SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA / Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)	–	–	–	**2(2)**
4.	Determinants	1(1)*	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)*	–	**3(9)**
7.	Integrals	2(2)#	1(2)	1(3)	–	**4(7)**
8.	Application of Integrals	–	1(2)*	1(3)	–	**2(5)**
9.	Differential Equations	1(1)*	1(2)	1(3)	–	**3(6)**
10.	Vector Algebra	1(1) + 1(4)	–	–	–	**2(5)**
11.	Three Dimensional Geometry	2(2)#	1(2)*	–	1(5)*	**4(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	4(4)#	2(4)#	–	–	**6(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Solve the differential equation $\frac{dy}{dx} = 1 - x + y - xy$.

OR

What is the degree of the differential equation $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$?

2. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, then find the values of x and y.

3. The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some number,

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'.

OR

Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

4. Check whether the relation R on the set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$ is reflexive, symmetric and transitive.

5. Evaluate : $\int \frac{\sqrt{x}}{\sqrt{x^2 - x}} dx$

OR

Evaluate : $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

6. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then find the value of k.

7. Find the vector equation of a plane which is at a distance of 6 units from the origin and which has $\hat{k}$ as the unit vector normal to it.

OR

Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ units from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$.

8. Prove that for any square matrix A, AA^T is a symmetric matrix.

9. If $\Delta(x) = \begin{vmatrix} f(x) & g(x) \\ a & b \end{vmatrix}$, then prove that $\int \Delta(x)dx = \begin{vmatrix} \int f(x)dx & \int g(x)dx \\ a & b \end{vmatrix}$.

OR

If A is invertible matrix of order 3×3, then prove that $|A^{-1}| = |A|^{-1}$.

10. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither 9 nor 11 ?

11. Let $f : [2, \infty) \rightarrow R$ be the function defined by $f(x) = x^2 - 4x + 5$, then find the range of f.

12. If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, then find the value of $P(A/B)$.

13. Evaluate : $\int_{\pi/6}^{\pi/4} (\sec^2 x + \text{cosec}^2 x)\, dx$

14. A bag contains 3 white and 6 black balls while another bag contains 6 white and 3 black balls. A bag is selected at random and a ball is drawn. Find the probability that the ball drawn is of white colour.

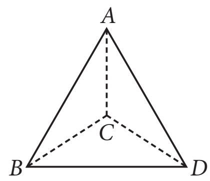
15. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of points A, B, C respectively such that $5\vec{a} - 3\vec{b} - 2\vec{c} = \vec{0}$, then find the ratio in which C divides AB externally.

16. For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Prove that the relation R is not transitive.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A building of a multinational company is to be constructed in the form of a triangular pyramid, $ABCD$ as shown in the figure.



Let its angular points are $A(3, 0, 1)$, $B(-1, 4, 1)$, $C(5, 2, 3)$ and $D(0, -5, 4)$ and G be the point of intersection of the medians of ΔBCD.
Based on the above answer the following.

(i) The coordinates of points G are

(a) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (b) $\left(0, \frac{1}{2}, \frac{1}{3}\right)$ (c) $\left(\frac{4}{3}, \frac{1}{3}, \frac{8}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{8}{3}, \frac{1}{3}\right)$

(ii) The length of vector $\overrightarrow{AG}$ is

(a) $\sqrt{17}$ units (b) $\frac{\sqrt{51}}{3}$ units (c) $\frac{3}{\sqrt{6}}$ units (d) $\frac{\sqrt{59}}{4}$ units

(iii) Area of triangle ABC (in sq. units) is

(a) 24 (b) $8\sqrt{6}$ (c) $4\sqrt{6}$ (d) $5\sqrt{6}$

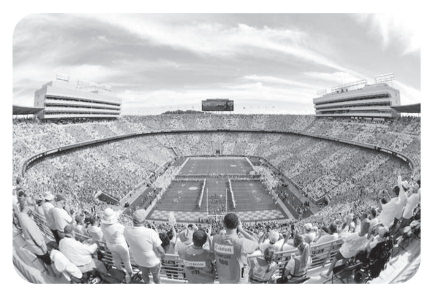
(iv) The sum of lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

(a) 4 units (b) 9.1 units (c) 8.7 units (d) 6 units

(v) The length of the perpendicular from the vertex D on the opposite face is

(a) $\frac{14}{\sqrt{6}}$ units (b) $\frac{2}{\sqrt{6}}$ units (c) $\frac{3}{\sqrt{6}}$ units (d) $8\sqrt{6}$ units

18. A concert is organised every year in the stadium that can hold 42000 spectators. With ticket price of ₹ 10, the average attendance has been 27000. Some financial expert estimated that price of a ticket should be determined by the function $p(x) = 19 - \frac{x}{3000}$, where x is the number of tickets sold.
Based on the above information, answer the following questions.



(i) The revenue, R as a function of x can be represented as

(a) $19x - \frac{x^2}{3000}$ (b) $19 - \frac{x^2}{3000}$ (c) $19x - \frac{1}{30000}$ (d) $19x - \frac{x}{3000}$

(ii) The range of x is

(a) [27000, 42000] (b) [0, 27000]
(c) [0, 42000] (d) none of these

(iii) The value of x for which revenue is maximum, is

(a) 20000 (b) 27000 (c) 28500 (d) 28000

(iv) When the revenue is maximum, the price of the ticket is

(a) ₹ 8 (b) ₹ 5 (c) ₹ 9 (d) ₹ 9.5

(v) How many spectators should be present to maximize the revenue?

(a) 25000 (b) 27000 (c) 22000 (d) 28500

PART - B

Section - III

19. Solve the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

20. Find the area of the larger part bounded by $y = \cos x$, $y = x + 1$ and $y = 0$.

OR

Find the area enclosed by the lines $y = 0$, $y = x$, $x = 1$, $x = 2$.

21. Consider $f(x) = \begin{cases} 3x - 8, & \text{if} \quad x \leq 5 \\ 2k, & \text{if} \quad x > 5 \end{cases}$

Find the value of k, if $f(x)$ is continuous at $x = 5$.

22. A random variable X has the following distribution.

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event $E = \{X \text{ is prime number}\}$ and $F = \{X < 4\}$, find $P(E \cup F)$.

23. Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.

OR

If O be the origin and the coordinates of P be $(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

24. Find cofactors of a_{21} and a_{31} of the matrix

$$A = [a_{ij}] = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}.$$

25. Evaluate : $\int_a^b \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a+b-x}}\, dx$

26. Let k and K be the minimum and the maximum values of the function, $f(x) = \frac{(1+x)^{0.6}}{1+x^{0.6}}$ defined on $[0, 1]$, respectively. Find the ordered pair (k, K).

27. Find the number of triplets (x, y, z) satisfying the equation $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$.

28. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if A and B are (i) mutually exclusive (ii) independent.

OR

A bag contains 12 white pearls and 18 black pearls. Two pearls are drawn in succession without replacement. Find the probability that the first pearl is white and the second is black.

Section - IV

29. Find all points of discontinuity of f, where f is defined as follows :

$$f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$$

30. Solve the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1, x\neq 0.$

31. If $y=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, find $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$.

OR

If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$.

32. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

33. Let A and B be non-empty sets. Show that $f: A\times B\to B\times A$ such that $f(a, b) = (b, a)$ is a bijective function.

34. If $f(x) = a\log|x| + bx^2 + x$ has extreme values at $x = -1$ and at $x = 2$, then find a and b.

OR

Show that $f(x) = \cos(2x + \pi/4)$ is an increasing function on $(3\pi/8, 7\pi/8)$.

35. Evaluate : $\int_0^{\pi/4}\frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

Section - V

36. Find the cartesian equations of the plane through the intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+24=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$, which are at a distance of 2 units from the origin.

OR

If the shortest distance between the lines $L_1:\frac{x-1}{1}=\frac{y}{-1}=\frac{z}{2}$ and $L_2:\frac{x+1}{2}=\frac{y}{2}=\frac{z-3}{\lambda}$ is unity, then find the value of λ.

37. Solve the following LPP graphically.
Maximize $Z = 50x + 40y$
Subject to constraints :
$1000x + 1200y \leq 7600$
$12x + 8y \leq 72$
$x, y \geq 0$

OR

Solve the following LPP graphically.
Minimize $Z = 5x + 7y$
Subject to constraints :
$2x + y \leq 8$
$x + 2y \geq 10$
and $x, y \geq 0$

38. If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$, then find AB. Hence, solve the system of equations :
$x - y = 6,\ 2x + 3y + 4z = 34,\ y + 2z = 14$

OR

Solve the system of the following equations : $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

SOLUTIONS

1. We have $\frac{dy}{dx}=(1-x)(1+y) \Rightarrow \frac{dy}{1+y}=(1-x)dx$

$\Rightarrow \int\frac{dy}{1+y}=\int(1-x)dx \Rightarrow \log|1+y|=x-\frac{x^2}{2}+C$

OR

Since greatest power of highest order derivative is 1, therefore degree of the given differential equation is 1.

2. Given $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix}=\begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$

$\Rightarrow$ $y = -1$ and $7 - x = 0 \Rightarrow x = 7, y = -1$

3. We have, $P(X=x)=\begin{cases} k, & \text{if } x=0 \\ 2k, & \text{if } x=1 \\ 3k, & \text{if } x=2 \\ 0, & \text{otherwise} \end{cases}$

Since, $\Sigma P(x_i) = 1$, therefore $k + 2k + 3k = 1$

$\Rightarrow 6k=1 \Rightarrow k=\frac{1}{6}$

OR

Total number of students = 8

The number of ways to select 4 students out of 8 students $= {}^8C_4=\frac{8!}{4!\,4!}=70$

The number of ways to select 2 boys and 2 girls

$= {}^3C_2\times{}^5C_2=\frac{3!}{2!1!}\times\frac{5!}{2!3!}=3\times10=30$

$\therefore$ Required probability $=\frac{30}{70}=\frac{3}{7}$.

4. Given, $R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$

Since, $(2, 2) \notin R$

Therefore, R is not reflexive.

But R is both symmetric and transitive as $(1, 2) \in R \Rightarrow (2, 1) \in R$ and $(1, 1) \in R, (2, 1) \in R \Rightarrow (1, 1) \in R$.

5. Let $I=\int\frac{\sqrt{x}}{\sqrt{x^2-x}}dx$

$=\int\frac{\sqrt{x}}{\sqrt{x(x-1)}}dx=\int\frac{dx}{\sqrt{x-1}}=2\sqrt{x-1}+C$

OR

Let $I=\int\frac{\cos 2x+2\sin^2 x}{\cos^2 x}dx$

$=\int\frac{\cos^2 x-\sin^2 x+2\sin^2 x}{\cos^2 x}dx$

$=\int\frac{\cos^2 x+\sin^2 x}{\cos^2 x}dx=\int\frac{1}{\cos^2 x}dx=\int\sec^2 x\,dx$

$= \tan x + C$

6. Lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ will be perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow -3(3k) + 2k + 2(-5) = 0 \Rightarrow -9k + 2k - 10 = 0$

$\Rightarrow k=-\frac{10}{7}$

7. Here $\hat{n}=\hat{k}$ and $d=6$

$\therefore$ $\vec{r}\cdot\hat{n}=d \Rightarrow \vec{r}\cdot\hat{k}=6$, which is the required equation of plane.

OR

Let $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Then,

$\hat{n}=\frac{\vec{n}}{|\vec{n}|}=\frac{2\hat{i}-3\hat{j}+4\hat{k}}{\sqrt{4+9+16}}=\frac{2\hat{i}-3\hat{j}+4\hat{k}}{\sqrt{29}}$

Hence, the required equation of the plane is

$\vec{r}\cdot\left(\frac{2}{\sqrt{29}}\hat{i}+\frac{(-3)}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$

or $r\cdot(2\hat{i}-3\hat{j}+4\hat{k})=6$

8. Clearly, $(AA^T)^T = (A^T)^T \cdot A^T$ $\quad [\because (AB)^T = B^TA^T]$

$\Rightarrow (AA^T)^T = AA^T$ $\quad [\because (X^T)^T = X]$

$\Rightarrow AA^T$ is a symmetric matrix.

9. Given, $\Delta(x) = \begin{vmatrix} f(x) & g(x) \\ a & b \end{vmatrix} = bf(x) - ag(x)$

$\therefore \int\Delta(x)dx = \int\{bf(x) - ag(x)\}dx$

$= b\int f(x)dx - a\int g(x)dx = \begin{vmatrix} \int f(x)dx & \int g(x)dx \\ a & b \end{vmatrix}$

OR

Given, A is a matrix of order 3×3, then

$|A^{-1}|=\left|\frac{1}{|A|}\text{adj}\,A\right|=\frac{1}{|A|^3}\cdot|A|^2$

[$\because$ If A is invertible matrix of order n, then $|\text{adj } A| = |A|^{n-1}$]

$=\frac{1}{|A|}=|A|^{-1}$

10. If two dice are thrown, then total number of cases = 36

Cases for total of 9 or 11 are {(3, 6), (4, 5), (6, 3), (5, 4), (6, 5), (5, 6)}, *i.e.*, 6 in number.

$\therefore\ P(\text{total 9 or 11}) = \frac{6}{36} = \frac{1}{6}$

P(sum is neither 9 nor 11) = 1 – P(sum is 9 or 11)

$= 1 - \frac{1}{6} = \frac{5}{6}$

11. Given, $f(x) = x^2 - 4x + 5$

Let $y = x^2 - 4x + 5 \Rightarrow y = (x-2)^2 + 1$

$\Rightarrow (x-2)^2 = y - 1 \Rightarrow x - 2 = \sqrt{y-1}\quad [\because x \in [2, \infty)]$

$\Rightarrow x = \sqrt{y-1} + 2$

For range $y - 1 \geq 0 \Rightarrow y \geq 1$

$\therefore$ Range is $[1, \infty)$.

12. We have, $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$

$\because\ P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.2 + 0.4 - 0.5 = 0.1$

$\therefore\ P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.4} = \frac{1}{4} = 0.25$

13. $\int_{\pi/6}^{\pi/4} (\sec^2 x + \operatorname{cosec}^2 x)dx = [\tan x - \cot x]_{\pi/6}^{\pi/4}$

$= (1 - 1) - \left(\frac{1}{\sqrt{3}} - \sqrt{3}\right) = \frac{2}{\sqrt{3}}.$

14. Let E_1 be the event that bag I is selected

E_2 be the event that bag II is selected

E be the event that the ball drawn is of white colour.

By rule of total probability,

$P(E) = P(E_1)\cdot P(E|E_1) + P(E_2)\cdot P(E|E_2)$

$= \frac{1}{2}\cdot\frac{3}{9} + \frac{1}{2}\cdot\frac{6}{9} = \frac{9}{18} = \frac{1}{2}$

15. Given, $5\vec{a} - 3\vec{b} - 2\vec{c} = \vec{0} \Rightarrow 2\vec{c} = 5\vec{a} - 3\vec{b}$

$\Rightarrow\ \vec{c} = \frac{5\vec{a} - 3\vec{b}}{2} = \frac{5\vec{a} - 3\vec{b}}{5-3} = \frac{3\vec{b} - 5\vec{a}}{3-5}$

So, C divides AB externally in the ratio 3 : 5

16. Clearly, $\sqrt{2}R1$ and $1R2\sqrt{2}$ but $\sqrt{2}\not R\ 2\sqrt{2}$, therefore R is not transitive.

17. (i) (c) : Clearly, G be the centroid of ΔBCD, therefore coordinates of G are

$\left(\frac{-1+5+0}{3}, \frac{4+2-5}{3}, \frac{1+3+4}{3}\right) = \left(\frac{4}{3}, \frac{1}{3}, \frac{8}{3}\right)$

(ii) (b) : Since, $A \equiv (3, 0, 1)$ and $G \equiv \left(\frac{4}{3}, \frac{1}{3}, \frac{8}{3}\right)$

$\therefore\ \overrightarrow{AG} = \left(\frac{4}{3} - 3\right)\hat{i} + \left(\frac{1}{3} - 0\right)\hat{j} + \left(\frac{8}{3} - 1\right)\hat{k}$

$= \frac{-5}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{5}{3}\hat{k}$

$\Rightarrow |\overrightarrow{AG}|^2 = \left(\frac{5}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{5}{3}\right)^2 = \frac{25}{9} + \frac{1}{9} + \frac{25}{9} = \frac{51}{9}$

$\Rightarrow |\overrightarrow{AG}| = \frac{\sqrt{51}}{3}$

(iii) (c) : Clearly, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Here, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1-3 & 4-0 & 1-1 \\ 5-3 & 2-0 & 3-1 \end{vmatrix}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 4 & 0 \\ 2 & 2 & 2 \end{vmatrix} = -8\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

$= -8[\hat{i}(-1-0) - \hat{j}(1-0) + \hat{k}(1+1)]$

$= -8[-\hat{i} - \hat{j} + 2\hat{k}] = 8(\hat{i} + \hat{j} - 2\hat{k})$

$\therefore\ |\overrightarrow{AB} \times \overrightarrow{AC}| = 8|\hat{i} + \hat{j} - 2\hat{k}| = 8\sqrt{1+1+4} = 8\sqrt{6}$

Hence, area of $\Delta ABC = \frac{1}{2} \times 8\sqrt{6} = 4\sqrt{6}$ sq. units

(iv) (b) : Here, $\overrightarrow{AB} = -4\hat{i} + 4\hat{j} + 0\hat{k}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$\overrightarrow{AC} = 2\hat{i} + 2\hat{j} + 2\hat{k} \Rightarrow |\overrightarrow{AC}| = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$

Now, $|\overrightarrow{AB}| + |\overrightarrow{AC}| = 4\sqrt{2} + 2\sqrt{3} = 9.1$ units

(v) (a) : The length of the perpendicular from the vertex D on the opposite face

$= |\text{Projection of } \overrightarrow{AD} \text{ on } \overrightarrow{AB} \times \overrightarrow{AC}|$

$= \left|\frac{(-3\hat{i} - 5\hat{j} + 3\hat{k})\cdot(\hat{i} + \hat{j} - 2\hat{k})}{\sqrt{1^2 + 1^2 + 2^2}}\right|$

$= \left|\frac{-3-5-6}{\sqrt{6}}\right| = \frac{14}{\sqrt{6}}$ units

18. (i) (a) : Let p be the price per ticket and x be the number of tickets sold.

Then, revenue function $R(x) = p \times x = \left(19 - \frac{x}{3000}\right)x$

$= 19x - \frac{x^2}{3000}$

(ii) (c) : Since, more than 42000 tickets cannot be sold. So, range of x is [0, 42000].

(iii) (c) : We have, $R(x) = 19x - \frac{x^2}{3000}$

$\Rightarrow R'(x) = 19 - \frac{x}{1500}$

For maxima/minima, put $R'(x) = 0$

$\Rightarrow\ 19-\dfrac{x}{1500}=0 \Rightarrow x = 28500$

Also, $R''(x) = -\dfrac{1}{1500}<0.$

(iv) (d) : Maximum revenue will be at $x = 28500$

$\therefore$ Price of a ticket $= 19-\dfrac{28500}{3000} = 19 - 9.5 = 9.5$

(v) (d) : Number of spectators will be equal to number of tickets sold when revenue is maximum.

$\therefore$ Required number of spectators = 28500

19. We have, $\dfrac{dy}{dx}=e^{x-y}+x^2e^{-y}$

$\Rightarrow\ dy = (e^{x-y} + x^2 e^{-y})dx \Rightarrow e^y\, dy = (e^x + x^2)dx$

$\Rightarrow\ \int e^y dy=\int\left(e^x+x^2\right)dx$ [Integrating both sides]

$\Rightarrow\ e^y=e^x+\dfrac{x^3}{3}+C,$ which is the required solution.

20. It can be observed that $y = \cos x$ and $y = x + 1$ meet at the point (0, 1).

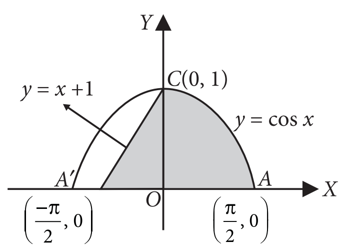
Also, $y=x+1$ passes through the points $(-1,0)$ and $(0,1)$, $y = \cos x$ meets X-axis at $\left(\dfrac{-\pi}{2},0\right)$ and $\left(\dfrac{\pi}{2},0\right)$.

$\therefore$ Required area $=\int_{-1}^{0}(x+1)\,dx+\int_{0}^{\frac{\pi}{2}}\cos x\,dx$

$=\left[\dfrac{x^2}{2}+x\right]_{-1}^{0}+[\sin x]_0^{\frac{\pi}{2}}$

$=0-\left(\dfrac{1}{2}-1\right)+1$

$=\dfrac{3}{2}$ sq. units.



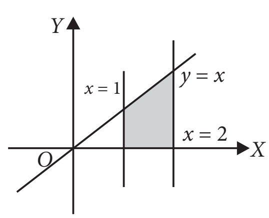
OR

Given lines are $y = x$, $x = 1$, $x = 2$ and $y = 0$.

$\therefore$ Required area $=\displaystyle\int_1^2 x\,dx$

$=\left[\dfrac{x^2}{2}\right]_1^2=\left[\dfrac{4}{2}-\dfrac{1}{2}\right]$

$=\dfrac{3}{2}$ sq. units



21. We have, L.H.L. (at $x = 5$)

$= \lim\limits_{x\to5^-} f(x) = \lim\limits_{x\to5}(3x-8) = 7$

and R.H.L. (at $x = 5$)

$\therefore\ \lim\limits_{x\to5^+} f(x) = \lim\limits_{x\to5}(2k) = 2k$ and $f(5)=7$

Since $f(x)$ is continuous at $x = 5$.

$\therefore\ \lim\limits_{x\to5^-} f(x) = \lim\limits_{x\to5^+} f(x) = f(5) \Rightarrow 7 = 2k$

$\therefore\ k=\dfrac{7}{2}$

22. Clearly, $P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$

$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

$P(F) = P(X = 1) + P(X = 2) + P(X = 3)$

$= 0.15 + 0.23 + 0.12 = 0.50$

$P(E \cap F) = P(X = 2) + P(X = 3) = 0.23 + 0.12 = 0.35$

$\therefore\ P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.50 - 0.35 = 0.77.$

23. We know that the direction cosines of the line passing through two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are given by

$\dfrac{x_2-x_1}{PQ},\dfrac{y_2-y_1}{PQ},\dfrac{z_2-z_1}{PQ}.$

where, $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here, P is $(-2, 4, -5)$ and Q is $(1, 2, 3)$.

So, $PQ=\sqrt{(1-(-2))^2+(2-4)^2+(3-(-5))^2}=\sqrt{77}$

Thus, the direction cosines of the line joining two points are $\dfrac{3}{\sqrt{77}},\dfrac{-2}{\sqrt{77}},\dfrac{8}{\sqrt{77}}.$

OR

The direction ratios of OP are $< 1 - 0, 2 - 0, - 3 - 0 >$ *i.e.* $<1, 2, -3>$

$\therefore$ The equation of the plane passing through P and perpendicular to OP is

$(1)(x-1)+(2)(y-2)+(-3)(z+3)=0$

$\Rightarrow\ x - 1 + 2y - 4 - 3z - 9 = 0$

$\Rightarrow\ x + 2y - 3z - 14 = 0$

24. Let M_{ij} and C_{ij} respectively denote the minor and cofactor of element a_{ij} in A. Then,

$M_{21}=\begin{vmatrix}3 & -2\\ 5 & 2\end{vmatrix}=6+10=16 \Rightarrow C_{21}=-M_{21}=-16$

$M_{31}=\begin{vmatrix}3 & -2\\ -5 & 6\end{vmatrix}=18-10=8 \Rightarrow C_{31}=M_{31}=8$

25. Let $I=\displaystyle\int_a^b\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a+b-x}}dx$ ...(i)

$\therefore\ I=\displaystyle\int_a^b\frac{\sqrt{a+b-x}}{\sqrt{a+b-x}+\sqrt{x}}dx$ ...(ii)

$\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$

On adding (i) and (ii), we get

$$2I=\int_a^b dx=[x]_a^b=b-a\Rightarrow I=\frac{b-a}{2}$$

26. We have, $f(x)=\dfrac{(1+x)^{3/5}}{1+x^{3/5}}$

$$f'(x)=\frac{\left(1+x^{3/5}\right)\frac{3}{5}(1+x)^{-2/5}-(1+x)^{3/5}\cdot\frac{3}{5}x^{-2/5}}{(1+x^{3/5})^2}$$

Clearly $f'(x)=0 \quad\Rightarrow\quad x=1$

Also, $f(0)=1$, and $f(1)=\dfrac{2^{0.6}}{2}=2^{-0.4}$

$\therefore\ f(x)\in(2^{-0.4},1)$

Thus, $(k,K)=(2^{-0.4},1)$

27. We have, $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\dfrac{3\pi}{2}$

$\because\quad -\dfrac{\pi}{2}\le\sin^{-1}x\le\dfrac{\pi}{2},\dfrac{-\pi}{2}\le\sin^{-1}y\le\dfrac{\pi}{2}$

and $\dfrac{-\pi}{2}\le\sin^{-1}z\le\dfrac{\pi}{2}$

$\therefore$ The above condition will true if

$\sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\dfrac{\pi}{2}\Rightarrow x=y=z=1$

Thus, there is only one triplet.

28. (i) When A and B are mutually exclusive, then

$A\cap B=\phi\Rightarrow P(A\cap B)=0\Rightarrow P(A\cup B)=P(A)+P(B)$

$\Rightarrow\ \dfrac{3}{5}=\dfrac{1}{2}+p\ \Rightarrow\ p=\dfrac{3}{5}-\dfrac{1}{2}=\dfrac{6-5}{10}=\dfrac{1}{10}$

(ii) When A and B are independent, then

$P(A\cap B)=P(A)\,P(B)$

$\Rightarrow P(A\cup B)=P(A)+P(B)-P(A)\,P(B)$

$\Rightarrow\ \dfrac{3}{5}=\dfrac{1}{2}+p-\dfrac{1}{2}\cdot p\ \Rightarrow\ \dfrac{3}{5}-\dfrac{1}{2}=\dfrac{2p-p}{2}$

$\Rightarrow\ \dfrac{p}{2}=\dfrac{6-5}{10}\ \Rightarrow\ p=\dfrac{2}{10}=\dfrac{1}{5}$

OR

Let A and B be the events of getting a white pearl in the first draw and a black pearl in the second draw respectively.

Now, $P(A)=P$(getting a white pearl in the first draw) $=\dfrac{12}{30}=\dfrac{2}{5}$

When second pearl is drawn without replacement, the probability that the second pearl is black is the conditional probability of the event B occurring when A has already occurred.

$\therefore\quad P(B\,|\,A)=\dfrac{18}{29}$

By multiplication rule of probability, we have

$$P(A\cap B)=P(A).P(B\,|\,A)=\frac{2}{5}\times\frac{18}{29}=\frac{36}{145}$$

29. We have, $f(x)=\begin{cases}-x+3, & x\le-3\\ -2x, & -3<x<3\\ 6x+2, & x\ge3\end{cases}$

Clearly, the possible points of discontinuity of f are 3 and -3.

[$\because$ For all other points $f(x)$ is a linear polynomial, which is continuous everywhere]

Continuity at $x = -3$:

$\lim\limits_{x\to-3^-}f(x)=\lim\limits_{x\to-3}(-x+3)=3+3=6$

$\lim\limits_{x\to-3^+}f(x)=\lim\limits_{x\to-3}(-2x)=6$

$f(-3)=-(-3)+3=3+3=6$

Thus, $\lim\limits_{x\to-3^-}f(x)=\lim\limits_{x\to-3^+}f(x)=f(-3)$

$\therefore\ f$ is continuous at $x=-3$.

Continuity at $x = 3$:

$\lim\limits_{x\to3^-}f(x)=\lim\limits_{x\to3}(-2x)=-6$

$\lim\limits_{x\to3^+}f(x)=\lim\limits_{x\to3}(6x+2)=6(3)+2=20$

Thus, $\lim\limits_{x\to3^-}f(x)\ne\lim\limits_{x\to3^+}f(x)$

$\therefore\ f(x)$ is discontinuous at $x=3$.

So, the only point of discontinuity of f is $x=3$.

30. We have, $\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}-\dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy}=1$

$\Rightarrow\ \dfrac{dy}{dx}=\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}-\dfrac{y}{\sqrt{x}}\Rightarrow\dfrac{dy}{dx}+\dfrac{y}{\sqrt{x}}=\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}$

This is a linear differential equation of the form

$\dfrac{dy}{dx}+Py=Q$, with $P=\dfrac{1}{\sqrt{x}}$ and $Q=\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}$

I.F. $=e^{\int P\,dx}=e^{\int\frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$

$\therefore$ The solution is given by

$$ye^{2\sqrt{x}}=\int e^{2\sqrt{x}}\cdot\frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx+C$$

$\Rightarrow\ ye^{2\sqrt{x}}=\int\dfrac{1}{\sqrt{x}}dx+C\ \Rightarrow\ ye^{2\sqrt{x}}=2\sqrt{x}+C$

$\Rightarrow\ y=\left(2\sqrt{x}+C\right)e^{-2\sqrt{x}}$, which is the required solution.

31. We have,

$$y=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$= \frac{\log \sin x}{\log \cos x}\left(\frac{\log \cos x}{\log \sin x}\right)^{-1} + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$= \left(\frac{\log \sin x}{\log \cos x}\right)^2 + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\therefore \frac{dy}{dx} = 2\left(\frac{\log \sin x}{\log \cos x}\right) \times \left\{\frac{(\log \cos x)\frac{1}{\sin x}\cos x - (\log \sin x)\frac{1}{\cos x}(-\sin x)}{(\log \cos x)^2}\right\}$$

$$+ \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \times \frac{(1+x^2)2 - 2x(0+2x)}{(1+x^2)^2}$$

$$= \frac{2\log(\sin x)}{\{\log(\cos x)\}^3}\{\cot x(\log \cos x) + \tan x(\log \sin x)\}$$

$$+ \frac{2(1-x^2)}{|1-x^2|(1+x^2)}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{2\log(1/\sqrt{2})}{\{\log(1/\sqrt{2})\}^3}\{\log(1/\sqrt{2}) + \log(1/\sqrt{2})\}$$

$$+ \frac{32}{\pi^2+16}$$

$$= \frac{4}{\log(1/\sqrt{2})} + \frac{32}{\pi^2+16} = \frac{32}{\pi^2+16} - \frac{8}{\log 2}$$

OR

We have, $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$

Here, $\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$

and $\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta}$$

$$= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta} = \frac{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}} = \tan\frac{3\theta}{2}$$

Differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \sec^2\frac{3\theta}{2}\cdot\frac{3}{2}\frac{d\theta}{dx} = \frac{3}{2}\cdot\frac{1}{\cos^2\left(\frac{3\theta}{2}\right)}\cdot\frac{1}{-2\sin\theta + 2\sin 2\theta}$$

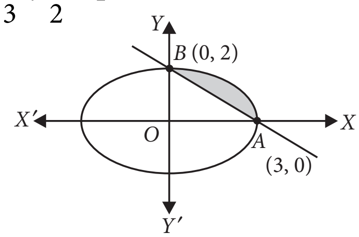
$$= \frac{3}{4}\cdot\frac{1}{\cos^2\left(\frac{3\theta}{2}\right)}\cdot\frac{1}{2\cos\left(\frac{3\theta}{2}\right)\sin\frac{\theta}{2}} = \frac{3}{8}\frac{1}{\cos^3\left(\frac{3\theta}{2}\right)\sin\frac{\theta}{2}}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}} = \frac{3}{8\cos^3\left(\frac{3\pi}{4}\right)\sin\frac{\pi}{4}} = \frac{3}{8\left(-\frac{1}{\sqrt{2}}\right)^3\frac{1}{\sqrt{2}}}$$

$$= -\frac{3\times 4}{8} = -\frac{3}{2}$$

32. Given ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$

and line is $\frac{x}{3} + \frac{y}{2} = 1$



Required area = Area of shaded region

$$= \int_0^3 \left[2\sqrt{1-\frac{x^2}{9}} - 2\left(1-\frac{x}{3}\right)\right]dx$$

$$= \int_0^3 \frac{2}{3}\sqrt{9-x^2}\,dx - \int_0^3 2\left(1-\frac{x}{3}\right)dx$$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 - 2\left[x - \frac{x^2}{6}\right]_0^3$$

$$= \frac{2}{3}\left[\frac{9}{2}\times\frac{\pi}{2}\right] - 2\left[3 - \frac{3}{2}\right]$$

$$= \frac{3}{2}\pi - 3 = \frac{3\pi - 6}{2} = \frac{3}{2}(\pi - 2) \text{ sq. units.}$$

33. Injectivity : Let (a_1, b_1) and $(a_2, b_2) \in A \times B$ such that, $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2) \Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

Thus, $f(a_1, b_1) = f(a_2, b_2) \Rightarrow (a_1, b_1) = (a_2, b_2)$ for all $(a_1, b_1), (a_2, b_2) \in A \times B$.

So, f is an injective function.

Surjectivity : Let (b, a) be an arbitrary element of $B \times A$, where, $b \in B$ and $a \in A \Rightarrow (a, b) \in A \times B$

Thus, for all $(b, a) \in B \times A$, their exists $(a, b) \in (A \times B)$ such that, $f(a, b) = (b, a)$

So, $f : A \times B \to B \times A$ is an onto function.

Hence, f is a bijective function.

34. Clearly, domain $(f) = R - \{0\}$

We have, $f(x) = a \log|x| + bx^2 + x$

$$\Rightarrow f'(x) = \frac{a}{x} + 2bx + 1$$

Since $f(x)$ has extreme values at $x = -1$ and $x = 2$.

Therefore, $f'(-1) = 0$ and $f'(2) = 0$

$\Rightarrow\ -a - 2b + 1 = 0$ and $\dfrac{a}{2}+4b+1=0$

$\Rightarrow\ a + 2b = 1$ ...(i)

and $a + 8b = -2$ ...(ii)

Solving (i) and (ii), we get

$$a=2 \text{ and } b=-\frac{1}{2}.$$

OR

We have, $f(x) = \cos(2x + \pi/4)$

$\Rightarrow\ f'(x) = -2\sin(2x + \pi/4)$

Now, $x \in (3\pi/8, 7\pi/8)$

$\Rightarrow\ 3\pi/8 < x < 7\pi/8 \ \Rightarrow\ 3\pi/4 < 2x < 7\pi/4$

$\Rightarrow\ \pi/4 + 3\pi/4 < 2x + \pi/4 < 7\pi/4 + \pi/4$

$\Rightarrow\ \pi < 2x + \pi/4 < 2\pi \ \Rightarrow\ \sin(2x + \pi/4) < 0$

[$\because$ sine function is negative in third and fourth quadrants]

$\Rightarrow\ -2\sin(2x + \pi/4) > 0 \ \Rightarrow\ f'(x) > 0$

Hence, $f(x)$ is increasing on $(3\pi/8, 7\pi/8)$.

35. Let $I=\displaystyle\int_0^{\pi/4}\frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$$=\int_0^{\pi/4}\frac{dx}{\cos^3 x\sqrt{2\cdot 2\sin x\cos x}}=\frac{1}{2}\int_0^{\pi/4}\frac{dx}{\cos^{\frac{7}{2}} x\cdot\sin^{\frac{1}{2}} x}$$

$$=\frac{1}{2}\int_0^{\pi/4}\frac{dx}{\cos^{\frac{7}{2}} x\cdot\tan^{\frac{1}{2}} x\cdot\cos^{\frac{1}{2}} x}$$

$$=\frac{1}{2}\int_0^{\pi/4}\frac{dx}{\cos^4 x\sqrt{\tan x}}=\frac{1}{2}\int_0^{\pi/4}\frac{\sec^4 x}{\sqrt{\tan x}}\,dx$$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

Also $x = 0 \Rightarrow t = 0$ and $x=\dfrac{\pi}{4}\Rightarrow t=1$

$$\therefore\ I=\frac{1}{2}\int_0^1\frac{(1+t^2)dt}{\sqrt{t}}=\frac{1}{2}\int_0^1(t^{-\frac{1}{2}}+t^{\frac{3}{2}})\,dt$$

$$=\frac{1}{2}\left[\frac{t^{1/2}}{1/2}+\frac{t^{5/2}}{5/2}\right]_0^1=\frac{6}{5}$$

36. Any plane through the line of intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+24=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$ is given by

$$\vec{r}\cdot(2\hat{i}+6\hat{j})+24+\lambda[\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})]=0$$

$$\vec{r}\cdot((2\hat{i}+6\hat{j})+\lambda(3\hat{i}-\hat{j}+4\hat{k}))+24=0$$

$\Rightarrow\ \vec{r}\cdot((2+3\lambda)\hat{i}+(6-\lambda)\hat{j}+4\lambda\hat{k})+24=0$ ...(i)

Given, distance of (i) from origin is 2.

$$\therefore\ \frac{24}{\sqrt{(2+3\lambda)^2+(6-\lambda)^2+16\lambda^2}}=2$$

$\Rightarrow\ 12=\sqrt{(2+3\lambda)^2+(6-\lambda)^2+16\lambda^2}$

Squaring both sides, we get

$144 = 4 + 9\lambda^2 + 12\lambda + 36 + \lambda^2 - 12\lambda + 16\lambda^2$

$\Rightarrow\ 26\lambda^2 = 104 \ \Rightarrow\ \lambda^2 = 4 \ \Rightarrow\ \lambda = \pm 2$

Now, from (i), we get

$\vec{r}\cdot(8\hat{i}+4\hat{j}+8\hat{k})+24=0$ and

$\vec{r}\cdot(-4\hat{i}+8\hat{j}-8\hat{k})+24=0$

$\Rightarrow\ \vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})+6=0$ and

$\vec{r}\cdot(-\hat{i}+2\hat{j}-2\hat{k})+6=0$

$\Rightarrow\ 2x + y + 2z + 6 = 0$ and $x - 2y + 2z - 6 = 0$

OR

The lines are $\dfrac{x-1}{1}=\dfrac{y}{-1}=\dfrac{z}{2}$ ...(i)

and $\dfrac{x+1}{2}=\dfrac{y}{2}=\dfrac{z-3}{\lambda}$ ...(ii)

Here, $x_1 = 1, y_1 = 0, z_1 = 0$

$a_1 = 1, b_1 = -1, c_1 = 2$

$x_2 = -1, y_2 = 0, z_2 = 3$

$a_2 = 2, b_2 = 2, c_2 = \lambda$

Now, $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} -2 & 0 & 3 \\ 1 & -1 & 2 \\ 2 & 2 & \lambda \end{vmatrix}$

$= -2(-\lambda - 4) + 0 + 3(2 + 2)$

$= 2\lambda + 8 + 12 = 2\lambda + 20$

and $(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2$

$= (-\lambda - 4)^2 + (4 - \lambda)^2 + (2 + 2)^2$

$= \lambda^2 + 16 + 8\lambda + 16 + \lambda^2 - 8\lambda + 16 = 2\lambda^2 + 48$

Shortest distance between lines

$=\dfrac{|2\lambda+20|}{\sqrt{2\lambda^2+48}}=1$ [Given]

$\Rightarrow\ 2\lambda+20=\sqrt{2\lambda^2+48}$

$\Rightarrow\ 4\lambda^2 + 400 + 80\lambda = 2\lambda^2 + 48$

$\Rightarrow\ 2\lambda^2 + 80\lambda + 352 = 0 \Rightarrow \lambda^2 + 40\lambda + 176 = 0$

$$\Rightarrow\ \lambda=\frac{-40\pm\sqrt{1600-4(1)(176)}}{2}$$

$$=\frac{-40\pm\sqrt{1600-704}}{2}=\frac{-40\pm\sqrt{896}}{2}$$

$$=\frac{-40\pm 2\sqrt{224}}{2}=-20\pm\sqrt{224}$$

37. The given problem can be written as

Maximize $Z = 50x + 40y$

subject to constraints :

$5x + 6y \le 38$

$3x + 2y \le 18$

$x \geq 0, y \geq 0$

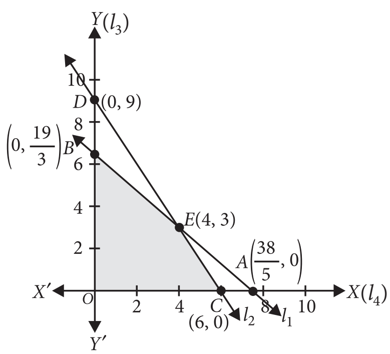
Now, let us draw the lines

$l_1 : 5x + 6y = 38$

or $\dfrac{x}{\left(\dfrac{38}{5}\right)} + \dfrac{y}{\left(\dfrac{19}{3}\right)} = 1$

$l_2 : 3x + 2y = 18$

or $\dfrac{x}{6} + \dfrac{y}{9} = 1$



$l_3 : x = 0$ and $l_4 : y = 0$

Lines l_1 and l_2 intersect at $E(4, 3)$.

The shaded region $OCEB$ is the feasible region which is bounded.

Corner points of the feasible region are $O(0, 0)$, $C(6, 0)$, $E(4, 3)$ and $B\left(0, \dfrac{19}{3}\right)$

The value of the objective function $Z = 50x + 40y$ at corner points are given below:

At O, $Z = 50 \times 0 + 40 \times 0 = 0$

At C, $Z = 50 \times 6 + 40 \times 0 = 300$

At E, $Z = 50 \times 4 + 40 \times 3 = 320$ (Maximum)

At B, $Z = 50 \times 0 + 40 \times \dfrac{19}{3} = 253.33$

Clearly, the maximum value is 320 at $E(4, 3)$.

OR

The given problem is

Minimize $Z = 5x + 7y$

subject to $2x + y \leq 8$

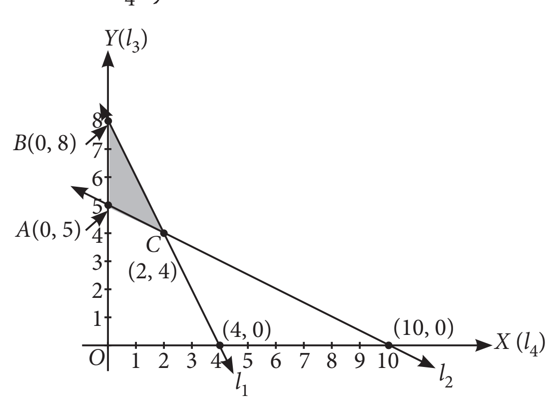
$x + 2y \geq 10$ and $x, y \geq 0$

To solve this LPP graphically, we first convert the inequations into equations to obtain the following line

$l_1 : 2x + y = 8$, or $\dfrac{x}{4} + \dfrac{y}{8} = 1$

$l_2 : x + 2y = 10$, or $\dfrac{x}{10} + \dfrac{y}{5} = 1$

$l_3 : x = 0$ and $l_4 : y = 0$



The coordinates of the corner points of the feasible region ABC are $A(0, 5)$, $B(0, 8)$ and $C(2, 4)$.

The values of the objective function $Z = 5x + 7y$ at the corner points of the feasible region are given in the following table.

Corner Points	Value of $Z = 5x + 7y$
$A(0, 5)$	$5 \times 0 + 7 \times 5 = 35$ (Minimum)
$B(0, 8)$	$5 \times 0 + 7 \times 8 = 56$
$C(2, 4)$	$5 \times 2 + 7 \times 4 = 38$

Thus, Z is minimum when $x = 0$ and $y = 5$.

38. $AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$= 6\, I_3$

$\Rightarrow A\left(\dfrac{1}{6}B\right) = I_3 \Rightarrow A^{-1} = \dfrac{1}{6}B$

(By definition of inverse)

$\Rightarrow A^{-1} = \dfrac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

The given system of equations is

$x - y + 0z = 6$

$2x + 3y + 4z = 34$

$0x + y + 2z = 14$

This system of equations can be written as

$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 34 \\ 14 \end{bmatrix}$ or $AX = C$

where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 6 \\ 34 \\ 14 \end{bmatrix}$

As A^{-1} exists, therefore $X = A^{-1}C$

$\Rightarrow X = \dfrac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 6 \\ 34 \\ 14 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12+68-56 \\ -24+68-56 \\ 12-34+70 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 24 \\ -12 \\ 48 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 8 \end{bmatrix}$$

$\Rightarrow \; x = 4, y = -2, z = 8$

Hence, the solution of the given system of equations is $x = 4, y = -2, z = 8$.

OR

The equations can be written in the form $AX = B$,

where, $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 2(75) - 3(-110) + 10(72) = 150 + 330 + 720$

$= 1200 \neq 0$

$\therefore \; A^{-1}$ exists.

$$\therefore \; \text{adj } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

As, $AX = B \quad \Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Thus, $\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$

Hence, $x = 2, y = 3, z = 5$

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