STATISTICS

IMPORTANT POINTS

- The word data means information (its exact dictionary meaning is: given facts). Statistical data are of two types :
  (i) Primary data
  (ii) Secondary data

- When an investigator collects data himself with a definite plan or design in his (her) mind, it is called **Primary data**.

- Data which are not originally collected rather obtained from published or unpublished sources are known as **Secondary data**.

- After collection of data, the investigator has to find ways to condense then in tabular form in order to study their silent features. Such an arrangement is called **Presentation of data**.

- Raw data when put in ascending or descending order of magnitude is called an array or arranged data.

- The number of times an observation occurs in the given data is called frequency of the observation.

- Classes/class intervals are the groups in which all the observations are divided.

- Suppose class-interval is 10-20, then 10 is called lower limit and 20 is called upper limit of the class

- Mid-value of class-interval is called **Class-mark**

  Class-mark= $\frac{\text{lower limit } + \text{ upper limit}}{2}$

  Class-mark = lower limit + $\frac{1}{2}$

  (difference between the upper and lower limits)

- If the frequency of first class interval is added to the frequency of second class and this sum is added to third class and so on then frequencies so obtained are known as **Cumulative Frequency (c.f.).**

- There are two types of cumulative frequencies
  (a) less than, (b) greater than

❖ EXAMPLES ❖

Ex.1 Given below are the ages of 25 students of class IX in a school. Prepare a discrete frequency distribution.

15, 16, 16, 14, 17, 17, 16, 15, 15, 16, 16, 17, 15, 16, 16, 14, 16, 15, 14, 15, 16, 16, 15, 14, 15.

Sol. Frequency distribution of ages of 25 students

Age	Tally marks	Frequency
14	||||	4
15	𝍸 |||	8
16	𝍸 𝍸	10
17	|||	3
Total		25

Ex.2 Form a discrete frequency distribution from the following scores:-

Sol. 15, 18, 16, 20, 25, 24, 25, 20, 16, 15, 18, 18, 16, 24, 15, 20, 28, 30, 27, 16, 24, 25, 20, 18, 28, 27, 25, 24, 24, 18, 18, 25, 20, 16, 15, 20, 27, 28, 29, 16.

Frequency Distribution of Scores

Variate	Tally marks	Frequency
15	||||	4
16	𝍸 |	6
18	𝍸 |	6
20	𝍸 |	6
24	𝍸	5
25	𝍸	5
27	|||	3
28	|||	3
29	|	1
30	|	1
Total		40

Ex.3 The water tax bills (in rupees) of 30 houses in a locality are given below. Construct a grouped frequency distribution with class size of 10.

30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88, 40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 96, 102, 110, 88, 74, 112, 14, 34, 44.

Sol. Here the maximum and minimum values of the variate are 112 and 14 respectively.

$\therefore$ Range = 112 – 14 = 98.

It is given that the class size is 10, and

$$\frac{\text{Range}}{\text{Class size}} = \frac{98}{10} = 9.8$$

So, we shoule have 10 classes each of size 10.

The minimum and maximum values of the variate are 14 and 112 respectively. So we have to make the classes in such a way that first class includes the minimum value and the last class includes the maximum value. If we take the first class as 14-24 it includes the minimum value 14. If the last class is taken as 104-114, then it includes the maximum value 112. Here, we form classes by exclusive method. In the class 14-24, 14 is included but 24 is excluded. Similarly, in other classes, the lower limit is included and the upper limit is excluded.

In the view of above discussion, we construct the frequency distribution table as follows:

Bill (in rupees)	Tally marks	Frequency
14-24	||||	4
24-34	||	2
34-44	|||	3
44-54	|||	3
54-64	|	1
64-74	||	2
74-84	卌	5
84-94	|||	3
94-104	|||	3
104-114	||||	4
Total		30

Ex.4 The marks obtained by 40 students of class IX in an examination are given below :

18, 8, 12, 6, 8, 16, 12, 5, 23, 2,16, 23, 2, 10, 20, 12, 9, 7, 6, 5, 3, 5, 13, 21, 13, 15, 20, 24, 1, 7, 21, 16, 13, 18, 23, 7, 3, 18, 17, 16.

Present the data in the form of a frequency distribution using the same class size, one such class being 15-20 (where 20 is not included)

Sol. The minimum and maximum marks in the given raw data are 0 and 24 respectively. It is given that 15-20 is one of the class intervals and the class size is same. So, the classes of equal size are

0-5, 5-10, 10-15, 15-20 and 20-25

Thus, the frequency distribution is as given under :

Frequency Distribution of Marks

Marks	Tally marks	Frequency
0-5	卌 I	6
5-10	卌 卌 I	10
10-15	卌 III	8
15-20	卌 III	8
20-25	卌 III	8
	Total	40

Ex.5 The class marks of a distribution are :

47, 52, 57, 62, 67, 72, 77, 82, 87, 92, 97, 102 Determine the class size, the class limits and the true class limits.

Sol. Here the class marks are uniformly spaced. So, the class size is the difference between any two consecutive class marks

$\therefore$ Class size = 52 – 47 = 5

We know that, if a is the class mark of a class interval and h is its class size, then the lower and upper limits of the class interval are $a - \frac{h}{2}$ and $a + \frac{h}{2}$ respectively.

$\therefore$ Lower limit of first class interval

$= 47 - \frac{5}{2} = 44.5$

And, upper limit of first class interval

$= 47 + \frac{5}{2} = 49.5$

So, first class interval is 44.5 – 49.5

Similarly, we obtain the other class limits as given under :

Class marks	Class limits
47	44.5-49.5
52	49.5-54.5
57	54.5-59.5
62	59.5-64.5
67	64.5-69.5
72	69.5-74.5
77	74.5-79.5
82	79.5-84.5
87	84.5-89.5
92	89.5-94.5
97	94.5-99.5
102	99.5-104.5

Since the classes are exclusive, so the true class limits are same as the class limits.

Ex.6 The class marks of a distribution are 26, 31, 36, 41, 46, 51, 56, 61, 66, 71. Find the true class limits.

Sol. Here the class marks are uniformly spaced. So, the class size is the difference between any two consecutive class marks.

$\therefore$ Class size = 31 – 26 = 5.

If a is the class mark of a class interval of size h, then the lower and upper limits of the class interval are $a - \frac{h}{2}$ and $a + \frac{h}{2}$ respectively.

Here h = 5

$\therefore$ Lower limit of first class interval

$= 26 - \frac{5}{2} = 23.5$

And, upper limit of first class interval

$= 26 + \frac{5}{2} = 28.5$

$\therefore$ First class interval is 23.5 – 28.5.

Thus, the class intervals are:

23.5 – 28.5, 28.5 – 33.5, 33.5–38.5, 38.5–43.5, 43.5–48.5, 48.5–53.5

Since the classs are formed by exclusive method. Therefore, these limits are true class limits.

Cumulative Frequency

A table which displays the manner in which cumulative frequencies are distributed over various classes is called a cumulative frequency distribution or cumulative frequency table.

There are two types of cumulative frequency.

(1) Less than type (2) Greater than type

❖ EXAMPLES ❖

Ex.7 Write down less than type cumulative frequency and greater than type cumulative frequency.

Height (in cm)	Frequency
140 – 145	10
145 – 150	12
150 – 155	18
155 – 160	5
160 – 165	5
165 – 170	38
170 – 175	22
175 – 180	20

Sol. We have

Height (in cm)	140–145	145–150	150–155	155–160	160–165	165–170	170–175	175–180
Frequency	10	12	18	35	45	38	22	20
Height Less than type	145	150	155	160	165	170	175	180
Cumulative frequency	10	22	40	75	120	158	180	200
Height Greater than type	140	145	150	155	160	165	170	175
Cumulative frequency	200	190	178	160	125	80	42	20

Ex.8 The distances (in km) covered by 24 cars in 2 hours are given below :

125, 140, 128, 108, 96, 149, 136, 112, 84, 123, 130, 120, 103, 89, 65, 103, 145, 97, 102, 87, 67, 78, 98, 126

Represent them as a cumulative frequency table using 60 as the lower limit of the first group and all the classes having the class size of 15.

Sol. We have, Class size = 15

Maximum distance covered = 149 km.

Minimum distance covered = 65 km.

$\therefore$ Range = (149 – 65) km = 84 km.

So, number of classes = 6 $\left[\because \frac{84}{15} = 5.6\right]$

Thus, the class intervals are 60-75, 75-90, 90-105, 105-120, 120-135, 135-50.

The cumulative frequency distribution is as given below **:**

Class interval	Tally marks	Frequency	Cumulative frequency
60-75	||	2	2
75-90	||||	4	6
90-105	~~||||~~ |	6	12
105-120	||	2	14
120-135	~~||||~~ |	6	20
135-150	||||	4	24

Ex.9 The following table gives the marks scored by 378 students in an entrance examination :

Marks	No. of students
0-10	3
10-20	12
20-30	36
30-40	76
40-50	97
50-60	85
60-70	39
70-80	12
80-90	12
90-100	6

From this table form (i) the less than series, and (ii) the more than series.

Sol. (i) Less than cumulative frequency table

Marks obtained	Number of students (Cumulative frequency)
Less than 10	3
Less than 20	15
Less than 30	51
Less than 40	127
Less than 50	224
Less than 60	309
Less than 70	348
Less than 80	360
Less than 90	372
Less than 100	378

(ii) More than cumulative frequency table

Marks obtained	Number of students (Cumulative frequency)
More than 0	378
More than 9	375
More than 19	363
More than 29	327
More than 39	257
More than 49	154
More than 59	69
More than 69	30
More than 79	18
More than 89	6

Ex.10 Find the unknown entries (a,b,c,d,e,f,g) from the following frequency distribution of heights of 50 students in a class :

Class intervals (Heights in cm)	Frequency	Cumulative frequency
150-155	12	a
155-160	b	25
160-165	10	c
165-170	d	43
170-175	e	48
175-180	2	f
Total	g	

Sol. Since the given frequency distribution is the frequency distribution of heights of 50 students. Therefore,

$g = 50.$

From the table, we have

$a = 12,\ b + 12 = 25,\ 12 + b + 10 = c,$

$12 + b + 10 + d = 43,$

$12 + b + 10 + d + e = 48$ and

$12 + b + 10 + d + e + 2 = f$

Now, $b + 12 = 25 \Rightarrow b = 13$

$12 + b + 10 = c$

$\Rightarrow 12 + 13 + 10 = c \qquad [\because b = 13]$

$\Rightarrow c = 35$

$12 + b + 10 + d = 43$

$\Rightarrow 12 + 13 + 10 + d = 43 \quad [\because b = 13]$

$\Rightarrow d = 8$

$12 + b + 10 + d + e = 48$

$\Rightarrow 12 + 13 + 10 + 8 + e = 48$

$[\because b = 13,\ d = 8]$

$\Rightarrow e = 5$

and, $12 + b + 10 + d + e + 2 = f$

$\Rightarrow 12 + 13 + 10 + 8 + 5 + 2 = f$

$\Rightarrow f = 50.$

Hence, $a = 12,\ b = 13,\ c = 35,\ d = 8,$

$e = 5,\ f = 50$ and $g = 50.$

Ex.11 The marks out of 10 obtained by 32 students are : 2, 4, 3, 1, 5, 4, 3, 8, 9, 7, 8, 5, 4, 3, 6, 7, 4, 7, 9, 8, 6, 4, 2, 1, 0, 0, 2, 6, 7, 8, 6, 1. Array the data and form the frequency distribution

Sol. An array of the given data is prepared by arranging the scores in ascending order as follows :

0, 0, 1, 1, 1, 2, 2, 2, 3,3,3, 4,4,4,4,4,

5,5, 6,6,6,6, 7,7,7,7, 8,8,8,8, 9,9.

Frequency distribution of the marks is shown below.

Marks	Tally marks	Frequency
0	||	2
1	|||	3
2	|||	3
3	|||	3
4	~~||||~~	5
5	||	2
6	||||	4
7	||||	4
8	||||	4
9	||	2

Ex.12 Prepare a discrete frequency distribution from the data given below, showing the weights in kg of 30 students of class VI.

39, 38, 42, 41, 39, 38, 39, 42, 41, 39, 38, 38 41, 40, 41, 42, 41, 39, 40, 38, 42, 43, 45, 43, 39, 38, 41, 40, 42, 39.

Sol. The discrete frequency distribution table for the weight (in kg) of 30 students is shown below.

Weights (in kg)	Tally marks	Frequency
38	~~||||~~ |	6
39	~~||||~~ ||	7
40	|||	3
41	~~||||~~ |	6
42	~~||||~~	5
43	||	2
45	|	1

Ex.13 The class marks of a distribution are 82, 88, 94, 100, 106, 112 and 118. Determine the class size and the classes.

Sol. The class size is the difference between two consecutive class marks. ∴ Class size = 88 – 82 = 6. Now 82 is the class mark of the first class whose width is 6. ∴ Class limits of the first class are $82 - \frac{6}{2}$ and $82 + \frac{6}{2}$ i.e. 79 and 85. Thus, the first class is 79-85. Similarly, the other classes are 85–91, 91– 97, 97–103, 103 –109, 109 –115 and 115 –121.

Ex.14 The class marks of a distribution are 13, 17, 21, 25 and 29. Find the true class limits.

Sol. The class marks are 13, 17, 21, 25 and 29.

The class marks are uniformly spaced.

Class size = difference between two consecutive class marks

$= 17 - 13 = 4$

Half of the class size $= \frac{4}{2} = 2$

To find the classes one has to subtract 2 from and add 2 to each of the class marks.

Hence, the classes are

11– 15

15 – 19

19 – 23

23 – 27

27 – 31

Since the classes are exclusive, the true class limits are the same as the class limits. So the lower class limits as well as the true lower class limits are 11, 15, 19, 23 and 27. The upper class limits as well as the true upper class limits are 15, 19, 23, 27 and 31.

Ex.15 Convert the given simple frequency series into a :

(i) Less than cumulative frequency series.

(ii) More than cumulative frequency series.

Marks	No. of students
0-10	3
10-20	7
20-30	12
30-40	8
40-50	5

Sol. (i) Less than cumulative frequency series

Marks	No. of students
Less than 10	3
Less than 20	10 (= 3 + 7)
Less than 30	22 (= 3 + 7 + 12)
Less than 40	30 (= 3 + 7 + 12 + 8)
Less than 50	35 (= 3 + 7 + 12 + 8 + 5)

(ii) More than cumulative frequency series

Marks	No. of students
More than 50	0
More than 40	5
More than 30	13
More than 20	25
More than 10	32
More than 0	35

Ex.16 Convert the following more than cumulative frequency series into simple frequency series.

Marks	No. of students
More than 0	40
More than 10	36
More than 20	29
More than 30	16
More than 40	5
More than 50	0

Sol. Simple frequency distribution table

Marks	Frequency
0-10	4 (= 40 – 36)
10-20	7 (= 36 – 29)
20-30	13 (= 29 – 16)
30-40	11 (= 16 – 5)
40-50	5 (= 5 – 0)

Ex.17 A test was conducted in a class of 30 students with maximum marks 25. The marks obtained are given below. Construct a frequency distribution table with a class size of 5, 28, 27, 20, 21, 17, 18, 18, 10, 12, 8, 7, 15, 10, 11, 25, 19, 21, 17, 12, 18, 4, 8, 12, 17, 9, 5, 7, 12, 18, 27

Sol.

Marks	Tally	Number of Students
0 – 5	I	1
5 – 10	~~IIII~~	5
10 – 15	~~IIII~~ III	8
15 – 20	~~IIII~~ IIII	9
20 – 25	III	3
25 – 30	IIII	4
		Total = 30

Ex.18 The marks obtained by 35 students in a class are given below. Construct the cumulative frequency table :

Marks obtained	Number of students
0	1
1	2
2	4
3	4
4	3
5	5
6	4
7	6
8	3
9	2
10	1

Sol.

Marks	Frequency	Number of students
0	1	1
1	2	3 (=1 + 2)
2	4	7 (=1 + 2 + 4)
3	4	11 (=1 + 2 + 4 + 4)
4	3	14 (=1 + 2 + 4 + 4 + 3)
5	5	19 (=1 + 2 + 4 + 4 + 3 + 5)
6	4	23 (=1 + 2 + 4 + 4 + 3 + 5 + 4)
7	6	29 (=1 + 2 + 4 + 4 + 3 + 5 + 4 + 6)
8	3	32 (=1 + 2 + 4 + 4 + 3 + 5 + 4 + 6 + 3)
9	2	34 (=1 + 2 + 4 + 4 + 3 + 5 + 4 + 6 + 3 + 2)
10	1	35 (=1 + 2 + 4 + 4 + 3 + 5 + 4 + 6 + 3 + 2 + 1)
	Total = 35	

Ex.19 The distribution of ages (in years) of 40 persons in a colony is given below.

Age (in years)	Number of Persons
20-25	7
25-30	10
30-35	8
35-40	6
40-45	4
45-50	5

(a) Determine the class mark of each class

(b) What is the upper class limit of 4th class

(c) Determine the class size

Sol. (a) Class marks are

$$\frac{20+25}{2}, \frac{25+30}{2}, \frac{30+35}{2}, \frac{35+40}{2}, \frac{40+45}{2}, \frac{45+50}{2}$$

= 22.5, 27.5, 32.5, 37.5, 42.5, 47.5

(b) The fourth class interval is 35–40. Its upper limit is 40

(c) The class size is 25 – 20 = 5

Ex.20 Following is the distribution of marks of 40 students in a class. Construct a cumulative frequency distribution table.

Marks	Number of students
0-10	3
10-20	8
20-30	9
30-40	15
40-50	5

Sol.

Class interval	Frequency	Cumulative Frequency
0–10	3	3
10–20	8	11 (= 3 + 8)
20–30	9	20 (= 3 + 8 + 9)
30–40	15	35 (= 3 + 8 + 9 + 15)
40–50	5	40 (= 3 + 8 + 9 + 15 + 5)
	Total = 40	

Ex.21 The class marks of a distribution are 25, 35, 45, 55, 65 and 75.
Determine the class size and class limit.

Sol. Class size = The difference between the class marks of two adjacent classes.
= 35 – 25 = 10
We need classes of size 10 with class marks as 25, 35, 45, 55, 65, 75
The class limits for the first class are $25 - \frac{10}{2}$ and $25 + \frac{10}{2}$
i.e. 20 and 30
First class is, therefore, 20–30
Similarly, the other classes are 30 – 40, 40 – 50, 50 – 60, 60 – 70, 70 – 80

Ex.22 Given below is the cumulative frequency distribution table showing the marks secured by 40 students.

Marks	Number of students
Below 20	5
Below 40	10
Below 60	25
Below 80	32
Below 100	40

Sol.

Marks	Cumulative frequency	Frequency
0-20	5	5
20-40	10	5 (= 10 – 5)
40-60	25	15 (= 25 – 10)
60-80	32	7 (= 32 – 25)
80-100	40	8 (= 40 – 32)

Mean

If $x_1, x_2, x_3, \ldots, x_n$ are n values of a variable X, then the arithmetic mean or simply the mean of these values is denoted by $\overline{X}$ and is defined as

$$\overline{X} = \frac{x_1 + x_2 + x_3 .... + x_n}{n} = \frac{1}{n}\left(\int_{i=1}^{n} x_i\right)$$

Here the symbol $\sum_{i=1}^{n} x_i$ denotes the sum $x_1 + x_2 + x_3 + ... + x_n$.

◈ If $\overline{X}$ is the mean of n observations $x_1, x_2..x_n$, then prove that $\sum_{i=1}^{n}(x_i - \overline{X}) = 0$ i.e. the algebraic sum of deviations from mean is zero.

◈ If $\overline{X}$ is the mean of n observations $x_1, x_2,...., x_n$, then the mean of the observations $x_1 + a, x_2 + a,..., x_n + a$ is $\overline{X} + a$ i.e. if each observation is increased by a, then the mean is also increased by a

◈ If $\overline{X}$ is the mean of $x_1, x_2,....x_n$ then the mean of $ax_1, ax_2,...ax_n$ is $a\overline{X}$, where a is any number different from zero i.e. if each observation is multiplied by a non-zero number a, then the mean is also multiplied by a.

◈ If $\overline{X}$ is the mean of n observations $x_1, x_2, x_3..., x_n$, then the mean of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, ..., \frac{x_n}{a}$ is $\frac{\overline{X}}{a}$, where a is any non-zero number

◈ If $\overline{X}$ is the mean of n observations $x_1, x_2 ...x_n$, then the mean of $x_1 - a, x_2 - a,..., x_n - a$ is $\overline{X} - a$, where a is any real number

◈ **Advantages**

(i) Arithmetic mean is simple to understand and easy to calculate.

(ii) It is rigidly defind.

(iii) It is suitable for further algebraic treatment.

(iv) It is least affected fluctuation of sampling.

(v) It is least affected fluctuation of sampling.

(vi) It takes into account all the values in the series.

◈ **Disadvantages**

(i) It is highly affected by the presence of a few abnormally high or abnormally low scores.

(ii) In absence of a single item, its value becomes inaccurate.

(iii) It can not be determined by inspection.

❖ EXAMPLES ❖

Ex.23 If the mean of n observations $ax_1, ax_2, ax_3,..., ax_n$ is $a\overline{X}$, show that

$$(ax_1 - a\overline{X}) + (ax_2 - a\overline{X}) + ... + (ax_n - a\overline{X}) = 0$$

Sol. We have

$$a\overline{X} = \frac{ax_1 + ax_2 + ... + ax_n}{n}$$

$\Rightarrow ax_1 + ax_2 + ... + ax_n = n(a\overline{X})$ ....(i)

Now, $(ax_1 - a\overline{X}) + (ax_2 - a\overline{X}) + ... +$

$(ax_n - a\overline{X})$

$= (ax_1 + ax_2 + ... + ax_n) - (\underbrace{a\overline{X} + a\overline{X} + ... + a\overline{X}}_{\textbf{n - terms}})$

$= n(a\overline{X}) - n(a\overline{X}) = 0.$

Ex.24 The mean of n observations $x_1, x_2, ..., x_n$ is $\overline{X}$. If (a – b) is added to each of the observations, show that the mean of the new set of observations is $\overline{X} + (a - b)$

Sol. We have,

$\overline{X} = \frac{x_1 + x_2 + ... + x_n}{n}$ ....(i)

Let $\overline{X}'$, be the mean of $x_1 + (a - b), x_2 + (a - b), ..., x_n + (a - b)$. Then,

$\overline{X}' = \frac{\{x_1 + (a-b)\} + \{x_2 + (a-b)\} + ... + \{x_n + (a-b)\}}{n}$

$\Rightarrow \overline{X}' = \frac{x_1 + x_2 + ... + x_n + n(a-b)}{n}$

$= \overline{X} + (a - b)$ (using (i))

Ex.25 Find the sum of the deviations of the variate values 3, 4, 6, 8, 14 from their mean.

Sol. Recall that the deviations of the values $x_1, x_2, x_3, ..., x_n$ about A are

$x_1 - A, x_2 - A, x_3 - A, ..., x_n - A.$

Let $\overline{X}$ be the mean of the values 3, 4, 6, 8, 14. Then,

$\overline{X} = \frac{3+4+6+8+14}{5} = \frac{35}{5} = 7$

Now, sum of the deviations of the values 3, 4, 6, 8, 14 from their mean $\overline{X} = 7$ is given by

$= (3 - 7) + (4 - 7) + (6 - 7) + (8 - 7)$

$+ (14 - 7) = -4 - 3 - 1 + 1 + 7 = 0.$

Ex.26 The mean of 40 observations was 160. It was detected on rechecking that the value of 165 was wrongly copied as 125 for computation of mean. Find the correct mean.

Sol. $\because$ Here, $n = 40$, $\overline{X} = 160$

So, $\overline{X} = \frac{1}{n}\left(\sum x_i\right) \Rightarrow 160 = \frac{1}{40}\left(\sum x_i\right)$

$\Rightarrow \sum x_i = 160 \times 40 = 6400$

$\Rightarrow$ Incorrect value of $\sum x_i = 6400$

Now,

Correct value of $\sum x_i$

= Incorrect value of $\sum x_i$ – Incorrect item + Correct item

$\Rightarrow$ Correct value of $\sum x_i = 6400 - 125 + 165 = 6440$

$\therefore$ Correctmean

$= \frac{\text{Correct value of} \sum x_i}{n} = \frac{6440}{40} = 161.$

Ex.27 The mean of 10 numbers is 20. If 5 is subtracted from every number, what will be the new mean ?

Sol. Let $x_1, x_2, ..., x_{10}$ be 10 numbers with their mean equal to 20. Then,

$$\overline{X} = \frac{1}{n}\left(\sum x_i\right)$$

$$\Rightarrow 20 = \frac{x_1 + x_2 + ... + x_{10}}{10}$$

$$\Rightarrow x_1 + x_2 + ... + x_{10} = 200 \qquad ....(i)$$

New numbers are $x_1 - 5, x_2 - 5, ..., x_{10} - 5$. Let $\overline{X}'$ be the mean of new numbers.

Then,

$$\overline{X}' = \frac{(x_1 - 5) + (x_2 - 5) + ... + (x_{10} - 5)}{10}$$

$$\overline{X}' = \frac{(x_1 + x_2 + ... + x_{10}) - 5 \times 10}{10} = \frac{200 - 50}{10}$$

[Using (i)]

$$\overline{X}' = 15.$$

Ex.28 The mean of 16 numbers is 8. If 2 is added to every number, what will be the new mean ?

Sol. Let $x_1, x_2, x_3, ..., x_{16}$ be 16 numbers with their mean equal to 8. Then,

$$\overline{X} = \frac{1}{n}\left(\sum x_i\right)$$

$$\Rightarrow 8 = \frac{x_1 + x_2 + ... + x_{16}}{16}$$

$\Rightarrow x_1 + x_2 + ... + x_{16} = 16 \times 8 = 128$...(i) New numbers are $x_1 + 2, x_2 + 2, x_3 + 2, ..., x_{16} + 2$. Let $\overline{X}'$ be the mean of new numbers. Then,

$$\overline{X}' = \frac{(x_1 + 2) + (x_2 + 2) + ... + (x_{16} + 2)}{16}$$

$$\Rightarrow \overline{X}' = \frac{(x_1 + x_2 + ... + x_{16}) + 2 \times 16}{16} = \frac{128 + 32}{16}$$

[Using(i)]

$$\Rightarrow \overline{X}' = \frac{160}{16} = 10$$

Ex.29 If $x_1, x_2, ..., x_n$ are n values of a variable X such that

$\sum_{i=1}^{n}(x_i - 2) = 110$ and $\sum_{i=1}^{n}(x_i - 5) = 20$. Find the value of n and the mean.

Sol. We have,

$$\sum_{i=1}^{n}(x_i - 2) = 110 \text{ and } \sum_{i=1}^{n}(x_i - 5) = 20$$

$$\Rightarrow (x_1 - 2) + (x_2 - 2) + ... + (x_n - 2) = 110$$

and $(x_1 - 5) + (x_2 - 5) + ... + (x_n - 5) = 20$

$$\Rightarrow (x_1 + x_2 + ... + x_n) - 2n = 110 \text{ and}$$

$$(x_1 + x_2 + ... + x_n) - 5n = 20$$

$\Rightarrow \sum_{i=1}^{n} x_i - 2n = 110$ and $\sum_{i=1}^{n} x_i - 5n = 20$

$\Rightarrow$ S – 2n = 110 and S – 5n = 20

Thus, we have

$S - 2n = 110$ ....(i)

and $S - 5n = 20$ ....(ii)

Subtracting (ii) from (i), we get

$3n = 90 \Rightarrow n = 30$

Putting n = 30 in (i), we get

$S - 60 = 110 \Rightarrow S = 170$

$\Rightarrow \sum_{i=1}^{n} x_i = 170$

$\therefore$ Mean $= \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{170}{30} = \frac{17}{3}$

Hence, n = 30 and mean $\frac{17}{3}$.

Arithmetic Mean of Ungrouped Data

Arithmetic mean of raw data (when frequency is not given) :

The arithmetic mean of a raw data is obtained by adding all the values of the variables and dividing the sum by total number of values that are added.

Arithmetic mean

$(\bar{x}) = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$

The symbol $\sum_{i=1}^{n} x_i$ denotes the sum $x_1 + x_2 + ... + x_n$.

❖ EXAMPLES ❖

Ex.30 Neeta and her four friends secured 65, 78, 82, 94 and 71 marks in a test of mathematics. Find the average (arithmetic mean) of their marks.

Sol. Arithmetic mean or average

$= \frac{65 + 78 + 82 + 94 + 71}{5} = \frac{390}{5} = 78$

Hence, arithmetic mean = 78

Ex.31 The marks obtained by 10 students in physics out of 40 are 24, 27, 29, 34, 32, 19, 26, 35, 18, 21. Compute the mean of the marks.

Sol. Mean of the marks is given by

$$\bar{x} = \frac{24+27+29+34+32+19+26+35+18+21}{10}$$

$$= \frac{265}{10} = 26.50$$

Ex.32 The mean of 20 observations was found to be 47. But later it was discovered that one observation 66 was wrongly taken as 86. Find the correct mean.

Sol. Here, $n = 20$, $\bar{x} = 47$

We have, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\therefore$ $47 = \frac{\sum_{i=1}^{n} x_i}{20}$

$$\sum_{i=1}^{n} x_i = 47 \times 20 = 940.$$

But the score 66 was wrongly taken as 86.

$\therefore$ Correct value of $\sum_{i=1}^{n} x_i = 940 + 66 - 86 = 920$

$\therefore$ Correct mean $= \frac{920}{20} = 46$

Ex.33 A car owner buys petrol at Rs. 20.00, Rs. 24.00 and Rs. 25.00 per litre for three successive years. Compute the average cost per litre of petrol when he spends Rs. 12000 on petrol each year.

Sol. Here, the amounts of petrol purchased in three successive years are $\frac{12000}{20.00}$ litres, $\frac{12000}{24.00}$ litres and $\frac{12000}{25.00}$ litres respectively i.e. 600 litres, 500 litres and 480 litres respectively.

Total amount of money spent on petrol in 3 years

= Rs 12000 × 3 = Rs 36000

$\therefore$ Average cost per litre of petrol

$$= \frac{\text{Total amount of money spent on petrol}}{\text{Total amount of petrol purchased}}$$

$$= \frac{\text{Rs } 36000}{(600+500+480)\text{ litres}} = \frac{\text{Rs } 36000}{1580 \text{ litres}}$$

= Rs 22.78 per litre (approx.)

Ex.34 Find the mean of the following numbers : 12, 14, 17, 25, 10, 11, 20, 8, 15, and 18.

Sol. Mean $= \frac{12+14+17+25+10+11+20+8+15+18}{10}$

$$= \frac{150}{10} = 15$$

Ex.35 The mean of 5, 7, p, 11, 15, 17, and 20 is 12, find p.

Sol. Mean $= \frac{5+7+p+11+15+17+20}{7}$

$\Rightarrow 12 = \frac{75+p}{7}$

$\Rightarrow p + 75 = 12 \times 7 \quad \Rightarrow p = 9$

Ex.36 If $\bar{x}$ denote the mean of $x_1, x_2, ..., x_n$, show that

$$\sum_{i=1}^{n} = (x_i - \bar{x})$$

Sol. $\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}$

$= x_1 + x_2 + ... + x_n = n\bar{x}$ (i)

$= \Sigma(x_1 - \bar{x}) = (x_1 - \bar{x}) + (x_2 - \bar{x}) + ...$

$... + (x_n - x_1) = (x_1 + x_2 + ... + x_n) - n\bar{x}$

$= n\bar{x} - n\bar{x}$

$= 0$ (from (i))

Ex.37 If the mean of 5 observations is 15 and that of another 10 observations is 20, find the mean of all 15 observations

Sol. Let first five observations be $x_1,...,x_5$

$\Rightarrow$ Mean $= \frac{x_1 + x_2 ... + x_5}{5}$

$\Rightarrow 15 = \frac{x_1 + x_2 ... + x_5}{5}$

$\Rightarrow x_1 + ... + x_5 = 75$ (i)

Let next ten observations be $y_1 + ... + y_{10}$.

$\Rightarrow$ Mean $= \frac{y_1 + ... + y_{10}}{10}$

$\Rightarrow 20 = \frac{y_1 + ... + y_{10}}{10}$

$y_1 + ... + y_{10} = 200$ (ii)

The mean of all 15 observations will be

$\frac{(x_1 + ... + x_5) + (y_1 + ... + y_{10})}{15}$

$= \frac{75 + 200}{15}$ (from (i) and (ii))

$= 18.33$

If a variate X takes values $x_1, x_2 ... x_n$ with corresponding frequencies $f_1, f_2, f_3, ..., f_n$ respectively, then arithmetic mean of these values is $\bar{X} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + ... + f_n}$

or $\bar{X} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$, where $N = \sum_{i=1}^{n} f_i = f_1 + f_2 + ... + f_n$

Ex.38 Find the mean of the following distribution :

x :	4	6	9	10	15
f :	5	10	10	7	8

Sol. Calculation of Arithmetic Mean

x_i	f_i	f_ix_i
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
	$N = \Sigma f_i = 40$	$\Sigma f_i x_i = 360$

$\therefore$ Mean $= \overline{X} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{360}{40} = 9.$

Ex.39 Find the mean of the following distribution :

x :	10	30	50	70	89
f :	7	8	10	15	10

Sol. Calculation of Mean

x_i	f_i	f_ix_i
10	7	70
30	8	240
50	10	500
70	15	1050
89	10	890
	$\Sigma f_i = N = 50$	$\Sigma f_i x_i = 2750$

$\therefore$ Mean $= \frac{\Sigma f_i x_i}{N} = \frac{2750}{50} = 55.$

Ex.40 Find the value of p, if the mean of following distribution is 7.5.

x :	3	5	7	9	11	13
y :	6	8	15	p	8	4

Sol. Calculation of Mean

x_i	f_i	f_ix_i
3	6	18
5	8	40
7	15	105
9	p	9p
11	8	88
13	4	52
	$N = Sf_i = 41 + p$	$Sf_ix_i = 303 + 9p$

We have, $\Sigma f_i = 41 + p$, $\Sigma f_i x_i = 303 + 9p$

$\therefore$ Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow 7.5 = \frac{303 + 9p}{41 + p}$

$\Rightarrow$ 7.5 × (41 + p) = 303 + 9p

$\Rightarrow$ 307.5 + 7.5p = 303 + 9p

$\Rightarrow$ 9p − 7.5p = 307.5 − 303

$\Rightarrow$ 1.5p = 4.5 $\Rightarrow$ p = 3

Ex.41 Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 1.46.

Number of accidents (x) :	0	1	2	3	4	5	Total
Frequency (f) :	46	?	?	25	10	5	200

Sol. Let the missing frequencies be f_t and f_2

Calculation of Mean

x_i	f_i	f_ix_i
0	46	0
1	f_1	f_1
2	f_2	75
3	25	$2f_2$
4	10	40
5	5	25
$N = 86 + f_1 + f_2$	$\Sigma f_ix_i = 140 + f_1 + 2f_2$	

We have : N = 200

$\therefore$ $200 = 86 + f_1 + f_2 \Rightarrow f_1 + f_2 = 114$ ....(i)

Also,

Mean = 1.46

$\Rightarrow 1.46 = \frac{\Sigma f_ix_i}{N}$

$\Rightarrow 1.46 = \frac{140 + f_2 + 2f_2}{200}$

$\Rightarrow 292 = 140 + f_1 + 2f_2$

$\Rightarrow f_1 + 2f_2 = 152$ ....(ii)

Solving (i) and (ii) we get $f_1 = 76$ and $f_2 = 38$.

Ex.42 If the mean of the following data be 9.2, find the value of p.

x	4	6	7	p + 4	12	12
f	5	6	4	10	8	7

Sol. The table is rewritten as below :

x	f	f.x
4	5	20
6	6	36
7	4	28
p+4	10	10p + 40
12	8	96
14	7	98
Total	40	318 + 10p

Now, Mean $\overline{x} = \frac{\Sigma f.x}{\Sigma f} = \frac{318 + 10.p}{40}$

$\therefore\ 9.2 = \frac{318 + 10.p}{40}$

$\Rightarrow 318 + 10.p = 368$

$\Rightarrow 10p = 50 \Rightarrow p = 5$

Ex.43 The marks of 30 students are given below, find the mean marks.

Marks	Number of Students
10	4
11	3
12	8
13	6
14	7
15	2

Sol.

x	f	fx
10	4	40
11	3	33
12	8	96
13	6	78
14	7	98
15	2	30
	$\Sigma f = 30$	$\Sigma fx = 375$

Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{375}{30} = 12.5$

Grouped Frequency Distribution

There are 3 methods for calculation of mean :

1. Direct Method
2. Assumed mean deviation method
3. Step deviation method.

Ex.48 A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
No. of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean and why ?

Sol.

Number of plants	Number of houses (f)	Mid value x	f x
0 – 2	1	1	1
2 – 4	2	3	6
4 – 6	1	5	5
6 – 8	5	7	35
8 – 10	6	9	54
10 – 12	2	11	22
12 – 14	3	13	39
	$\Sigma f = 20$		$\Sigma fx = 162$

Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{162}{20} = 8.1$

Here, we have used direct method because numerical values of x and f are small.

Ex.49 Find the mean of the following distribution by direct method.

Class interval	0 – 10	11 – 20	21 – 30	31 – 40	41 – 50
Frequency	3	4	2	5	6

Sol.

Class interval	Frequency f	Mid value x	f x
0 – 10	3	5.0	15.0
11 – 20	4	15.5	62.0
21 – 30	2	25.5	51.0
31 – 40	5	35.5	177.5
41 – 50	6	45.5	273.0
	$\Sigma f = 20$		$\Sigma fx = 578.5$

$\therefore$ Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{578.5}{20} = 28.9$

Ex.50 For the following distribution, calculate mean using all the suitable methods.

Size of Item	1 – 4	4 – 9	9 – 16	16 – 27
Frequency	6	12	26	20

Sol.

Size of item	Mid value (x_i)	Frequency (f_i)	$f_i x_i$
1– 4	2.5	6	15
4– 9	6.5	12	78
9– 16	12.5	26	325
16– 27	21.5	20	430
		$\Sigma f_i = 64$	$\Sigma f_i x_i = 848$

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{848}{64} = 13.25$

❖ Assumed Mean Method

$$\text{Arithmetic mean} = a + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

Note : The assumed mean is chosen, in such a manner, that

1. It should be one of the central values.
2. The deviation are small.
3. One deviation is zero.

Working Rule :

Step 1: Choose a number 'a' from the central values of x of the first column, that will be our assumed mean.

Step 2: Obtain deviations d_i by subtracting 'a' from x_i. Write down hese deviations against the corresponding frequencies in the third column.

Step 3: Multiply the frequencies of second column with corresponding deviations d_i in the third column to prepare a fourth column of $f_i d_i$.

Step 4: Find the sum of all the entries of fourth column to obtain Σfidi and also, find the sum of all the frequencies in the second column to obtain Σf_i.

❖ EXAMPLES ❖

Ex.51 The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

Expenditure (in rupees)	100-150	150-200	200-250	250-300	300-350	350-400	400-450	450-500
Frequency	24	40	33	28	30	22	16	7

Sol. Let assumed mean = 275

Expenditure (in rupees)	Frequency (f^i)	Mid value (x^i)	$d^i = x - 275$	$f^i d^i$
100–150	24	125	–150	–3600
150–200	40	175	–100	–4000
200–250	33	225	–50	–1650
250– 300	28	275	0	0
300– 350	30	325	50	1500
350– 400	22	375	100	2200
400– 450	16	425	150	2400
450– 500	7	475	200	1400
	$\Sigma f^i = 200$			$\Sigma f^i d^i = -1750$

$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 275 + \frac{-1750}{200}$ = Rs 266.25

Ex.52 Calculate the arithmetic mean of the following distribution :

Class Interval	Frequency
0 – 50	17
50 –100	35
100 –150	43
150–200	40
200– 250	21
250– 300	24

Sol. Let assumed mean = 175 i.e. a = 175

Class	Mid value (x_i)	$d_i = x_i - 175$	frequency f_i	$f_i d_i$
0–50	25	–150	17	–2550
50–100	75	–100	35	–3500
100–150	125	–50	43	–2150
150–200	175	0	40	0
200-250	225	50	21	1050
250–300	275	100	24	2400
			$\Sigma f_i = 180$	$\Sigma f_i d_i = -4750$

Now , a = 175

$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 175 + \frac{-4750}{180} = 175 - 26.39$

= 148.61 approx.

Ex.53 Calculate the arithmetic mean of the following frequency distribution :

Class interval	50– 60	60–70	70–80	80–90	90– 100
Frequency	8	6	12	11	13

Sol. Let assumed mean = 75 i.e., a = 75

Class	frequency f_i	Mid value (x_i)	$d_i = x_i - 75$	$f_i d_i$
50–60	8	55	–20	–160
60–70	6	65	–10	–60
70–80	12	75	0	0
80–90	11	85	10	110
90–100	13	95	20	260
	$\Sigma f = 50$			$\Sigma f_i d_i = 150$

a = 75, $\Sigma f_i d_i = 150$, $\Sigma f_i = 50$

Mean $(\bar{x}) = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 75 + \frac{150}{50} = 78$

$\Rightarrow$ 18 (44 + f) = 752 + 20f

$\Rightarrow$ 752 + 20f = 792 + 18f

$\Rightarrow$ 2f = 40 $\Rightarrow$ f = 20

Hence, the missing frequency is 20.

Ex.60 The arithmetic mean of the following frequency distribution is 50. Find the value of p.

Class-Interval	0–20	20–40	40–60	60–80	80–100
Frequency	17	P	32	24	19

Sol.

Class-interval	Frequency (f)	Mid value (x)	fx
0 – 20	17	10	170
20 – 40	P	30	30 P
40 – 60	32	50	1600
60 – 80	24	70	1680
80 – 100	19	90	1710
	$\Sigma f = 92 + P$		$\Sigma fx = 5160 + 30P$

Mean $\bar{x} = \frac{\Sigma fx}{\Sigma f}$ $\Rightarrow 50 = \frac{5160 + 30P}{92 + P}$

$\Rightarrow$ 50 (92 + P) = 5160 + 30 P

$\Rightarrow$ 4600 + 50 P = 5160 + 30P

$\Rightarrow$ 20 P = 560 $\Rightarrow$ P = 28

Ex.61 The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2:

Class-Interval	0–20	20–40	40–60	60–80	80–100	100–120	Total
Frequency	5	f1	10	f2	7	8	50

Sol.

Class-interval	Frequency (f)	Mid value (x)	fx
0 – 20	5	10	50
20 – 40	f_1	30	$30f_1$
40 – 60	10	50	500
60 – 80	f_2	70	$70f_2$
80 – 100	7	90	630
100 – 120	8	110	880
	$\Sigma f = 30 + f_1 + f_2 = 50$		$\Sigma fx = 2060 + 30f_1 + 70f_2$

$30 + f_1 + f_2 = 50 \Rightarrow f_1 + f_2 = 20$ ....(1)

Mean $\bar{x} = \frac{\Sigma fx}{\Sigma f}$ $\Rightarrow 62.8 = \frac{2060 + 30f_1 + 70f_2}{50}$

$\Rightarrow 62.8 = \frac{206 + 3f_1 + 7f_2}{5}$

$\Rightarrow 206 + 3f_1 + 7f_2 = 314$

$\Rightarrow 3f_1 + 7f_2 = 108$ ....(2)

$3f_1 + 3f_2 = 60$ ....(3) [Multiplying (1) by 3] On Subtracting (3) from (2), we get

$4f_2 = 48 \quad \Rightarrow f_2 = 12$

Putting $f_2 = 12$ in (1), we get

$f_1 = 8$

Median

Median of a distribution is the value of the variable which divides the distribution into two equal parts i.e. it is the value of the variable such that the number of observations above it is equal to the number of observations below it.

- If the values x_i in the raw data. are arranged in order of increasing or decreasing magnitude, then the middle, most value in the arrangement is called the median.

Algorithm :

Step I : Arrange the observations (values of the variate) in ascending or descending order of magnitude.

Step II : Determine the total number of observations, say, n.

Step III : If n is odd, then

Median = value of $\left(\frac{n+1}{2}\right)^{th}$ observation

If n is even, then

Median

$$= \frac{\text{Value of } \left(\frac{n}{2}\right)^{th} \text{ observation} + \text{Value of } \left(\frac{n}{2}+1\right)^{th} \text{ observation}}{2}$$

- The median can be calculated graphically, while mean cannot be.
- The sum of the absolute deviations taken from the median is less than the sum of the absolute deviations taken from any other observation in the data.
- Median is not affected by extreme values.

❖ EXAMPLES ❖

Ex.62 Find the median of the following data :

25, 34, 31, 23, 22, 26, 35, 28, 20, 32

Sol. Arranging the data in ascending order, we get20, 22, 23, 25, 26, 28, 31, 32, 34, 35

Here, the number of observations n = 10 (even).

$$\therefore \text{ Median} = \frac{\text{Value of } \left(\frac{10}{2}\right)^{th} \text{ observatio n} + \text{Value of } \left(\frac{10}{2}+1\right)^{th} \text{ observatio n}}{2}$$

$$\Rightarrow \text{Median} = \frac{\text{Value of } 5^{th} \text{ observation} + \text{value of } 6^{th} \text{ observation}}{2}$$

$$\therefore \text{ Median} = \frac{26+28}{2} = 27$$

Hence, median of the given data is 27.

Ex.63 Find the median of the following values :

37, 31, 42, 43, 46, 25, 39, 45, 32

Sol. Arranging the data in ascending order, we have 25, 31, 32, 37, 39, 42, 43, 45, 46

Here, the number of observations n = 9 (odd)

$\therefore$ Median = Value of $\left(\frac{9+1}{2}\right)^{th}$ observation

= Value of 5th observation = 39.

Ex.64 The median of the observations 11, 12, 14, 18, x + 2, x + 4, 30, 32, 35, 41 arranged in ascending order is 24. Find the value of x.

Sol. Here, the number of observations n = 10. Since n is even, therefore

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th}\text{ observation} + \left(\frac{n}{2}+1\right)^{th}\text{ observation}}{2}$$

$$\Rightarrow 24 = \frac{5^{th}\text{ observation} + 6^{th}\text{ observation}}{2}$$

$$\Rightarrow 24 = \frac{(x+2)+(x+4)}{2}$$

$$\Rightarrow 24 = \frac{2x+6}{2}$$

$\Rightarrow 24 = x + 3 \quad \Rightarrow x = 21.$

Hence, x = 21.

Ex.65 Find the median of the following data : 19, 25, 59, 48, 35, 31, 30, 32, 51. If 25 is replaced by 52, what will be the new median.

Sol. Arranging the given data in ascending order, we have 19, 25, 30, 31, 32, 35, 48, 51, 59

Here, the number of observations n = 9 (odd)

Since the number of observations is odd. Therefore.

Median = Value of $\left(\frac{9+1}{2}\right)$ the observations

$\Rightarrow$ Median = value of 5^{th} observation = 32.

Hence, Median = 32

If 25 is replaced by 52, then the new observations arranged in ascending order are :

19, 30, 31, 32, 35, 48, 51, 52, 59

$\therefore$ New median = Value of 5^{th} observation = 35.

Ex.66 Find the median of the following data

(i) 17, 27, 37, 13, 18, 25, 32, 34, 23

(ii) 24, 37, 19, 41, 28, 32, 29, 31, 33, 21

Sol. (i) The scores when arranged in ascending order are

13, 17, 18, 23, 25, 27, 32, 34, 37

Here, the number of scores n = 9 (odd)

$\therefore$ Median = $t_{\frac{9+1}{2}} = t_5 = 25$

(ii) The scores when arraged in ascending order are

19, 21, 24, 28, 29, 31, 33, 34, 37, 41.

Total number of scores = 10, which is even. So there will be two middle-terms which are $t_5 = 29$ and $t_6 = 31$.

$\therefore$ Median = $\frac{t_5 + t_6}{2} = \frac{29+31}{2} = 30$

Ex.67 Calculate the median for the following distribution

Weight (in kg)	Number of sudent
46	3
47	2
48	4
49	6
50	5
51	2
52	1

Sol. The cumulative frequency table is constructed as shown below :

Weights x_i	Number of students f_i	Cumulative frequency
46	3	3
47	2	5
48	4	9
49	6	15
50	5	20
51	2	22
52	1	23

Here, n = 23, which is odd

Median= $t_{\frac{23+1}{2}} = t_{12}$

= 49

(i.e. weight of the 12th student when the weights have been arranged in order)

Ex.68 Find the median of the following data :

(i) 8, 10, 5, 7, 12, 15, 11

(ii) 12, 14, 10, 7, 15, 16

Sol. (i) 8, 10, 5, 7, 12, 15, 11

These numbers are arranged in an order

5, 7, 8, 10, 11, 12, 15

The number of observations = 7 (odd)

$\Rightarrow$ Median = $\frac{7+1}{2}$ = 4th term

$\Rightarrow$ Median = 10

(ii) 12, 14, 10, 7, 15, 16

These numbers are arranged in an order

7, 10, 12, 14, 15, 16

The number of observations = 6 (even)

The medians will be mean of $\frac{6}{2}$ = 3rd and 4th terms i.e., 12 and 14

$\Rightarrow$ The median = $\frac{12+14}{2} = 13$

Ex.68 The following data have been arranged in desending orders of magnitude 75, 70, 68, x + 2, x − 2, 50, 45, 40

If the median of the data is 60, find the value of x.

Sol. The number of observations are 8, the median will be the average of 4th and 5th number

$\Rightarrow$ Median = $\frac{(x+2)+(x-2)}{2}$

$\Rightarrow 60 = \frac{2x}{2}$

$\Rightarrow x = 60$

Ex.69 Find the median of 6, 8, 9, 10, 11, 12 and 13.

Sol. Total number of terms = 7

The middle terms = $\frac{1}{2}(7 + 1) = 4\text{th}$

Median = Value of the 4th term = 10.

Hence, the median of the given series is 10.

Ex.70 Find the mean of 21, 22, 23, 24, 25, 26, 27 and 28.

Sol. Total number of terms = 8

Median = Value of $\frac{1}{2}\left[\frac{8}{2}\text{th term} + \left(\frac{8}{2}+1\right)\text{th term}\right]$

= Value of $\frac{1}{2}$[4th term + 5th term]

= $\frac{1}{2}[24 + 25] = \frac{49}{2} = 24.5$

Mode

Mode is also known as norm.

◈ Mode is the value which occurs most frequently in a set of observations and around which the other items of the set cluster density.

Algorithm

Step I : Obtain the set of observations.

Step II : Count the number of times the various values repeat themselves. In other words, prepare the frequency distribution.

Step III: Find the value which occurs the maximum number of times i.e. obtain the value which has the maximum frequency.

Step IV: The value obtained in step III is the mode.

Ex.71 Find the mode from the following data :

110, 120, 130, 120, 110, 140, 130, 120, 140, 120.

Sol. Arranging the data in the form of a frequency table, we have

Value	Tally bars	Frequency
110	| |	2
120	| | | |	4
130	| |	2
140	| |	2

Since the value 120 occurs maximum number of times i.e. 4. Hence, the modal value is 120.

Ex.72 Find the mode for the following series :

2.5, 2.3, 2.2, 2.2, 2.4, 2.7, 2.7, 2.5, 2.3, 2.2, 2.6, 2.2

Sol. Arranging the data in the form of a frequency table, we have

Value	Tally bars	Frequency
2.2	| | | |	4
2.3	| |	2
2.4	|	1
2.5	| |	2
2.6	|	1
2.7	| |	2

We see that the value 2.2 has the maximum frequency i.e. 4.

So, 4 is the mode for the given series.

Ex.73 Compute mode for the following data

7, 7, 8, 8, 8, 9, 9, 10, 10, 10, 11, 11, 12, 13, 13

Sol. Here, both the scores 8 and 10 occurs thrice (maximum number of times). So, we apply the empirical formula.

Here,

Weights x	No. of men f	Cumulative frequency	Product f.x
54	6	6	324
56	4	10	224
58	5	15	290
60	5	20	300
62	6	26	372
63	5	31	315
64	2	33	128
72	6	39	432
80	1	40	80
Total	40		2465

$$\text{Mean} = \frac{\Sigma f.x}{\Sigma f} = \frac{2465}{40} = 61.625$$

Here, No. of scores = 40 (even)

$$\text{Median} = \frac{t_{20} + t_{21}}{2} = \frac{60 + 62}{2} = 61$$

$\therefore$ Mode = 3 median – 2 mean

$= 3 \times 61 - 2 \times 61.625$

$= 183 - 123.25 = 59.75$

Thus, modal weight = 59.75 kg

Ex.74 If the heights of 5 persons are 144 cm, 153 cm, 150 cm, 158 cm and 155 cm respectively, then mean height is -

(A) 150 cm (B) 151 cm

(C) 152 cm (D) None of these

Sol. Mean Height $= \frac{144 + 153 + 150 + 158 + 155}{5}$

$= \frac{760}{5} = 152$ cm.

Ex.75 Arithmetic mean of the following frequency distribution :

x : 4 7 10 13 16 19

f : 7 10 15 20 25 30

is -

(A) 13.6 (B) 13.8

(C) 14.0 (D) None of these

Sol. The given frequency distribution is -

x_i	f_i	f_ix_i
4	7	28
7	10	70
10	15	150
13	20	260
16	25	400
19	30	570

$\Sigma f_i = 107$ $\Sigma f_i\ x_i = 1478$

$$\bar{x} = \frac{\Sigma f_i\ x_i}{\Sigma f_i} = \frac{1478}{107} = 13.81$$

Ex.76 The mean income of a group of persons is Rs.400. Another group of persons has mean income Rs.480. If the mean income of all the persons in the two groups together is Rs.430, then ratio of the number of persons in the groups:

(A) $\frac{4}{3}$ (B) $\frac{5}{4}$

(C) $\frac{5}{3}$ (D) None of these

Sol. $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$\because$ $\bar{x}_1 = 400$, $\bar{x}_2 = 480$, $\bar{x} = 430$

$\therefore$ $430 = \frac{n_1(400) + n_2(480)}{n_1 + n_2}$

$\Rightarrow 30n_1 = 50n_2$

$\Rightarrow \frac{n_1}{n_2} = \frac{5}{3}$

Ex.77 The mean of a set of number is $\bar{x}$ if each number is increased by λ, then mean of the new set is-

(A) $\bar{x}$ (B) $\bar{x} + \lambda$

(C) $\lambda\ \bar{x}$ (D) None of these

Sol. $\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\sum x_i}{n}$

$\therefore \sum x_i = n\bar{x}$

New mean $= \frac{\sum(x_i + \lambda)}{n} = \frac{\sum x_i + n\lambda}{n}$

$= \bar{x} + \lambda$

Ex.78 The number of runs scored by 11 players of a cricket team of school are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27. The median is-

(A) 21 (B) 27

(C) 30 (D) None of these

Sol. Let us arrange the value in ascending order

0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

$\therefore$ Median M = $\left(\frac{n+1}{2}\right)^{th}$ value

= $\left(\frac{11+1}{2}\right)^{th}$ value = 6^{th} value

Now 6^{th} value in data is 27

$\therefore$ Median = 27 runs.

Ex.79 Mode of the data 3, 2, 5, 2, 3, 5, 6, 6, 5, 3, 5, 2, 5 is -

(A) 6 (B) 4 (C) 5 (D) 3

Sol. Since 5 is repeated maximum number of times, therefore mode of the given data is 5.

Ex.80 If the value of mode and mean is 60 and 66 respectively, then the value of median is-

(A) 60 (B) 64

(C) 68 (D) None of these

Sol. Mode = 3 Median – 2 mean

$\therefore$ Median = $\frac{1}{3}$(mode + 2 mean)

= $\frac{1}{3}(60 + 2 \times 66) = 64$

Ex.81 Mean of 25 observations was found to be 78.4. But later on it was found that 96 was misread 69. The correct mean is

(A) 79.24

(B) 79.48

(C) 80.10

(D) None of these

Sol. Mean $\bar{x} = \frac{\sum x}{n}$

or $\sum x = n\bar{x}$

$\sum x = 25 \times 78.4 = 1960$

But this $\sum x$ is incorrect as 96 was misread as 69.

$\therefore$ correct $\sum x = 1960 + (96 - 69) = 1987$

$\therefore$ correct mean $= \frac{1987}{25} = 79.47$

Ex.82 If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$ then mean of $x_1 + a, x_2 + a, \ldots, x_n + a$ where a is any number positive or negative is -

(A) $\bar{x} + a$ (B) $\bar{x}$

(C) $a\bar{x}$ (D) None of these

Sol. We have $\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$

Let $\bar{x}'$ be the mean of $x_1 + a, x_2 + a, \ldots, x_n + a$ then

$\bar{x}' = \frac{(x_1 + a) + (x_2 + a) + \ldots + (x_n + c)}{n}$

$= \frac{(x_1 + x_2 + \ldots + x_n) \ldots + na}{n}$

$= \frac{x_1 + x_2 + \ldots + x_n}{n} + a = \bar{x} + a$

EXERCISE

Q.1 In a frequency distribution, the mid value of a class is 15 and the class size is 4. The lower limit of the class is :

(A) 10 (B) 12 (C) 13 (D) 14

Q.2 The mid value of a class interval is 42. If the class size is 10, then the upper and lower limits of the class are :

(A) 47 & 37 (B) 37 & 47

(C) 37.5 & 47.5 (D) 47.5 & 37.5

Q.3 If the arithmetic mean of 7, 5, 13, x and 9 be 10, then the value of x is :

(A) 10 (B) 12 (C) 14 (D) 16

Q.4 Consider the table given below :

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students	12	18	27	20	17	6

The arithmetic mean of the marks given above, is :

(A) 18 (B) 28 (C) 27 (D) 6

Q.5 The arithmetic mean of 5 numbers is 27. If one of the numbers be excluded, their mean is 25. The excluded number is :

(A) 28 (B) 26 (C) 25 (D) 35

Q.6 Mode is :

(A) Least frequent value

(B) Middle most value

(C) Most frequent value

(D) None of these

Q.7 The following is the data of wages per day

5, 4, 7, 5, 8, 8, 8, 5, 7, 9, 5, 7, 9, 10, 8

The mode of the data is :

(A) 7 (B) 5 (C) 8 (D) 10

Q.8 The mode of the given distribution is :

Weight (in kg)	40	43	46	49	52	55
Number of Children	5	8	16	9	7	3

(A) 40 (B) 46

(C) 55 (D) None of these

Q.9 The median of

0, 2, 2, 2, –3, 5, –1, 5, 5, –3, 6, 6, 5, 6 is :

(A) 0 (B) –1.5

(C) 2 (D) 3.5

Q.10 The median of the following distribution is :

Class-interval	35-45	45-55	55-65	65-70
Frequency	8	12	20	10

(A) 56.5 (B) 57.5

(C) 58.7 (D) 59

Q.11 The mode of the following frequency distribution is :

Class interval	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	21	23	10	12	3

(A) 11.5 (B) 12.4 (C) 12 (D) 11.8

Q.12 The average value of the median of 2, 8, 3, 7, 4, 6, 7 and the mode of 2, 9, 3, 4, 9, 6, 9 is :

(A) 9 (B) 8 (C) 7.5 (D) 6

Q.13 The average weight of a group of 20 boys was calculated to be 89.4 kg and it was later discovered that one weight was misread as 78 kg instead of the correct one of 87 kg, then the correct average weight is :

(A) 88.95 kg (B) 89.25 kg

(C) 89.55 kg (D) 89.85 kg

Q.14 Mean of first n natural numbers is -

(A) $\frac{n(n-1)}{2}$ (B) $\frac{n(n+1)}{2}$

(C) $\frac{(n+1)}{2n}$ (D) $\frac{n+1}{2}$

Q.15 $(x_1 - \bar{x}) + (x_2 - \bar{x}) + ... + (x_n - \bar{x}) =$

(A) 0 (B) 1

(C) $\bar{x}$ (D) None of these

Q.16 A factory employs 100 workers of whom 60 work in the first shift and 40 work in the second shift. The average wage of all the 100 workers is Rs.38. If the average wage of 60 workers of the first shift is Rs.40, then the average wage of the remaining 40 workers of the second shift is -

(A) 35 (B) 40

(C) 45 (D) None of these

Q.17 The median of the items 6, 10, 4, 3, 9, 11, 22, 18 is -

(A) 9 (B) 10 (C) 9.5 (D) 11

Q.18 If the mean of 3, 4, x , 7, 10 is 6, then the value of x is -

(A) 4 (B) 5 (C) 6 (D) 7

Q.19 The mean of a set of numbers is $\bar{x}$. If each number is increased by λ, the mean of the net set is

-

(A) $\overline{x}$ (B) $\overline{x} + \lambda$ (C) $\lambda\ \overline{x}$ (D) None

Q.20 The mean of a set of numbers is $\overline{x}$. If such number is multiplied by λ, then the mean of the new set is -

(A) $\overline{x}$ (B) $\lambda + \overline{x}$ (C) $\lambda\overline{x}$ (D) None

Q.21 The mean of first n natural numbers is -

(A) $\frac{n(n+1)}{2}$ (B) n (n + 1)

(C) $\frac{n+1}{2}$ (D) (n + 1)

Q.22 If the mean of first n natural numbers is equal to $\frac{n+7}{3}$, then n is equal to -

(A) 10 (B) 11 (C) 12 (D) None

Q.23 If the mean of numbers 27, 31, 89, 107, 156 is 82, then the mean of 130, 126, 68, 50, 1 is-

(A) 75 (B) 157 (C) 82 (D) 80

Q.24 The mean of first three terms is 14 and mean of next two terms is 18. The mean of all the five terms is -

(A) 14.5 (B) 15.0 (C) 15.2 (D) 15.6

Q.25 In an arranged series of n observations (n being an odd number), the median is the value of -

(A) $\left(\frac{n}{2}\right)$th item (B) $\left(\frac{n+1}{2}\right)$ th item

(C) $\left(\frac{n}{2}+1\right)$ th item (D) $\left(n+\frac{1}{2}\right)$ th item

Q.26 The median of 10, 14, 11, 9, 8, 12, 6 is

(A) 10 (B) 12 (C) 14 (D) 11

Q.27 If a variable takes the discrete values $\alpha + 4$, $\alpha - \frac{7}{2}$, $\alpha - \frac{5}{2}$, $\alpha - 3$, $\alpha - 2$, $\alpha + \frac{1}{2}$, $\alpha - \frac{1}{2}$, $\alpha + 5$ ($\alpha > 0$), then the median is-

(A) $\alpha - \frac{5}{4}$ (B) $\alpha - \frac{1}{2}$

(C) $\alpha - 2$ (D) $\alpha + \frac{5}{4}$

Q.28 In an arranged discrete series in which total number of observations 'n' is even, median is -

(A) $\frac{n}{2}$th item (B) $\left(\frac{n}{2}+1\right)$ th item

(C) the mean of $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$ th item (D) None of these

Q.29 The mode of the following items is 0, 1, 6, 7, 2, 3, 7, 6, 6, 2, 6, 0, 5, 6, 0 is -

(A) 0 (B) 5 (C) 6 (D) 2

Q.30 If the mode of a data is 18 and the mean is 24, then median is -

(A) 18 (B) 24 (C) 22 (D) 21

Q.31 If the mean of the first n odd natural numbers be n itself, then n is -

(A) 1 (B) 2

(C) 3 (D) any natural number

Q.32 The mean of 50 observations is 36. If two observations 30 and 42 are deleted, then the mean of the remaining observations is-

(A) 48 (B) 36

(C) 38 (D) None of these

Q.33 A group of 10 items has mean 6. If the mean of 4 of these items is 7.5, then the mean of the remaining items is -

(A) 6.5 (B) 5.5 (C) 4.5 (D) 5.0

Q.34 The mean of a set of observations is $\overline{x}$. If each observation is divided by α, $\alpha \neq 0$, and then is increased by 10 then the mean of the new set is -

(A) $\frac{\overline{x}}{\alpha}$ (B) $\frac{\overline{x}+10}{\alpha}$

(C) $\frac{\overline{x}+10\alpha}{\alpha}$ (D) $a\overline{x}+10$

Q.35 Ram spends equal amounts on purchasing three kinds of pens being sold at Rs.5, Rs.10, Rs.15 per piece. Average cost of each pen is -

(A) Rs.10 (B) Rs.$\frac{90}{11}$ (C) Rs.9 (D) None

Q.36 If a, b, c are any three positive numbers, then the least value of $(a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is -

(A) 3 (B) 6 (C) 9 (D) None

Q.37 The median of the data 13, 14, 16, 18, 20, 22 is-

(A) 17 (B) 16 (C) 18 (D) None

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	D	B	D	C	C	B	D	B	B	C	D	D	A	A	C	C	B	C
Q.No	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37			
Ans.	C	B	A	D	B	A	A	C	C	C	D	B	D	C	B	C	A			