10

Thermal Properties of Matter

1. Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is :

   [Sep. 05, 2020 (II)]

   (a) $\dfrac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$ (b) $2\sqrt{\alpha_1 \alpha_2}$

   (c) $\dfrac{\alpha_1 + \alpha_2}{2}$ (d) $4\dfrac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}\dfrac{L_2 L_1}{(L_2 + L_1)^2}$

2. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}$ °C^{-1} and $\gamma_{(mercury)} = 1.5 \times 10^{-4}$ °C^{-1}, where γ is the coefficient of volume expansion, then V_m (in cc) is close to _____.

   [NA Sep. 03, 2020 (I)]

3. When the temperature of a metal wire is increased from 0°C to 10°C, its length increased by 0.02%. The percentage change in its mass density will be closest to :

   [Sep. 02, 2020 (II)]

   (a) 0.06 (b) 2.3
   (c) 0.008 (d) 0.8

4. A non-isotropic solid metal cube has coefficients of linear expansion as: 5×10^{-5}/°C along the x-axis and 5×10^{-6}/°C along the y and the z-axis. If the coefficient of volume expansion of the solid is C $\times 10^{-6}$/°C then the value of C is _____.

   [NA 7 Jan. 2020 I]

5. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to:

   [12 April 2019 I]

   (Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2, respectively; $g = 10$ ms^{-2})

   (a) 9 kg (b) 0.5 kg (c) 1.5 kg (d) 0.9 kg

6. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is :

   [11 Jan. 2019 II]

   (a) 230°C (b) 270°C
   (c) 200°C (d) 250°C

7. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$?

   [11 Jan. 2019 II]

   (a) 25 (b) 60 (c) 40 (d) 35

8. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is:

   [9 Jan. 2019 I]

   (a) $\dfrac{F}{A \alpha \Delta T}$ (b) $\dfrac{F}{A \alpha (\Delta T - 273)}$

   (c) $\dfrac{F}{2A \alpha \Delta T}$ (d) $\dfrac{2F}{A \alpha \Delta T}$

9. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by :

   [2017]

   (a) $\dfrac{3\alpha}{PK}$ (b) $3PK\alpha$

   (c) $\dfrac{P}{3\alpha K}$ (d) $\dfrac{P}{\alpha K}$

10. A steel rail of length 5 m and area of cross-section 40 cm^2 is prevented from expanding along its length while the temperature rises by 10°C. If coefficient of linear expansion and Young's modulus of steel are 1.2×10^{-5} K^{-1} and 2×10^{11} Nm^{-2} respectively, the force developed in the rail is approximately:

    [Online April 9, 2017]

    (a) 2×10^7 N (b) 1×10^5 N
    (c) 2×10^9 N (d) 3×10^{-5} N

11. A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT. The net change in its length is zero. Let l be the length of the rod, A its area of cross–section, Y its Young's modulus, and α its coefficient of linear expansion. Then, F is equal to :

[Online April 8, 2017]

(a) $l^2 Y\alpha \Delta T$ (b) $lAY\alpha \Delta T$

(c) $AY\alpha \Delta T$ (d) $\dfrac{AY}{\alpha \Delta T}$

12. The ratio of the coefficient of volume expansion of a glass container to that of a viscous liquid kept inside the container is 1 : 4. What fraction of the inner volume of the container should the liquid occupy so that the volume of the remaining vacant space will be same at all temperatures ?

[Online April 23, 2013]

(a) 2 : 5   (b) 1 : 4   (c) 1 : 64   (d) 1 : 8

13. On a linear temperature scale Y, water freezes at $-160°$ Y and boils at $-50°$ Y. On this Y scale, a temperature of 340 K would be read as : (water freezes at 273 K and boils at 373 K)

[Online April 9, 2013]

(a) $-73.7°$ Y   (b) $-233.7°$ Y

(c) $-86.3°$ Y   (d) $-106.3°$ Y

14. A wooden wheel of radius R is made of two semicircular part (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is : **[2012]**

(a) $2\pi SY\alpha\Delta T$

(b) $SY\alpha\Delta T$

(c) $\pi SY\alpha\Delta T$

(d) $2SY\alpha\Delta T$
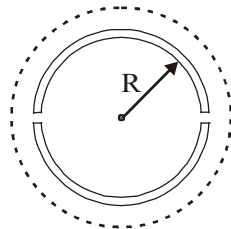
TOPIC **2** **Calorimetry and Heat Transfer**

15. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100ºC and the other at 0ºC (see figure). If the joints of the rod are at 70ºC and 20ºC in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is :

[Sep. 06, 2020 (II)]
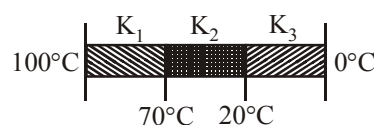
(a) $K_1 : K_3 = 2 : 3$, $K_1 < K_3 = 2 : 5$
(b) $K_1 < K_2 < K_3$
(c) $K_1 : K_2 = 5 : 2$, $K_1 : K_3 = 3 : 5$
(d) $K_1 > K_2 > K_3$

16. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetics energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is $0.030 \text{ cal}/(g-°C)$ ($1 \text{ cal} = 4.2 \times 10^7$ ergs) close to :

[Sep. 05, 2020 (I)]

(a) 87.5ºC   (b) 83.3ºC
(c) 119.2ºC   (d) 38.4ºC

17. The specific heat of water $= 4200$ J kg^{-1} K^{-1} and the latent heat of ice $= 3.4 \times 10^5$ J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams) : **[Sep. 04, 2020 (I)]**

(a) 61.7   (b) 63.8
(c) 69.3   (d) 64.6

18. A calorimter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of 'm' is close to (Latent heat of water $= 540$ cal g^{-1}, specific heat of water $= 1$ cal g^{-1} °C^{-1}) **[Sep. 03, 2020 (II)]**

(a) 2   (b) 4
(c) 3.2   (d) 2.6

19. Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each container and mixed (assume no loss of heat during the process) **[8 Jan. 2020 II]**
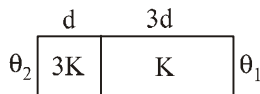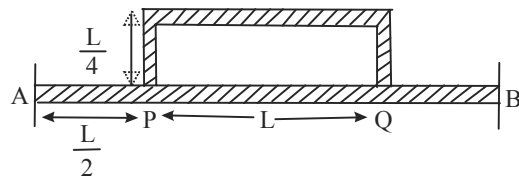
C_1	C_2	C_3	T
1 l	2 l	—	60°C
–	1 l	2 l	30°C
2 l	—	1 l	60°C
1 l	1 l	1 l	θ

The value of θ (in °C to the nearest integer) is_____.

20. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/ g and heat of fusion of ice is 80 cal/g], the value of M is _____ **[NA 7 Jan. 2020 II]**

21. When M_1 gram of ice at -10 ºC (Specific heat $= 0.5$ cal g^{-1}ºC^{-1}) is added to M_2 gram of water at 50ºC, finally no ice is left and the water is at 0ºC. The value of latent heat of ice, in cal g^{-1} is: **[12 April 2019 I]**

(a) $\dfrac{50M_2}{M_1} - 5$ (b) $\dfrac{5M_1}{M_2} - 50$

(c) $\dfrac{50M_2}{M_1}$ (d) $\dfrac{5M_2}{M_1} - 5$

22. A massless spring (K = 800 N/m), attached with a mass (500 g) is completely immersed in 1kg of water. The spring is stretched by 2cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K) **[9 April 2019 II]**
(a) 10^{-4} K   (b) 10^{-5} K   (c) 10^{-1} K   (d) 10^{-3} K

23. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and '3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is: **[9 April 2019 II]**

(a) $\dfrac{\theta_1}{10} + \dfrac{9\theta_2}{10}$ (b) $\dfrac{\theta_2 + \theta_1}{2}$

(c) $\dfrac{\theta_1}{6} + \dfrac{5\theta_2}{6}$ (d) $\dfrac{\theta_1}{3} + \dfrac{2\theta_2}{3}$

24. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is: **[12 Jan. 2019 I]**

(a) $\dfrac{K_1 + K_2}{2}$ (b) $K_1 + K_2$

(c) $\dfrac{2K_1 + 3K_2}{5}$ (d) $\dfrac{K_1 + 3K_2}{4}$

25. Ice at –20°C is added to 50 g of water at 40°C, When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to **[11 Jan. 2019 I]**
(Specific heat of water = 4.2J/g/°C
Specific heat of Ice = 2.1 J/g/°C
Heat of fusion of water at 0°C = 334J/g)
(a) 50g   (b) 100 g
(c) 60 g   (d) 40 g

26. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be : **[11 Jan. 2019 II]**
(a) 85°C   (b) 60°C
(c) 80°C   (d) 70°C

27. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water? [Specific Heat Capacities of water and metal are, respectively, 4200 Jkg^{-1}K^{-1} and 400 Jkg^{-1} K^{-1}] **[11 Jan. 2019 II]**
(a) 15%   (b) 30%
(c) 25%   (d) 20%

28. A heat source at T = 10^3 K is connected to another heat reservoir at T = 10^2 K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 WK^{-1} m^{-1}, the energy flux through it in the steady state is: **[10 Jan. 2019 I]**
(a) 90 Wm^{-2} (b) 120 Wm^{-2}
(c) 65 Wm^{-2} (d) 200 Wm^{-2}

29. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stablizes at 21.5°C. (Specific heat of brass is 394 J kg^{-1} K^{-1}) **[10 Jan. 2019 II]**
(a) 458 J kg^{-1} K^{-1} (b) 1232 J kg^{-1} K^{-1}
(c) 916 J kg^{-1} K^{-1} (d) 654 J kg^{-1} K^{-1}

30. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\dfrac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to: **[9 Jan. 2019 I]**

(a) 45°C   (b) 75°C   (c) 60°C   (d) 35°C

31. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C **[2017]**
(a) 1250°C   (b) 825°C   (c) 800°C   (d) 885° C

32. In an experiment a sphere of aluminium of mass 0.20 kg is heated upto 150°C. Immediately, it is put into water of volume 150 cc at 27°C kept in a calorimeter of water equivalent to 0.025 kg. Final temperature of the system is 40°C. The specific heat of aluminium is :
(take 4.2 Joule=1 calorie) **[Online April 8, 2017]**
(a) 378 J/kg – °C   (b) 315 J/kg – °C
(c) 476 J/kg – °C   (d) 434 J/kg – °C

33. An experiment takes 10 minutes to raise the temperature of water in a container from 0°C to 100°C and another 55 minutes to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of water to be 1 cal / g °C, the heat of vapourization according to this experiment will come out to be :

[Online April 11, 2015]

(a) 560 cal/ g (b) 550 cal/ g
(c) 540 cal/ g (d) 530 cal/ g

34. Three rods of Copper, Brass and Steel are welded together to form a Y shaped structure. Area of cross - section of each rod = 4 cm². End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings excepts at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is: **[2014]**

(a) 1.2 cal/s (b) 2.4 cal/s
(c) 4.8 cal/s (d) 6.0 cal/s

35. A black coloured solid sphere of radius R and mass M is inside a cavity with vacuum inside. The walls of the cavity are maintained at temperature T_0. The initial temperature of the sphere is $3T_0$. If the specific heat of the material of the sphere varies as αT^3 per unit mass with the temperature T of the sphere, where α is a constant, then the time taken for the sphere to cool down to temperature $2T_0$ will be (σ is Stefan Boltzmann constant) **[Online April 19, 2014]**

(a) $\dfrac{M\alpha}{4\pi R^2 \sigma} \ln\left(\dfrac{3}{2}\right)$ (b) $\dfrac{M\alpha}{4\pi R^2 \sigma} \ln\left(\dfrac{16}{3}\right)$

(c) $\dfrac{M\alpha}{16\pi R^2 \sigma} \ln\left(\dfrac{16}{3}\right)$ (d) $\dfrac{M\alpha}{16\pi R^2 \sigma} \ln\left(\dfrac{3}{2}\right)$

36. Water of volume 2 L in a closed container is heated with a coil of 1 kW. While water is heated, the container loses energy at a rate of 160 J/s. In how much time will the temperature of water rise from 27°C to 77°C? (Specific heat of water is 4.2 kJ/kg and that of the container is negligible).

[Online April 9, 2014]

(a) 8 min 20 s (b) 6 min 2 s
(c) 7 min (d) 14 min

37. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization. **[2013]**

(a) ρL/T (b) $\sqrt{T/\rho L}$ (c) T/ρL (d) 2T/ρL

38. A mass of 50g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from 30°C to 25°C. A mass of 100g of another liquid in an identical vessel with identical surroundings takes the same time to cool from 30° C to 25° C. The specific heat of the liquid is :

(The water equivalent of the vessel is 30g.)

[Online April 25, 2013]

(a) 2.0 kcal/kg (b) 7 kcal/kg
(c) 3 kcal/kg (d) 0.5 kcal/kg

39. 500 g of water and 100 g of ice at 0°C are in a calorimeter whose water equivalent is 40 g. 10 g of steam at 100°C is added to it. Then water in the calorimeter is : (Latent heat of ice = 80 cal/g, Latent heat of steam = 540 cal/ g)

[Online April 23, 2013]

(a) 580 g (b) 590 g (c) 600 g (d) 610 g

40. Given that 1 g of water in liquid phase has volume 1 cm³ and in vapour phase 1671 cm³ at atmospheric pressure and the latent heat of vaporization of water is 2256 J/g; the change in the internal energy in joules for 1 g of water at 373 K when it changes from liquid phase to vapour phase at the same temperature is : **[Online April 22, 2013]**

(a) 2256 (b) 167 (c) 2089 (d) 1

41. A large cylindrical rod of length L is made by joining two identical rods of copper and steel of length $\left(\dfrac{L}{2}\right)$ each.

The rods are completely insulated from the surroundings. If the free end of copper rod is maintained at 100°C and that of steel at 0°C then the temperature of junction is (Thermal conductivity of copper is 9 times that of steel)

[Online May 19, 2012]

(a) 90°C (b) 50°C (c) 10°C (d) 67°C

42. The heat radiated per unit area in 1 hour by a furnace whose temperature is 3000 K is ($\sigma = 5.7 \times 10^{-8}$ W m⁻² K⁻⁴)

[Online May 7, 2012]

(a) 1.7×10^{10} J (b) 1.1×10^{12} J
(c) 2.8×10^{8} J (d) 4.6×10^{6} J

43. 100g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K): **[2011]**

(a) 8.4 kJ (b) 84 kJ (c) 2.1 kJ (d) 4.2 kJ

44. The specific heat capacity of a metal at low temperature (T) is given as

$$C_p(kJK^{-1}kg^{-1}) = 32\left(\dfrac{T}{400}\right)^3$$

A 100 gram vessel of this metal is to be cooled from 20°K to 4°K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is **[2011 RS]**

(a) greater than 0.148 kJ
(b) between 0.148 kJ and 0.028 kJ
(c) less than 0.028 kJ
(d) equal to 0.002 kJ

45. A long metallic bar is carrying heat from one of its ends to the other end under steady–state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures? **[2009]**

(a)

(b)

(c)

(d)
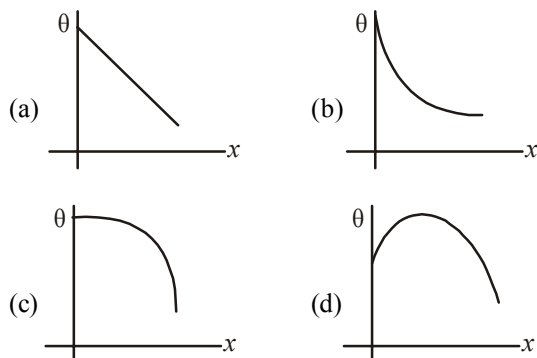
46. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of length l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two section is **[2007]**
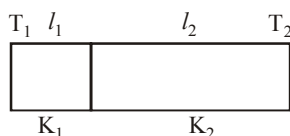
(a) $\dfrac{(K_1 l_1 T_1 + K_2 l_2 T_2)}{(K_1 l_1 + K_2 l_2)}$

(b) $\dfrac{(K_2 l_2 T_1 + K_1 l_1 T_2)}{(K_1 l_1 + K_2 l_2)}$

(c) $\dfrac{(K_2 l_1 T_1 + K_1 l_2 T_2)}{(K_2 l_1 + K_1 l_2)}$

(d) $\dfrac{(K_1 l_2 T_1 + K_2 l_1 T_2)}{(K_1 l_2 + K_2 l_1)}$

47. Assuming the Sun to be a spherical body of radius R at a temperature of $T K$, evaluate the total radiant powerd incident of Earth at a distance r from the Sun **[2006]**

(a) $4\pi r_0^2 R^2 \sigma \dfrac{T^4}{r^2}$

(b) $\pi r_0^2 R^2 \sigma \dfrac{T^4}{r^2}$

(c) $r_0^2 R^2 \sigma \dfrac{T^4}{4\pi r^2}$

(d) $R^2 \sigma \dfrac{T^4}{r^2}$

where r_0 is the radius of the Earth and σ is Stefan's constant.

48. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box contains one mole of helium at temperature $\left(\dfrac{7}{3}\right) T_0$. The boxes are then put into thermal contact with each other, and heat flows between them until the gases reach a common final temperature (ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f in terms of T_0 is **[2006]**

(a) $T_f = \dfrac{3}{7} T_0$

(b) $T_f = \dfrac{7}{3} T_0$

(c) $T_f = \dfrac{3}{2} T_0$

(d) $T_f = \dfrac{5}{2} T_0$

49. The figure shows a system of two concentric spheres of radii r_1 and r_2 are kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to **[2005]**
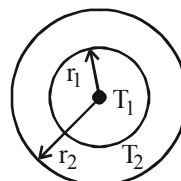
(a) $\ell n\left(\dfrac{r_2}{r_1}\right)$

(b) $\dfrac{(r_2 - r_1)}{(r_1 r_2)}$

(c) $(r_2 - r_1)$

(d) $\dfrac{r_1 r_2}{(r_2 - r_1)}$

50. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be **[2004]**

(a) 32   (b) 16   (c) 4   (d) 64

51. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\dfrac{A(T_2 - T_1)K}{x}\right) f$, with f equal to **[2004]**
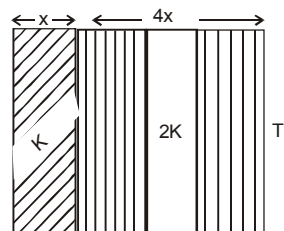

(a) $\dfrac{2}{3}$

(b) $\dfrac{1}{2}$

(c) 1

(d) $\dfrac{1}{3}$

52. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by **[2003]**

(a) Rayleigh Jeans law
(b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law

53. Heat given to a body which raises its temperature by 1°C is **[2002]**

(a) water equivalent
(b) thermal capacity
(c) specific heat
(d) temperature gradient

54. Infrared radiation is detected by **[2002]**
(a) spectrometer (b) pyrometer
(c) nanometer (d) photometer

55. Which of the following is more close to a black body?
[2002]
(a) black board paint (b) green leaves
(c) black holes (d) red roses

56. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should **[2002]**
(a) increase
(b) remain unchanged
(c) decrease
(d) first increase then decrease

57. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is **[2002]**
(a) $1:1$ (b) $16:1$
(c) $4:1$ (d) $1:9$.

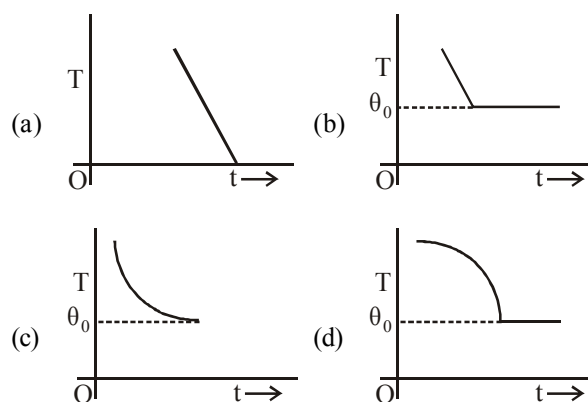**TOPIC 3 Newton's Law of Cooling**

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58. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to :
[Sep. 03, 2020 (II)]
(a) 31℃ (b) 33℃ (c) 28℃ (d) 35℃

59. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m³ and specific heat of 2000 J kg⁻¹ K⁻¹ while liquid in B has density of 10^3 kg m⁻³ and specific heat of 4000 J kg⁻¹ K⁻¹. Which of the following best describes their temperature versus time graph schematically ? (assume the emissivity of both the beakers to be the same) **[8 April 2019 I]**
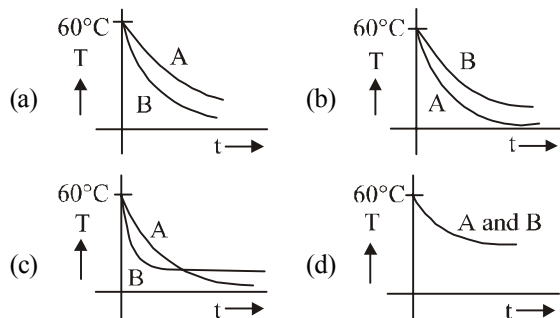

60. A body takes 10 minutes to cool from 60°C to 50°C. The temperature of surroundings is constant at 25°C. Then, the temperature of the body after next 10 minutes will be approximately **[Online April 15, 2018]**
(a) 43°C (b) 47°C (c) 41°C (d) 45°C

61. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is: **[Online April 12, 2014]**
(a) 25°C (b) 10°C (c) 15°C (d) 20°C

62. A hot body, obeying Newton's law of cooling is cooling down from its peak value 80°C to an ambient temperature of 30°C. It takes 5 minutes in cooling down from 80°C to 40°C. How much time will it take to cool down from 62°C to 32°C?
(Given In 2 = 0.693, In 5 = 1.609) **[Online April 11, 2014]**
(a) 3.75 minutes (b) 8.6 minutes
(c) 9.6 minutes (d) 6.5 minutes

63. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to **[2013]**

64. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is : **[2012]**
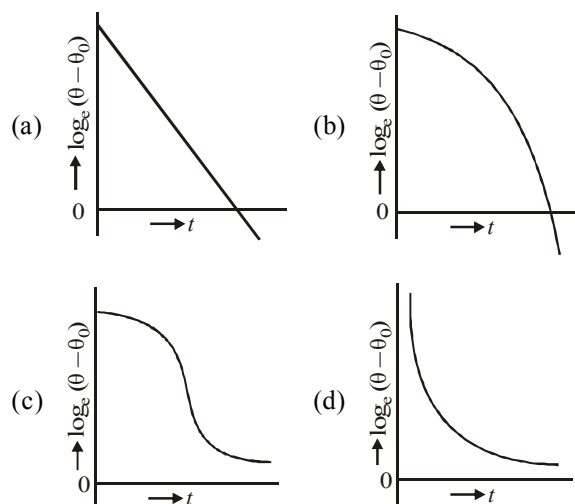
65. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to **[2003]**
(a) two (b) three
(c) four (d) one

Hints & Solutions

1. **(a)** Let L_1' and L_2' be the lengths of the wire when temperature is changed by $\Delta T°C$.

At $T°C$,

$L_{eq} = L_1 + L_2$

At $T + \Delta °C$

$L_{eq}' = L_1' + L_2'$

$\therefore\ L_{eq}(1 + \alpha_{eq}\Delta T) = L_1(1 + \alpha_1\Delta T) + L_2(1 + \alpha_2\Delta T)$

$$[\because L' = L(1 + \alpha\,\Delta T)]$$

$\Rightarrow (L_1 + L_2)(1 + \alpha_{eq}\Delta T) = L_1 + L_2 + L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$

$\Rightarrow \alpha_{eq} = \dfrac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$

2. **(20.00)**

Volume capacity of beaker, $V_0 = 500$ cc

$V_b = V_0 + V_0\gamma_{\text{beaker}}\Delta T$

When beaker is partially filled with V_m volume of mercury,

$V_b^1 = V_m + V_m\gamma_m\Delta T$

Unfilled volume $(V_0 - V_m) = (V_b - V_m^1)$

$\Rightarrow V_0\gamma_{\text{beaker}} = V_m\gamma_M$

$\therefore V_m = \dfrac{V_0\gamma_{\text{beaker}}}{\gamma_M}$

or, $V_m = \dfrac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}} = 20$ cc.

3. **(a)** Change in length of the metal wire (Δl) when its temperature is changed by ΔT is given by

$\Delta l = l\alpha\Delta T$

Here, α = Coefficient of linear expansion

Here, $\Delta l = 0.02\%$, $\Delta T = 10°C$

$\therefore \alpha = \dfrac{\Delta l}{l\Delta T} = \dfrac{0.02}{100 \times 10}$

$\Rightarrow \alpha = 2 \times 10^{-5}$

Volume coefficient of expansion, $\gamma = 3\alpha = 6 \times 10^{-5}$

$\because \rho = \dfrac{M}{V}$

$\dfrac{\Delta V}{V} \times 100 = \gamma\Delta T = (6 \times 10^{-5} \times 10 \times 100) = 6 \times 10^{-2}$

Volume increase by 0.06% therefore density decrease by 0.06%.

4. **(60.00)** Volume, $V = lbh$

$\therefore \gamma = \dfrac{\Delta V}{V} = \dfrac{\Delta \ell}{\ell} + \dfrac{\Delta b}{b} + \dfrac{\Delta h}{h}$

(γ = coefficient of volume expansion)

$\Rightarrow \gamma = 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$

$= 60 \times 10^{-6}/°C$

$\therefore$ Value of $C = 60.00$

5. **(Bonus)** $\Delta_{\text{temp}} = \Delta_{\text{load}}$ and $A = \pi r^2 = \pi(10^{-3})^2 = \pi \times 10^{-6}$

$L\,\alpha\,\Delta T = \dfrac{FL}{AY}$

or $\quad 0.2 \times 10^{-5} \times 20 = \dfrac{F \times 0.2}{(\pi \times 10^{-6}) \times 10^{11}}$

$\therefore\ F = 20\pi\,N \therefore m = \dfrac{f}{g} = 2\pi = 6.28$ kg

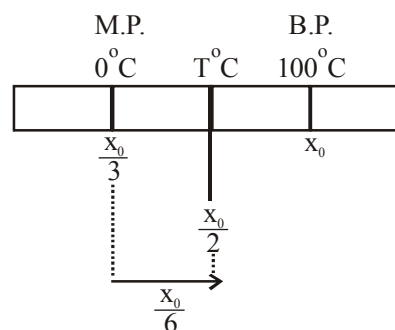**6.** **(a)** Change in length in both rods are same i.e.

$\Delta \ell_1 = \Delta \ell_2$

$\ell\alpha_1\Delta\theta_1 = \ell\alpha_2\Delta\theta_2$

$\dfrac{\alpha_1}{\alpha_2} = \dfrac{\Delta\theta_2}{\Delta\theta_1} \qquad \left[\because \dfrac{\alpha_1}{\alpha_2} = \dfrac{4}{3}\right]$

$\dfrac{4}{3} = \dfrac{\theta - 30}{180 - 30}$

$\boxed{\theta = 230°C}$

7. **(a)** Let required temperature $= T°C$

$\Rightarrow T°C = \dfrac{x_0}{2} - \dfrac{x_0}{3} = \dfrac{x_0}{6}$

$\&\ \left(x_0 - \dfrac{x_0}{3}\right) = (100 - 0°C)$

$$\Rightarrow \frac{2x_0}{3}=100 \Rightarrow x_0 = \frac{300}{2}$$

$$\Rightarrow T°C = \frac{x_0}{6} = \frac{150}{6} = 25°C$$

8. **(a)** Young's modulus $Y = \dfrac{stress}{strain} = \dfrac{F/A}{A(\Delta \ell / \ell)}$
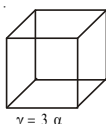
   Using, coefficient of linear expansion,

   $$\alpha = \frac{\Delta \ell}{\ell \Delta T} \Rightarrow \frac{\Delta \ell}{\ell} = \alpha \Delta T$$

   $$\therefore Y = \frac{F}{A(\alpha \Delta T)}$$

9. **(c)** As we know, Bulk modulus

   $$K = \frac{\Delta P}{\left(\dfrac{-\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = \frac{P}{K}$$

   $\gamma = 3 \alpha$

   $$V = V_0(1 + \gamma \Delta t)$$

   $$\frac{\Delta V}{V_0} = \gamma \Delta t$$

   $$\therefore \frac{P}{K} = \gamma \Delta t \Rightarrow \Delta t = \frac{P}{\gamma K} = \frac{P}{3\alpha K}$$

10. **(b)** Young's modulus $= \dfrac{\text{Thermal stress}}{\text{Strain}} = \dfrac{F/A}{\Delta L / L}$

    $$Y = \frac{F}{A.\alpha.\Delta \theta} \qquad \left(Q \frac{\Delta L}{L} = \alpha \Delta \theta\right)$$

    Force developed in the rail $F = YA\alpha \Delta t$
    $= 2 \times 10^{11} \times 40 \times 10^{-4} \times 1.2 \times 10^{-5} \times 10$
    $= 9.6 \times 10^4 = 1 \times 10^5 \, N$

11. **(c)** Due to thermal exp., change in length $(\Delta l) = l \, \alpha \, \Delta T$ ... (i)

    Young's modulus $(Y) = \dfrac{\text{Normal stress}}{\text{Longitudinal strain}}$

    $$Y = \frac{F/A}{\Delta l / l} \Rightarrow \frac{\Delta l}{l} = \frac{F}{AY}$$

    $$\Delta l = \frac{Fl}{AY}$$

    From eqⁿ (i), $\dfrac{Fl}{AY} = l \, \alpha \, \Delta T$

    $F = AY\alpha \Delta T$

12. **(b)** When there is no change in liquid level in vessel then $\gamma'_{real} = \gamma'_{vessel}$

    Change in volume in liquid relative to vessel

    $$\Delta V_{app} = V\gamma'_{app} \Delta \theta = V(\gamma'_{real} - \gamma'_{vessel})$$

13. **(c)** $\dfrac{\text{Reading on any scale} - \text{LFP}}{\text{UFP} - \text{LFP}}$

    = constant for all scales

$$\frac{340 - 273}{373 - 273} = \frac{°Y - (-160)}{-50 - (-160)}$$

$$\Rightarrow \frac{67}{100} = \frac{y + 160}{110}$$

$$\therefore \quad Y = -86.3° Y$$

14. **(d)** The Young modulus is given as

    $$Y = \frac{stress}{strain} = \frac{F/S}{\Delta L / L}$$

    Here, $\Delta L = 2\pi \, \Delta R \quad L = 2\pi \, R$

    $$Y = \frac{F}{S 2\pi \Delta R} \times 2\pi R$$

    $$\Rightarrow \quad Y = \frac{FR}{S.\Delta R} \qquad\qquad ...(i)$$

    The coefficient of linear expansion $\alpha = \dfrac{\Delta R}{R\Delta T}$

    $$\Rightarrow \frac{\Delta R}{R} = \alpha.\Delta T \Rightarrow \frac{R}{\Delta R} = \frac{1}{\alpha \Delta T} \qquad ... (ii)$$

    From equation (i) and (ii)

    $$Y = \frac{F}{S.\alpha \Delta T} \Rightarrow F = Y.S.\alpha \Delta T$$

    $\therefore$ The ring is pressing the wheel from both sides, Thus
    $F_{net} = 2F = 2YS\alpha \Delta T$

15. **(a)** As the rods are identical, so they have same length (l) and area of cross-section (A). They are connected in series. So, heat current will be same for all rods.

    $$\text{Heat current} = \left(\frac{\Delta Q}{\Delta t}\right)_{AB} = \left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CD}$$

    $$\Rightarrow \frac{(100 - 70)K_1 A}{l} = \frac{(70 - 20)K_2 A}{l} = \frac{(20 - 0)K_3 A}{l}$$

    $$\Rightarrow K_1(100 - 70) = K_2(70 - 20) = K_3(20 - 0)$$

    $$\Rightarrow K_1(30) = K_2(50) = K_3(20)$$

    $$\Rightarrow \frac{K_1}{10} = \frac{K_2}{6} = \frac{K_3}{15}$$

    $$\Rightarrow K_1 : K_2 : K_3 = 10 : 6 : 15$$

    $$\Rightarrow K_1 : K_3 = 2 : 3.$$

16. **(a)** According to question, one half of its kinetic energy is converted into heat in the wood.

    $$\frac{1}{2}mv^2 \times \frac{1}{2} = ms\Delta T$$

    $$\Rightarrow \Delta T = \frac{v^2}{4 \times s} = \frac{210 \times 210}{4 \times 4.2 \times 0.3 \times 1000} = 87.5°C$$

17. **(a)** Here ice melts due to water.

    Let the amount of ice melts $= m_{ice}$

    $$m_w s_w \Delta \theta = m_{ice} L_{ice}$$

$\therefore m_{\text{ice}} = \dfrac{m_w s_w \Delta\theta}{L_{\text{ice}}}$

$= \dfrac{0.2 \times 4200 \times 25}{3.4 \times 10^5} = 0.0617$ kg $= 61.7$ g

18. (a) Heat given by water $= m_w C_w (T_{\text{mix}} - T_w)$

$\qquad\qquad\qquad = 200 \times 1 \times (31 - 25)$

Heat taken by steam $= m L_{\text{stem}} + m C_w (T_s - T_{\text{mix}})$

$= m \times 540 + m(1) \times (100 - 31)$

$= m \times 540 + m(1) \times (69)$

From the principal of calorimeter,

Heat lost = Heat gained

$\therefore (200)(31 - 25) = m \times 540 + m(1)(69)$

$\Rightarrow 1200 = m(609) \Rightarrow m \approx 2.$

19. (50.00)

Let Q_1, Q_2, Q_3 be the temperatures of container C_1, C_2 and C_3 respectively.

Using principle of calorimetry in container C_1, we have

$(\theta_1 - 60) = 2 ms(60 - \theta)$

$\Rightarrow \theta_1 - 60 = 120 - 2\theta$

$\Rightarrow \theta_1 = 180 - 2\theta$ \qquad\qquad ...(i)

For container C_2

$ms(\theta_2 - 30) = 2ms(30 - \theta)$

$\Rightarrow \theta_2 = 90 - 2\theta 3$ \qquad\qquad ...(ii)

For container C_3

$2ms(\theta_1 - 60) = ms(60 - \theta)$

$\Rightarrow 2\theta_1 - 120 = 60 - \theta$

$\Rightarrow 2\theta_1 + \theta = 180$ \qquad\qquad ...(iii)

Also, $\theta_1 + \theta_2 + \theta_3 = 3\theta$ \qquad\qquad ...(iv)

Adding (i), (ii) and (iii)

$3\theta_1 + 3\theta_2 + 3\theta_3 = 450$

$\Rightarrow \quad \theta_1 + \theta_2 + \theta_3 = 150$

$\Rightarrow \quad 3\theta = 150 \Rightarrow \theta = 50\,°C$

20. (40) Using the principal of calorimetry

$M_{\text{ice}} L_f + m_{\text{ice}} (40 - 0) C_w$

$= m_{\text{stream}} L_v + m_{\text{stream}} (100 - 40) C_w$

$\Rightarrow M(540) + M \times 1 \times (100 - 40)$

$= 200 \times 80 + 200 \times 1 \times 40$

$\Rightarrow 600\,M = 24000$

$\Rightarrow M = 40g$

21. (a) $M_1 C_{\text{ice}} \times (10) + M_1 L = M_2 C_\omega (50)$

or $M_1 \times C_{\text{ice}} (=0.5) \times 10 + M_1 L = M_2 \times 1 \times 50$

$\Rightarrow L = \dfrac{50 M_2}{M_1} - 5$

22. (b) $\dfrac{1}{2} . kx^2 = mC(\Delta T) + m_\omega C_\omega \Delta T$

or $\dfrac{1}{2} \times 800 \times 0.02^2 = 0.5 \times 400 \times \Delta T + 1 \times 4184 \times \Delta T$
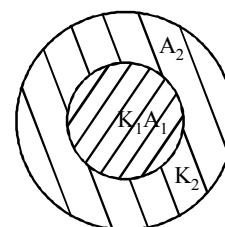
$\therefore \Delta T = 1 \times 10^{-5} K$

23. (a) $H_1 = H_2$ $\;\theta_2 \boxed{\;3k\;|\;k\;}\theta_1$ \quad (with labels d, θ, $3d$ above)

or $(3k)A\left(\dfrac{\theta_2 - \theta}{d}\right) = kA\left(\dfrac{\theta - \theta_1}{3d}\right)$

or $\theta = \left(\dfrac{\theta_1 + 9\theta_2}{10}\right)$

24. (d) Effective thermal conductivity of system

$K_{eq} = \dfrac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$

$= \dfrac{K_1 \pi R^2 + K_2 [\pi (2R)^2 - \pi R^2]}{\pi (2R)^2}$

$= \dfrac{K_1 (\pi R^2) + K_2 (3\pi R^2)}{4\pi R^2} = \dfrac{K_1 + 3K_2}{4}$

25. (d) Let m gram of ice is added.

From principal of calorimeter

heat gained (by ice) = heat lost (by water)

$\therefore \; 20 \times 2.1 \times m + (m - 20) \times 334$

$= 50 \times 4.2 \times 40$

$376\,m = 8400 + 6680$

$m = 40.1$

26. (c) Heat loss = Heat gain $= mS\Delta\theta$

So, $m_A S_A \Delta\theta_A = m_B S_B \Delta\theta_B$

$\Rightarrow 100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5 S_B \Rightarrow S_A = \dfrac{3}{4} S_B$

Now, $100 \times S_A \times (100 - \theta) = 50 \times S_B \times (\theta - 50)$

$2 \times \left(\dfrac{3}{4}\right) \times (100 - \theta) = (\theta - 50)$

$300 - 3\theta = 2\theta - 100$

$400 = 5\theta \Rightarrow \theta = 80°C$

27. (d) Assume final temperature $= T°C$

Heat lass = Heat gain $= ms\Delta T$

$\Rightarrow m_B s_B \Delta T_B = m_w s_w \Delta T_w$

$0.1 \times 400 \times (500 - T)$

$= 0.5 \times 4200 \times (T - 30) + 800\,(T - 30)$

$\Rightarrow 40\,(500 - T) = (T - 30)(2100 + 800)$

$\Rightarrow 20000 - 40T = 2900\,T - 30 \times 2900$

$\Rightarrow 20000 + 30 \times 2900 = T(2940)$

$T = 30.4°C$

$\dfrac{\Delta T}{T} \times 100 = \dfrac{6.4}{30} \times 100 = 21\%,$

so the closest answer is 20%.

28. (a)

Temp. of heat source   Temp. of heat reservoir
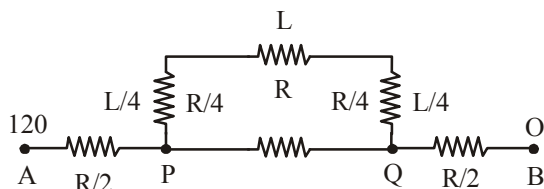

$10^3 K \xleftarrow{\text{1 m}} 10^2 K$

$\left(\dfrac{dQ}{dt}\right) = \dfrac{kA\Delta T}{\ell}$

Energy flux, $\dfrac{1}{A}\left(\dfrac{dQ}{dt}\right) = \dfrac{k\Delta T}{\ell}$

$= \dfrac{(0.1)(900)}{1} = 90 \text{ W/m}^2$

29. (c) Let specific heat of unknown metal be 's' According to principle of calorimetry, Heat lost

= Heat gain $m \times s\Delta\theta = m_1 s_{brass}(\Delta\theta_1 + m_2 s_{water} + \Delta\theta_2)$

$\Rightarrow 192 \times S \times (100 - 21.5)$

$= 128 \times 394 \times (21.5 - 8.4)$

Solving we get,$+ 240 \times 4200 \times (21.5 - 8.4)$

$S = 916 \text{ Jkg}^{-1}\text{k}^{-1}$

30. (a) $\dfrac{\Delta T_{AB}}{R_{AB}} = \dfrac{120}{\dfrac{8}{5}R} = \dfrac{120 \times 5}{8R}$

In steady state temperature difference between P and Q,

$\Delta T_{PQ} = \dfrac{120 \times 5}{8R} \times \dfrac{3}{5}R = \dfrac{360}{8} = 45°C$

31. (d) According to principle of calorimetry,

Heat lost = Heat gain

$100 \times 0.1(T - 75) = 100 \times 0.1 \times 45 + 170 \times 1 \times 45$

$10\,T - 750 = 450 + 7650 = 8100$

$\Rightarrow T - 75 = 810$

$T = 885°C$

32. (d) According to principle of calorimetry,

$Q_{given} = Q_{used}$

$0.2 \times S \times (150 - 40) = 150 \times 1 \times (40 - 27) + 25 \times (40 - 27)$

$0.2 \times S \times 110 = 150 \times 13 + 25 \times 13$

Specific heat of aluminium

$S = \dfrac{13 \times 25 \times 7}{0.2 \times 110} = 434 \text{ J/kg-}°C$

33. (b) As $Pt = mC\Delta T$

So, $P \times 10 \times 60 = mC\,100$ ...(i)

and $P \times 55 \times 60 = mL$ ...(ii)
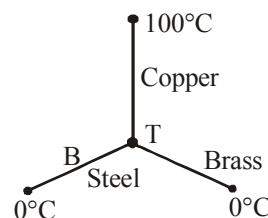
Dividing equation (i) by (ii) we get

$\dfrac{10}{55} = \dfrac{C \times 100}{L}$

$\therefore L = 550 \text{ cal./g.}$

34. (c) Rate of heat flow is given by,

$Q = \dfrac{KA(\theta_1 - \theta_2)}{l}$

Where, K = coefficient of thermal conductivity

l = length of rod and A = area of cross-section of rod

If the junction temperature is T, then

$Q_{Copper} = Q_{Brass} + Q_{Steel}$

$\dfrac{0.92 \times 4(100 - T)}{46}$

$= \dfrac{0.26 \times 4 \times (T - 0)}{13} + \dfrac{0.12 \times 4 \times (T - 0)}{12}$

$\Rightarrow 200 - 2T = 2T + T$

$\Rightarrow T = 40°C$

$\therefore Q_{Copper} = \dfrac{0.92 \times 4 \times 60}{46} = 4.8 \text{ cal/s}$

35. (c) In the given problem, fall in temperature of sphere,

$dT = (3T_0 - 2T_0) = T_0$

Temperature of surrounding, $T_{surr} = T_0$

Initial temperature of sphere, $T_{initial} = 3T_0$

Specific heat of the material of the sphere varies as,

$c = \alpha T^3$ per unit mass (α = a constant)

Applying formula,

$\dfrac{dT}{dt} = \dfrac{\sigma A}{McJ}\left(T^4 - T_{surr}^4\right)$

$\Rightarrow \dfrac{T_0}{dt} = \dfrac{\sigma 4\pi R^2}{M\alpha(3T_0)^3 J}\left[(3T_0)^4 - (T_0)^4\right]$

$\Rightarrow dt = \dfrac{M\alpha 27 T_0^4 J}{\sigma 4\pi R^2 \times 80 T_0^4}$

Solving we get,

Time taken for the sphere to cool down temperature $2T_0$,

$t = \dfrac{M\alpha}{16\pi R^2 \sigma}\ln\left(\dfrac{16}{3}\right)$

36. (a) From question,

In 1 sec heat gained by water

$= 1\,KW - 160\,J/s$

$= 1000\,J/s - 160\,J/s$

$= 840\,J/s$

Total heat required to raise the temperature of water (volume 2L) from 27°c to 77°c

$= m_{water} \times sp.\ ht \times \Delta\theta$

$= 2 \times 10^3 \times 4.2 \times 50\ [\because\ mass = density \times volume]$

And, $840 \times t = 2 \times 10^3 \times 4.2 \times 50$

or, $t = \dfrac{2 \times 10^3 \times 4.2 \times 50}{840}$

$= 500\,s = 8\ min\ 20s$

37. (d) When radius is decrease by ΔR,

$4\pi R^2 \Delta R \rho L = 4\pi T[R^2 - (R - \Delta R)^2]$

$\Rightarrow \rho R^2 \Delta R L = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$

$\Rightarrow \rho R^2 \Delta R L = T 2R \Delta R \qquad [\ \Delta R\ is\ very\ small]$

$\Rightarrow R = \dfrac{2T}{\rho L}$

38. (d) As the surrounding is identical, vessel is identical time taken to cool both water and liquid (from 30°C to 25°C) is same 2 minutes, therefore

$\left(\dfrac{dQ}{dt}\right)_{water} = \left(\dfrac{dQ}{dt}\right)_{liquid}$

or, $\dfrac{(m_w C_w + W)\Delta T}{t} = \dfrac{(m_\ell C_\ell + W)\Delta T}{t}$

(W = water equivalent of the vessel)

or, $m_w C_w = m_\ell C_\ell$

$\therefore$ Specific heat of liquid , $C_\ell = \dfrac{m_w C_w}{m_\ell}$

$= \dfrac{50 \times 1}{100} = 0.5\ kcal / kg$

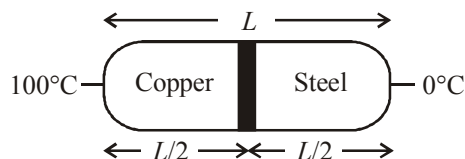**39. (b)** As 1g of steam at 100°C melts 8g of ice at 0°C.

10 g of steam will melt 8× 10 g of ice at 0°C

Water in calorimeter = 500 + 80 + 10g = 590g

40. (c)

41. (a)

Let conductivity of steel $K_{steel} = k$ then from question Conductivity of copper $K_{copper} = 9k$

$\theta_{copper} = 100°C$

$\theta_{steel} = 0°C$

$l_{steel} = l_{copper} = \dfrac{L}{2}$

From formula temperature of junction;

$\theta = \dfrac{K_{copper}\theta_{copper}\,l_{steel} + K_{steel}\,\theta_{steel}\,l_{copper}}{K_{copper}\,l_{steel} + K_{steel}\,l_{copper}}$

$= \dfrac{9k \times 100 \times \dfrac{L}{2} + k \times 0 \times \dfrac{L}{2}}{9k \times \dfrac{L}{2} + k \times \dfrac{L}{2}}$

$= \dfrac{\dfrac{900}{2}kL}{\dfrac{10kL}{2}} = 90°C$

42. (a) According to Stefan's law

$E = \sigma T^4$

Heat radiated per unit area in 1 hour (3600s) is

$= 5.7 \times 10^{-8} \times (300)^4 \times 3600 = 1.7 \times 10^{10}\,J$

43. (a) $\Delta U = \Delta Q = mc\Delta T$

$= \dfrac{100}{1000} \times 4184\,(50 - 30) \approx 8.4\,kJ$

44. (d) Required work = energy released

Here, $Q = \displaystyle\int mc\,dT$

$= \displaystyle\int_{20}^{4} 0.1 \times 32 \times \left(\dfrac{T^3}{400^3}\right)dT = \int_{20}^{4} \dfrac{3.2}{64 \times 10^6}T^3 dT$

$= 5 \times 10^{-8} \displaystyle\int_{20}^{4} T^3 dT = 0.002kJ$

Therefore, required work = 0.002 kJ

45. (a) Let Q be the temperature at a distance x from hot end of bar. Let Q is the temperature of hot end.

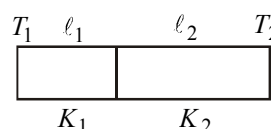The heat flow rate is given by

$\dfrac{dQ}{dt} = \dfrac{kA(\theta_1 - \theta)}{x}$

$\Rightarrow \theta_1 - \theta = \dfrac{x}{kA}\dfrac{dQ}{dt} \qquad \Rightarrow \theta = \theta_1 - \dfrac{x}{kA}\dfrac{dQ}{dt}$

Thus, the graph of Q versus x is a straight line with a positive intercept and a negative slope.

The above equation can be graphically represented by option (a).

46. (d) Let T be the temperature of the interface. In the steady state, $Q_1 = Q_2$

$\therefore \dfrac{K_1 A(T_1 - T)}{\ell_1} = \dfrac{K_2 A(T - T_2)}{\ell_2},$

where A is the area of cross-section.

$\Rightarrow \quad K_1 A(T_1 - T)\ell_2 = K_2 A(T - T_2)\ell_1$

$\Rightarrow \quad K_1 T_1 \ell_2 - K_1 T \ell_2 = K_2 T \ell_1 - K_2 T_2 \ell_1$

$\Rightarrow \quad (K_2 \ell_1 + K_1 \ell_2)T = K_1 T_1 \ell_2 + K_2 T_2 \ell_1$

$\Rightarrow \quad T = \dfrac{K_1 T_1 \ell_2 + K_2 T_2 \ell_1}{K_2 \ell_1 + K_1 \ell_2}$

$= \dfrac{K_1 \ell_2 T_1 + K_2 \ell_1 T_2}{K_1 \ell_2 + K_2 \ell_1}.$

47. **(b)** From stefan's law, total power radiated by Sun, $E = \sigma T^4 \times 4\pi R^2$

The intensity of power Per unit area incident on earth's surface

$= \dfrac{\sigma T^4 \times 4\pi R^2}{4\pi r^2}$

Total power received by Earth

$E' = \dfrac{E}{4\pi r^2} \times$ Cross – Section area of earth facing the

sun $= \dfrac{\sigma T^4 R^2}{r^2}(\pi r_0^2)$

48. **(c)** When two gases are mixed to gether then
Heat lost by He gas = Heat gained by N_2 gas

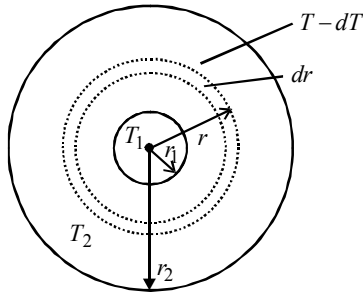$n_1 C_{v_1} \Delta T_1 = n_2 C_{v_2} \Delta T_2$

$\dfrac{3}{2} R \left[\dfrac{7}{3} T_0 - T_f \right] = \dfrac{5}{2} R \left[T_f - T_0 \right]$

$7T_0 - 3T_f = 5T_f - 5T_0$

$\Rightarrow 12 T_0 = 8 T_f \Rightarrow T_f = \dfrac{12}{8} T_0$

$\Rightarrow T_f = \dfrac{3}{2} T_0 \,..$

49. **(d)**

Consider a thin concentric shell of thickness (dr) and of radius (r) and let the temperature of inner and outer surfaces of this shell be T and ($T - dT$) respectively.

The radial rate of flow of heat through this elementary shell will be

$\dfrac{dQ}{dt} = \dfrac{KA[(T - dT) - T]}{dr} = \dfrac{-KA dT}{dr}$

$= -4\pi K r^2 \dfrac{dT}{dr} \qquad (\because A = 4\pi r^2)$

Since the area of the surface through which heat will flow is not constant. Integrating both sides between the limits of radii and temperatures of the two shells, we get

$\left(\dfrac{dQ}{dt} \right) \displaystyle\int_{r_1}^{r_2} \dfrac{1}{r^2} dr = -4\pi K \int_{T_1}^{T_2} dT$

$\left(\dfrac{dQ}{dt} \right) \displaystyle\int_{r_1}^{r_2} r^{-2} dr = -4\pi K \int_{T_1}^{T_2} dT$

$\dfrac{dQ}{dt} \left[\dfrac{1}{r_1} - \dfrac{1}{r_2} \right] = -4\pi K \left[T_2 - T_1 \right]$

or $\dfrac{dQ}{dt} = \dfrac{-4\pi K r_1 r_2 (T_2 - T_1)}{(r_2 - r_1)}$

$\therefore \quad \dfrac{dQ}{dt} \propto \dfrac{r_1 r_2}{(r_2 - r_1)}$

50. **(d)** From stefan's law, energy radiated by sun per second

$E = \sigma A T^4 ;$

$\therefore A \propto R^2$

$\therefore E \propto R^2 T^4$

$\therefore \dfrac{E_2}{E_1} = \dfrac{R_2^2 \, T_2^4}{R_1^2 \, T_1^4}$

put $R_2 = 2R, R_1 = R$; $T_2 = 2T, T_1 = T$

$\Rightarrow \dfrac{E_2}{E_1} = \dfrac{(2R)^2 (2T)^4}{R^2 T^4} = 64$

51. **(d)** The thermal resistance is given by

$\dfrac{x}{KA} + \dfrac{4x}{2KA} = \dfrac{x}{KA} + \dfrac{2x}{KA} = \dfrac{3x}{KA}$

Amount of heat flow per second,

$\dfrac{dQ}{dt} = \dfrac{\Delta T}{\dfrac{3x}{KA}} = \dfrac{(T_2 - T_1)KA}{3x}$

$= \dfrac{1}{3} \left\{ \dfrac{A(T_2 - T_1)K}{x} \right\} \qquad \therefore f = \dfrac{1}{3}$

52. **(d)** Wein's law correctly explains the spectrum

53. **(b)** Heat required for raising the temperature of a body through 1°C is called its thermal capacity.

54. **(b)** Pyrometer is used to detect infra-red radiation.

55. **(a)** Black body is one which absorb all incident radiation. Black board paint is quite approximately equal to black bodies.

56. **(c)** When water is cooled at 0°C to form ice, energy is released from water in the form of heat. As energy is equivalent to mass, therefore, when water is cooled to ice, its mass decreases.

57. **(a)** From stefan's law, the energy radiated per second is given by $E = e\sigma T^4 A$

Here, T = temperature of the body

A = surface area of the body

For same material e is same. σ is stefan's constant

Let T_1 and T_2 be the temperature of two spheres. A_1 and A_2 be the area of two spheres.

$$\therefore \frac{E_1}{E_2} = \frac{T_1^4 A_1}{T_2^4 A_2} = \frac{T_1^4\, 4\pi r_1^2}{T_2^4\, 4\pi r_2^2}$$

$$= \frac{(4000)^4 \times 1^2}{(2000)^4 \times 4^2} = \frac{1}{1}$$

58. **(b)** From Newton's Law of cooling,

$$\frac{T_1 - T_2}{t} = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

Here, $T_1 = 50°C$, $T_2 = 40°C$

and $T_o = 20°C$, t = 600S = 5 minutes

$$\Rightarrow \frac{50-40}{5\,\text{Min}} = K\left(\frac{50+40}{2} - 20\right) \qquad \text{...(i)}$$

Let T be the temperature of sphere after next 5 minutes. Then

$$\frac{40-T}{5} = K\left(\frac{40+T}{2} - 20\right) \qquad \text{...(ii)}$$

Dividing eqn. (ii) by (i), we get

$$\frac{40-T}{10} = \frac{40+T-40}{50+40-40} = \frac{T}{50}$$

$$\Rightarrow 40-T = \frac{T}{5} \Rightarrow 200 - 5T = T$$

$$\therefore T = \frac{200}{6} = 33.3°C$$

59. **(b)** Rate of Heat loss $= mS\left(\dfrac{dT}{dt}\right) = e\sigma A T^4$

$$-\frac{dT}{dt} = \frac{e\sigma \times A \times T^4}{\rho \times Vol. \times S} \Rightarrow -\frac{dT}{dt} \propto \frac{1}{\rho S}$$

$$\frac{\left(-\dfrac{dT}{dt}\right)_A}{\left(-\dfrac{dT}{dt}\right)_B} = \frac{\rho_B}{\rho_A} \times \frac{S_B}{S_A} = \frac{10^3}{8\times 10^2} \times \frac{4000}{2000}$$

$$\Rightarrow \left(-\frac{dT}{dt}\right)_A > \left(-\frac{dT}{dt}\right)_B$$

So, A cools down at faster rate.

60. **(a)** According to Newton's law of cooling,

$$\left(\frac{\theta_1 - \theta_2}{t}\right) = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

$$\left(\frac{60-50}{10}\right) = K\left(\frac{60+50}{2} - 25\right) \qquad \text{..... (i)}$$

and, $\left(\dfrac{50-\theta}{10}\right) = K\left(\dfrac{50+\theta}{2} - 25\right) \qquad \text{..... (ii)}$

Dividing eq. (i) by (ii),

$$\frac{10}{(50-\theta)} = \frac{60}{\theta} \Rightarrow \theta = 42.85°C \cong 43°C$$

61. **(b)** By Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = -K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

where θ_0 is the temperature of surrounding.

Now, hot water cools from 60°C to 50°C in 10 minutes,

$$\frac{60-50}{10} = -K\left[\frac{60+50}{2} - \theta_0\right] \qquad \text{...(i)}$$

Again, it cools from 50°C to 42°C in next 10 minutes.

$$\frac{50-42}{10} = -K\left[\frac{50+42}{2} - \theta_0\right] \qquad \text{...(ii)}$$

Dividing equations (i) by (ii) we get

$$\frac{1}{0.8} = \frac{55-\theta_0}{46-\theta_0}$$

$$\frac{10}{8} = \frac{55-\theta_0}{46-\theta_0}$$

$$460 - 10\theta_0 = 440 - 8\theta_0$$

$$2\theta_0 = 20$$

$$\theta_0 = 10°C$$

62. **(b)** From Newton's law of cooling,

$$t = \frac{1}{k} \log_e \left(\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} \right)$$

From question and above equation,

$$5 = \frac{1}{k} \log_e \frac{(40-30)}{(80-30)} \qquad \qquad ...(1)$$

And, $t = \frac{1}{k} \log_e \frac{(32-30)}{(62-30)} \qquad \qquad ...(2)$

Dividing equation (2) by (1),

$$\frac{t}{5} = \frac{\frac{1}{k} \log_e \frac{(32-30)}{(62-30)}}{\frac{1}{k} \log_e \frac{(40-30)}{(80-30)}}$$

On solving we get, time taken to cool down from 62°C to 32°C, $t = 8.6$ minutes.

63. **(c)** According to Newton's law of cooling, the temperature goes on decreasing with time non-linearly.

64. **(a)** According to newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\Rightarrow \frac{d\theta}{(\theta - \theta_0)} = -kdt$$

$$\Rightarrow \int_{\theta_0}^{\theta} \frac{d\theta}{(\theta - \theta_0)} = -k \int_{\theta}^{t} dt$$

$$\Rightarrow \log(\theta - \theta_0) = -kt + c$$

Which represents an equation of straight line.

Thus the option (a) is correct.

65. **(d)** From Newton's law of cooling $-\dfrac{dQ}{dt} \propto (\Delta\theta)$