Mathematics Sample Paper - 1

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 1

f(x) is a continuous function that takes only rational values. If f(0) = 3, then f(2) is equal to ____

A) 5 B) 0 C) 1 D) None of these

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 2

Let f(x + y) = f(x) f(y) for all x and y. Suppose that f(3) = 3 and f '(0) = 11, then f ' (3) is given by

A) 28 B) 22 C) None of these D) 33

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 3

Let A = {p,q,r}. Which of the following is an equivalence relation on A ?

A) R^1= {(p,q), (q,r), (p,r), (p,p)} B) R^2= {(r,q), (r,p), (r,r), (q,q)} C) R^3= {(p,p), (q,q), (r,r), (p,q)} D) None of these
E) All of these

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 4

If A = {a, b, c, d}, then a relation R = {(a, b), (b, a), (a, a)} on A is

A) symmetric and transitive only B) Reflexive and transitive only C) symmetric only D) Transitive only
E) none of these

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 5

Match the following

Function	Range
(1) Sin^{-1}	(i) $[0, \pi]$
(2) $\cos^{-1}$	(ii) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
(3) $\tan^{-1}$	(iii) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
(4) $\cot^{-1}$	(iv) $(0, \pi)$

A) (1)-(iii), (2)-(ii), (3)-(i), (4)-(iv) B) (1)-(iii), (2)-(ii), (3)-(iv), (4)-(i) C) (1)-(ii), (2)-(i), (3)-(iii), 4-(iv)
D) (1)-(iv), (2)-(i), (3)-(iii), 4-(ii)

DIRECTIONS for the question: *Mark the best option:*

Question No. : 6

Which of the following is a possible value of

$\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$ is

A) $\frac{3\pi}{5}$ B) $\frac{-7\pi}{5}$ C) $\frac{\pi}{10}$ D) $-\frac{\pi}{10}$

DIRECTIONS for the question: *Mark the best option:*

Question No. : 7

The value of cot ($\sin^{-1}x$) is

A) $\sqrt{\frac{1+x^2}{x}}$ B) $\frac{x}{\sqrt{1+x^2}}$ C) $\frac{1}{x}$ D) $\frac{\sqrt{1-x^2}}{x}$

DIRECTIONS for the question: *Mark the best option:*

Question No. : 8

The domain of $\sin^{-1}$ 2x is

A) [0, 1] B) [– 1, 1] C) $\left[-\frac{1}{2},\frac{1}{2}\right]$ D) [–2, 2]

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 9

Find the value of determinant

$\begin{vmatrix} 7579 & 7589 \\ 7581 & 7591 \end{vmatrix} =$

A) 20 B) -2 C) -20 D) 4

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 10

The inverse of matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is

A) A B) A^T C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 11

A and B are two non-zero square matrices such that AB = O. Then

A) Both A and B are singular B) either A or B is singular C) neither matrix is singular D) None of these

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 12

If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{pmatrix}$, then A^2 is equal to

A) A B) –A C) null matrix D) *I*

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 13

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ equals

A) x + y B) xy C) x- y D) 1 + x+ y

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 14

If D = diag (d_1, d_2, d_3, ..., d_n), where $d_i \neq 0$ for all i = 1, 2, ... , n, then D^{-1} is equal to

A) D B) diag (d^{1-1}, d^{2-1}, ...d^{n-1}) C) I^n D) None of these

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 15

If K is a scalar and A is an n × n square matrix. Then |KA| =

A) K $|A|^n$ B) K | A| C) $K^n|A^n|$ D) $K^n|A|$

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 16

If I_3 is the identity matrix of order 3, then $(I_3)^{-1}$ =

A) 0 B) 3 I^3 C) I^3 D) Not necessarily exists

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 17

The value of $\lim_{x \to 0} \frac{|x|}{x}$ is

A) 1 B) - 1 C) 0 D) None of these

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 18

Let f(x) = |x| + | x – 1|, then

A) f(x) is continuous at x = 0, as well as at x = 1 B) f(x) is continuous at x = 0, but not at x = 1
C) f(x) is continuous at x = 1, but not at x = 0 D) None of these

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 19

The value of $\lim_{x \to e} \frac{\log x - 1}{x - e}$ is

A) e B) 1 C) e^{-1} D) None of these

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 20

$f(x) = \begin{cases} 9x+2 & \text{for} \quad x > 1 \\ 15x-4 & \text{for} \quad x \leq 1 \end{cases}$ find $\lim_{x \to 1} f(x)$.

A) 9 B) 15 C) 11 D) 18

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 21

The derivative of the function log(sinx) is

A) cotx B) tanx C) cosecx D) sin2x

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 22

Differentiation of (logx).(sinx)

A) (sinx).(1/x) B) (sinx).(1/x) + (logx).(cosx) C) (cosx).(sinx) + logx D) (cosx).(-1/x) + 1/logx

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 23

Find $\frac{dy}{dx}$ of $y = e^{x+e^{x+e^{x+\cdots\cdots\infty}}}$

A) $\frac{y}{1+y}$ B) $\frac{y}{y-1}$ C) $\frac{y}{1-y}$ D) 1 – y

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 24

If f (x) = tan x + e^{-2x} – $7x^3$, then what is the value of f '(0)?

A) -2 B) -1 C) 0 D) 3

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 25

Which one of the following differential equations is ***not*** linear?

A) $\frac{d^2y}{dx^2} + 4y = 0$ B) $x\frac{dy}{dx} + y = x^3$ C) $(x-y)^2\frac{dy}{dx} = 9$ D) $\cos^2 x\frac{dy}{dx} + y = \tan x$

DIRECTIONS for the question: *Solve the following question and mark the best possible option.*

Question No. : 26

The order and degree of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{8/2}$ are respectively

A) 1, 2 B) 2, 2 C) 3, 2 D) 1, 3

DIRECTIONS for the question: *Solve the following question and mark the best possible option.*

Question No. : 27

What is the solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = 0$?

A) xy = c B) x = cy C) y = cx D) None of the above

DIRECTIONS for the question: *Solve the following question and mark the best possible option.*

Question No. : 28

What is the degree of the following differential equation?

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$$

A) 1 B) 2 C) 3 D) 4

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 29

The area of the region enclosed by $y = x^2$ and $2y = x$ is:

A) 1/48 B) 1/24 C) 1/16 D) 1/12

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 30

The value of $\int_0^8 |x - 5|\, dx$ is _____

A) 17 B) 15 C) 21 D) 12

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 31

If f(x) is an odd function then integration of f(x) on [-a, a] =

A) 2 B) 0 C) 1 D) Does not exist

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 32

The anti derivative F of *f* defined by $f(x) = 4x^3 - 6$, where F (0) = 3 is:

A) $x^4 + 6x + 3$ B) $x^4 - 6x + 3$ C) $x^4 - 6x - 3$ D) $x^4 - 3x + 3$

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 33

What is $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$?

A) π B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) 0

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 34

The area bounded by the curve x = f(y) the y-axis and two lines y = a and y = b is equal to:

A) $\int_a^b y \, dx$ B) $\int_a^b y^2 \, dx$ C) $\int_a^b x \, dy$ D) None of the above

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 35

What is $\int_0^1 \frac{e^{\tan^{-1} x} dx}{1+x^2}$ equal to ?

A) $e^{\frac{\pi}{4}} - 1$ B) $e^{\frac{\pi}{4}} + 1$ C) e - 1 D) e

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 36

If $f(x) = a + bx + cx^2$, then what is $\int_0^1 f(x)\, dx$

A) [f(0) + 4f(1/2) + f(1)]/6 B) [f(0) + 4f(1/2) + f(1)]/3 C) [f(0) + 4f(1/2) + f(1)] D) [f(0) + 2f(1/2) + f(1)]/6

DIRECTIONS*: Solve the following question and mark the best possible option*

Question No. : 37

A problem on mathematics is given to three students whose chances of solving it are 1/2, 1/3 and 1/4 respectively. What is the chance that the problem will be solved?

A) 2/3 B) 3/4 C) 1/3 D) 1/2

DIRECTIONS for the question: *Solve the following question and mark the best possible option.*
Question No. : 38

A candidate is selected for interview for 3 posts. For the first post there are 5 candidates, for the second there are 8 and for the third there are 7. What are the chances for his getting at least one post?

A) $\frac{1}{5}$ B) $\frac{3}{5}$ C) $\frac{2}{5}$ D) $\frac{4}{5}$

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 39

A can hit a target 4 times in 5 shots, B hits 3 times in 4 shots, and C hits twice in 3 shots. They fire together. The probability that at least two shots hit the target is

A) 5/6 B) $(1/3)^2$ C) 13/30 D) None of these

DIRECTIONS for the question: *Answer the question based on the following information:*

An urn contains 6 red, 5 blue and 2 green marbles

Question No. : 40

If 2 marbles are picked up at random, what is the probability that both are red?

A) $\frac{6}{13}$ B) $\frac{5}{26}$ C) $\frac{5}{13}$ D) $\frac{7}{26}$ E) None of these

DIRECTIONS for the question: *Solve the following question and mark the best possible option.*

Question No. : 41

The angle between the lines whose direction ratios are 5, -12, 13 and -3, 4, 5 is –

A) $\cos^{-1}\left(\frac{3}{65}\right)$ B) $\cos^{-1}\left(\frac{2}{65}\right)$ C) $\cos^{-1}\left(\frac{1}{65}\right)$ D) $\frac{\pi}{6}$

DIRECTIONS for the question: *Solve the following question and mark the best possible option.*

Question No. : 42

The length of perpendicular from $(1, 2, 3)$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

A) 3 B) $\sqrt{5}$ C) 7 D) $\sqrt{66}$

DIRECTIONS for the question: *Solve the following question and mark the best possible option.*

Question No. : 43

Find the direction ratios of the line determined by the planes 2x - y - z = 2 and x + 2y - 3z = 11 .

A) 1, 2, 2 B) 1, 1, 1 C) 2, 2, 3 D) 2, 1, 1

DIRECTIONS for the question: *Solve the following question and mark the best possible option.*

Question No. : 44

Find the value of m for which the points A (m, - 1), B(2, 1) and C (4, 5) are collinear

A) M = 1 B) M = 2 C) M = 3 D) M = 4

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 45

$\left|\vec{a}+\vec{b}\right|^2 - \left|\vec{a}-\vec{b}\right|^2$ is equal to

A) 0 B) $4\ \vec{a}\ .\ \vec{b}$ C) $-4\ \vec{a}\ .\ \vec{b}$ D) $\vec{a}\ .\ \vec{b}$

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 46

The coordinates of the tip of the position vector which is equivalent to *AB* where the coordinates of A and B are (3,1) and (5,0) respectively is _____.

A) (2,1) B) (1,-2) C) (1,2) D) (2,-1)

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 47

If the points with position vectors $60\,\hat{i}+3\,\hat{j}$, $40\hat{i}-8\,\hat{j}$ and $a\,\hat{i}-52\,\hat{j}$ are collinear, then the value of a is:

A) - 40 B) 20 C) 40 D) - 20

DIRECTION for the question: *Solve the following question and mark the best possible option.*

Question No. : 48

What unit vector is parallel to the vector $-6\,\hat{i}+8\hat{j}$?

A) $-\frac{3}{2}\hat{i}+\frac{4}{3}\hat{j}$ B) $-\frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}$ C) $\frac{3}{2}\hat{i}+\frac{8}{3}\hat{j}$ D) $\frac{3}{10}\hat{i}+\frac{8}{5}\hat{j}$

DIRECTIONS for the question: *Mark the best option:*

Question No. : 49

An optimisation problem may involve finding

A) Maximum profit B) Minimum cost C) Minimum use of resources D) All of the above are true

DIRECTIONS for the question: *Mark the best option:*

Question No. : 50

The variables of a linear programming problem are called

A) decision variables B) Contrvints C) Objective D) None of these

QNo:- 1 ,Correct Answer:- *D*

Explanation:- *f(x) is a continuous function that takes only rational values this is only possible if f(x) is constant. So if f(0) = 3 then f(2) = 3*

QNo:- 2 ,Correct Answer:- *D*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{f(x)f(h) - f(x)f(o)}{h}$$
$$= f(x) \lim_{h \to o} \frac{f(h) - f(o)}{h}$$

Explanation:-

f'(x) = f(x) f'(o)
now put x = 3
f ' (3) = f(3) f '(o)

= 3 × 11
= 33

QNo:- 3 ,Correct Answer:- *D*

Explanation:-

R_1 is neither reflexive nor symmetric.
R_2 is neither symmetric nor transitive.
R_3 is not symmetric.
So, none of the given relations are equivalence relations.

QNo:- 4 ,Correct Answer:- *C*

Explanation:- *Since (b,b) is not in R so R fails to be reflexive and transitive. So, it is only symmetric*

QNo:- 5 ,Correct Answer:- *C*

Explanation:-

The range of the trigonometric functions is as follows:

$$Sin^{-1}(x) = \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$
$$Cos^{-1}(x) = [0, \pi]$$
$$\tan^{-1}(x) = \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$
$$\cot^{-1}(x) = (0, \pi)$$

Hence the combination is (1)-(ii), (2)-(i), (3)-(iii), 4-(iv)

QNo:- 6 ,Correct Answer:- *D*

Explanation:-

$$\sin^{-1}\left(\cos\frac{40\pi + 3\pi}{5}\right) = \sin^{-1}\cos\left(8\pi + \frac{3\pi}{5}\right)$$
$$= \sin^{-1}\left(\cos\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} + \frac{3\pi}{5}\right)\right)$$
$$= \sin^{-1}\left(\sin\left(-\frac{\pi}{10}\right)\right) = -\frac{\pi}{10}$$

QNo:- 7 ,Correct Answer:- *D*

Explanation:- *Let $\sin^{-1} x = q$, then $\sin q = x$*

$$\Rightarrow \operatorname{cosec}\theta = \frac{1}{x} \Rightarrow \operatorname{cosec}^2\theta = \frac{1}{x^2}$$

$$\Rightarrow 1 + \cot^2\theta = \frac{1}{x^2} \Rightarrow \cot\theta = \sqrt{\frac{1-x^2}{x}}$$

QNo:- 8 ,Correct Answer:- *C*

Explanation:- *Let $\sin^{-1} 2x = \theta$ so that $2x = \sin\theta$.*
Now $-1 \le \sin\theta \le 1$, i.e.,$-1 \le 2x \le 1$ which gives

$$-\frac{1}{2} \le x \le \frac{1}{2}.$$

QNo:- 9 ,Correct Answer:- *C*

Explanation:- *Applying $R_2 \to R_2 - R_1$ and expanding we get $\Delta = -20$*

QNo:- 10 ,Correct Answer:- *A*

Explanation:-

$$A^{-1} = \operatorname{adj}\frac{A}{|A|} \text{ and } A^2 = AA$$

= I for the given matrix.
Alternate explanation
$|A| = -1$

$$\operatorname{adj} A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$\therefore A^{-1} = A$

QNo:- 11 ,Correct Answer:- *B*

Explanation:- *Definition: A matrix is said to be singular if its determinant is zero.*
Since $|AB| = |A|\,|B|$. Therefore $AB = 0 \Rightarrow |AB|$
$= 0 \Rightarrow |A|\,|B| = 0 \Rightarrow$either $|A| = 0$ or $|B| = 0$

QNo:- 12 ,Correct Answer:- *D*

Explanation:- *$A^2 = A$*

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

QNo:- 13 ,Correct Answer:- *B*

Explanation:- *Apply R2 → R2 - R1, R3 → R3 – R1*

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{bmatrix}$$

and then by expanding the result would be xy

QNo:- 14 ,Correct Answer:- *B*

Explanation:-

$$\text{Let D} = \begin{bmatrix} d_1 & 0 & 0 \ldots . 0 \\ 0 & d_2 & 0 \ldots 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \ldots d_n \end{bmatrix}.$$

Then $|D| = d_1 d_2 \ldots d_n$
and cofactor of $d_{11} = d_2 d_3 \ldots d_n$
cofactor of $d_{22} = d_1 d_3 \ldots d_n$ etc
cofactor of $D_{ij} = 0$ when $i \neq j$

$$\therefore D^{-1} = \frac{1}{|D|} adj\, D$$

$$= \frac{1}{d_1 d_2 \ldots d_n} \begin{bmatrix} d_2 d_3 \ldots d_n & 0 & 0 & \ldots. & 0 \\ 0 & d_1 d_3 \ldots d_n & 0 & \ldots. & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \ldots d_1 d_2 \ldots d_{n-1} & \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{d_1} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{d_2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \cdots & \frac{1}{d_n} \end{bmatrix} = diag(d_1^{-1}, d_2^{-1}, \cdots, d_n^{-1})$$

QNo:- 15 ,Correct Answer:- *D*

Explanation:- *Since k is common from each row of |KA| and there are n rows in A, therefore $|kA| = k^n |A|$.*

QNo:- 16 ,Correct Answer:- *C*

Explanation:- *The inverse of identity matrix I_3 is I_3 itself.*

QNo:- 17 ,Correct Answer:- *D*

Explanation:-

$$\text{We have R.H.L } \lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = 1$$

$$\text{And L.H.L } \lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{|-x|}{-x} = \lim_{x \to 0^+} \frac{x}{-x} = -1$$

$$\text{Hence } \lim_{x \to 0} \frac{|x|}{x} \text{ does not exit.}$$

QNo:- 18 ,Correct Answer:- A

Explanation:- We have $f(x) = |x| + |x-1|$

$$= \begin{cases} -2x+1, & x<0 \\ x-x+1, & 0 \le x<1 \\ x+x-1, & x \ge 1 \end{cases} = \begin{cases} -2x+1, & x<0 \\ 1 & 0 \le x<1 \\ 2x-1, & x \ge 1 \end{cases}$$

Clearly

$$\lim_{x\to 0^-} f(x)=1, \lim_{x\to 0^+} f(x)=1, f(0)=1 \;\&\; \lim_{x\to 1^-} f(x)=1, \lim_{x\to 1^+} f(x)=1, f(1)=1.$$

So f (x) is continuous at x = 0 as well as at x = 1.

QNo:- 19 ,Correct Answer:- C

Explanation:- $\lim_{x\to e} \frac{\log x - 1}{x-e}$

Put $x = e + h$ where $h \to 0$

$$= \lim_{h\to 0} \frac{\log(e+h)-\log e}{e+h-e}$$

$$= \frac{1}{e} \lim_{h\to 0} \frac{\log\left(1+\frac{h}{e}\right)}{\frac{h}{e}} = \frac{1}{e} = e^{-1}.$$

QNo:- 20 ,Correct Answer:- C

Explanation:- $\lim_{x\to 1^+} 9x+2 = 11$

$\lim_{x\to 1^-} 15x-4 = 11$

$\Rightarrow \lim_{x\to 1} f(x) = 11$

QNo:- 21 ,Correct Answer:- A

Explanation:- $y = \log \sin x$

$$\frac{dy}{dx} = \frac{1}{\sin x}\frac{d}{dx}\sin x = \frac{\cos x}{\sin x} = \cot x$$

QNo:- 22 ,Correct Answer:- B

Here $y = \log x . \sin x$

Differentiating both sides w. r. t. x, we get

$$\frac{dy}{dx} = \log x \frac{d(\sin x)}{dx} + \sin x \frac{d(\log x)}{dx} = (\log x)(\cos x) + \frac{\sin x}{x}$$

Explanation:-

QNo:- 23 ,Correct Answer:- C

Explanation:- $y = e^{(x+y)}$
ln y = (x + y) ln e
ln y = (x + y)
differentiating both sides, we get (1/y) dy/dx = 1 + dy/dx
So, (1/y - 1) dy/dx = 1
So, dy/dx = y / (1 - y)

QNo:- 24 ,Correct Answer:- B

Explanation:- We have

$$f(x) = \tan x + e^{-2x} - 7x^3$$
$$\Rightarrow f'(x) = \sec^2 x - 2e^{-2x} - 21x^2$$
$$\Rightarrow f'(0) = \sec^2 0 - 2e^{0} - 0 = 1 - 2 = -1$$

QNo:- 25 ,Correct Answer:- A

Explanation:- A ifferential eqaution is said to be linear if dependent variable and its derivative ocurs only in first degree and not multiplied together.
In a differential equation, if the variables and derivatives multiplied by constant, then the equation in linear
So only option 1 is linear differential equation

QNo:- 26 ,Correct Answer:- B

Explanation:- Squareing on both sides:

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left[1+\left(\frac{dy}{dx}\right)^2\right]^3$$

So order & degree both are 2.

QNo:- 27 ,Correct Answer:- A

$$\frac{dy}{dx} + \frac{y}{x} = 0$$
$$\frac{dy}{dx} = \frac{-y}{x}$$
$$\Rightarrow \frac{dy}{y} = \frac{-dx}{x} \Rightarrow \frac{dy}{y} + \frac{dx}{x} = 0$$

Explanation:-
Integrating both sides
Log y + log x = c
xy = c so answer is option 1

QNo:- 28 ,Correct Answer:- *B*

Explanation:-

The given differential equation is $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$

$$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} = -4 + 3\frac{d^2y}{dx^2} - 5\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^{2} = \left[-4 + 3\frac{d^2y}{dx^2} - 5\frac{dy}{dx}\right]^3$$

As degree of a differential equation is the highest power of the highest order derivative that occurs in the equation after all the derivatives are converted into radical and radical free form. Hence, the degree of above differential equation is 2. D

QNo:- 29 ,Correct Answer:- *A*

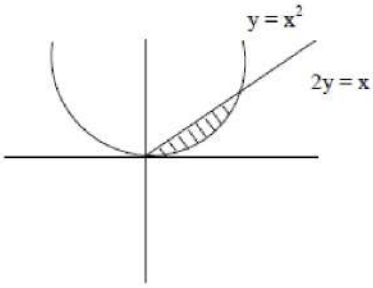
Explanation:- $y = x^2$ and $2y = x$

$$y = \frac{x}{2}$$

So, $\frac{x}{2} = x^2$

Both given curve will intersect at x = 0, ½

So area $\int \left|\frac{x}{2} - x^2\right| dx$



$$= \left[\frac{x^2}{4} - \frac{x^3}{3}\right]_0^{1/2}$$

$$= \left(\frac{1}{16} - \frac{1}{24}\right) - (0 - 0)$$

$$= \frac{3-2}{48} = \frac{1}{48}$$

QNo:- 30 ,Correct Answer:- *A*

Explanation:- $|x - 5| = 5 - x$ *if* $x < 5$

$|x - 5| = x - 5$ *if* $x > 5$

So $\int_0^8 |x-5|\,dx = \int_0^5 (5-x)dx + \int_5^8 (x-5)dx =$

$$\left(5x - \frac{x^2}{2}\right)_0^5 + \left(\frac{x^2}{2} - 5x\right)_5^8$$

$$\left(25 - \frac{25}{2}\right) + \left(\frac{64}{2} - 40 - \frac{25}{2} + 25\right) = 17$$

QNo:- 31 ,Correct Answer:- *B*

Explanation:- *For an odd fn, the integral over a symmetric interval equals zero, because half the area is negative.*

QNo:- 32 ,Correct Answer:- B

Explanation:- $F(x) = \int f(x)\,dx = \int (4x^3 - 6)\,dx = x^4 - 6x + c$

$F(0) = c = 3$

So, $F(x) = x^4 - 6x + 3$

QNo:- 33 ,Correct Answer:- C

$$I = \int_0^{\Pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\,dx \text{.....(1)}$$

$$I = \int_0^{\Pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin^3\left(\frac{\pi}{2} - x\right) + cox^3\left(\frac{\pi}{2} - x\right)} \frac{dx}{\left(\frac{\pi}{2} - x\right)} . = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}\,dx \text{.....(2)}$$

Explanation:-

Adding (1) *and* (2)

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2I = \int_0^{\pi/2} dx \Rightarrow 2I = [x]^{\frac{\pi}{2}}$$

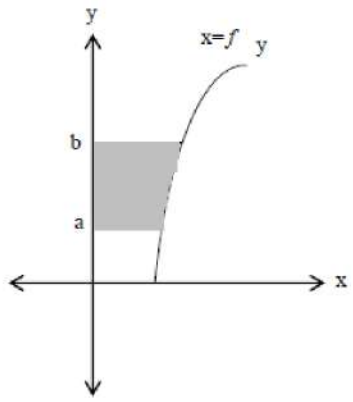
$$\Rightarrow 2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

QNo:- 34 ,Correct Answer:- C

$$\int_a^b f(y)\,dy = \int_a^b x\,dy$$

Explanation:-



QNo:- 35 ,Correct Answer:- A

$$\int_0^1 \frac{e^{\tan^{-1}x}}{1+x^2}dx$$

$$let\ \tan^{-1}x = t \Rightarrow \frac{dx}{1+x^2} = dt$$

Explanation:-

Also $\tan^{-1} 1 = \pi/4$

$\tan^{-1} 0 = 0$

So, new limits are 0 & $\pi/4$

$$\int_0^{\pi/4} e^t\,dt = \left[e^t\right]_0^{\pi/4}$$

$$= e^{\pi/4} - e^0$$

$$= e^{\pi/4} - 1$$

QNo:- 36 ,Correct Answer:- A

Explanation:- We have

$$\int_0^1 f(x)dx = \int_0^1 (a+bx+cx^2)dx$$

$$= \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1 = a + \frac{b}{2} + \frac{c}{3}$$

Now taking option 1

$$\frac{f(0)+4f\left(\frac{1}{2}\right)+f(1)}{6}$$

$$= \frac{a+4\left(a+\frac{b}{2}+\frac{c}{4}\right)+(a+b+c)}{6}$$

$$= \frac{a+4a+2b+c+a+b+c}{6} = \frac{6a+3b+2c}{6}$$

$$= a + \frac{b}{2} + \frac{c}{3}$$

Similarly we can check for option (B), (C) & (D)

$$\therefore \int_0^1 f(x)\,dx = \frac{f(0)+4f\left(\frac{1}{2}\right)+f(1)}{6}$$

QNo:- 37 ,Correct Answer:- B

Explanation:-

We will solve this question by complimentary method, i.e reverse probability.

The chances of not solving the question would be subtracted from 1.

Let A, B and C be the three students and P(A),P(B),P(C) be their probabilities of solving a problem respectively.

P (A) = 1/2, P (B) = 1/3, P(C) = 1/4

P [problem will be solved at least by 1] = 1−P (not solved by A) P (not solved by B) P (not solved by C)

= 1− [1−P (A)][1−P (B)][1−P(C)]

= 1− (1/2) (2/3) (3/4)

=1− (1/4) = 3/4

QNo:- 38 ,Correct Answer:- C

Explanation:-

For the 1st post chances of being not selected is = $\frac{4}{5}$

For the 2nd post chances of being not selected is = $\frac{7}{8}$

For the 3rd post chances of being not selected is = $\frac{6}{7}$

Being not selected at all = $\frac{4}{5}\times\frac{7}{8}\times\frac{6}{7}=\frac{3}{5}$

Being selected at least in one post = $1-\frac{3}{5}=\frac{2}{5}$

So, answer is option C

QNo:- 39 ,Correct Answer:- A

Explanation:-

Probability of A hitting the target = $\frac{4}{5}\Rightarrow P(\overline{A})=\frac{1}{5}$

Probability of B hitting the target = $\frac{3}{4}\Rightarrow P(\overline{B})=\frac{1}{4}$

Probability of C hitting the target = $\frac{2}{3}\Rightarrow P(\overline{C})=\frac{1}{3}$

We have 2 cases: - 1. All 3 hit 2. Exactly 2 persons hit

1. P (all 3hit) = $\frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}=\frac{2}{5}$

2. Exactly 2 hit = $\frac{4}{5}\times\frac{3}{4}\times\frac{1}{3}+\frac{4}{5}\times\frac{2}{3}\times\frac{1}{4}+\frac{3}{4}\times\frac{2}{3}\times\frac{1}{5}$

$\Rightarrow\frac{1}{5}+\frac{2}{15}+\frac{1}{10}\Rightarrow\frac{6+4+3}{30}=\frac{13}{30}$

So required probability = $\frac{2}{5}+\frac{13}{30}\Rightarrow\frac{25}{30}=\frac{5}{6}$.

QNo:- 40 ,Correct Answer:- B

Explanation:-

Required probability = $\frac{{}^{6}C_{2}}{{}^{13}C_{2}}=\frac{5}{26}$

QNo:- 41 ,Correct Answer:- C

Explanation:- *Angle b/w the lines with direction ratios a, b, c, & a_2, b_2, c_2 is given by*

$$\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+x_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

So, $\cos\theta=\left|\frac{-15-48+65}{\sqrt{5^2+12^2+13^2}\sqrt{3^2+4^2+5^2}}\right|=\left|\frac{1}{65}\right|$

$\Theta = \cos^{-1}(1/65$

QNo:- 42 ,Correct Answer:- D

Explanation:-

QNo:- 43 ,Correct Answer:- B

Explanation:- Let the equation of line is ax+by+cz=d
Then, 2a-b-c=0 and a+2b-3c=0
Solving for b and c in terms of a, we get,
a=c and a=b
So, the direction ratios are 1,1,1

QNo:- 44 ,Correct Answer:- A

Explanation:- Area of ΔABC = 0

$$\Delta = \begin{vmatrix} m & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

⇒ m (1 - 5) + (2 - 4) + 1 (10 - 4) = 0
⇒ - 4m - 2 + 6 = 0
⇒ - 4 m = - 4 ⇒ m = 1

QNo:- 45 ,Correct Answer:- B

Explanation:-

$$\left|\vec{a}+\vec{b}\right|^2 - \left|\vec{a}-\vec{b}\right|^2$$
$$= \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\left|\vec{a}\right|\left|\vec{b}\right| - \left(\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\left|\vec{a}\right|\left|\vec{b}\right|\right)$$
$$= 4\,\vec{a}.\vec{b}$$

QNo:- 46 ,Correct Answer:- D

Explanation:- A (3, 1) and B(5, 0)
So AB = [(5-3), (0-1)]
(2, -1)

QNo:- 47 ,Correct Answer:- A

Explanation:-

$$\text{Let } \vec{A} = 60\hat{i} + 3\hat{j}, \vec{B} = 40\hat{i} - 8\hat{j} \text{ and } \vec{C} = a\hat{i} - 52\hat{j}$$
$$\therefore \vec{AB} = (40-60)\hat{i} + (-8-3)\hat{j} = -20\hat{i} - 11\hat{j}$$
$$\vec{AC} = (a-60)\hat{i} + (-52-3)\hat{j} = (a-60)\hat{i} - 55\hat{j}$$
$$As\ \vec{AB}\ \&\ \vec{AC}\ are\ collinear \Rightarrow \frac{-20}{a-60} = \frac{-11}{-55}$$
$$\Rightarrow \frac{-20}{a-60} = \frac{1}{5} \Rightarrow a - 60 = -100$$
$$\Rightarrow a = -40$$

QNo:- 48 ,Correct Answer:- *B*

Explanation:- *The unit vector parallel to the vector* $-6\hat{i}+8\hat{j}$ *is given by*

$$\frac{-6\hat{i}+8\hat{j}}{\sqrt{36+64}}=\frac{-6\hat{i}+8\hat{j}}{10}$$
$$=\frac{-3\hat{i}}{5}+\frac{4\hat{j}}{5}$$

QNo:- 49 ,Correct Answer:- *D*

Explanation:- *An optimisation problem can deal with maximistion of profit, minimization of cost or usage of resources*

QNo:- 50 ,Correct Answer:- *A*

Explanation:- *The variables of a linear programming problem are called decision variables*