Total No. of Questions - 24

Total No. of Printed Pages - 4

Regd. No.

Part - III

MATHEMATICS, Paper - II (B)

(Coordinate Geometry and Calculus)

(English Version)

Time : 3 hours

Max. Marks : 75

Note : This question paper consists of **three** sections A, B and C.

SECTION A

$10 \times 2 = 20$

I.   **Very short** answer type questions.

   i)   Attempt **all** questions.

   ii) Each question carries **two** marks.

1.   If the length of the tangent from $(5, 4)$ to the circle $x^2 + y^2 + 2ky = 0$ is 1, then find k.

2.   Find the equation of the polar of $(1, -2)$ with respect to circle $x^2 + y^2 - 10x - 10y + 25 = 0$.

3.   Find the angle between the circles $x^2 + y^2 - 12x - 6y + 41 = 0$ and $x^2 + y^2 + 4x + 6y - 59 = 0$.

4.   Find the equation of parabola whose focus is $S(1, -7)$ and vertex is $A(1, -2)$.

5.   If the eccentricity of a hyperbola is $\dfrac{5}{4}$, then find the eccentricity of its conjugate hyperbola.

6. Evaluate $\int \dfrac{Log(1+x)}{1+x} dx$ on $(-1, \infty)$.

7. Evaluate $\int \dfrac{1}{1+Cos\,x} dx$ on $I \subset R - \{(2n+1)\pi : n \in z\}$.

8. Evaluate $\displaystyle\int_1^5 \dfrac{dx}{\sqrt{2x-1}}$.

9. Find the value of $\displaystyle\int_0^{2\pi} Sin^2 x \cdot Cos^4 x \; dx$.

10. Find the order and degree of the differential equation

$$x^{1/2}\left(\dfrac{d^2 y}{dx^2}\right)^{1/3} + x \cdot \dfrac{dy}{dx} + y = 0$$

SECTION B $\hspace{4cm}$ 5 × 4 = 20

II. Short answer type questions.

i) Attempt any five questions.

ii) Each question carries four marks.

11. Find the equation of the circle whose center lies on the X-axis and passing through the points $(-2, 3)$ and $(4, 5)$.

12. Show that the circles $S \equiv x^2 + y^2 - 2x - 4y - 20 = 0$, $S' \equiv x^2 + y^2 + 6x + 2y - 90 = 0$ touch each other internally. Find their point of contact.

13. Find the equation of the ellipse in the standard form whose distance between foci is 2 and the length of latus rectum is $\dfrac{15}{2}$.

14. Find the eccentricity and length of latus rectum of the ellipse $9x^2 + 16y^2 - 36x + 32y - 92 = 0$.

15. Find the equation of the tangents to the hyperbola $x^2 - 4y^2 = 4$ which are :

    i) Parallel   ii) Perpendicular to the line $x + 2y = 0$.

16. Obtain the reduction formula for $\displaystyle\int_{0}^{\pi/2} Sin^n x\, dx$ for an integer $n \geq 2$.

17. Solve the differential equation $\left(1 + x^2\right)\dfrac{dy}{dx} + y = e^{Tan^{-1}x}$.

SECTION C

5 × 7 = 35

III. **Long answer** type questions.

 i)   Attempt **any five** questions.

 ii)   Each question carries **seven** marks.

18. If $(2, 0)$, $(0, 1)$, $(4, 5)$ and $(0, c)$ are concyclic, then find c.

19. Find the direct common tangents of the circles $x^2 + y^2 + 22x - 4y - 100 = 0$ and $x^2 + y^2 - 22x + 4y + 100 = 0$.

20. Prove that the area of the triangle formed by the tangents at (x_1, y_1), (x_2, y_2) and (x_3, y_3) to the parabola $y^2 = 4ax\ (a > 0)$

    is $\dfrac{1}{16a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$ sq. units.

21. Evaluate $\int \dfrac{1}{1 + Sin\,x + Cos\,x}\,dx$.

22. Evaluate $\int \dfrac{2x + 5}{\sqrt{x^2 - 2x + 10}}\,dx$.

23. Evaluate $\int\limits_{0}^{\pi} \dfrac{x\,Sin^3 x}{1 + Cos^2 x}\,dx$.

24. Solve the differential equation
$$(2x + y + 1)dx + (4x + 2y - 1)dy = 0.$$

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