac{4}{8}, \frac{7}{8}$

The pictorial representation of these fractions are given below:



By looking at the shaded portion in these pictures, we can easily say that

shaded area of D > shaded area of C > shaded area of B > shaded area of A

$\frac{7}{8} > \frac{4}{8} > \frac{3}{8} > \frac{1}{8}$

Thus, we conclude that if two or more fractions have the same denominator, then the fraction with the greater numerator is the greater fraction.

3. Fractions with different numerators and denominators

To compare the fractions with different numerators and denominators, first we find the LCM of their denominators. Then, we make the denominator of each fraction equal to the LCM by multiplying with a suitable number.

Example 1: Compare the fractions $\frac{3}{4}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}$

Solution: LCM of denominators 4,6,7, and 8 = 168
Hence, the given fractions can be written as

$$\frac{3}{4} = \frac{3 \times 42}{4 \times 42} = \frac{126}{168}$$

$$\frac{5}{6} = \frac{5 \times 28}{6 \times 28} = \frac{140}{168}$$

$$\frac{6}{7} = \frac{6 \times 24}{7 \times 24} = \frac{144}{168}$$

$$\frac{7}{8} = \frac{7 \times 21}{8 \times 21} = \frac{147}{168}$$

given fractions can be written as

$$\frac{147}{168} > \frac{144}{168} > \frac{140}{168} > \frac{126}{168}$$

$$\frac{7}{8} > \frac{6}{7} > \frac{5}{6} > \frac{3}{4}$$

Example 2: Arrange the following fractions in ascending order:

Solution: LCM of denominators 4,2,8, and 16

Hence, the given fractions can be written as

$$\frac{1}{4} = \frac{1\times 4}{4\times 4} = \frac{4}{16}$$

$$\frac{3}{2} = \frac{3\times 8}{2\times 8} = \frac{24}{16}$$

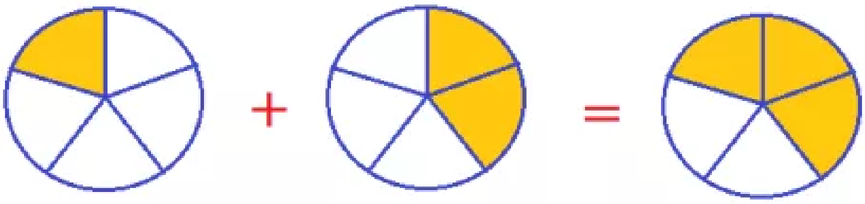
$$\frac{7}{8} = \frac{7\times 2}{8\times 2} = \frac{14}{16}$$

$$\frac{1}{16} = \frac{1\times 1}{16\times 1} = \frac{1}{16}$$

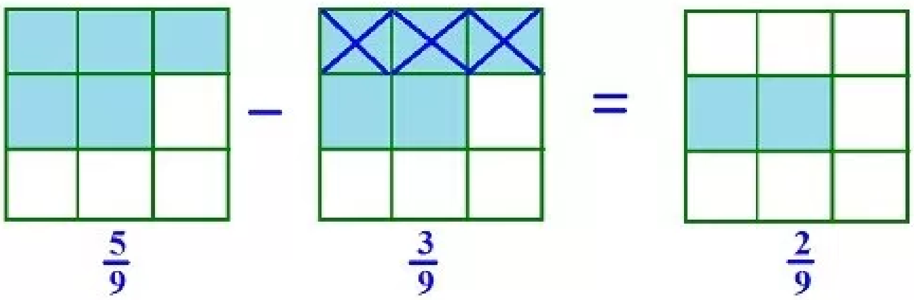
$\therefore$ Ascending order is $\frac{1}{16} < \frac{4}{16} < \frac{14}{16} < \frac{24}{16}$

or, $\frac{1}{16} < \frac{1}{4} < \frac{7}{8} < \frac{3}{2}$

$$\text{Addition/ subtraction of fractions} = \frac{\text{Sum/Difference of numerators}}{\text{Common denominator}}$$



$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$



What are the Operations on Fractions

Now, we have to learn, how to add and subtract the fractions. Certain methods are to be followed for doing these operations.

Addition and subtraction of like fractions

For adding and subtracting like fractions, we follow these steps:
Step 1. Add/subtract the numerators with common denominator.
Step 2. Reduce the fraction to its lowest term.
Step 3. If the result is an improper fraction, convert it into a mixed fraction.

Example 1: Find the sum of

$$(a)\ \frac{6}{11} \text{ and } \frac{9}{11} \qquad (b)\ 1\frac{1}{7} \text{ and } 2\frac{2}{7}$$

Solution:

$$(a)\quad \frac{6}{11}+\frac{9}{11}=\frac{6+9}{11}=\frac{15}{11}=1\frac{4}{11}$$

$$(b)\quad 1\frac{1}{7} \text{ and } 2\frac{2}{7}=\frac{8}{7}+\frac{16}{7}=\frac{8+16}{7}$$

$$=\frac{24}{7}=3\frac{3}{7}$$

Example 2: Subtract

$$(a)\ \frac{8}{15} \text{ from } \frac{13}{15} \qquad (b)\ 1\frac{1}{5} \text{ from } 2\frac{3}{5}$$

Solution:

$$(a)\quad \frac{13}{15}-\frac{8}{15}=\frac{13-8}{15}$$

$$=\frac{5}{15}=\frac{1}{3}$$

$$(b)\quad 2\frac{3}{5}-1\frac{1}{5}=\frac{13}{5}-\frac{6}{5}$$

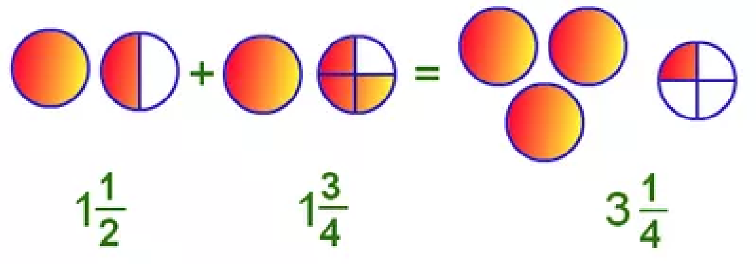
$$=\frac{13-6}{5}$$

$$=\frac{7}{5}=1\frac{2}{5}$$

Addition and subtraction of unlike fractions

For adding/subtracting unlike fractions, we follow these steps:
1. Find the LCM of denominators of the given fractions.
2. Convert unlike fractions into like fractions by making LCM as its denominator.

3. Add/ subtract the like fractions.



Example 3: Add $\frac{9}{5}$ and $\frac{5}{6}$

Solution: LCM of 10 and 6 = 30

$$\frac{9}{10} = \frac{9\times 3}{10\times 3} = \frac{27}{30}$$

$$\frac{5}{6} = \frac{5\times 5}{6\times 5} = \frac{25}{30}.$$

$$\therefore \frac{9}{10}+\frac{5}{6} = \frac{27}{30}+\frac{25}{30} = \frac{27+25}{30} = \frac{52}{30} = \frac{26}{15} = 1\frac{11}{15}$$

Example 4:

$$Add\ \ 2\frac{1}{8},\ \ 2\frac{1}{2}\ and\ \frac{7}{16}.$$

Solution:

$$\text{We have } 2\frac{1}{8}+2\frac{1}{2}+\frac{7}{16}$$

$$= \frac{17}{8}+\frac{5}{2}+\frac{7}{16} \quad \left(\begin{array}{l}\text{Convert mixed fractions}\\ \text{to improper fractions.}\end{array}\right)$$

$$= \frac{17\times 2}{8\times 2}+\frac{5\times 8}{2\times 8}+\frac{7\times 1}{16\times 1} \quad \left(\begin{array}{l}\because \text{LCM of}\\ 8, 2 \text{ and } 16 = 16\end{array}\right)$$

$$= \frac{34}{16}+\frac{40}{16}+\frac{7}{16}$$

$$= \frac{34+40+7}{16}$$

$$= \frac{81}{16}$$

$$= 5\frac{1}{16}$$

Example 5: Find $\frac{13}{5} - \frac{4}{5}$

Solution: LCM of 15 and 5 = 15

$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3}$$

$$= \frac{12}{15}$$

$$\therefore \frac{13}{15} - \frac{4}{5} = \frac{13}{15} - \frac{12}{15}$$

$$= \frac{13-12}{15}$$

$$= \frac{1}{15}$$

Example 6: Simplify $6\frac{1}{2} + 2\frac{2}{3} - \frac{1}{4}$

Solution: $6\frac{1}{2} + 2\frac{2}{3} - \frac{1}{4}$

$$= \frac{13}{2} + \frac{8}{3} - \frac{1}{4} \left(\begin{array}{l}\text{Converting mixed fractions} \\ \text{into impoper fractions.}\end{array}\right)$$

$$= \frac{13 \times 6}{2 \times 6} + \frac{8 \times 4}{3 \times 4} - \frac{1 \times 3}{4 \times 3} \left(\begin{array}{l}\because \text{LCM of 2,} \\ \text{3 and 4 = 12}\end{array}\right)$$

$$= \frac{78}{12} + \frac{32}{12} - \frac{3}{12}$$

$$= \frac{78 + 32 - 3}{12}$$

$$= \frac{110 - 3}{12}$$

$$= \frac{107}{12}$$

$$= 8\frac{11}{12}$$

Multiplication of Fractions

Rule:

$$\text{Product of fractions} = \frac{\text{Product of their Numerators}}{\text{Product of their Denominators}}$$

(i) Whole number by a fraction

(ii) Fraction by a fraction
(iii) Whole number by a mixed fraction
(iv) Multiplication of two mixed fractions

Whole number by a fraction:

To multiply a whole number by a fraction, we simply multiply the numerator of the fraction by the whole number, keeping the denominator same.

Example 1: Find the product

(i) $3 \times \frac{2}{7}$ (ii) $3 \times \frac{1}{8}$ (iii) $\frac{7}{9} \times 6$

Solution:

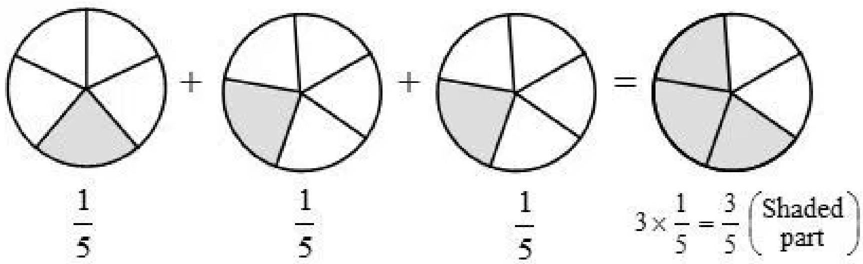
(i) $3 \times \frac{2}{7} = \frac{3}{1} \times \frac{2}{7} = \frac{3 \times 2}{1 \times 7} = \frac{6}{7}$

(ii) $3 \times \frac{1}{8} = \frac{3}{1} \times \frac{1}{8} = \frac{3 \times 1}{1 \times 8} = \frac{3}{8}$

(iii) $\frac{7}{9} \times 6 = \frac{7}{9} \times \frac{6}{1} = \frac{14}{3} = 4\frac{2}{3}$

Example 2: Show $3 \times \frac{1}{5}$ by picture.

Solution:



Note : Multiplication is commutative i.e. ab = ba

Fraction by a fraction :

Example 3: Find the product

(i) $\frac{5}{8} \times \frac{3}{7}$ (ii) $\frac{6}{14} \times \frac{7}{9}$ (iii) $2\frac{4}{7} \times 2\frac{3}{4} \times 1\frac{2}{5}$

Solution:

(i) $\frac{5}{8} \times \frac{3}{7} = \frac{5 \times 3}{8 \times 7} = \frac{15}{56}$

(ii) $\frac{6}{14} \times \frac{7}{9} = \frac{6}{14} \times \frac{7}{9} = \frac{2 \times 1}{2 \times 3} = \frac{1 \times 1}{1 \times 3} = \frac{1}{3}$

(iii) $2\frac{4}{7} \times 2\frac{3}{4} \times 1\frac{2}{5} = \frac{18}{7} \times \frac{11}{4} \times \frac{7}{5}$

$= \frac{18 \times 11 \times 7}{7 \times 4 \times 5} = \frac{9 \times 11}{2 \times 5} = \frac{99}{10} = 9\frac{9}{10}$

Whole Number by a Mixed Fraction :

To multiply a whole number by a mixed fraction, we follow the following steps:

1. Convert the mixed fraction into an improper fraction.
2. Multiply the numerator by the whole number keeping the denominator same.
3. After multiplication, the fraction should be converted in its lowest form.
4. Convert the improper fraction (product so obtained) into a mixed numeral.

Example 4: Find $8 \times 5\frac{1}{6}$

Solution:

$= 8 \times \frac{31}{6}$ (Converting the mixed fraction into an improper fraction).

$= \frac{248}{6}$ (Multiplying the numerator by the whole number)

$= \frac{124}{3}$ (Simplifying into lowest term).

$= 41\frac{1}{3}$ (Converting the improper fraction into a mixed numeral).

Example 5: Find $6 \times 3\frac{1}{2}$

Solution:

$$\text{Step 1. } 3\frac{1}{2} = \frac{3\times 2+1}{2} = \frac{7}{2}$$

$$\text{Step 2. } 6\times 3\frac{1}{2} = 6\times \frac{7}{2} = \frac{6\times 7}{2} = \frac{42}{2}$$

$$\text{Step 3. } \frac{42}{2} = 21;$$

$$\text{Hence, } 6\times 3\frac{1}{2} = 21$$

Multiplication of two Mixed Fractions:

1. To multiply two or more mixed numerals, we follow the following steps :
2. Convert the mixed fractions into improper fractions.
3. Multiply the improper fractions.
4. Reduce to lowest form.
5. If the product is an improper fraction, convert it into mixed fraction.

Example 6: Find the product of

$$\text{(i) } 3\frac{4}{5}\times\frac{10}{21} \quad \text{(ii) } \frac{15}{22}\times 4\frac{5}{7} \quad \text{(iii) } 5\frac{2}{15}\times 3\frac{4}{7}$$

Solution:

$$\text{(i) } 3\frac{4}{5}\times\frac{10}{21} = \frac{19}{5}\times\frac{10}{21} = \frac{38}{21} = 1\frac{17}{21}$$

$$\text{Thus, } 3\frac{4}{5}\times\frac{10}{21} = 1\frac{17}{21}$$

$$\text{(ii) } \frac{15}{22}\times 4\frac{5}{7} = \frac{15}{22}\times\frac{33}{7} = \frac{15}{2}\times\frac{3}{7} = \frac{45}{14} = 3\frac{3}{14}$$

$$\text{(iii) } 5\frac{2}{15}\times 3\frac{4}{7} = \frac{77}{15}\times\frac{25}{7} = \frac{11\times 5}{3\times 1} = \frac{55}{3} = 18\frac{1}{3}$$

Facts:

1. It is not necessary first to multiply the fractions and then simplify. We may simplify first then multiply. For example,

$$\text{(i) } \frac{21}{25}\times\frac{45}{68} = \frac{21\times 45}{25\times 68} = \frac{21\times 9}{5\times 68} = \frac{189}{340}$$

$$\text{(ii) } \frac{25}{7}\times\frac{12}{5}\times\frac{7}{4} = \frac{25\times 12\times 7}{7\times 5\times 4} = \frac{5\times 3\times 1}{1\times 1\times 1} = 15$$

2. Cancellation could use only for fractions are multiplied and could not use for addition & subtraction of fractions.

3. Double of 3 or half of 7 can be written as 2 × 3 and 1/2 × 7 respectively.
   If word 'OF' is in between two fractions then multiply those fractions.
4. Product of two proper fractions < Each proper fraction.

   $$\text{Ex. } \frac{2}{7} \times \frac{1}{3} = \frac{2}{21} \quad \therefore \quad \frac{2}{21} < \frac{2}{7} \text{ and } \frac{2}{21} < \frac{1}{3}$$

5. Product of two improper fractions > Each improper fraction.

   $$\text{Eg. } \frac{9}{4} \times \frac{7}{3} = \frac{63}{12} \quad \therefore \quad \frac{63}{12} > \frac{9}{4} \ \& \ \frac{63}{12} > \frac{7}{3};$$

6. Proper fraction < Product of proper and improper fraction < Improper fraction

   $$\text{Eg. } \frac{1}{7} \times \frac{5}{2} = \frac{5}{14} \quad \therefore \quad \frac{1}{7} < \frac{5}{14} < \frac{5}{2}$$

7. When the product of two fractional numbers or a fractional number and a whole number is 1, then either of them is the multiplicative inverse (or reciprocal) of the other. So the reciprocal of a fraction (or a whole number) is obtained by interchanging its numerator and denominator.
   Note : Reciprocal of zero (0) is not possible.

Division of Fractional Numbers

∵ We know Division = Dividend ÷ Divisor
When a fraction number (or whole no.) divide by fractional number (or whole no.) then we multiply dividend to reciprocal of divisor.

Example 1: Find the value of

$$\text{(i) } \frac{5}{7} \div \frac{25}{21} \quad \text{(ii) } \frac{7}{8} \div \frac{15}{8} \quad \text{(iii) } 1\frac{2}{7} \div 2\frac{1}{14}$$

Solution:

$$\text{(i) } \frac{5}{7} \div \frac{25}{21} = \frac{5}{7} \times \frac{21}{25} = \frac{3}{5}$$

$$\text{(ii) } \frac{7}{8} \div \frac{15}{8} = \frac{7}{8} \times \frac{8}{15} = \frac{7}{15}$$

$$\text{(iii) } 1\frac{2}{7} \div 2\frac{1}{14} = \frac{9}{7} \div \frac{29}{14} = \frac{9}{7} \times \frac{14}{29} = \frac{18}{29}$$

Facts:

1. (Fractional number) ÷ 1 = same fractional number

   $$\frac{2}{3} \div 1 = \frac{2}{3} \times \frac{1}{1} = \frac{2}{3}$$

2. 0 ÷ Fractional number = 0 (always)
3. non zero fractional number ÷ same number = 1 (always)

   $$\frac{2}{3} \div \frac{2}{3} = \frac{2}{3} \times \frac{3}{2} = 1$$

4. '0' cannot be a divisor (∵ reciprocal of zero is not possible)

Example 2:

$$\text{Simplify : } \frac{2\frac{3}{4}}{1\frac{5}{7}}$$

Solution:

$\frac{2\frac{3}{4}}{1\frac{5}{7}}$ is same as $2\frac{3}{4} \div 1\frac{5}{7}$

Now, $2\frac{3}{4} \div 1\frac{5}{7}$

$= \frac{11}{4} \div \frac{12}{7} \leftarrow$ (Rewrite the mixed numerals as improper fractions)

$= \frac{11}{4} \times \frac{7}{12} \leftarrow$ (Change ÷ to × and replace the divisor by its reciprocal.)

$= \frac{77}{48} \leftarrow$ (Reduce to lowest form and multiply the numerators and multiply the denominators)

$= 1\frac{29}{48} \leftarrow$(Rewrite the improper fraction as mixed numeral)

Example 3:

(i) $12 \div \frac{3}{4}$ (ii) $2\frac{1}{5} \div 1\frac{1}{5}$

(iii) $\frac{2}{5} \div 1\frac{1}{2}$ (iv) $3\frac{1}{2} \div 4$

Solution:

(i) $12 \div \frac{3}{4} = \frac{12}{1} \times \frac{4}{3} = \frac{4 \times 4}{1 \times 1} = \frac{16}{1} = 16$

(ii) $2\frac{1}{5} \div 1\frac{1}{5} = \frac{11}{5} \div \frac{6}{5} = \frac{11}{5} \times \frac{5}{6}$

$= \frac{11 \times 5}{5 \times 6} = \frac{11}{6} = 1\frac{5}{6}$

(iii) $\frac{2}{5} \div 1\frac{1}{2} = \frac{2}{5} \div \frac{3}{2} = \frac{2}{5} \times \frac{2}{3} = \frac{2 \times 2}{5 \times 3} = \frac{4}{15}$

(iv) $3\frac{1}{2} \div 4 = \frac{7}{2} \div \frac{4}{1} = \frac{7}{2} \times \frac{1}{4} = \frac{7 \times 1}{2 \times 4} = \frac{7}{8}$

Simplifying brackets in fractions

Example 1:

$$\text{Simplify}: \frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{1}{5}+\overline{\frac{3}{7}-\frac{1}{5}}\right)\right\}\right]$$

Solution:

Let us first solve bar brackets :

$$\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{1}{5}+\overline{\frac{3}{7}-\frac{1}{5}}\right)\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{1}{5}+\overline{\frac{15-7}{35}}\right)\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{1}{5}+\frac{8}{35}\right)\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\left(\frac{7+8}{35}\right)\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\frac{15}{35}\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{3}{4}-\frac{3}{7}\right\}\right]$$

$$=\frac{4}{7}+\left[\frac{1}{2}-\left\{\frac{21-12}{28}\right\}\right]=\frac{4}{7}+\left[\frac{1}{2}-\frac{9}{28}\right]$$

$$=\frac{4}{7}+\left[\frac{14-9}{28}\right]=\frac{4}{7}+\left[\frac{5}{28}\right]$$

$$=\frac{4}{7}+\frac{5}{28}=\frac{4\times4+5}{28}=\frac{21}{28}$$

Example 2:

$$\text{Simplify : } \frac{\frac{2}{3}-\overline{\frac{1}{2}-\frac{1}{3}} \text{ of } \frac{1}{2}}{\overline{\frac{5}{6}-\frac{2}{3}} \text{ of } \frac{1}{3}+\frac{1}{9}}$$

Solution:

$$\text{We have } \frac{\frac{2}{3}-\overline{\frac{1}{2}-\frac{1}{3}} \text{ of } \frac{1}{2}}{\overline{\frac{5}{6}-\frac{2}{3}} \text{ of } \frac{1}{3}+\frac{1}{9}}$$

$$= \frac{\frac{2}{3}-\overline{\frac{3-2}{6}} \text{ of } \frac{1}{2}}{\overline{\frac{5}{6}-\frac{2}{3}} \text{ of } \frac{1}{3}+\frac{1}{9}}$$

$$= \frac{\frac{2}{3}-\frac{1}{6} \text{ of } \frac{1}{2}}{\overline{\frac{5-4}{6}} \text{ of } \frac{1}{3}+\frac{1}{9}}$$

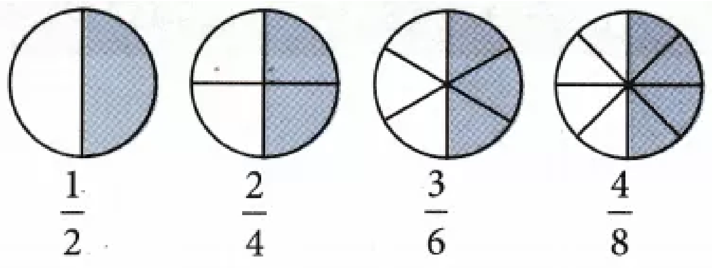
$$= \frac{\frac{2}{3}-\frac{1}{6}\times\frac{1}{2}}{\frac{1}{6}\times\frac{1}{3}+\frac{1}{9}} = \frac{\frac{2}{3}-\frac{1}{12}}{\frac{1}{18}+\frac{1}{9}}$$

$$= \frac{\frac{8-1}{12}}{\frac{1+2}{18}} = \frac{\frac{7}{12}}{\frac{3}{18}} = \frac{7}{12}\times\frac{18}{3} = \frac{7\times 6}{12\times 1} = \frac{7}{2}$$

What is an Equivalent Fraction

Equivalent Fraction

To understand the concept of equivalent fractions, let us take an example – Rama gave four cakes to his four children. The first child cut his cake into two equal halves and ate the first half. The second child cut his cake into four equal parts and ate two pieces out of four. The third child cut his cake into six equal parts and ate three of them, and the fourth child cut the cake into eight equal pieces and ate four of them.



Do you think, they have eaten equal parts of the cake? Yes.

1st child ate $\frac{1}{2}$ of the cake.

2nd child ate $\frac{2}{4}$th of the cake. $\left(=\frac{1}{2}\text{ of the cake}\right)$.

3rd child ate $\frac{3}{6}$th of the cake. $\left(=\frac{1}{2}\text{ of the cake}\right)$.

4th child ate $\frac{4}{8}$th of the cake. $\left(=\frac{1}{2}\text{ of the cake}\right)$.

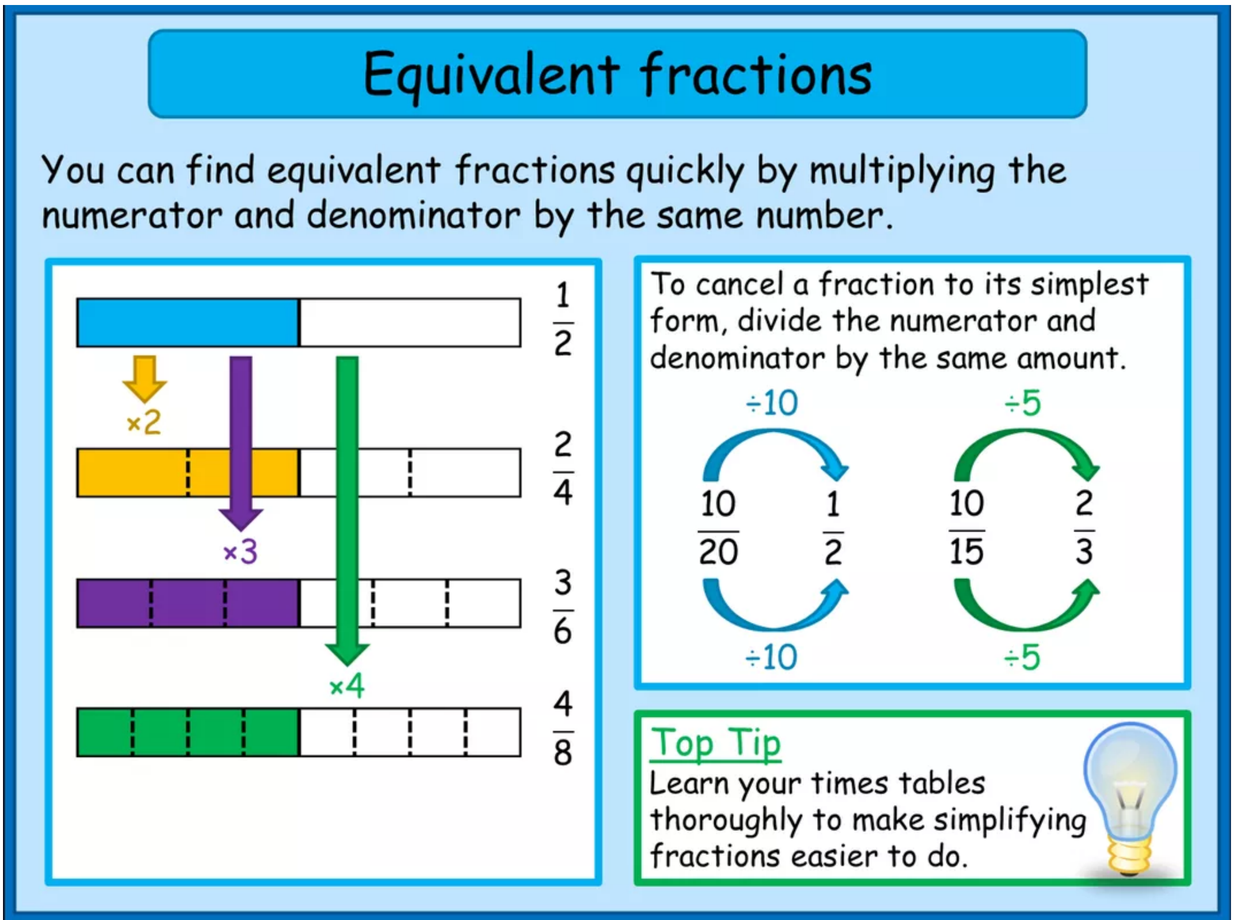
This means they all ate $\frac{1}{2}$ of the cake. Thus, fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ represent the same fraction $\frac{1}{2}$. These are called equivalent fractions. So, two or more fractions representing the same part (value) of the whole are called equivalent fractions.

To check if the fractions are equivalent or not, we do cross-multiplication.

For the fractions $\frac{a}{b}$ and $\frac{c}{d}$, if

(i) $a \times d = b \times c$ then $\frac{a}{b} = \frac{c}{d}$

(ii) $a \times d \neq b \times c$ then $\frac{a}{b} \neq \frac{c}{d}$



Note: We can get as many equivalent fractions as we want, by multiplying/dividing the numerator and denominator of the given fraction by the same number.

Example 1: Write five equivalent fractions of $\frac{3}{5}$

Solution:

$$\frac{3}{5}=\frac{3\times 2}{5\times 2}=\frac{3\times 3}{5\times 3}=\frac{3\times 4}{5\times 4}=\frac{3\times 5}{5\times 5}=\frac{3\times 6}{5\times 6}$$

$$\frac{3}{5}=\frac{6}{10}=\frac{9}{15}=\frac{12}{20}=\frac{15}{25}=\frac{18}{30}$$

Hence, the five equivalent fractions of $\frac{3}{5}$ are

$$\frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \frac{18}{30}.$$

Example 2: Which ofthefollowing pairs of fractions are equivalent:

$(i)\ \frac{5}{12}, \frac{30}{72}$ $\qquad$ $(ii)\ \frac{6}{7}, \frac{14}{12}$

Solution:

(i) By cross-multiplying the terms of the fractions,

$$\frac{5}{12} \times \frac{30}{72}$$

5 × 72 = 12 × 30
360 = 360

$\therefore$ $\frac{5}{12}$ is equivalent to $\frac{30}{72}$

(ii) By cross-multiplying the terms of the fractions,

$$\frac{6}{7} \times \frac{14}{12}$$

$$6\times 12 \neq 14\times 7$$

$$72 \neq 98$$

$\therefore$ $\frac{6}{7}$ is equivalent to $\frac{12}{14}$

Simplest Form of a Fraction

A fraction is said to be in the lowest term or the simplest form, if its numerator and denominator do not have any common factor other than 1.

Methods to reduce a fraction into the simplest form:

(i) Find the HCF of the numerator and the yg denominator of the given fraction.
(ii) Divide its numerator and denominator by their HCF.

$$\frac{162 \div 18}{90 \div 18} = \frac{9}{5}$$

Example: $\frac{162}{90}$

HCF of 162 and 90 = 18
Dividing the numerator and denominator of the fraction by their HCF,
So, $\frac{9}{5}$ is the lowest form of the given fraction because common factor of 9 and 5 is only 1.