Algebraic Expressions and Identities

Multiplication of Monomials with Polynomials

Let us discuss another example based on the above concept.

Suppose we have two monomials $4x$ and $5y$. By multiplying them, we get

$4x \times 5y = (4 \times x) \times (5 \times y)$

$= (4 \times 5) \times (x \times y)$

$= 20xy$

Hence, we can say that $4x \times 5y = 20xy$

Now, $10x \times 5x^2z = (10 \times x) \times (5 \times x^2 \times z)$

This becomes,

$(10 \times 5) \times (x \times x^2 \times z) = 50x^3z$

Now, what if you have three monomials and you want to multiply them? How will you do so?

Suppose you want to multiply $10x$, $2xy$, and $5z$.

$10x \times 2xy \times 5z = (10 \times x) \times (2 \times x \times y) \times (5 \times z)$

First, we multiply the first two monomials.

$= \{(10 \times 2) \times (x \times x \times y)\} \times (5 \times z)$

$= (20 \times x^2 \times y) \times (5 \times z)$ [$\because\because x \times x = x^2$]

$= (20 \times 5) \times (x^2 \times y \times z)$

$= 100x^2yz$

This method of multiplication of three monomials can be extended to find out the product of any number of monomials.

Let us discuss some more examples based on multiplication of monomials.

Example 1:

Find the product of the following:

$(\mathbf{a})\ -2x^2y$ **and** $15\ xy^2z^3$

$(\mathbf{b})\ 7ap,\ 2qa^2x^3$ **and** $-\ 5rx$

$(\mathbf{c})\ ab,\ -2bc,\ -3cd$ **and** $4ad$

Solution:

$(\mathbf{a})\ -2x^2y \text{ and } 15\ xy^2z^3$

$= (-2x^2y) \times (15\ xy^2z^3)$

$= (-2 \times 15) \times (x^2y \times xy^2z^3)$

$= -30x^3y^3z^3 \qquad \{\text{as } x^2 \times x = x^3 \text{ and } y \times y^2 = y^3\}$

$(\mathbf{b})\ 7ap,\ 2qa^2x^3, \text{ and } -5rx$

$= (7ap) \times (2qa^2x^3) \times (-5rx)$

$= \{(7 \times 2) \times (ap \times qa^2x^3)\} \times (-5rx)$

$= (14a^3pqx^3) \times (-5rx)$

$= (14 \times -5) \times (a^3pqx^3 \times rx)$

$= -70a^3pqrx^4 \qquad [\text{as } x^3 \times x = x^4]$

$(\mathbf{c})\ ab,\ -2bc,\ -3cd, \text{ and } 4ad$

$= (ab) \times (-2bc) \times (-3cd) \times (4ad)$

$= [(1 \times -2) \times (ab \times bc)] \times [(-3 \times 4) \times (cd \times ad)]$

$\{\text{Multiplying the 1st to 2nd and 3rd to 4th term}\}$

$= (-2ab^2c) \times (-12cd^2a)$

$= (-2 \times -12) \times (ab^2c \times cd^2a)$

$= 24a^2b^2c^2d^2$

Example 2.

If the side of a square is 4*x* cm, what is its area?

Answer:

Side of square = 4*x* cm

∴ Area of square = side × side = $4x \times 4x = 16x^2$ cm^2

Example 3:

The length, breadth, and height of three cuboids are given below in the table. Find the volume and area of the base of these cuboids.

	Length	Breadth	Height
(i)	$3ab$	$2bx$	$5xy$
(ii)	a^2b	b^2c	c^2a
(iii)	$3x$	$9x^2$	$27x^3$

Solution:

We know that, area of the base = Length × Breadth

Volume of cuboid = Length × Breadth × Height

(i) Area of the base = $3ab \times 2bx = (3 \times 2) \times (ab \times bx) = 6ab^2x$

Volume of the cuboid = $3ab \times 2bx \times 5xy$

$= \{(3 \times 2) \times (ab \times bx)\} \times (5xy)$
$= (6ab^2x) \times (5xy)$
$= (6 \times 5) \times (ab^2x \times xy)$
$= 30\ ab^2x^2y$

(ii) Area of the base = $a^2b \times b^2c = a^2b^3c$

Volume of the cuboid = $\left(a^2b \times b^2c\right) \times c^2a = a^2b^3c \times c^2a = a^3\, b^3\, c^3$

(iii) Area of the base = $3x \times 9x^2 = (3 \times 9) \times \left(x \times x^2\right) = 27x^3$

Volume of the cuboid = $3x \times 9x^2 \times 27x^3$

$= \left[(3 \times 9) \times \left(x \times x^2\right)\right] \times 27x^3$

$= 27x^3 \times 27x^3$

$= (27 \times 27)\left(x^3 \times x^3\right)$

$= 729\ x^6$

So far, we know how two multiply any number of monomials. But, what if we need to multiply a monomial with a binomial or a trinomial, etc.?

Can we multiply them?

We can multiply them easily.

The method discussed in the above video shows the **horizontal arrangement** of multiplying monomials with polynomials.

Let us now learn about the **vertical arrangement** for the same by performing the multiplication of ($4x^2$ + $2x$) and $3x$.

This is similar to vertical method of multiplication of whole numbers.

Here, we will first multiply $3x$ with $2x$ and write the product with sign at the bottom. After doing this, we will multiply $3x$ with $4x^2$ and write the product with sign at the bottom. The expression obtained at the bottom will be the required product.

This can be done as follows:

$$\begin{array}{r} 4x^2 + 2x \\ \underline{\times \quad\quad 3x} \\ \underline{12x^3 + 6x^2} \end{array}$$

Similarly, we can multiply a trinomial with monomial as follows:

$$\begin{array}{r} 2y^3 - 5y + 1 \\ \times \qquad\qquad 2y \\ \hline 4y^4 - 10y^2 + 2y \\ \hline \end{array}$$

Let us discuss some more examples based on the multiplication of a monomial with polynomials.

Example 4:

Multiply the following in horizontal and vertical arrangements:

(a) $\frac{3}{5}$ ***p* and *p* − 6*q***

(b) (1.5*x* + *y* + 3*z*) and 7*z*

Also find the values of the above expressions if *p* = − 5, *q* = − 1, *x* = 2, *y* = − 3, and *z* = − 1.

Solution:

(a) Horizontal arrangement:

$$\frac{3}{5}p \times (p - 6q) = \frac{3}{5}p \times p - \frac{3}{5}p \times 6q = \frac{3}{5}p^2 - \frac{18}{5}pq$$

Vertical arrangement:

$$\begin{array}{r} p - 6q \\ \times \qquad \frac{3}{5}p \\ \hline \frac{3}{5}p^2 - \frac{18}{5}pq \\ \hline \end{array}$$

Substituting the values of *p* and *q*, we get

$$= \frac{3}{5} \times (-5)^2 - \frac{18}{5} \times (-5)(-1)$$

$= \frac{3}{5} \times 25 - 18$
$= 15 - 18$
$= -3$

(b) Horizontal arrangement:

$(1.5x + y + 3z) \times 7z = 1.5x \times 7z + y \times 7z + 3z \times 7z = 10.5xz + 7yz + 21z^2$

Vertical arrangement:

$$\begin{array}{r} 1.5x + y + 3z \\ \times \quad 7z \\ \hline 10.5xz + 7yz + 21z^2 \\ \hline \end{array}$$

Substituting the values of *x*, *y*, and *z*, we get

$10.5xz + 7yz + 21z^2$
$= 10.5 \times 2 \times (-1) + 7 \times (-3) \times (-1) + 21 \times (-1)^2$

$= -21 + 21 + 21$

= 21

Example 5:

(a) Add $\boldsymbol{a(b-c)}$**,** $\boldsymbol{b(c-a)}$**, and** $\boldsymbol{c(a-b)}$

(b) **Subtract** $\boldsymbol{5x(x - y + z) - 2z(-3x + 4y + 5z)}$ **from** $\boldsymbol{3y(4x + 3y - 2z)}$

Solution:

(a) Addition of *a* (*b* − *c*), *b* (*c* − *a*), and *c* (*a* − *b*)

$= a(b-c) + b(c-a) + c(a-b)$
$= ab - ac + bc - ab + ac - bc$
$= 0$

(b) Subtraction of $5x(x - y + z) - 2z(-3x + 4y + 5z)$ from $3y(4x + 3y - 2z)$

$= 3y(4x + 3y - 2z) - \{5x(x - y + z) - 2z(-3x + 4y + 5z)\}$

$$= (3y)(4x) + (3y)(3y) + (3y)(-2z) - \left\{\begin{matrix}(5x)(x) + (5x)(-y) + (5x)(z) \\ -2z(-3x) - 2z(4y) - 2z(5z)\end{matrix}\right\}$$

$= 12xy + 9y^2 - 6yz - \{5x^2 - 5xy + 5zx + 6zx - 8yz - 10z^2\}$

$= 12xy + 9y^2 - 6yz - \{5x^2 - 5xy + 11zx - 8yz - 10z^2\}$

$= 12xy + 9y^2 - 6yz - 5x^2 + 5xy - 11zx + 8yz + 10z^2$

$= -5x^2 + 9y^2 + 10z^2 + 17xy + 2yz - 11zx$

Multiplication of Two Polynomials

Suppose you want to buy $(2x + y)$ metres of rope at the rate of Rs $(a - 3b)$ per metre.

Can you calculate the amount of money you require?

The amount you require is $(2x + y) \times (a - 3b)$.

Now, how will you carry out this type of multiplication?

In the video, we have multiplied binomial with binomial in **horizontal arrangement**. Let us now multiply the binomials $(3x - y)$ and $(x + 3y)$ in **vertical arrangement**.

Here, first multiply $3x - y$ with $3y$ and then multiply $3x - y$ with x. After doing so, add the like terms as shown below:

$$\begin{array}{r} 3x - y \\ \times \quad x + 3y \\ \hline 9xy - 3y^2 \\ +3x^2 - xy \quad\quad \\ \hline 3x^2 + 8xy - 3y^2 \\ \hline \end{array}$$

The process of multiplying a binomial with a trinomial is not too different from that of multiplying two binomials.

In the video, we have multiplied binomial with trinomial in **horizontal arrangement**. Let us now multiply the binomial ($x + y$) with trinomial ($2x + 3y + 1$) in **vertical arrangement**.

$$\begin{array}{r} 2x + 3y + 1 \\ \times \quad x + y \\ \hline 2xy + 3y^2 + y \\ +2x^2 + 3xy + x \quad\quad \\ \hline 2x^2 + 5xy + 3y^2 + x + y \\ \hline \end{array}$$

Thus, we can perform multiplication of binomials with binomials and trinomials using any of the horizontal or vertical arrangement method.

Let us now solve examples based on the above concepts.

Example 1:

Multiply the following using horizontal and vertical arrangement:

(a) 2 ($x + y$) and $x - 3y$

(b) ($l + 3m$) and ($l + 6m + 7n$)

Solution:

(a) H**orizontal arrangement:**

$2\ (x + y) = 2x + 2y$

Now, we have to multiply ($2x + 2y$) and $x - 3y$.

$(2x + 2y) \times (x - 3y) = 2x \times (x - 3y) + 2y \times (x - 3y)$

(Using distributive property)

$= 2 \times x \times x - 2 \times 3 \times x \times y + 2 \times x \times y - 2 \times 3 \times y \times y$

$= 2x^2 - 6xy + 2xy - 6y^2$

$= 2x^2 - 4xy - 6y^2$ (Combining the like terms)

Vertical arrangement:

$$\begin{array}{r} x + y \\ \times \quad 2 \\ \hline 2x + 2y \\ \hline \end{array} \qquad \begin{array}{r} x - 3y \\ \times \quad 2x + 2y \\ \hline 2xy - 6y^2 \\ +2x^2 - 6xy \quad \\ \hline 2x^2 - 4xy - 6y^2 \\ \hline \end{array}$$

(b) H**orizontal arrangement:**

$(l + 3m) \times (l + 6m + 7n)$

$= l \times (l + 6m + 7n) + 3m \times (l + 6m + 7n)$

(Using distributive property)

$= l \times l + l \times 6m + l \times 7n + 3m \times l + 3m \times 6m + 3m \times 7n$

$= l^2 + 6lm + 7ln + 3ml + 18m^2 + 21mn$

$= l^2 + 9ml + 7ln + 21mn + 18m^2$

[Combining the like terms $6lm$ and $3ml$]

Vertical arrangement:

$$\begin{array}{r} l + 6m + 7n \\ \times \qquad\qquad l + 3m \\ \hline 3lm + 18m^2 + 21mn \\ +l^2 + 6lm + 7nl \qquad\qquad \\ \hline l^2 + 9lm + 18m^2 + 21mn + 7nl \\ \hline \end{array}$$

Example 2:

Simplify the following:

(a) $(x^2 + y^2) \times (x^3 + y + z^2) + 2\ (z^2 + 5z)$

(b) $(l - m)\ (l + m) + (m - n)\ (m + n) - (l - n)\ (n + l)$

(c) $(x - 4)\ (y - 4) - 16$

(d) $(a + b + c)\ (a - b + c)$

Solution:

(a) $(x^2 + y^2) \times (x^3 + y + z^2) + 2\ (z^2 + 5z)$

$= x^2\ (x^3 + y + z^2) + y^2\ (x^3 + y + z^2) + 2(z^2 + 5z)$

(Using distributive property)

$= x^2 \times x^3 + x^2 \times y + x^2 \times z^2 + y^2 \times x^3 + y^2 \times y + y^2 \times z^2 + 2 \times z^2 + 2 \times 5z$

$= x^5 + x^2y + x^2z^2 + x^3y^2 + y^3 + y^2z^2 + 2z^2 + 10z$

(b) $(l - m)\ (l + m) + (m - n)\ (m + n) - (l - n)\ (n + l)$

$= l\ (l + m) - m\ (l + m) + m\ (m + n) - n\ (m + n) - l\ (n + l) + n\ (n + l)$

(Using distributive property)

$= l^2 + lm - ml - m^2 + m^2 + mn - nm - n^2 - ln - l^2 + n^2 + nl$

$=(l^2 - l^2) + (lm - lm) + (- m^2 + m^2) + (mn - mn) + (- n^2 + n^2) + (- ln + ln)$

{lm and ml, ln and nl, mn and nm are like terms}

= 0

(c) $(x - 4) \times (y - 4) - 16$

$= x \times (y - 4) - 4(y - 4) - 16$ (Using distributive property)

$= xy - 4x - 4y + 16 - 16$

$= xy - 4x - 4y$

(d) $(a + b + c)\ (a - b + c)$

$= a\ (a - b + c) + b\ (a - b + c) + c\ (a - b + c)$ (Using distributive property)

$= a^2 - ab + ac + ba - b^2 + bc + ca - cb + c^2$ (Combining the like terms)

$= a^2 + c^2 - b^2 + 2ac$

Using Identities for "Square of Sum or Difference of Two Terms"

Let us try to find the square of the number 102. The square of a number, as we know, is the product of the number with itself. One way to do this is by writing the numbers one below the other, and then multiplying them as we normally do. The other way is to break the numbers and then apply distributive property. This will make our work much easier.

Let us see how.

$102^2 = 102 \times 102$

$= (100 + 2)\ (100 + 2)$

$= 100\ (100 + 2) + 2\ (100 + 2)$

$= 100 \times 100 + 100 \times 2 + 2 \times 100 + 2 \times 2$

$= 10000 + 200 + 200 + 4$

$= 10404$

Observing the similar expressions as above, we obtain the following identities.

$(a + b)^2 = a^2 + 2ab + b^2$

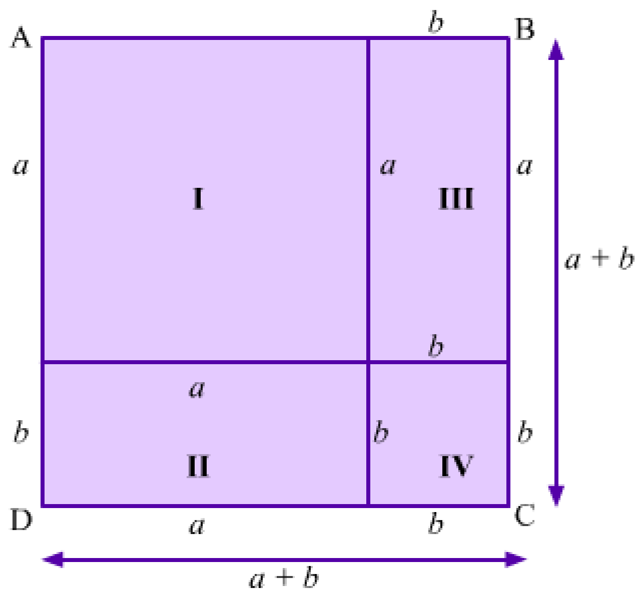
$(a - b)^2 = a^2 - 2ab + b^2$

Deriving the identities geometrically:

These identities can be derived by geometrical construction as well. Let us learn the same.

(1) $(a + b)^2 = a^2 + 2ab + b^2$:

Let us consider a square ABCD whose each side measures $(a + b)$ unit.



It can be seen that, we have drawn two line segments at a distance of a unit from A such that one is parallel to AB and other is parallel to AD.

Also, the figure is divided into four regions named as I, II, III and IV.

Now,

∴ Area of square ABCD = $(a + b)^2$ sq. unit ...(i)

Region I is a square of side measuring a unit.

∴ Area of region I = a^2 sq. unit ...(ii)

Each of regions II and III is a rectangle having length and breadth as a unit and b unit respectively.

∴ Area of region II = ab sq. unit ...(iii)

And,

Area of region III = ab sq. unit ...(iv)

Region IV is a square of side measuring b unit.

∴ Area of region IV = b^2 sq. unit ...(v)

From the figure, we have

Area of square ABCD = Area of region I + Area of region II + Area of region III + Area of region IV

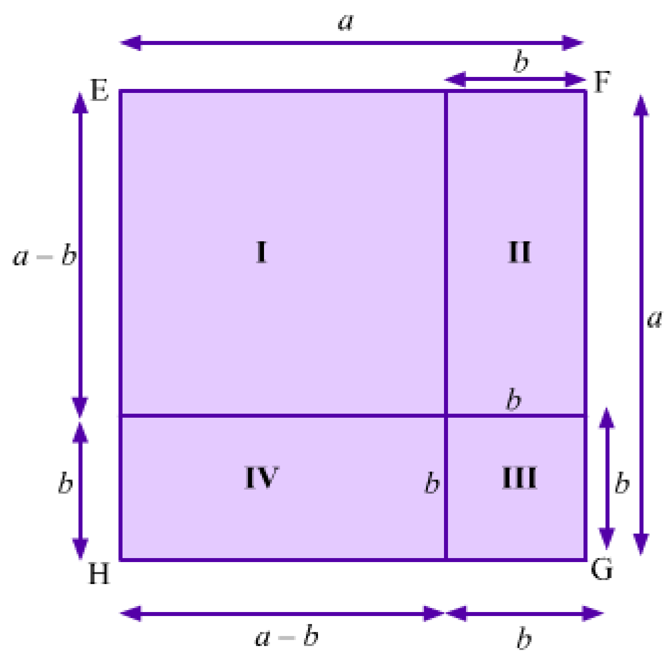
On substituting the values from (i), (ii), (iii), (iv) and (v), we get

$(a + b)^2 = a^2 + ab + ab + b^2$

∴ $(a + b)^2 = a^2 + 2ab + b^2$

(2) $(a - b)^2 = a^2 - 2ab + b^2$:

Let us consider a square EFGH whose each side measures a unit.



It can be seen that, we have drawn two line segments at a distance of b unit from G such that one is parallel to GH and other is parallel to FG.

Also, the figure is divided into four regions named as I, II, III and IV.

Now,

∴ Area of square EFGH = a^2 sq. unit ...(i)

Region I is a square of side measuring $(a - b)$ unit.

∴ Area of region I = $(a - b)^2$ sq. unit ...(ii)

Each of regions II and IV is a rectangle having length and breadth as $(a - b)$ unit and b unit respectively.

∴ Area of region II = $b(a - b)$ sq. unit ...(iii)

And,

Area of region IV = $b(a - b)$ sq. unit ...(iv)

Region III is a square of side measuring b unit.

∴ Area of region III = b^2 sq. unit ...(v)

From the figure, we have

Area of region I = Area of square ABCD − (Area of region II + Area of region III + Area of region IV)

On substituting the values from (i), (ii), (iii), (iv) and (v), we get

$(a-b)^2 = a^2 - [b(a-b) + b^2 + b(a-b)]$

$(a-b)^2 = a^2 - [ab - b^2 + b^2 + ab - b^2]$

$(a-b)^2 = a^2 - [2ab - b^2]$

$$\therefore (a-b)^2 = a^2 - 2ab + b^2$$

The identities we have proved above are known as identity because for any value of a and b, the LHS is always equal to the RHS. The difference between an identity and an equation is that for an equation, its LHS and RHS are equal only for some values of the variable. On the other hand, as we discussed, for an identity, the LHS equals the RHS for any value of the variable.

Many a times, these identities help in shortening our calculations. Let us discuss some examples using the above identities to understand this better.

Example 1:
Simplify the following expressions using suitable identities:
(a) $(2m + 3n)^2$
(b) $(4p - 7q)^2$

Solution:
(a)
On comparing the given expression $(2m + 3n)^2$ with $(a + b)^2$, we get
$a = 2m$ and $b = 3n$.
Now,
$(a + b)^2 = a^2 + 2ab + b^2$
Thus,

$$\begin{aligned}(2m+3n)^2 &= (2m)^2 + 2(2m)(3n) + (3n)^2 \\ &= 4m^2 + 12mn + 9n^2\end{aligned}$$

(b)

On comparing the given expression $(4p - 7q)^2$ with $(a - b)^2$, we get
$a = 4p$ and $b = 7q$.

Now,
$(a - b)^2 = a^2 - 2ab + b^2$
Thus,
$(4p - 7q)^2 = (4p)^2 - 2(4p)(7q) + (7q)^2$

$= 16p^2 - 56pq + 49q^2$

Example 2:

Simplify the following expressions using suitable identities:

(a) $\mathbf{(3ax + 5by)^2}$

(b) $\mathbf{(0.6a^2 - 0.04b^3)^2}$

(c) $\left(\frac{3}{7}l + \frac{4}{5}m\right)^2$

Solution:

(a) The given expression is $(3ax + 5by)^2$, which is of the form $(a+ b)^2$.

Thus, we can use the identity $(a + b)^2 = a^2 + 2ab + b^2$.

$\therefore (3ax + 5by)^2 = (3ax)^2 + 2\,(3ax)\,(5by) + (5by)^2$

$= 9a^2x^2 + 30abxy + 25b^2y^2$

(b) The given expression is $(0.6a^2 - 0.04b^3)^2$, which is of the form $(a - b)^2$.

Thus, we can use the identity $(a - b)^2 = a^2 - 2ab + b^2$.

$\therefore (0.6a^2 - 0.04b^3)^2 = (0.6a^2)^2 - 2\,(0.6a^2)\,(0.04b^3) + (0.04b^3)^2$

$= 0.36a^4 - 0.048a^2b^3 + 0.0016b^6$

(c) The given expression is $\left(\frac{3}{7}l + \frac{4}{5}m\right)^2$, which is of the form $(a + b)^2$.

Thus, we can use the identity $(a + b)^2 = a^2 + 2ab + b^2$.

$$\therefore\left(\frac{3}{7}l+\frac{4}{5}m\right)^2=\left(\frac{3}{7}l\right)^2+2\left(\frac{3}{7}l\times\frac{4}{5}m\right)+\left(\frac{4}{5}m\right)^2$$

$$=\frac{9}{49}l^2+\frac{24}{35}lm+\frac{16}{25}m^2$$

Example 3:

Find the value of $(208)^2$ using a suitable identity.

Solution:

208 = 200 + 8

$\therefore (208)^2 = (200 + 8)^2$

Thus, we can use the identity $(a + b)^2 = a^2 + 2ab + b^2$.

$\therefore (208)^2 = (200 + 8)^2$

$= (200)^2 + 2\ (200)\ (8) + (8)^2$

= 40000 + 3200 + 64

= 43264

Example 4:

Find the value of $(99)^2$ using a suitable identity.

Solution:

99 = 100 − 1

$\therefore (99)^2 = (100 - 1)^2$

Thus, we can use the identity $(a - b)^2 = a^2 - 2ab + b^2$.

$\therefore (99)^2 = (100 - 1)^2 = (100)^2 - 2\ (100)\ (1) + (1)^2$

= 10000 − 200 + 1

= 9800 + 1

= 9801

Example 5:

(a) If $x - \frac{1}{x} = 3$, then find the value of the expressions $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$.

(b) If $2y + \frac{3}{y} = 5$, then find the value of the expression $4y^2 + \frac{9}{y^2}$.

(c) If 3*x* − 5*y* = −1 and *xy* = 6, then find the value of the expression $9x^2 + 25y^2$.

Solution:

(a) It is given that $x - \frac{1}{x} = 3$.

On squaring both sides, we get

$$\left(x - \frac{1}{x}\right)^2 = 3^2$$

$$\Rightarrow (x)^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = 9 \qquad \left[\text{Using the identity } (a-b)^2 = a^2 - 2ab + b^2\right]$$

$$\Rightarrow x^2 - 2 + \frac{1}{x^2} = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 9 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 11$$

Now, on squaring both sides again, we get

$$\left(x^2+\frac{1}{x^2}\right)^2=11^2$$

$$\Rightarrow\left(x^2\right)^2+2\left(x^2\right)\left(\frac{1}{x^2}\right)+\left(\frac{1}{x^2}\right)^2=121 \qquad \left[\text{Using the identity } (a+b)^2=a^2+2ab+b^2\right]$$

$$\Rightarrow x^4+2+\frac{1}{x^4}=121$$

$$\Rightarrow x^4+\frac{1}{x^4}=119$$

Thus, the value of the expression $\left(x^2+\frac{1}{x^2}\right)$ is 11 and the value of the expression $\left(x^4+\frac{1}{x^4}\right)$ is 119.

(b) It is given that $2y+\frac{3}{y}=5$.

On squaring both sides, we get

$$\left(2y+\frac{3}{y}\right)^2=5^2$$

$$\Rightarrow(2y)^2+2(2y)\left(\frac{3}{y}\right)+\left(\frac{3}{y}\right)^2=25$$

$$\Rightarrow 4y^2+12+\frac{9}{y^2}=25$$

$$\Rightarrow 4y^2+\frac{9}{y^2}=13$$

Thus, the value of the expression $\left(4y^2+\frac{9}{y^2}\right)$ is 13.

(c) It is given that $3x-5y=-1$.

On squaring both sides, we get

$$(3x-5y)^2=(-1)^2$$
$$\Rightarrow (3x)^2-2(3x)(5y)+(5y)^2=1$$
$$\Rightarrow 9x^2-30xy+25y^2=1$$
$$\Rightarrow 9x^2-30\times 6+25y^2=1 \qquad (xy=6)$$
$$\Rightarrow 9x^2+25y^2=1+180$$
$$\Rightarrow 9x^2+25y^2=181$$

Thus, the value of the expression $(9x^2 + 25y^2)$ is 181.

Example 6:

Prove that

(a) $(a+b)^2+(a-b)^2=2(a^2+b^2)$

(b) $\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2=ab$

Solution:

(a) We know that $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$. Thus,

LHS = $(a + b)^2 + (a - b)^2$

= $a^2 + 2ab + b^2 + a^2 - 2ab + b^2$

= $(a^2 + a^2) + (b^2 + b^2) + (2ab - 2ab)$

= $2a^2 + 2b^2$

= $2(a^2 + b^2)$ = RHS

Hence, proved.

(b) We know that $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$. Thus,

$$\text{LHS}=\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2$$

$$= \left(\frac{a^2+2ab+b^2}{4}\right) - \left(\frac{a^2-2ab+b^2}{4}\right)$$

$$= \frac{a^2+2ab+b^2-a^2+2ab-b^2}{4}$$

$$= \frac{4ab}{4}$$

$$= ab = \text{RHS}$$

Hence, proved.

Using Identity (x + a)(x + b)

A very important identity that we have to learn is regarding the expression $(x + a)$ $(x + b)$.

We know how to multiply binomials. By finding the product of these binomials, we can find the identity.

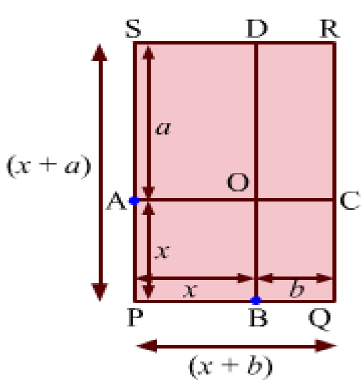
This identity can be derived by geometrical construction as well. Let us learn the same.

Deriving the identity geometrically:

Let us draw a rectangle PQRS of length and breadth $(x + a)$ and $(x + b)$ units respectively.

Also, let us take two points A and B on sides SP and PQ respectively, such that PA = PB = x unit.

Now, let us draw two line segments AC and BD such that AC || PQ and BD || PS. AC intersects QR at point C and BD intersects RS at point D.



Now, we have

l(PS) = $(x + a)$ unit and l(PQ) = $(x + b)$ unit

∴ Area of rectangle PQRS = length × breadth

⇒ Area of rectangle PQRS = l(PS) × l(PQ)

⇒ Area of rectangle PQRS = $(x + a)(x + b)$ sq. unit ...(i)

Also,

Area of square PBOA = x^2 sq. unit ...(ii)

Area of rectangle BQCO = bx sq. unit ...(iii)

Area of rectangle OCRD = ab sq. unit ...(iv)

Area of rectangle AODS = ax sq. unit ...(v)

From the figure, it can be observed that

Area of rectangle PQRS = Area of square PBOA + Area of rectangle BQCO + Area of rectangle OCRD + Area of rectangle AODS

On substituting the values from (i), (ii), (iii), (iv) and (v), we get

$(x + a)(x + b) = x^2 + bx + ab + ax$

⇒ $(x + a)(x + b) = x^2 + ax + bx + ab$

$\therefore (x + a)(x + b) = x^2 + (a + b)x + ab$

Now, let us solve some examples in which this identity is used.

Example 1:
Find the product of $(m + 3)$ and $(m - 5)$.

Solution:

This expression is of the form $(x + a)(x + b)$.

Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$\therefore (m+3)(m-5) = m^2 + (3-5)m + (3)(-5)$$
$$= m^2 - 2m - 15$$

Example 2:

Use the appropriate identity to simplify the following expressions.

(a) $(3p+5q)(3p-7z)$

(b) $\left(z^2 - \frac{4}{3}\right)\left(z^2 - \frac{3}{2}\right)$

Solution:

(a) The given expression is $(3p+5q)(3p-7z)$.

This expression is of the form $(x + a)$ $(x + b)$.

Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$\therefore (3p+5q)(3p-7z) = (3p)^2 + (5q-7z)(3p) + (5q)(-7z)$$
$$= 9p^2 + (5q)(3p) - (7z)(3p) - 35qz$$
$$= 9p^2 + 15pq - 21pz - 35qz$$

(b) The given expression is $\left(z^2 - \frac{4}{3}\right)\left(z^2 - \frac{3}{2}\right)$.

This expression is of the form $(x + a)$ $(x + b)$.

Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$\therefore \left(z^2 - \frac{4}{3}\right)\left(z^2 - \frac{3}{2}\right) = (z^2)^2 + \left\{\left(-\frac{4}{3}\right) + \left(-\frac{3}{2}\right)\right\}z^2 + \left(-\frac{4}{3}\right) \times \left(-\frac{3}{2}\right)$$
$$= z^4 + \left(\frac{-8-9}{6}\right)z^2 + 2$$
$$= z^4 - \frac{17}{6}z^2 + 2$$

Example 3:

Find the products of the following pairs of numbers using suitable identities.

(a) 105 × 102

(b) 98 × 103

Solution:

(a) 105 × 102 = (100 + 5) × (100 + 2)

Now, we can use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here, $x = 100$, $a = 5$, $b = 2$

∴105 × 102 = (100 + 5) × (100 + 2)

$= (100)^2 + (5 + 2) \times 100 + 5 \times 2$

= 10000 + 700 + 10

= 10710

Thus, the product of the numbers 105 and 102 is 10710.

(b) 98 × 103 = (100 − 2) × (100 + 3)

Now, we can use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here, $x = 100$, $a = -2$, $b = 3$

∴98 × 103 = (100 − 2) × (100 + 3)

$= (100)^2 + (-2 + 3) \times 100 + (-2)(3)$

= 10000 + 1 × 100 − 6

= 10000 + 100 − 6

= 10100 − 6

= 10094

Thus, the product of the numbers 98 and 103 is 10094.

Using Identity for "Difference of Two Squares"

Suppose we need to find the product of the numbers 79 and 81. Instead of multiplying these two numbers, we can use the identity $(a + b)(a - b)$. This identity is very important and is applicable in various situations.

Let us first understand this identity.

$(a + b)(a - b) = a(a - b) + b(a - b)$ (By distributive property)

$= a^2 - ab + ba - b^2$

$= a^2 - ab + ab - b^2$ ($ab = ba$)

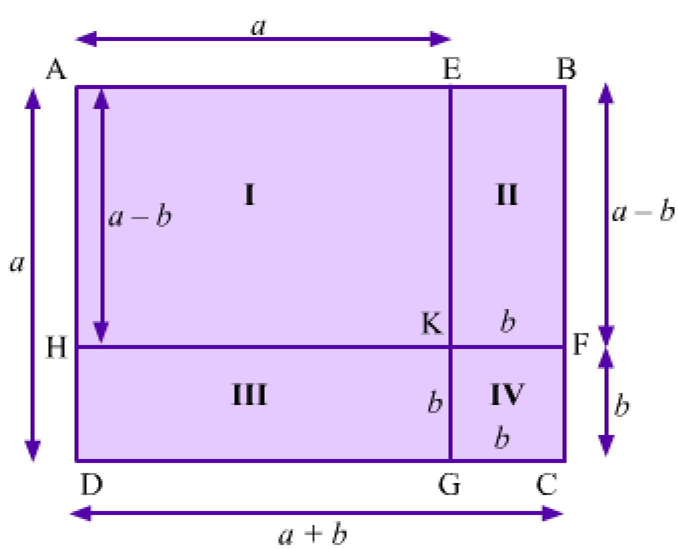
$= a^2 - b^2$

$$\therefore (a + b)(a - b) = a^2 - b^2$$

Deriving the identity geometrically:

This identity can be derived from geometric construction as well.

For this, let us consider a square AEGD whose each side measures a units.



It can be seen that, a segment BC is drawn outside the square AEGD such that BC is parallel to EG and at a distance of *b* unit from G.

Another segment HF is drawn inside the square AEGD such that HF is parallel to GD and at a distance of *b* unit from G.

From the figure, it can be observed that

Area of rectangle ABFH = Area of rectangle ABCD – (Area of rectangle HKGD + Area of square KFCG)

$\Rightarrow (a + b)(a - b) = a(a + b) - [(a \times b) + (b \times b)]$

$\Rightarrow (a + b)(a - b) = a^2 + ab - ab - b^2$

$$\Rightarrow (a + b)(a - b) = a^2 - b^2$$

Now, let us solve some examples in which the above identity can be applied.

Example 1:

Simplify the following expressions.

(a) $(x + 3)(x - 3)$

(b) $(11 + y)(11 - y)$

Solution:

(a) $(x + 3)(x - 3)$

This expression is of the form $(a + b)(a - b)$.

Hence, we can use the identity $(a + b)(a - b) = a^2 - b^2$.

$(x + 3)(x - 3) = x^2 - 3^2 = x^2 - 9$

(b) $(11 + y)(11 - y)$

This expression is of the form $(a + b)(a - b)$.

Hence, we can use the identity $(a + b)(a - b) = a^2 - b^2$.

$(11 + y)(11 - y) = 11^2 - y^2 = 121 - y^2$

Example 2:

Simplify the following expressions.

(a) $\left(\frac{3}{7}l+\frac{4}{5}m\right)\left(\frac{3}{7}l-\frac{4}{5}m\right)$

(b) $\left(x^2-y^3\right)\left(x^2+y^3\right)+\left(y^3-z^4\right)\left(y^3+z^4\right)+\left(z^4-x^2\right)\left(z^4+x^2\right)$

Solution:

(a) The given expression is $\left(\frac{3}{7}l+\frac{4}{5}m\right)\left(\frac{3}{7}l-\frac{4}{5}m\right)$.

Using the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\therefore \left(\frac{3}{7}l+\frac{4}{5}m\right)\left(\frac{3}{7}l-\frac{4}{5}m\right)=\left(\frac{3}{7}l\right)^2-\left(\frac{4}{5}m\right)^2$$
$$=\frac{9}{49}l^2-\frac{16}{25}m^2$$

(b) $\left(x^2-y^3\right)\left(x^2+y^3\right)+\left(y^3-z^4\right)\left(y^3+z^4\right)+\left(z^4-x^2\right)\left(z^4+x^2\right)$

Using the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\left\{\left(x^2\right)^2-\left(y^3\right)^2\right\}+\left\{\left(y^3\right)^2-\left(z^4\right)^2\right\}+\left\{\left(z^4\right)^2-\left(x^2\right)^2\right\}$$
$$=x^4-y^6+y^6-z^8+z^8-x^4$$
$$=\left(x^4-x^4\right)+\left(-y^6+y^6\right)+\left(-z^8+z^8\right)$$
$$=0$$

Example 3:

Find the values of the following expressions using suitable identities.

(a) 195 × 205

(b) $(993)^2 - (7)^2$

(c) 24.5 × 25.5

Solution:

(a) 195 = 200 − 5
and, 205 = 200 + 5

∴ 195 × 205 = (200 − 5) × (200 + 5)

$= (200)^2 - (5)^2$ [∵ $(a + b)(a - b) = a^2 - b^2$]

= 40000 − 25

= 39975

(b) $(993)^2 - (7)^2$
= (993 + 7) (993 − 7) [∵ $(a + b)(a - b) = a^2 - b^2$]

= (1000) (986)

= 986000

(c) 24.5 × 25.5

= (25 − 0.5) (25 + 0.5)

= $(25)^2 - (0.5)^2$ [∵ $(a + b)(a - b) = a^2 - b^2$]

= 625 − 0.25

= 624.75