

## PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

### (A) Main Concepts and Results

- Two linear equations in the same two variables are said to form a pair of linear equations in two variables.
- The most general form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0,$$

where  $a_1, a_2, b_1, b_2, c_1, c_2$  are real numbers, such that  $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$ .

- A pair of linear equations is consistent if it has a solution – either a unique or infinitely many.

In case of infinitely many solutions, the pair of linear equations is also said to be dependent. Thus, in this case, the pair of linear equations is dependent and consistent.

- A pair of linear equations is inconsistent, if it has no solution.
- Let a pair of linear equations in two variables be  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$ .

(I) If  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ , then

- (i) the pair of linear equations is consistent,
  - (ii) the graph will be a pair of lines intersecting at a unique point, which is the solution of the pair of equations.
- (II) If  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ , then
- (i) the pair of linear equations is inconsistent,
  - (ii) the graph will be a pair of parallel lines and so the pair of equations will have no solution.
- (III) If  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , then
- (i) the pair of linear equations is dependent, and consistent,
  - (ii) the graph will be a pair of coincident lines. Each point on the lines will be a solution, and so the pair of equations will have infinitely many solutions.
- A pair of linear equations can be solved algebraically by any of the following methods:
    - (i) Substitution Method
    - (ii) Elimination Method
    - (iii) Cross- multiplication Method
  - The pair of linear equations can also be solved geometrically/graphically.

### (B) Multiple Choice Questions

Choose the correct answer from the given four options:

**Sample Question 1 :** The pair of equations  $5x - 15y = 8$  and  $3x - 9y = \frac{24}{5}$  has

- (A) one solution    (B) two solutions    (C) infinitely many solutions  
(D) no solution

**Solution :** Answer (C)

**Sample Question 2 :** The sum of the digits of a two-digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is

- (A) 25                      (B) 72                      (C) 63                      (D) 36

**Solution :** Answer (D)

### EXERCISE 3.1

Choose the correct answer from the given four options:

1. Graphically, the pair of equations

$$6x - 3y + 10 = 0$$

$$2x - y + 9 = 0$$

represents two lines which are

- (A) intersecting at exactly one point.                      (B) intersecting at exactly two points.  
(C) coincident.                      (D) parallel.
2. The pair of equations  $x + 2y + 5 = 0$  and  $-3x - 6y + 1 = 0$  have  
(A) a unique solution                      (B) exactly two solutions  
(C) infinitely many solutions                      (D) no solution
3. If a pair of linear equations is consistent, then the lines will be  
(A) parallel                      (B) always coincident  
(C) intersecting or coincident                      (D) always intersecting
4. The pair of equations  $y = 0$  and  $y = -7$  has  
(A) one solution                      (B) two solutions  
(C) infinitely many solutions                      (D) no solution
5. The pair of equations  $x = a$  and  $y = b$  graphically represents lines which are  
(A) parallel                      (B) intersecting at  $(b, a)$   
(C) coincident                      (D) intersecting at  $(a, b)$
6. For what value of  $k$ , do the equations  $3x - y + 8 = 0$  and  $6x - ky = -16$  represent coincident lines?  
(A)  $\frac{1}{2}$                       (B)  $-\frac{1}{2}$                       (C) 2                      (D) -2

7. If the lines given by  $3x + 2ky = 2$  and  $2x + 5y + 1 = 0$  are parallel, then the value of  $k$  is
- (A)  $-\frac{5}{4}$  (B)  $\frac{2}{5}$  (C)  $\frac{15}{4}$  (D)  $\frac{3}{2}$
8. The value of  $c$  for which the pair of equations  $cx - y = 2$  and  $6x - 2y = 3$  will have infinitely many solutions is
- (A) 3 (B)  $-3$  (C)  $-12$  (D) no value
9. One equation of a pair of dependent linear equations is  $-5x + 7y = 2$ . The second equation can be
- (A)  $10x + 14y + 4 = 0$  (B)  $-10x - 14y + 4 = 0$   
(C)  $-10x + 14y + 4 = 0$  (D)  $10x - 14y = -4$
10. A pair of linear equations which has a unique solution  $x = 2, y = -3$  is
- (A)  $x + y = -1$  (B)  $2x + 5y = -11$   
 $2x - 3y = -5$   $4x + 10y = -22$   
(C)  $2x - y = 1$  (D)  $x - 4y - 14 = 0$   
 $3x + 2y = 0$   $5x - y - 13 = 0$
11. If  $x = a, y = b$  is the solution of the equations  $x - y = 2$  and  $x + y = 4$ , then the values of  $a$  and  $b$  are, respectively
- (A) 3 and 5 (B) 5 and 3  
(C) 3 and 1 (D)  $-1$  and  $-3$
12. Aruna has only Re 1 and Rs 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs 75, then the number of Re 1 and Rs 2 coins are, respectively
- (A) 35 and 15 (B) 35 and 20  
(C) 15 and 35 (D) 25 and 25
13. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively
- (A) 4 and 24 (B) 5 and 30  
(C) 6 and 36 (D) 3 and 24

**(C) Short Answer Questions with Reasoning**

**Sample Question 1 :** Is it true to say that the pair of equations

$$-x + 2y + 2 = 0 \text{ and } \frac{1}{2}x - \frac{1}{4}y - 1 = 0$$

has a unique solution? Justify your answer.

**Solution :** Yes.

$$\text{Here, } \frac{a_1}{a_2} = \frac{-1}{\frac{1}{2}} = -2, \quad \frac{b_1}{b_2} = \frac{2}{-\frac{1}{4}} = -8$$

As  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ , the pair of equations has a unique solution.

**Sample Question 2 :** Do the equations  $4x + 3y - 1 = 5$  and  $12x + 9y = 15$  represent a pair of coincident lines? Justify your answer.

**Solution :** No.

We may rewrite the equations as

$$4x + 3y = 6$$

$$12x + 9y = 15$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{1}{3} \text{ and } \frac{c_1}{c_2} = \frac{2}{5}$$

As  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ , the given equations do not represent a pair of coincident lines.

**Sample Question 3 :** Is the pair of equations  $x + 2y - 3 = 0$  and  $6y + 3x - 9 = 0$  consistent? Justify your answer.

**Solution :** Yes.

Rearranging the terms in the equations, we get

$$x + 2y - 3 = 0$$

$$3x + 6y - 9 = 0$$

Here,  $\frac{a_1}{a_2} = \frac{1}{3}$ ,  $\frac{b_1}{b_2} = \frac{1}{3}$ ,  $\frac{c_1}{c_2} = \frac{1}{3}$ . As  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ , the pair of equations is consistent.

### EXERCISE 3.2

1. Do the following pair of linear equations have no solution? Justify your answer.

(i)  $2x + 4y = 3$

(ii)  $x = 2y$

$12y + 6x = 6$

$y = 2x$

(iii)  $3x + y - 3 = 0$

$2x + \frac{2}{3}y = 2$

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i)  $3x + \frac{1}{7}y = 3$

(ii)  $-2x - 3y = 1$

$7x + 3y = 7$

$6y + 4x = -2$

(iii)  $\frac{x}{2} + y + \frac{2}{5} = 0$

$4x + 8y + \frac{5}{16} = 0$

3. Are the following pair of linear equations consistent? Justify your answer.

(i)  $-3x - 4y = 12$

(ii)  $\frac{3}{5}x - y = \frac{1}{2}$

$4y + 3x = 12$

$\frac{1}{5}x - 3y = \frac{1}{6}$

(iii)  $2ax + by = a$

(iv)  $x + 3y = 11$

$4ax + 2by - 2a = 0; a, b \neq 0$

$2(2x + 6y) = 22$

4. For the pair of equations

$\lambda x + 3y = -7$

$2x + 6y = 14$

to have infinitely many solutions, the value of  $\lambda$  should be 1. Is the statement true? Give reasons.

5. For all real values of  $c$ , the pair of equations

$$x - 2y = 8$$

$$5x - 10y = c$$

have a unique solution. Justify whether it is true or false.

6. The line represented by  $x = 7$  is parallel to the  $x$ -axis. Justify whether the statement is true or not.

### (D) Short Answer Questions

**Sample Question 1 :** For which values of  $p$  and  $q$ , will the following pair of linear equations have infinitely many solutions?

$$4x + 5y = 2$$

$$(2p + 7q)x + (p + 8q)y = 2q - p + 1.$$

**Solution :**

Here,  $\frac{a_1}{a_2} = \frac{4}{2p + 7q}$

$$\frac{b_1}{b_2} = \frac{5}{p + 8q}$$

$$\frac{c_1}{c_2} = \frac{2}{2q - p + 1}$$

For a pair of linear equations to have infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So,  $\frac{4}{2p + 7q} = \frac{5}{p + 8q} = \frac{2}{2q - p + 1}$

So,  $\frac{4}{2p + 7q} = \frac{5}{p + 8q}$  and  $\frac{4}{2p + 7q} = \frac{2}{2q - p + 1}$

$$\text{i.e., } 4p + 32q = 10p + 35q \text{ and } 8q - 4p + 4 = 4p + 14q$$

$$\text{i.e., } 6p + 3q = 0 \quad \text{and } 8p + 6q = 4$$

$$\text{i.e., } q = -2p \quad (1) \quad \text{and } 4p + 3q = 2 \quad (2)$$

Substituting the value of  $q$  obtained from Equation(1) in Equation(2), we get

$$4p - 6p = 2$$

$$\text{or } p = -1$$

Substituting the value of  $p$  in Equation (1), we get

$$q = 2$$

So, for  $p = -1, q = 2$ , the given pair of linear equations will have infinitely many solutions.

**Sample Question 2:** Solve the following pair of linear equations:

$$21x + 47y = 110$$

$$47x + 21y = 162$$

**Solution:** We have

$$21x + 47y = 110 \quad (1)$$

$$47x + 21y = 162 \quad (2)$$

Multiplying Equation (1) by 47 and Equation (2) by 21, we get

$$987x + 2209y = 5170 \quad (3)$$

$$987x + 441y = 3402 \quad (4)$$

Subtracting Equation (4) from Equation (3), we get

$$1768y = 1768$$

$$\text{or } y = 1$$

Substituting the value of  $y$  in Equation (1), we get

$$21x + 47 = 110$$

$$\text{or } 21x = 63$$

$$\text{or } x = 3$$

$$\text{So, } x = 3, y = 1$$

**Alternative Solution:** We have

$$21x + 47y = 110 \quad (1)$$



$$47x + 21y = 162 \quad (2)$$

Adding Equations (1) and (2), we have

$$68x + 68y = 272$$

or  $x + y = 4 \quad (5)$

Subtracting Equation (1) from Equation (2), we have

$$26x - 26y = 52$$

or  $x - y = 2 \quad (6)$

On adding and subtracting Equations (5) and (6), we get

$$x = 3, \quad y = 1$$

**Sample Question 3 :** Draw the graphs of the pair of linear equations  $x - y + 2 = 0$  and  $4x - y - 4 = 0$ . Calculate the area of the triangle formed by the lines so drawn and the  $x$ -axis.

**Solution :**

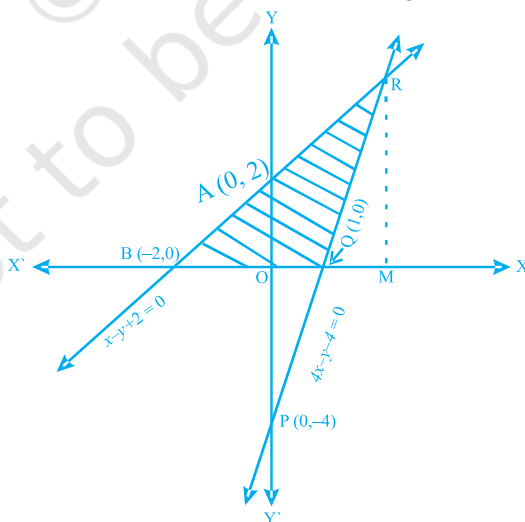
For drawing the graphs of the given equations, we find two solutions of each of the equations, which are given in Table 3.1

**Table 3.1**

|             |   |    |
|-------------|---|----|
| $x$         | 0 | -2 |
| $y = x + 2$ | 2 | 0  |

|              |    |   |
|--------------|----|---|
| $x$          | 0  | 1 |
| $y = 4x - 4$ | -4 | 0 |

Plot the points A (0, 2), B (-2, 0), P (0, -4) and Q (1, 0) on the graph paper, and join the points to form the lines AB and PQ as shown in Fig 3.1



**Fig. 3.1**

We observe that there is a point R (2, 4) common to both the lines AB and PQ. The triangle formed by these lines and the  $x$ -axis is BQR.

The vertices of this triangle are B (−2, 0), Q (1, 0) and R (2, 4).

We know that;

$$\text{Area of triangle} = \frac{1}{2} \text{Base} \times \text{Altitude}$$

Here, Base = BQ = BO + OQ = 2 + 1 = 3 units.

Altitude = RM = Ordinate of R = 4 units.

$$\text{So, area of } \triangle \text{ BQR} = \frac{1}{2} \times 3 \times 4 = 6 \text{ sq. units.}$$

### EXERCISE 3.3

1. For which value(s) of  $\lambda$ , do the pair of linear equations

$$\lambda x + y = \lambda^2 \text{ and } x + \lambda y = 1 \quad \text{have}$$

- (i) no solution?
- (ii) infinitely many solutions?
- (iii) a unique solution?

2. For which value(s) of  $k$  will the pair of equations

$$kx + 3y = k - 3$$

$$12x + ky = k$$

have no solution?

3. For which values of  $a$  and  $b$ , will the following pair of linear equations have infinitely many solutions?

$$x + 2y = 1$$

$$(a - b)x + (a + b)y = a + b - 2$$

4. Find the value(s) of  $p$  in (i) to (iv) and  $p$  and  $q$  in (v) for the following pair of equations:

- (i)  $3x - y - 5 = 0$  and  $6x - 2y - p = 0$ ,  
if the lines represented by these equations are parallel.

- (ii)  $-x + py = 1$  and  $px - y = 1$ ,  
if the pair of equations has no solution.
- (iii)  $-3x + 5y = 7$  and  $2px - 3y = 1$ ,  
if the lines represented by these equations are intersecting at a unique point.
- (iv)  $2x + 3y - 5 = 0$  and  $px - 6y - 8 = 0$ ,  
if the pair of equations has a unique solution.
- (v)  $2x + 3y = 7$  and  $2px + py = 28 - qy$ ,  
if the pair of equations have infinitely many solutions.
5. Two straight paths are represented by the equations  $x - 3y = 2$  and  $-2x + 6y = 5$ . Check whether the paths cross each other or not.
6. Write a pair of linear equations which has the unique solution  $x = -1$ ,  $y = 3$ . How many such pairs can you write?
7. If  $2x + y = 23$  and  $4x - y = 19$ , find the values of  $5y - 2x$  and  $\frac{y}{x} - 2$ .
8. Find the values of  $x$  and  $y$  in the following rectangle [see Fig. 3.2].

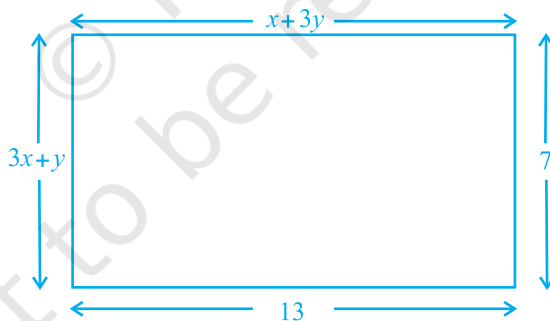


Fig. 3.2

9. Solve the following pairs of equations:

(i)  $x + y = 3.3$

$$\frac{0.6}{3x-2y} = -1, \quad 3x-2y \neq 0$$

(ii)  $\frac{x}{3} + \frac{y}{4} = 4$

$$\frac{5x}{6} - \frac{y}{8} = 4$$

$$(iii) \quad 4x + \frac{6}{y} = 15$$

$$6x - \frac{8}{y} = 14, y \neq 0$$

$$(v) \quad 43x + 67y = -24$$

$$67x + 43y = 24$$

$$(vii) \quad \frac{2xy}{x+y} = \frac{3}{2}$$

$$\frac{xy}{2x-y} = \frac{-3}{10}, \quad x+y \neq 0, 2x-y \neq 0$$

$$(iv) \quad \frac{1}{2x} - \frac{1}{y} = -1$$

$$\frac{1}{x} + \frac{1}{2y} = 8, \quad x, y \neq 0$$

$$(vi) \quad \frac{x}{a} + \frac{y}{b} = a+b$$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2, \quad a, b \neq 0$$

- 10.** Find the solution of the pair of equations  $\frac{x}{10} + \frac{y}{5} - 1 = 0$  and  $\frac{x}{8} + \frac{y}{6} = 15$ .

Hence, find  $\lambda$ , if  $y = \lambda x + 5$ .

- 11.** By the graphical method, find whether the following pair of equations are consistent or not. If consistent, solve them.

$$(i) \quad 3x + y + 4 = 0$$

$$(ii) \quad x - 2y = 6$$

$$6x - 2y + 4 = 0$$

$$3x - 6y = 0$$

$$(iii) \quad x + y = 3$$

$$3x + 3y = 9$$

- 12.** Draw the graph of the pair of equations  $2x + y = 4$  and  $2x - y = 4$ . Write the vertices of the triangle formed by these lines and the  $y$ -axis. Also find the area of this triangle.
- 13.** Write an equation of a line passing through the point representing solution of the pair of linear equations  $x+y=2$  and  $2x-y=1$ . How many such lines can we find?
- 14.** If  $x+1$  is a factor of  $2x^3 + ax^2 + 2bx + 1$ , then find the values of  $a$  and  $b$  given that  $2a-3b=4$ .
- 15.** The angles of a triangle are  $x$ ,  $y$  and  $40^\circ$ . The difference between the two angles  $x$  and  $y$  is  $30^\circ$ . Find  $x$  and  $y$ .

16. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?
17. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.
18. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.
19. There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.
20. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid Rs 22 for a book kept for six days, while Anand paid Rs 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.
21. In a competitive examination, one mark is awarded for each correct answer while  $\frac{1}{2}$  mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?
22. The angles of a cyclic quadrilateral ABCD are  
 $\angle A = (6x + 10)^\circ$ ,  $\angle B = (5x)^\circ$   
 $\angle C = (x + y)^\circ$ ,  $\angle D = (3y - 10)^\circ$   
Find  $x$  and  $y$ , and hence the values of the four angles.

### (E) Long Answer Questions

**Sample Question 1 :** Draw the graphs of the lines  $x = -2$  and  $y = 3$ . Write the vertices of the figure formed by these lines, the  $x$ -axis and the  $y$ -axis. Also, find the area of the figure.

#### Solution :

We know that the graph of  $x = -2$  is a line parallel to  $y$ -axis at a distance of 2 units to the left of it.

So, the line  $l$  is the graph of  $x = -2$  [see Fig. 3.3]

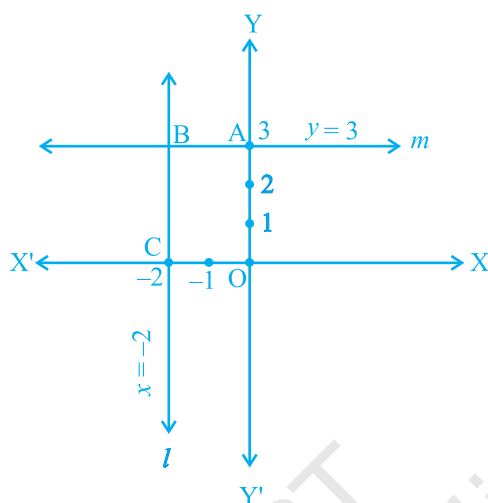


Fig. 3.3

The graph of  $y = 3$  is a line parallel to the  $x$ -axis at a distance of 3 units above it.

So, the line  $m$  is the graph of  $y = 3$ .

The figure enclosed by the lines  $x = -2$ ,  $y = 3$ , the  $x$ -axis and the  $y$ -axis is OABC, which is a rectangle. (Why?)

A is a point on the  $y$ -axis at a distance of 3 units above the  $x$ -axis. So, the coordinates of A are  $(0, 3)$ ;

C is a point on the  $x$ -axis at a distance of 2 units to the left of  $y$ -axis. So, the coordinates of C are  $(-2, 0)$

B is the solution of the pair of equations  $x = -2$  and  $y = 3$ . So, the coordinates of B are  $(-2, 3)$

So, the vertices of the rectangle OABC are O  $(0, 0)$ , A  $(0, 3)$ , B  $(-2, 3)$ , C  $(-2, 0)$

The length and breadth of this rectangle are 2 units and 3 units, respectively.

As the area of a rectangle = length  $\times$  breadth,

the area of rectangle OABC =  $2 \times 3 = 6$  sq. units.

**Sample Question 2:** Determine, algebraically, the vertices of the triangle formed by the lines

$$5x - y = 5, \quad x + 2y = 1 \text{ and } 6x + y = 17.$$

**Solution:**

The vertex of a triangle is the common solution of the two equations forming its two sides. So, solving the given equations pairwise will give the vertices of the triangle.

From the given equations, we will have the following three pairs of equations:

$$5x - y = 5 \quad \text{and} \quad x + 2y = 1$$

$$x + 2y = 1 \quad \text{and} \quad 6x + y = 17$$

$$5x - y = 5 \quad \text{and} \quad 6x + y = 17$$

Solving the pair of equations

$$5x - y = 5$$

$$x + 2y = 1$$

we get,  $x = 1, y = 0$

So, one vertex of the triangle is  $(1, 0)$

Solving the second pair of equations

$$x + 2y = 1$$

$$6x + y = 17$$

we get  $x = 3, y = -1$

So, another vertex of the triangle is  $(3, -1)$

Solving the third pair of equations

$$5x - y = 5$$

$$6x + y = 17,$$

we get  $x = 2, y = 5$ .

So, the third vertex of the triangle is  $(2, 5)$ . So, the three vertices of the triangle are  $(1, 0), (3, -1)$  and  $(2, 5)$ .

**Sample Question 3 :** Jamila sold a table and a chair for Rs 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got Rs 1065. Find the cost price of each.

**Solution :** Let the cost price of the table be Rs  $x$  and the cost price of the chair be Rs  $y$ .

The selling price of the table, when it is sold at a profit of 10%

$$= \text{Rs } x + \frac{10}{100}x = \text{Rs } \frac{110}{100}x$$

The selling price of the chair when it is sold at a profit of 25%

$$= \text{Rs } y + \frac{25}{100}y = \text{Rs } \frac{125}{100}y$$

$$\text{So, } \frac{110}{100}x + \frac{125}{100}y = 10500 \quad (1)$$

When the table is sold at a profit of 25%, its selling price =  $\text{Rs } \left(x + \frac{25}{100}x\right) = \text{Rs } \frac{125}{100}x$

When the chair is sold at a profit of 10%, its selling price =  $\text{Rs } \left(y + \frac{10}{100}y\right) = \text{Rs } \frac{110}{100}y$

$$\text{So, } \frac{125}{100}x + \frac{110}{100}y = 10650 \quad (2)$$

From Equations (1) and (2), we get

$$110x + 125y = 105000$$

$$\text{and } 125x + 110y = 106500$$

On adding and subtracting these equations, we get

$$235x + 235y = 211500$$

$$\text{and } 15x - 15y = 1500$$

$$\text{i.e., } x + y = 900 \quad (3)$$

$$\text{and } x - y = 100 \quad (4)$$

Solving Equations (3) and (4), we get

$$x = 500, y = 400$$

So, the cost price of the table is Rs 500 and the cost price of the chair is Rs 400.



**Sample Question 4:** It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. How long would it take for each pipe to fill the pool separately?

**Solution:**

Let the time taken by the pipe of larger diameter to fill the pool be  $x$  hours and that taken by the pipe of smaller diameter pipe alone be  $y$  hours.

In  $x$  hours, the pipe of larger diameter fills the pool.

So, in 1 hour the pipe of larger diameter fills  $\frac{1}{x}$  part of the pool, and so, in 4 hours, the pipe of larger diameter fills  $\frac{4}{x}$  parts of the pool.

Similarly, in 9 hours, the pipe of smaller diameter fills  $\frac{9}{y}$  parts of the pool.

According to the question,

$$\frac{4}{x} + \frac{9}{y} = \frac{1}{2} \quad (1)$$

Also, using both the pipes, the pool is filled in 12 hours.

$$\text{So, } \frac{12}{x} + \frac{12}{y} = 1 \quad (2)$$

Let  $\frac{1}{x} = u$  and  $\frac{1}{y} = v$ . Then Equations (1) and (2) become

$$4u + 9v = \frac{1}{2} \quad (3)$$

$$12u + 12v = 1 \quad (4)$$

Multiplying Equation (3) by 3 and subtracting Equation (4) from it, we get

$$15v = \frac{1}{2} \text{ or } v = \frac{1}{30}$$

Substituting the value of  $v$  in Equation (4), we get  $u = \frac{1}{20}$

So,  $u = \frac{1}{20}, v = \frac{1}{30}$

So,  $\frac{1}{x} = \frac{1}{20}, \frac{1}{y} = \frac{1}{30}$

or,  $x = 20, y = 30$ .

So, the pipe of larger diameter alone can fill the pool in 20 hours and the pipe of smaller diameter alone can fill the pool in 30 hours.

### EXERCISE 3.4

1. Graphically, solve the following pair of equations:

$$2x + y = 6$$

$$2x - y + 2 = 0$$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the  $x$ -axis and the lines with the  $y$ -axis.

2. Determine, graphically, the vertices of the triangle formed by the lines

$$y = x, \quad 3y = x, \quad x + y = 8$$

3. Draw the graphs of the equations  $x = 3$ ,  $x = 5$  and  $2x - y - 4 = 0$ . Also find the area of the quadrilateral formed by the lines and the  $x$ -axis.

4. The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

5. Determine, algebraically, the vertices of the triangle formed by the lines

$$3x - y = 3$$

$$2x - 3y = 2$$

$$x + 2y = 8$$

6. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw, and the remaining distance by bus.

On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

7. A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.
8. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.
9. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.
10. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs Rs 2530. Also, one reserved first class ticket and one reserved first class half ticket from A to B costs Rs 3810. Find the full first class fare from station A to B, and also the reservation charges for a ticket.
11. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum Rs 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs 1028. Find the cost price of the saree and the list price (price before discount) of the sweater.
12. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received Rs 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received Rs 20 more as annual interest. How much money did she invest in each scheme?
13. Vijay had some bananas, and he divided them into two lots A and B. He sold the first lot at the rate of Rs 2 for 3 bananas and the second lot at the rate of Re 1 per banana, and got a total of Rs 400. If he had sold the first lot at the rate of Re 1 per banana, and the second lot at the rate of Rs 4 for 5 bananas, his total collection would have been Rs 460. Find the total number of bananas he had.